

## Chapter 9. Indices

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### Ex 9.1

#### Answer 1.

$$(i) 6^0 = 1$$

$$(ii) \left(\frac{1}{2}\right)^{-3} = (2)^3 = 8$$

$$(iii) 2^{3^2} = 2^6 = 64$$

$$(iv) 3^{2^3} = 3^6 = 729$$

$$(v) (0.008)^{\frac{2}{3}} = (0.2^3)^{\frac{2}{3}} = (0.2)^{3 \times \frac{2}{3}} = (0.2)^2 = 0.04$$

$$(vi) (0.00243)^{\frac{-3}{5}} = \frac{1}{(0.00243)^{\frac{3}{5}}} = \frac{1}{(0.3^5)^{\frac{3}{5}}} = \frac{1}{(0.3)^3} = \frac{1}{0.027}$$

$$(vii) \sqrt[6]{25^3} = \sqrt[6]{(5^2)^3} = \sqrt[6]{5^6} = 5^{6 \times \frac{1}{6}} = 5$$

$$(viii) \left(2\frac{10}{27}\right)^{\frac{2}{3}} = \left(\frac{64}{27}\right)^{\frac{2}{3}} = \left(\frac{4}{3}\right)^{3 \times \frac{2}{3}} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

#### Answer 2A.

$$\begin{aligned} 9^4 \div 27^{-\frac{2}{3}} &= [(3)^2]^4 \div [(3)^3]^{-\frac{2}{3}} \\ &= (3)^{2 \times 4} \div (3)^{3 \times \left(-\frac{2}{3}\right)} \dots \dots (\text{Using } (a^m)^n = a^{mn}) \\ &= (3)^8 \div (3)^{-2} \\ &= (3)^{8 - (-2)} \dots \dots (\text{Using } a^m \div a^n = a^{m-n}) \\ &= (3)^{8+2} \\ &= 3^{10} \\ &= (3)^{2 \times 5} \\ &= [(3)^2]^5 \\ &= [9]^5 \\ &= 59049 \end{aligned}$$

**Answer 2B.**

$$\begin{aligned}
& 7^{-4} \times (343)^{\frac{2}{3}} \div (49)^{-\frac{1}{2}} \\
&= 7^{-4} \times (7^3)^{\frac{2}{3}} \div (7^2)^{-\frac{1}{2}} \\
&= 7^{-4} \times 7^{3 \times \frac{2}{3}} \div 7^{2 \times (-\frac{1}{2})} \\
&= 7^{-4} \times 7^2 \div 7^{-1} \\
&= 7^{-4+2-(-1)} \quad \dots \dots \left( \text{Using } a^m \times a^n = a^{m+n} \text{ and } a^m \div a^n = a^{m-n} \right) \\
&= 7^{-4+2+1} \\
&= 7^{-1} \\
&= \frac{1}{7} \quad \dots \dots \left( \text{Using } a^{-m} = \frac{1}{a^m} \right)
\end{aligned}$$

**Answer 2C.**

$$\begin{aligned}
\left( \frac{64}{216} \right)^{\frac{2}{3}} \times \left( \frac{16}{36} \right)^{-\frac{3}{2}} &= \left( \frac{2^6}{6^3} \right)^{\frac{2}{3}} \times \left( \frac{2^4}{6^2} \right)^{-\frac{3}{2}} \\
&= \frac{(2^6)^{\frac{2}{3}}}{(6^3)^{\frac{2}{3}}} \times \frac{(2^4)^{-\frac{3}{2}}}{(6^2)^{-\frac{3}{2}}} \\
&= \frac{(2)^{6 \times \frac{2}{3}}}{(6)^{3 \times \frac{2}{3}}} \times \frac{(2)^{4 \times (-\frac{3}{2})}}{(6)^{2 \times (-\frac{3}{2})}} \quad \dots \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
&= \frac{(2)^{2 \times 2}}{(6)^2} \times \frac{(2)^{2 \times (-3)}}{(6)^{-3}} \\
&= \frac{(2)^4}{(6)^2} \times \frac{(2)^{-6}}{(6)^{-3}} \\
&= \frac{(2)^4}{(6)^2} \times \frac{(6)^3}{(2)^6} \quad \dots \dots \left( \text{Using } a^{-m} = \frac{1}{a^m} \right) \\
&= \frac{(2)^4}{(2)^6} \times \frac{(6)^3}{(6)^2} \\
&= (2)^{4-6} \times (6)^{3-2} \quad \dots \dots \left( \text{Using } a^m \div a^n = a^{m-n} \right) \\
&= (2)^{-2} \times (6)^1 \\
&= \frac{1}{2^2} \times 6 \\
&= \frac{1}{4} \times 6 \\
&= \frac{3}{2}
\end{aligned}$$

**Answer 3A.**

$$\begin{aligned}
 (a^3)^5 \times a^4 &= (a)^{3 \times 5} \times a^4 \quad \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
 &= (a)^{15} \times a^4 \\
 &= a^{15+4} \quad \dots \left( \text{Using } a^m \times a^n = a^{m+n} \right) \\
 &= a^{19}
 \end{aligned}$$

**Answer 3B.**

$$\begin{aligned}
 a^2 \times a^3 \div a^4 &= a^{2+3-4} \quad \dots \left( \text{Using } a^m \times a^n = a^{m+n} \text{ and } a^m \div a^n = a^{m-n} \right) \\
 &= a^1 \\
 &= a
 \end{aligned}$$

**Answer 3C.**

$$\begin{aligned}
 a^{\frac{1}{3}} \div a^{-\frac{2}{3}} &= a^{\frac{1}{3} - \left(-\frac{2}{3}\right)} \quad \dots \left( \text{Using } a^m \div a^n = a^{m-n} \right) \\
 &= a^{\frac{1}{3} + \frac{2}{3}} \\
 &= a^{\frac{1+2}{3}} \\
 &= a^1 \\
 &= a
 \end{aligned}$$

**Answer 3D.**

$$\begin{aligned}
 a^{-3} \times a^2 \times a^0 &= a^{-3+2+0} \quad \dots \left( \text{Using } a^m \times a^n = a^{m+n} \right) \\
 &= a^{-1} \\
 &= \frac{1}{a}
 \end{aligned}$$

**Answer 3E.**

$$\begin{aligned}
 (b^{-2} \times a^{-2}) \div (b^{-1} \times a^{-1}) &= a^{-3+2+0} \quad \dots \left( \text{Using } a^m \times a^n = a^{m+n} \right) \\
 &= a^{-1} \\
 &= \frac{1}{a}
 \end{aligned}$$

**Answer 4.**

$$\begin{aligned}
 \text{(i)} \quad & \frac{2^3 \times 3^5 \times 24^2}{12^2 \times 18^3 \times 27} \\
 &= \frac{2^3 \times 3^5 \times (2^3 \times 3)^2}{(2^2 \times 3)^2 \times (2 \times 3^2)^3 \times (3^3)} \\
 &= \frac{2^3 \times 3^5 \times 2^6 \times 3^2}{2^4 \times 3^2 \times 2^3 \times 3^6 \times 3^3} \\
 &= \frac{2^3 \times 3^5 \times 2^6 \times 3^2}{2^4 \times 3^2 \times 2^3 \times 3^6 \times 3^3} \\
 &= \frac{2^9 \times 3^7}{2^7 \times 3^{11}} = \frac{2^{9-7}}{3^{11-7}} = \frac{2^2}{3^4} = \frac{4}{81}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \frac{4^3 \times 3^7 \times 5^6}{5^8 \times 2^7 \times 3^3} \\
 &= \frac{(2^2)^3 \times 3^{7-3}}{5^{8-6} \times 2^7} \\
 &= \frac{2^6 \times 3^4}{5^2 \times 2^7} \\
 &= \frac{3^4}{5^2 \times 2^{7-6}} = \frac{81}{5^2 \times 2^1} = \frac{81}{50}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \frac{12^2 \times 75^{-2} \times 35 \times 400}{48^2 \times 15^{-3} \times 525} \\
 &= \frac{(2^2 \times 3)^2 \times (7 \times 5) \times (2^4 \times 5^2) \times (3 \times 5)^3}{(2^4 \times 3)^2 \times (3 \times 5^2 \times 7) \times (3 \times 5^2)^2} \\
 &= \frac{2^4 \times 3^2 \times 7 \times 5 \times 2^4 \times 5^2 \times 3^3 \times 5^3}{2^8 \times 3^2 \times 3 \times 5^2 \times 7 \times 3^2 \times 5^4} \\
 &= \frac{2^{4+4} \times 3^{2+3} \times 5^{1+2+3} \times 7}{2^8 \times 3^{2+1+2} \times 5^{4+2} \times 7} \\
 &= \frac{2^8 \times 3^5 \times 5^6 \times 7}{2^8 \times 3^5 \times 5^6 \times 7} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & \frac{2^6 \times 5^{-4} \times 3^{-3} \times 4^2}{8^3 \times 15^{-3} \times 25^{-1}} \\
 &= \frac{2^6 \times (2^2)^2 \times (3 \times 5)^3 \times (5^2)^1}{(2^3)^3 \times 5^4 \times 3^3} \\
 &= \frac{2^{6+4} \times 3^3 \times 5^{3+2}}{2^9 \times 3^3 \times 5^4} = 2^{10-9} \times 5^{5-4} = 2 \times 5 = 10
 \end{aligned}$$

**Answer 5A.**

$$\begin{aligned}
3p^{-2}q^3 \div 2p^3q^{-2} &= \frac{3p^{-2}q^3}{2p^3q^{-2}} \\
&= \frac{3}{2} \left[ \frac{p^{-2}}{p^3} \times \frac{q^3}{q^{-2}} \right] \\
&= \frac{3}{2} [(p^{-2} \div p^3) \times (q^3 \div q^{-2})] \\
&= \frac{3}{2} [(p^{-2-3}) \times (q^{3-(-2)})] \quad \dots \dots (\text{Using } a^m \div a^n = a^{m-n}) \\
&= \frac{3}{2} [(p^{-5}) \times (q^5)] \\
&= \frac{3}{2} \left[ \left( \frac{1}{p^5} \right) \times (q^5) \right] \\
&= \frac{3q^5}{2p^5}
\end{aligned}$$

**Answer 5B.**

$$\begin{aligned}
\left[ (p^{-3})^{\frac{2}{3}} \right]^{\frac{1}{2}} &= p^{-3 \times \frac{2}{3} \times \frac{1}{2}} \quad \dots \dots (\text{Using } (a^m)^n = a^{mn}) \\
&= p^{-1} \\
&= \frac{1}{p}
\end{aligned}$$

**Answer 6.**

$$(i) \left( 1 - \frac{15}{64} \right)^{-\frac{1}{2}} = \left( \frac{64-15}{64} \right)^{-\frac{1}{2}} = \left( \frac{49}{64} \right)^{-\frac{1}{2}} = \left( \frac{64}{49} \right)^{\frac{1}{2}} = \frac{8}{7}$$

$$(ii) \left( \frac{8}{27} \right)^{-\frac{2}{3}} - \left( \frac{1}{3} \right)^{-2} - 7^0$$

$$= \left( \frac{27}{8} \right)^{\frac{2}{3}} - (3)^2 - 1$$

$$= \left( \frac{3}{2} \right)^{3 \times \frac{2}{3}} - 9 - 1$$

$$= \left( \frac{3}{2} \right)^2 - 10$$

$$= \frac{9}{4} - 10 = \frac{9-40}{4} = \frac{-31}{4}$$

$$(iii) 9^{\frac{5}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{\frac{-1}{2}}$$

$$= 3^{2 \times \frac{5}{2}} - 3 \times 1 - \left(\frac{1}{81}\right)^{\frac{-1}{2}}$$

$$= 3^5 - 3 - 9^{2 \times \frac{1}{2}}$$

$$= 243 - 3 - 9$$

$$= 231$$

$$(iv) (27)^{\frac{2}{3}} \times 8^{\frac{-1}{6}} \div 18^{\frac{-1}{2}}$$

$$= 3^{3 \times \frac{2}{3}} \times \frac{1}{2^{3 \times \frac{1}{6}}} \div \left(\frac{1}{18}\right)^{\frac{1}{2}}$$

$$= \frac{3^2}{2^{\frac{1}{2}}} \times (2 \times 3^2)^{\frac{1}{2}}$$

$$= \frac{3^2}{2^{\frac{1}{2}}} \times 2^{\frac{1}{2}} \times 3$$

$$= 3^{2+1} = 3^3 = 27$$

$$(v) 16^{\frac{3}{4}} + 2\left(\frac{1}{2}\right)^{-1} \times 3^0$$

$$= 2^{4 \times \frac{3}{4}} + 2 \times 2 \times 1$$

$$= 2^3 + 4$$

$$= 2^3 + 4 = 8 + 4 = 12$$

$$(vi) \sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}}$$

$$= \left(\frac{1}{2}\right)^{\frac{1}{2}} + (0.1)^{-1} - 3^2$$

$$= \frac{1}{2} + (0.1)^{-1} - 3^2$$

$$= \frac{1}{2} + \frac{1}{0.1} - 9$$

$$= \frac{1}{2} + \frac{10}{1} - 9$$

$$= \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

**Answer 7A.**

$$\begin{aligned}
(27x^9)^{\frac{2}{3}} &= (3^3x^9)^{\frac{2}{3}} \\
&= (3^3)^{\frac{2}{3}}(x^9)^{\frac{2}{3}} \quad \dots \left( \text{Using } (a \times b)^n = a^n \times b^n \right) \\
&= (3)^{3 \times \frac{2}{3}}(x)^{9 \times \frac{2}{3}} \quad \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
&= (3)^2x^{3 \times 2} \\
&= 9x^6
\end{aligned}$$

**Answer 7B.**

$$\begin{aligned}
(8x^6y^3)^{\frac{2}{3}} &= (2^3x^6y^3)^{\frac{2}{3}} \\
&= (2^3)^{\frac{2}{3}}(x^6)^{\frac{2}{3}}(y^3)^{\frac{2}{3}} \quad \dots \left( \text{Using } (a \times b)^n = a^n \times b^n \right) \\
&= (2)^{3 \times \frac{2}{3}}(x)^{6 \times \frac{2}{3}}(y)^{3 \times \frac{2}{3}} \quad \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
&= (2)^2(x)^4(y)^2 \\
&= 4x^4y^2
\end{aligned}$$

**Answer 7C.**

$$\begin{aligned}
\left( \frac{64a^{12}}{27b^6} \right)^{-\frac{2}{3}} &= \left( \frac{2^6a^{12}}{3^3b^6} \right)^{-\frac{2}{3}} \\
&= \left( \frac{2^{6 \times (-\frac{2}{3})}a^{12 \times (-\frac{2}{3})}}{3^{3 \times (-\frac{2}{3})}b^{6 \times (-\frac{2}{3})}} \right) \quad \dots \left( \text{Using } (a \times b)^n = a^n \times b^n \text{ and } \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \right) \\
&= \frac{2^{-4}a^{-8}}{3^{-2}b^{-4}} \\
&= \frac{3^2b^4}{2^4a^8} \quad \dots \left( \text{Using } a^{-n} = \frac{1}{a^n} \right) \\
&= \frac{9b^4}{16a^8}
\end{aligned}$$

**Answer 7D.**

$$\begin{aligned}
\left(\frac{36m^{-4}}{49n^{-2}}\right)^{-\frac{3}{2}} &= \left(\frac{6^2m^{-4}}{7^2n^{-2}}\right)^{-\frac{3}{2}} \\
&= \left(\frac{6^{2 \times \left(-\frac{3}{2}\right)} m^{-4 \times \left(-\frac{3}{2}\right)}}{7^{2 \times \left(-\frac{3}{2}\right)} n^{-2 \times \left(-\frac{3}{2}\right)}}\right) \dots\dots \left(\text{Using } (a \times b)^n = a^n \times b^n \text{ and } \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right) \\
&= \frac{6^{-3}m^6}{7^{-3}n^3} \\
&= \frac{7^3m^6}{6^3n^3} \dots\dots \left(\text{Using } a^{-n} = \frac{1}{a^n}\right) \\
&= \frac{343m^6}{216n^3}
\end{aligned}$$

**Answer 7E.**

$$\begin{aligned}
&\left(a^{\frac{1}{3}} + a^{-\frac{1}{3}}\right)\left(a^{\frac{2}{3}} - 1 + a^{-\frac{2}{3}}\right) \\
&= a^{\frac{1}{3}}\left(a^{\frac{2}{3}} - 1 + a^{-\frac{2}{3}}\right) + a^{-\frac{1}{3}}\left(a^{\frac{2}{3}} - 1 + a^{-\frac{2}{3}}\right) \\
&= \left(a^{\frac{1}{3}} \times a^{\frac{2}{3}} - a^{\frac{1}{3}} \times 1 + a^{\frac{1}{3}} \times a^{-\frac{2}{3}}\right) + \left(a^{-\frac{1}{3}} \times a^{\frac{2}{3}} - a^{-\frac{1}{3}} \times 1 + a^{-\frac{1}{3}} \times a^{-\frac{2}{3}}\right) \\
&= \left(a^{\frac{1}{3}+\frac{2}{3}} - a^{\frac{1}{3}} \times 1 + a^{\frac{1}{3}-\frac{2}{3}}\right) + \left(a^{-\frac{1}{3}+\frac{2}{3}} - a^{-\frac{1}{3}} + a^{-\frac{1}{3}-\frac{2}{3}}\right) \\
&\quad \dots\dots \left(\text{Using } a^m \times a^n = a^{m+n}\right) \\
&= \left(a^1 - a^{\frac{1}{3}} + a^{-\frac{1}{3}}\right) + \left(a^{\frac{1}{3}} - a^{-\frac{1}{3}} + a^{-1}\right) \\
&= a - a^{\frac{1}{3}} + a^{-\frac{1}{3}} + a^{\frac{1}{3}} - a^{-\frac{1}{3}} + \frac{1}{a} \\
&= a + \frac{1}{a}
\end{aligned}$$



**Answer 7F.**

$$\begin{aligned}
& \sqrt[3]{x^4 y^2} \div \sqrt[6]{x^5 y^{-5}} \\
&= (x^4 y^2)^{\frac{1}{3}} \div (x^5 y^{-5})^{\frac{1}{6}} \\
&= \left( x^{4 \times \frac{1}{3}} y^{2 \times \frac{1}{3}} \right) \div \left( x^{5 \times \frac{1}{6}} y^{-5 \times \frac{1}{6}} \right) \dots \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
&= \left( x^{\frac{4}{3}} y^{\frac{2}{3}} \right) \div \left( x^{\frac{5}{6}} y^{-\frac{5}{6}} \right) \\
&= \frac{x^{\frac{4}{3}} y^{\frac{2}{3}}}{x^{\frac{5}{6}} y^{-\frac{5}{6}}} \\
&= x^{\frac{4}{3} - \frac{5}{6}} y^{\frac{2}{3} - \left(-\frac{5}{6}\right)} \dots \dots \left( \text{Using } a^m \div a^n = a^{m-n} \right) \\
&= x^{\frac{1}{2}} y^{\frac{3}{2}} \\
&= x^{\frac{1}{2}} (y^3)^{\frac{1}{2}} \dots \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
&= \sqrt{x} \sqrt{y^3} \\
&= \sqrt{xy^3}
\end{aligned}$$

**Answer 7G.**

$$\begin{aligned}
& \left\{ (a^m)^{m-\frac{1}{m}} \right\}^{\frac{1}{m+1}} = (a)^{m \times \left(m-\frac{1}{m}\right) \times \left(\frac{1}{m+1}\right)} \dots \dots \left( \text{Using } a^m \div a^n = a^{m-n} \right) \\
& \text{Consider, } m \times \left(m - \frac{1}{m}\right) \times \left(\frac{1}{m+1}\right) \\
&= (m^2 - 1) \times \left(\frac{1}{m+1}\right) \\
&= m^2 \times \left(\frac{1}{m+1}\right) - 1 \times \left(\frac{1}{m+1}\right) \\
&= \frac{m^2}{m+1} - \frac{1}{m+1} \\
&= \frac{m^2 - 1}{m+1} \\
&= \frac{(m-1)(m+1)}{m+1} \\
&= m-1 \\
& (a)^{m \times \left(m-\frac{1}{m}\right) \times \left(\frac{1}{m+1}\right)} = a^{m-1}
\end{aligned}$$

**Answer 7H.**

$$\begin{aligned}
& x^{m+2n} \cdot x^{3m-8n} \div x^{5m-60} \\
&= x^{m+2n+3m-8n-5m-(-60)} \quad \dots \left( \text{Using } a^m \times a^n = a^{m+n} \text{ and } a^m \div a^n = a^{m-n} \right) \\
&= x^{m+2n+3m-8n-5m+60} \\
&= x^{-m-6n+60}
\end{aligned}$$

**Answer 7I.**

$$\begin{aligned}
& (81)^{\frac{3}{4}} - \left( \frac{1}{32} \right)^{-\frac{2}{5}} + 8^{\frac{1}{3}} \cdot \left( \frac{1}{2} \right)^{-1} \cdot 2^0 \\
&= (3^4)^{\frac{3}{4}} - \left( \frac{1}{2^5} \right)^{-\frac{2}{5}} + (2^3)^{\frac{1}{3}} \cdot \left( \frac{1}{2} \right)^{-1} \times 1 \quad \dots \left( \text{Using } a^0 = 1 \right) \\
&= 3^{4 \times \frac{3}{4}} - \frac{1}{2^{5 \times \left( -\frac{2}{5} \right)}} + 2^{3 \times \frac{1}{3}} \cdot (2)^1 \quad \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
&= 3^3 - \frac{1}{2^{-2}} + 2^1 \cdot (2)^1 \\
&= 3^3 - 2^2 + 2^{1+1} \quad \dots \left( \text{Using } a^m \times a^n = a^{m+n} \text{ and } a^{-n} = \frac{1}{a^n} \right) \\
&= 3^3 - 2^2 + 2^2 \\
&= 27
\end{aligned}$$

**Answer 7J.**

$$\begin{aligned}
& \left( \frac{27}{343} \right)^{\frac{2}{3}} \div \frac{1}{\left( \frac{625}{1296} \right)^{\frac{1}{4}}} \times \frac{536}{\sqrt[3]{27}} \\
&= \left( \frac{3^3}{7^3} \right)^{\frac{2}{3}} \div \frac{1}{\left( \frac{5^4}{2^4 \times 3^4} \right)^{\frac{1}{4}}} \times \frac{2^3 \times 67}{\sqrt[3]{3^3}} \\
&= \left( \frac{3^3}{7^3} \right)^{\frac{2}{3}} \div \frac{1}{\left( \frac{5^4}{2^4 \times 3^4} \right)^{\frac{1}{4}}} \times \frac{2^3 \times 67}{(3^3)^{\frac{1}{3}}} \\
&= \left( \frac{3^{3 \times \frac{2}{3}}}{7^{3 \times \frac{2}{3}}} \right) \div \left( \frac{5^{4 \times \frac{1}{4}}}{2^{4 \times \frac{1}{4}} \times 3^{4 \times \frac{1}{4}}} \right) \times \frac{2^3 \times 67}{3^{3 \times \frac{1}{3}}} \quad \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
&= \left( \frac{3^2}{7^2} \right) \div \left( \frac{5^1}{2^1 \times 3^1} \right) \times \frac{2^3 \times 67}{3^1}
\end{aligned}$$

$$\begin{aligned}
&= \left(\frac{3^2}{7^2}\right) \times \left(\frac{5^1}{2^1 \times 3^1}\right) \times \left(\frac{2^3 \times 67}{3^1}\right) \\
&= 3^{2-1-1} \times 2^{3-1} \times 5^1 \times 7^2 \times 67 \\
&= 3^0 \times 2^2 \times 5^1 \times 7^2 \times 67 \\
&= 1 \times 4 \times 5 \times 49 \times 67 \quad \dots\dots(\text{Using } a^0 = 1) \\
&= 65660
\end{aligned}$$

**Answer 8.**

$$\begin{aligned}
\text{(i)} \quad \frac{5^x \times 7 - 5^x}{5^{x+2} - 5^{x+1}} &= \frac{5^x (7 - 1)}{5^{x+1} (5 - 1)} \\
&= \frac{5^{x-x-1} \times 6}{4} \\
&= \frac{5^{-1} \times 6}{4} \\
&= \frac{6}{5 \times 4} = \frac{3}{10}
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad \frac{3^{x+1} + 3^x}{3^{x+3} - 3^{x+1}} &= \frac{3^x (3 + 1)}{3^x (3^3 - 3)} \\
&= \frac{4}{27 - 3} = \frac{4}{24} = \frac{1}{6}
\end{aligned}$$

$$\begin{aligned}
\text{(iii)} \quad \frac{2^m \times 3 - 2^m}{2^{m+4} - 2^{m+1}} &= \frac{2^m (3 - 1)}{2^m (2^4 - 2)} \\
&= \frac{2}{16 - 2} = \frac{2}{14} = \frac{1}{7}
\end{aligned}$$

$$\text{(iv)} \quad \frac{5^n + 2 - 6.5^n + 1}{13.5^n - 2.5^n + 1} = \frac{5^n (5^2 - 6 \times 5)}{5^n (13 - 2 \times 5)} = \frac{25 - 30}{13 - 10} = \frac{-5}{3}$$

**Answer 9A.**

$$\begin{aligned}
2^{2x+1} &= 8 \\
\Rightarrow 2^{2x+1} &= 2^3 \\
\Rightarrow 2x + 1 &= 3 \\
\Rightarrow 2x &= 2 \\
\Rightarrow x &= 1
\end{aligned}$$

**Answer 9B.**

$$3 \times 7^x = 7 \times 3^x$$

$$\Rightarrow \frac{7^x}{7} = \frac{3^x}{3}$$

$$\Rightarrow 7^{x-1} = 3^{x-1} \dots\dots (\text{Using } a^m \div a^n = a^{m-n})$$

$$\Rightarrow 7^{x-1} = 3^{x-1} \times 1$$

$$\Rightarrow 7^{x-1} = 3^{x-1} \times 7^0 \dots\dots (\text{Using } a^0 = 1)$$

$$\Rightarrow x - 1 = 0$$

$$\Rightarrow x = 1$$

**Answer 9C.**

$$2^{x+3} + 2^{x+1} = 320$$

$$\Rightarrow 2^{x+3} + 2^{x+1} = 2^6 \times 5$$

$$\Rightarrow 2^x \cdot 2^3 + 2^x \cdot 2^1 = 2^6 \times 5$$

$$\Rightarrow 2^x (2^3 + 2^1) = 2^6 \times 5$$

$$\Rightarrow 2^x (8 + 2) = 2^6 \times 5$$

$$\Rightarrow 2^x (10) = 2^6 \times 5$$

$$\Rightarrow 2^x \left( \frac{10}{5} \right) = 2^6$$

$$\Rightarrow 2^x \cdot 2 = 2^6$$

$$\Rightarrow \frac{2^x \cdot 2}{2^6} = 1$$

$$\Rightarrow 2^{x+1-6} = 1 \times 2^0$$

$$\Rightarrow 2^{x-5} = 1 \times 2^0$$

$$\Rightarrow x - 5 = 0$$

$$\Rightarrow x = 5$$

**Answer 9D.**

$$9 \times 3^x = (27)^{2x-5}$$

$$\Rightarrow 3^2 \times 3^x = (3^3)^{2x-5}$$

$$\Rightarrow 3^2 \times 3^x = 3^{3(2x-5)}$$

$$\Rightarrow 3^{2+x} = 3^{6x-15}$$

$$\Rightarrow 1 = \frac{3^{6x-15}}{3^{2+x}}$$

$$\Rightarrow 1 = 3^{6x-15-2-x}$$

$$\Rightarrow 3^0 = 3^{5x-17}$$

$$\Rightarrow 5x - 17 = 0$$

$$\Rightarrow x = \frac{17}{5}$$

**Answer 9E.**

$$2^{2x+3} - 9 \times 2^x + 1 = 0$$

$$2^{2x} \cdot 2^3 - 9 \times 2^x + 1 = 0$$

$$\text{Put } 2^x = t, \text{ so, } 2^{2x} = t^2$$

$$\text{So, } 2^{2x} \cdot 2^3 - 9 \times 2^x + 1 = 0 \text{ becomes } 8t^2 - 9t + 1 = 0$$

$$\Rightarrow 8t^2 - 8t - t + 1 = 0$$

$$\Rightarrow 8t(t-1) - (t-1) = 0$$

$$\Rightarrow t-1 = 0 \text{ or } 8t-1 = 0$$

$$\Rightarrow t = 1 \text{ or } t = \frac{1}{8}$$

$$\Rightarrow 2^x = 1 \text{ or } 2^x = \frac{1}{2^3}$$

$$\Rightarrow 2^x = 2^0 \text{ or } 2^x = 2^{-3}$$

$$\Rightarrow x = 0 \text{ or } x = -3$$

**Answer 9F.**

$$1 = p^x$$

$$\Rightarrow p^0 = p^x \quad \dots\dots (\text{Using } a^0 = 1)$$

$$\Rightarrow x = 0$$

**Answer 9G.**

$$p^3 \times p^{-2} = p^x$$

$$\Rightarrow p^{3+(-2)} = p^x \quad \dots\dots (\text{Using } a^m \times a^n = a^{m+n})$$

$$\Rightarrow p^1 = p^x$$

$$\Rightarrow x = 1$$

**Answer 9H.**

$$p^{-5} = \frac{1}{p^{x+1}}$$

$$\Rightarrow p^{-5} \times p^{x+1} = 1$$

$$\Rightarrow p^{-5+x+1} = 1$$

$$\Rightarrow p^{x-4} = p^0$$

$$\Rightarrow x-4 = 0$$

$$\Rightarrow x = 4$$

**Answer 9I.**

$$2^{2x} + 2^{x+2} - 4 \times 2^3 = 0$$

$$\Rightarrow 2^{2x} + 2^{x+2} - 2^2 \times 2^3 = 0$$

$$\Rightarrow 2^{2x} + 2^x \cdot 2^2 - 2^{2+3} = 0 \quad \dots \left( \text{Using } a^m \times a^n = a^{m+n} \right)$$

$$\Rightarrow 2^{2x} + 2^x \cdot 2^2 - 2^5 = 0$$

$$\Rightarrow 2^{2x} + 2^x \cdot 4 - 32 = 0$$

$$\text{Put } 2^x = t$$

$$\text{So, } 2^{2x} = t^2$$

$$2^{2x} + 2^{x+2} - 32 = 0 \text{ becomes } t^2 + 4t - 32 = 0$$

$$\Rightarrow (t+8)(t-4) = 0$$

$$\Rightarrow t+8=0 \text{ or } t-4=0$$

$$\Rightarrow t=-8 \text{ or } t=4$$

$$\Rightarrow 2^x = -8 \text{ or } 2^x = 4$$

$$\Rightarrow 2^x = -2^3 \text{ or } 2^x = 2^2$$

Using the second equation  $2^x = 2^2$ , we get  $x = 2$ .

**Answer 9J.**

$$9 \times 81^x = \frac{1}{27^{x-3}}$$

$$\Rightarrow 3^2 \times 3^{4x} = \frac{1}{3^{3(x-3)}}$$

$$\Rightarrow 3^2 \times 3^{4x} = \frac{1}{3^{3x-9}} \quad \dots \left( \text{Using } (a^m)^n = a^{mn} \right)$$

$$\Rightarrow 3^2 \times 3^{4x} \times 3^{3x-9} = 1$$

$$\Rightarrow 3^{2+4+3x-9} = 1 \times 3^0$$

$$\Rightarrow 2+4+3x-9=0$$

$$\Rightarrow 3x-3=0$$

$$\Rightarrow x=1$$

**Answer 9K.**

$$2^{2x-1} - 9 \times 2^{x-2} + 1 = 0$$

$$2^{2x} \cdot 2^{-1} - 9 \times 2^x \cdot 2^{-2} + 1 = 0$$

$$\text{Let } 2^x = t, \text{ so } 2^{2x} = t^2$$

$$\text{So, } 2^{2x} \cdot 2^{-1} - 9 \times 2^x \cdot 2^{-2} + 1 = 0 \text{ becomes } \frac{t^2}{2} - 9 \times \frac{t}{2^2} + 1 = 0$$

$$\Rightarrow \frac{t^2}{2} - \frac{9t}{4} + 1 = 0$$

$$\Rightarrow 2t^2 - 9t + 4 = 0$$

$$\Rightarrow 2t^2 - 8t - t + 4 = 0$$

$$\Rightarrow 2t(t-4) - 1(t-4) = 0$$

$$\Rightarrow (t-4)(2t-1) = 0$$

$$\Rightarrow t-4 = 0 \text{ or } 2t-1 = 0$$

$$\Rightarrow t = 4 \text{ or } t = \frac{1}{2}$$

$$\text{So, } 2^x = 4 \text{ or } 2^x = \frac{1}{2}$$

$$\Rightarrow 2^x = 2^2 \text{ or } 2^x = 2^{-1}$$

$$\Rightarrow x = 2 \text{ or } x = -1$$

**Answer 9L.**

$$5^{x^2} : 5^x = 25 : 1$$

$$\Rightarrow \frac{5^{x^2}}{5^x} = \frac{25}{1}$$

$$\Rightarrow \frac{5^{x^2}}{5^x} = \frac{5^2}{1}$$

$$\Rightarrow 5^{x^2} = 5^2 \times 5^x$$

$$\Rightarrow 5^{x^2} = 5^{2+x}$$

$$\Rightarrow x^2 = 2 + x$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x-2)(x+1) = 0$$

$$\Rightarrow x-2 = 0 \text{ or } x+1 = 0$$

$$\Rightarrow x = 2 \text{ or } x = -1$$

**Answer 9M.**

$$\begin{aligned}\sqrt{\left(8^0 + \frac{2}{3}\right)} &= (0.6)^{2-3x} \\ \Rightarrow \left(1 + \frac{2}{3}\right)^{\frac{1}{2}} &= \left(\frac{6}{10}\right)^{2-3x} \\ \Rightarrow \left(\frac{5}{3}\right)^{\frac{1}{2}} &= \left(\frac{3}{5}\right)^{2-3x} \\ \Rightarrow \left(\frac{3}{5}\right)^{-\frac{1}{2}} &= \left(\frac{3}{5}\right)^{2-3x} \\ \Rightarrow -\frac{1}{2} &= 2-3x \\ \Rightarrow -1 &= 4-6x \\ \Rightarrow -5 &= -6x \\ \Rightarrow x &= \frac{5}{6}\end{aligned}$$

**Answer 9N.**

$$\begin{aligned}\sqrt{\left(\frac{3}{5}\right)^{x+3}} &= \frac{27^{-1}}{125^{-1}} \\ \Rightarrow \left(\frac{3}{5}\right)^{(x+3)\left(\frac{1}{2}\right)} &= \frac{(3^3)^{-1}}{(5^3)^{-1}} \\ \Rightarrow \left(\frac{3}{5}\right)^{\frac{x+3}{2}} &= \left(\frac{3}{5}\right)^{-3} \\ \Rightarrow \frac{x+3}{2} &= -3 \\ \Rightarrow x+3 &= -6 \\ \Rightarrow x &= -9\end{aligned}$$

**Answer 9O.**

$$\begin{aligned}9^{x+4} &= 3^2 \times (27)^{x+1} \\ \Rightarrow 9^{x+4} &= 3^2 \times (3^3)^{x+1} \\ \Rightarrow 3^{2(x+4)} &= 3^2 \times 3^{3x+3} \\ \Rightarrow 3^{2x+8} &= 3^{2+3x+3} \\ \Rightarrow 2x+8 &= 2+3x+3 \\ \Rightarrow 2x+8 &= 3x+5 \\ \Rightarrow x &= 3\end{aligned}$$



**Answer 10.**

$$(i) (\sqrt[3]{8})^{\frac{-1}{2}} = 2^k$$

$$\Rightarrow 8^{\frac{1}{3} \times \frac{-1}{2}} = 2^k$$

$$\Rightarrow (2^3)^{\frac{1}{3} \times \frac{-1}{2}} = 2^k$$

$$\Rightarrow (2^3)^{\frac{1}{3} \times \frac{-1}{2}} = 2^k$$

$$\Rightarrow 2^{\frac{-1}{2}} = 2^k$$

$$= k = -\frac{1}{2}$$

$$(ii) \sqrt[4]{\sqrt[3]{x^2}} = x^k$$

$$\Rightarrow \left\{ \left( x^2 \right)^{\frac{1}{3}} \right\}^{\frac{1}{4}} = x^k$$

$$\Rightarrow \left( x^2 \right)^{\frac{1}{12}} = x^k$$

$$\Rightarrow x^{\frac{2}{12}} = x^k$$

$$\Rightarrow x^{\frac{1}{6}} = x^k$$

$$\Rightarrow k = \frac{1}{6}$$

$$(iii) (\sqrt{9})^{-7} \times (\sqrt{3})^{-5} = 3^k$$

$$\Rightarrow \left\{ (3^2)^{\frac{1}{2}} \right\}^{-7} \times \left\{ (3)^{\frac{1}{2}} \right\}^{-5} = 3^k$$

$$\Rightarrow 3^{-7} \times 3^{\frac{-5}{2}} = 3^k$$

$$\Rightarrow 3^{-7 - \frac{5}{2}} = 3^k$$

$$\Rightarrow 3^{\frac{-14-5}{2}} = 3^k$$

$$\Rightarrow 3^{\frac{-19}{2}} = 3^k$$

$$= k = \frac{-19}{2}$$

$$(iv) \left( \frac{1}{3} \right)^{-4} \div 9^{\frac{-1}{3}} = 3^k$$

$$\Rightarrow (3^{-1})^{-4} \div (3^2)^{\frac{-1}{3}} = 3^k$$

$$\Rightarrow 3^4 \div 3^{\frac{-2}{3}} = 3^k$$

$$\Rightarrow 3^{4 + \frac{2}{3}} = 3^k$$

$$\Rightarrow 3^{\frac{14}{3}} = 3^k$$

$$\Rightarrow k = \frac{14}{3}$$

**Answer 11.**

$$a = 2^{\frac{1}{3}} - 2^{\frac{-1}{3}}$$

$$\Rightarrow a = 2^{\frac{1}{3}} - \frac{1}{2^{\frac{1}{3}}}$$

$$\Rightarrow a^3 = \left( 2^{\frac{1}{3}} - \frac{1}{2^{\frac{1}{3}}} \right)^3 = 2 - \frac{1}{2} - 3 \left( 2^{\frac{1}{3}} - \frac{1}{2^{\frac{1}{3}}} \right)$$

$$\Rightarrow a^3 = \frac{4-1}{2} - 3a$$

$$\Rightarrow a^3 = \frac{3}{2} - 3a$$

$$\Rightarrow 2a^3 + 6a = 3$$

**Answer 12.**

$$x = 3^{\frac{2}{3}} + 3^{\frac{1}{3}}$$

$$\Rightarrow x^3 = 3^2 + 3 + 3 \times 3^{\frac{2}{3}} \times 3^{\frac{1}{3}} \left( 3^{\frac{2}{3}} + 3^{\frac{1}{3}} \right)$$

$$\Rightarrow x^3 = 9 + 3 + 3 \times 3^{\frac{2}{3} + \frac{1}{3}} (x)$$

$$\Rightarrow x^3 = 12 + 9x$$

$$\Rightarrow x^3 - 9x - 12 = 0$$

**Answer 13.**

$$\text{Let } \sqrt[x]{a} = \sqrt[y]{b} = \sqrt[z]{c}$$

$$\Rightarrow a^{\frac{1}{x}} = k, b^{\frac{1}{y}} = k, c^{\frac{1}{z}} = k$$

$$\Rightarrow a = k^x, b = k^y, c = k^z$$

It is also given that  $abc = 1$

$$\Rightarrow k^x \times k^y \times k^z = 1$$

$$\Rightarrow k^{x+y+z} = k^0$$

$$\Rightarrow x + y + z = 0$$

**Answer 14.**

$$\text{Let } a^x = b^y = c^z = k$$

$$\Rightarrow a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}$$

It is also given that  $b^2 = ac$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x}} \times k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x} + \frac{1}{z}}$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\Rightarrow y = \frac{2zx}{z+x}$$

**Answer 15.**

$$\begin{aligned} \text{LHS} &= \frac{1}{1+a^{p-q}} + \frac{1}{1+a^{q-p}} \\ &= \frac{1+a^{q-p} + 1+a^{p-q}}{(1+a^{p-q})(1+a^{q-p})} \\ &= \frac{2+a^{-(p-q)} + a^{p-q}}{(1+a^{p-q})(1+a^{-(p-q)})} \\ &= \frac{2+a^{-(p-q)} + a^{p-q}}{1+a^{-(p-q)} + a^{p-q} + a^{p-q} \cdot a^{-(p-q)}} \\ &= \frac{2+a^{-(p-q)} + a^{p-q}}{1+a^{-(p-q)} + a^{p-q} + a^{p-q-p+q}} \\ &= \frac{2+a^{-(p-q)} + a^{p-q}}{1+a^{-(p-q)} + a^{p-q} + a^0} \\ &= \frac{2+a^{-(p-q)} + a^{p-q}}{1+a^{-(p-q)} + a^{p-q} + 1} \\ &= \frac{2+a^{-(p-q)} + a^{p-q}}{2+a^{-(p-q)} + a^{p-q}} \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

Hence proved.

**Answer 16.**

$$9^{p+2} - 9^p = 240$$

$$\Rightarrow 9^p (9^2 - 1) = 240$$

$$\Rightarrow 9^p (80) = 240$$

$$\Rightarrow 9^p = 3$$

$$\Rightarrow 3^{2p} = 3$$

$$\Rightarrow 2p = 1$$

$$\Rightarrow p = \frac{1}{2}$$

$$\begin{aligned} (8p)^p &= (2^3 p)^p \\ &= \left(2^3 \cdot \frac{1}{2}\right)^{\frac{1}{2}} \\ &= (2^{3-1})^{\frac{1}{2}} \\ &= (2^2)^{\frac{1}{2}} \\ &= 2 \end{aligned}$$

**Answer 17.**

$$a^x = b^y = c^z$$

$$\text{So, } a^x = b^y \Rightarrow a = b^{\frac{y}{x}} \quad \dots \left( \text{Using } a^{\frac{1}{n}} = \sqrt[n]{a} \right)$$

$$b^y = c^z \Rightarrow c = b^{\frac{y}{z}} \quad \dots \left( \text{Using } a^{\frac{1}{n}} = \sqrt[n]{a} \right)$$

$$\text{and } abc = 1$$

$$\Rightarrow b^{\frac{y}{x}} \cdot b \cdot b^{\frac{y}{z}} = 1$$

$$\Rightarrow b^{\frac{y}{x}} \cdot b \cdot b^{\frac{y}{z}} = 1$$

$$\Rightarrow b^{\frac{y}{x} + 1 + \frac{y}{z}} = 1$$

$$\Rightarrow b^{\frac{y}{x} + 1 + \frac{y}{z}} = b^0 \quad \dots \left( \text{Using } a^0 = 1 \right)$$

$$\Rightarrow \frac{y}{x} + 1 + \frac{y}{z} = 0$$

Divide throughout by y.

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{y}{z} = 0$$

Hence proved.

**Answer 18.**

$$x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$$

$$\Rightarrow \left( x^{\frac{1}{3}} + y^{\frac{1}{3}} \right) + z^{\frac{1}{3}} = 0 \text{ cubing both sides, we get :}$$

$$\Rightarrow \left( x^{\frac{1}{3}} + y^{\frac{1}{3}} \right)^3 + z + 3 \left( x^{\frac{1}{3}} + y^{\frac{1}{3}} \right) z^{\frac{1}{3}} \left( x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} \right) = 0$$

$$\Rightarrow x + y + 3 x^{\frac{1}{3}} y^{\frac{1}{3}} \left( x^{\frac{1}{3}} + y^{\frac{1}{3}} \right) + z + 0 = 0$$

$$\Rightarrow x + y + 3 x^{\frac{1}{3}} y^{\frac{1}{3}} \left( -z^{\frac{1}{3}} \right) + z = 0 \quad \text{(Using the given condition again)}$$

$$\Rightarrow x + y + z = 3 x^{\frac{1}{3}} y^{\frac{1}{3}} z^{\frac{1}{3}}$$

$$\Rightarrow (x + y + z)^3 = 27xyz$$

**Answer 19.**

$$\text{Given } 2250 = 2^a \cdot 3^b \cdot 5^c$$

$$\Rightarrow 3^2 \times 5^3 \times 2 = 2^a \cdot 3^b \cdot 5$$

$$\Rightarrow a = 1, b = 2, c = 3$$

$$3^a \times 2^{-b} \times 5^{-c}$$

$$= 3^1 \times 2^{-2} \times 5^{-3}$$

$$= \frac{3}{2^2 \times 5^3}$$

$$= \frac{3}{500}$$

**Answer 20.**

$$2400 = 2^x \times 3^y \times 5^z$$

$$2400 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5$$

$$\therefore 2^x \times 3^y \times 5^z = 2^5 \times 3^1 \times 5^2$$

$$\Rightarrow x = 5, y = 1, z = 2$$

$$\begin{aligned} \therefore 2^{-x} \times 3^y \times 5^z &= 2^{-5} \times 3^1 \times 5^2 \\ &= \frac{1}{32} \times 3 \times 25 = \frac{75}{32} \end{aligned}$$

**Answer 21.**

$$\text{Let } 2^x = 3^y = 12^z = k$$

$$\Rightarrow 2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 12 = k^{\frac{1}{z}}$$

$$\text{Now, } 12 = 2 \times 2 \times 3$$

$$\Rightarrow k^{\frac{1}{z}} = k^{\frac{1}{x}} \times k^{\frac{1}{x}} \times k^{\frac{1}{y}}$$

$$\Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{1}{x} + \frac{1}{y}$$

$$\Rightarrow \frac{1}{z} = \frac{2}{x} + \frac{1}{y}$$

**Answer 22A.**

$$9^{2a} = (\sqrt[3]{81})^{\frac{-6}{b}} = (\sqrt{27})^2$$

$$\Rightarrow 9^{2a} = (\sqrt[3]{3^4})^{\frac{-6}{b}} = (\sqrt{3^3})^2$$

$$\Rightarrow (3^2)^{2a} = \left(3^{4 \times \frac{1}{3}}\right)^{\frac{-6}{b}} = \left(3^{3 \times \frac{1}{2}}\right)^2$$

$$\Rightarrow 3^{4a} = (3^1)^{\frac{-8}{b}} = (3^1)^3$$

$$\Rightarrow 3^{4a} = \frac{-8}{b} = 3$$

$$\Rightarrow 3^{4a} = 3 \text{ and } \frac{-8}{b} = 3$$

$$\Rightarrow 3^{4a} = 3 \text{ and } \frac{-8}{b} = 3$$

$$\Rightarrow 4a = 3 \text{ and } b = \frac{-8}{3}$$

$$\Rightarrow a = \frac{3}{4} \text{ and } b = \frac{-8}{3}$$

**Answer 22B.**

$$(\sqrt{243})^a \div 3^{b+1} = 1 \text{ and } 27^b - 81^{4-\frac{a}{2}} = 0$$

$$\Rightarrow (\sqrt{3^5})^a \div 3^{b+1} = 1 \text{ and } (3^3)^b - (3^4)^{4-\frac{a}{2}} = 0$$

$$\Rightarrow (3^5)^{\frac{a}{2}} \div 3^{b+1} = 1 \text{ and } 3^{3b} - (3^4)^{4-\frac{a}{2}} = 0$$

$$\Rightarrow 3^{\left(\frac{5a}{2}\right)} \div 3^{b+1} = 1 \text{ and } 3^{(3b)} - 3^{4\left(4-\frac{a}{2}\right)} = 0$$

$$\Rightarrow 3^{\left(\frac{5a}{2}-b-1\right)} = 1 \text{ and } 3^{(3b)} - 3^{16-2a} = 0$$

$$\Rightarrow 3^{\left(\frac{5a}{2}-b-1\right)} = 3^0 \text{ and } 3^{3b} = 3^{16-2a}$$

$$\Rightarrow \frac{5a}{2} - b - 1 = 0 \text{ and } 3b = 16 - 2a$$

$$\Rightarrow \frac{5a}{2} - b = 1 \text{ and } 2a + 3b = 16$$

$$\Rightarrow 5a - 2b = 2 \text{ and } 2a + 3b = 16$$

Multiply the equations by 3 and 2 respectively.

$$\Rightarrow 15a - 6b = 6 \text{ and } 4a + 6b = 32$$

Adding the equations,

$$19a = 38$$

$$\Rightarrow a = 2$$

Substitute the value of  $a$  in  $5a - 2b = 2$  to find  $b$ .

$$5a - 2b = 2$$

$$\Rightarrow 5(2) - 2b = 2$$

$$\Rightarrow 10 - 2b = 2$$

$$\Rightarrow b = 4$$

Hence,  $a = 2$  and  $b = 4$ .

### Answer 23A.

$$\begin{aligned} \text{LHS} &= \sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x} \\ &= \sqrt{\frac{y}{x}} \cdot \sqrt{\frac{z}{y}} \cdot \sqrt{\frac{x}{z}} \quad \dots \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\ &= \sqrt{\left( \frac{y}{x} \right) \left( \frac{z}{y} \right) \left( \frac{x}{z} \right)} \\ &= \sqrt{x^{1-1} \cdot y^{1-1} \cdot z^{1-1}} \\ &= \sqrt{x^0 \cdot y^0 \cdot z^0} \\ &= \sqrt{1 \cdot 1 \cdot 1} \\ &= 1 \quad \dots \dots \left( \text{Using } a^0 = 1 \right) \\ &= \text{RHS} \end{aligned}$$

Hence proved.

### Answer 23B.

$$\begin{aligned} \text{LHS} &= \left( \frac{a^m}{a^n} \right)^{m+n-1} \cdot \left( \frac{a^n}{a^1} \right)^{n+1-m} \cdot \left( \frac{a^1}{a^m} \right)^{1+m-n} \\ &= \frac{a^{m(m+n-1)}}{a^{n(m+n-1)}} \cdot \frac{a^{n(n+1-m)}}{a^{1(n+1-m)}} \cdot \frac{a^{1(1+m-n)}}{a^{m(1+m-n)}} \quad \dots \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\ &= \frac{a^{m^2+mn-m}}{a^{n^2+mn-n}} \cdot \frac{a^{n^2-mn+n}}{a^{n+1-m}} \cdot \frac{a^{1+m-n}}{a^{m^2-mn+m}} \\ &= a^{m^2+mn-m-(n^2+mn-n)} \cdot a^{n^2-mn+n-(n+1-m)} \cdot a^{1+m-n-(m^2-mn+m)} \quad \dots \dots \left( \text{Using } a^m \div a^n = a^{m-n} \right) \\ &= a^{m^2+mn-m-n^2-mn+n} \cdot a^{n^2-mn+n-n-1+m} \cdot a^{1+m-n-m^2+mn-m} \\ &= a^{m^2+mn-m-n^2-mn+n+n^2-mn+n-n-1+m+1+m-n-m^2+mn-m} \quad \dots \dots \left( \text{Using } a^m \times a^n = a^{m+n} \right) \\ &= a^0 \\ &= 1 \quad \dots \dots \left( \text{Using } a^0 = 1 \right) \\ &= \text{RHS} \end{aligned}$$

Hence proved.

**Answer 23C.**

$$\begin{aligned}
\text{LHS} &= \left(\frac{a^m}{a^n}\right)^{m+n-1} \cdot \left(\frac{a^n}{a^1}\right)^{n+1-m} \cdot \left(\frac{a^1}{a^m}\right)^{1+m-n} \\
&= \frac{a^{m(m+n-1)}}{a^{n(m+n-1)}} \cdot \frac{a^{n(n+1-m)}}{a^{1(n+1-m)}} \cdot \frac{a^{1(1+m-n)}}{a^{m(1+m-n)}} \dots \left(\text{Using } (a^m)^n = a^{mn}\right) \\
&= \frac{a^{m^2+mn-m}}{a^{n^2+mn-n}} \cdot \frac{a^{n^2-mn+n}}{a^{n+1-m}} \cdot \frac{a^{1+m-n}}{a^{m^2-mn+m}} \\
&= a^{m^2+mn-m-(n^2+mn-n)} \cdot a^{n^2-mn+n-(n+1-m)} \cdot a^{1+m-n-(m^2-mn+m)} \dots \left(\text{Using } a^m \div a^n = a^{m-n}\right) \\
&= a^{m^2+mn-m-n^2-mn+n} \cdot a^{n^2-mn+n-n-1+m} \cdot a^{1+m-n-m^2+mn-m} \\
&= a^{m^2+mn-m-n^2-mn+n+n^2-mn+n-n-1+m+1+m-n-m^2+mn-m} \dots \left(\text{Using } a^m \times a^n = a^{m+n}\right) \\
&= a^0 \\
&= 1 \dots \dots \left(\text{Using } a^0 = 1\right) \\
&= \text{RHS}
\end{aligned}$$

Hence proved.

**Answer 23D.**

$$\begin{aligned}
\text{LHS} &= \sqrt[ab]{\frac{x^a}{x^b}} \cdot \sqrt[bc]{\frac{x^b}{x^c}} \cdot \sqrt[ca]{\frac{x^c}{x^a}} \\
&= \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \\
&= \frac{x^{\frac{1}{b}}}{x^{\frac{1}{a}}} \cdot \frac{x^{\frac{1}{c}}}{x^{\frac{1}{b}}} \cdot \frac{x^{\frac{1}{a}}}{x^{\frac{1}{c}}} \dots \dots \left(\text{Using } (a^m)^n = a^{mn}\right) \\
&= x^{\frac{1}{b}-\frac{1}{a}} \cdot x^{\frac{1}{c}-\frac{1}{b}} \cdot x^{\frac{1}{a}-\frac{1}{c}} \dots \dots \left(\text{Using } a^m \div a^n = a^{m-n}\right) \\
&= x^{\frac{a-b}{ab}} \cdot x^{\frac{b-c}{bc}} \cdot x^{\frac{c-a}{ac}} \\
&= x^{\frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c-a}{ac}} \dots \dots \left(\text{Using } a^m \times a^n = a^{m+n}\right) \\
&= x^{\frac{ac-bc+ab-ac+bc-ab}{abc}} \\
&= x^{\frac{0}{abc}} \\
&= x^0 \\
&= 1 \dots \dots \left(\text{Using } a^0 = 1\right) \\
&= \text{RHS}
\end{aligned}$$

Hence proved.



**Answer 23E.**

$$\begin{aligned}
\text{LHS} &= (x^a)^{b-c} \times (x^b)^{c-a} \times (x^c)^{a-b} \\
&= x^{a(b-c)} \times x^{b(c-a)} \times x^{c(a-b)} \quad \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
&= x^{ab-ac} \times x^{bc-ab} \times x^{ac-bc} \\
&= x^{ab-ac+bc-ab+ac-bc} \quad \dots \left( \text{Using } a^m \times a^n = a^{m+n} \right) \\
&= x^0 \\
&= 1 \\
&= \text{RHS}
\end{aligned}$$

Hence proved.

**Answer 23F.**

$$\begin{aligned}
\text{LHS} &= \frac{x^{p(q-r)}}{x^{q(p-r)}} \div \left( \frac{x^q}{x^p} \right)^r \\
&= \frac{x^{p(q-r)}}{x^{q(p-r)}} \div \frac{x^{qr}}{x^{pr}} \quad \dots \left( \text{Using } (a^m)^n = a^{mn} \right) \\
&= \frac{x^{p(q-r)}}{x^{q(p-r)}} \times \frac{x^{pr}}{x^{qr}} \\
&= \frac{x^{pq-pr}}{x^{pq-qr}} \times \frac{x^{pr}}{x^{qr}} \\
&= \frac{x^{pq-pr+pr}}{x^{pq-qr+qr}} \quad \dots \left( \text{Using } a^m \times a^n = a^{m+n} \right) \\
&= \frac{x^{pq}}{x^{pq}} \\
&= 1 \\
&= \text{RHS}
\end{aligned}$$

Hence proved.