☆ Polar Plots :-

* **Purpose:**

⇒ To draw the Freq. response of the OLTF.

⇒ To find the CL system stability.

⇒ To find the gain Margin and phase Margin.

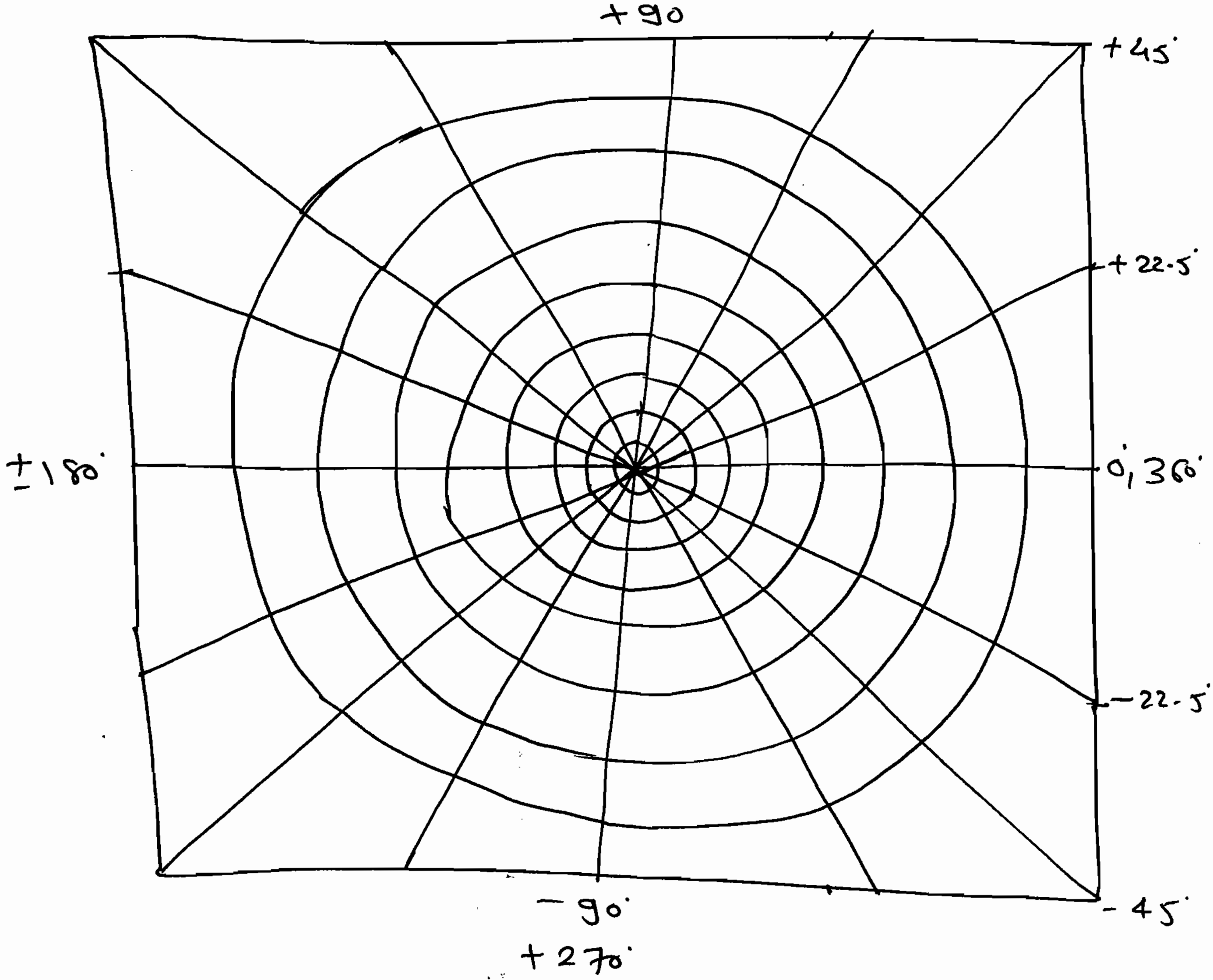
⇒ The Polar plots are used in the Nyquist plots.

⇒ The Polar plots is not a Complete freq. response plots. The Complete freq. response plots are Nyquist plots.

⇒ The freq. range for Polar plot is 0 to ∞ where as for Nyquist plot the freq. range is $-\infty$ to $+\infty$.

⇒ The Polar plot is nothing but the Mag. verses Phase plot.

⇒



Q. Draw the Polar plot for $GH(s) = 1/s$.

Soln:

$$GH(s) = \frac{1}{s}.$$

$s \to j\omega$

$$GH(j\omega) = \frac{1}{j\omega}.$$

$$M = |GH(j\omega)| = \frac{1}{\omega}. \quad (2).$$

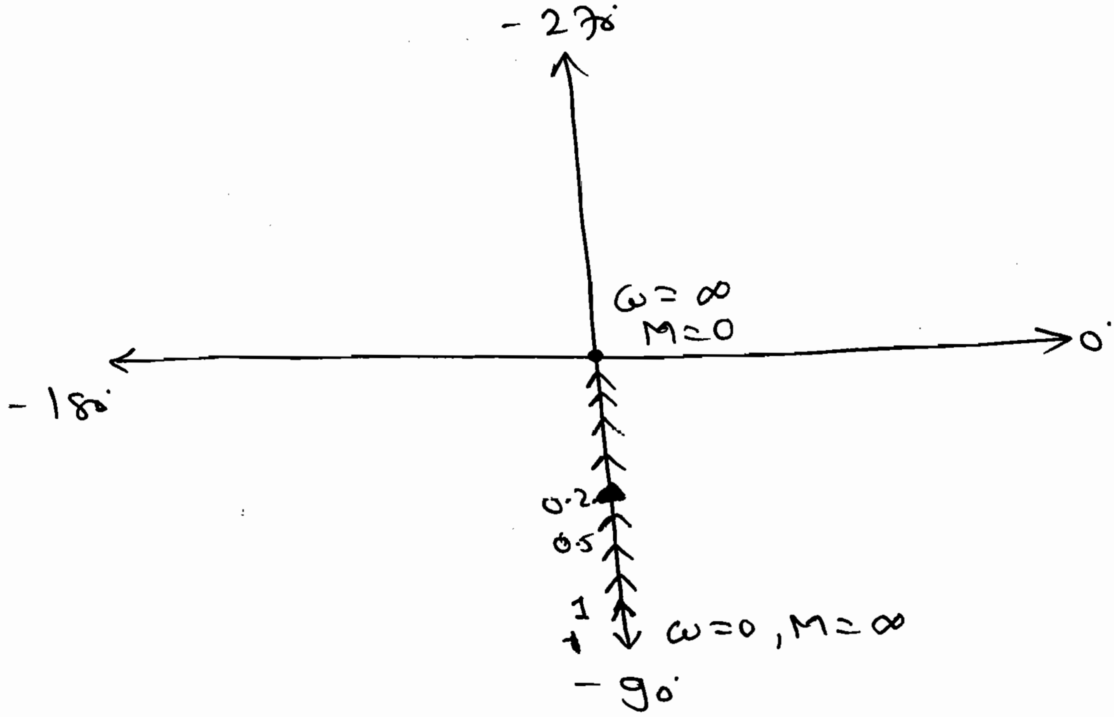
$$M_{in\,dB} = -20\log\omega.$$

$$\angle GH(j\omega) = -90°.$$

⇒

ω	M	ϕ
0	∞	$-90°$
1	1	$-90°$
2	0.5	$-90°$
5	0.2	$-90°$
10	0.1	$-90°$
⋮	⋮	⋮
∞	0	$-90°$

⇒



$\boxed{a}$ $GH(s) = \dfrac{1}{(s\tau+1)}$.

Sol^n:

$$GH(j\omega) = \frac{1}{(j\omega\tau+1)}$$

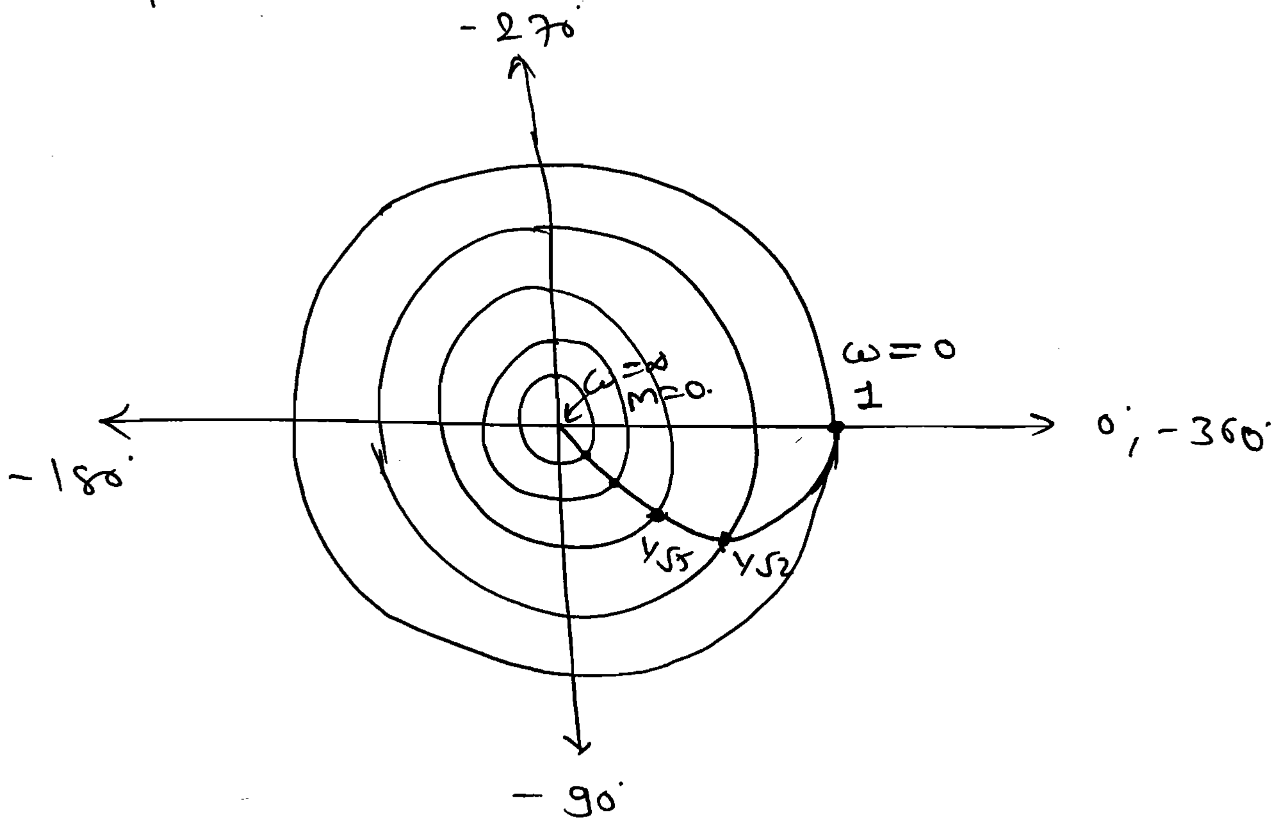
$$M = |GH(j\omega)| = \frac{1}{\sqrt{(\omega\tau)^2+1}}$$

$$\angle GH = \phi = -\tan^{-1}(\omega\tau).$$

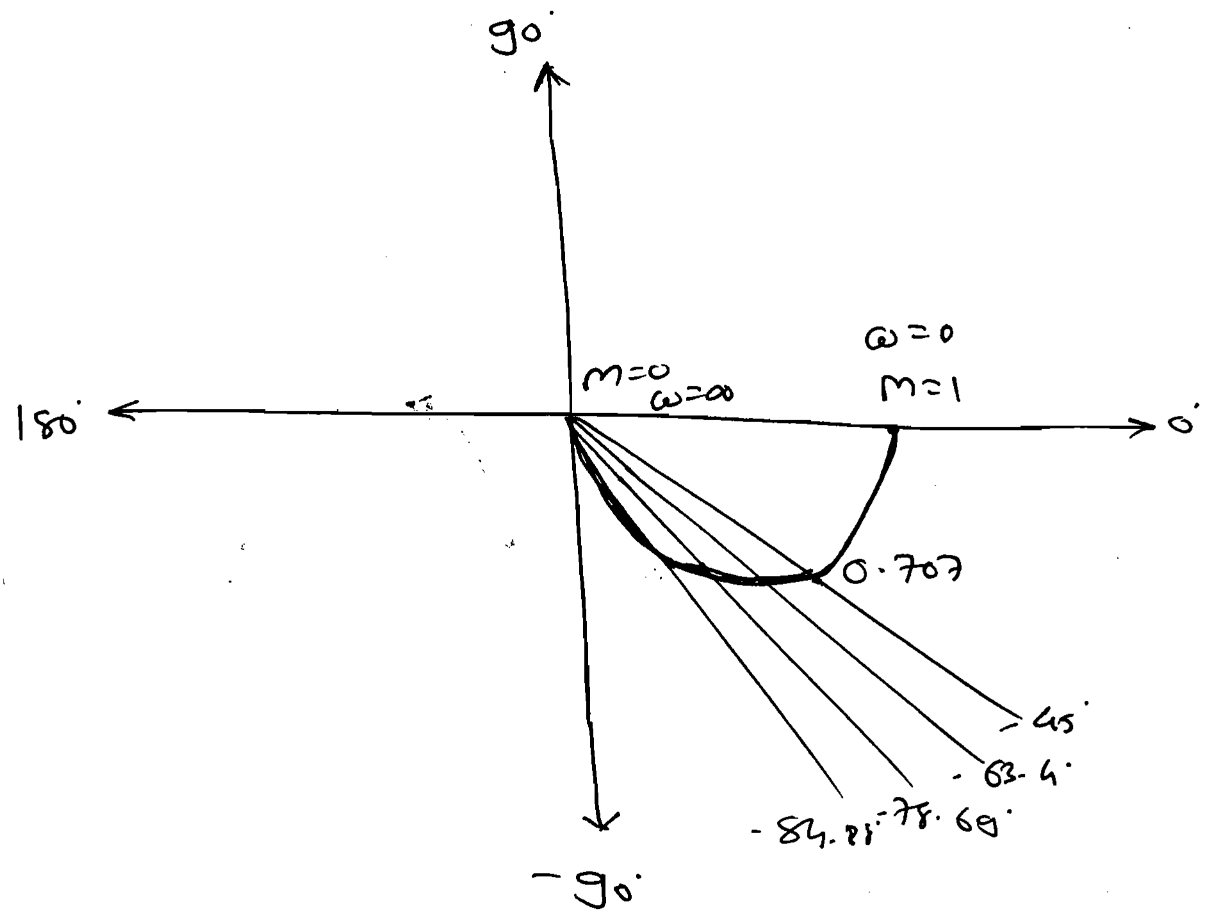
ω	M	ϕ
0	1	0°
$1/\tau$	$1/\sqrt{2}$	135° (-45°).
$2/\tau$	$1/\sqrt{5}$	-63.43

ω	M	ϕ
$5/\tau$	$1/\sqrt{26}$	$-78.69°$
$10/\tau$	$1/\sqrt{101}$	$-84.28°$
⋮	⋮	⋮
∞	0	$-90°$

⇒



⇒



Q Draw the Polar plots:

① $GH(s) = \dfrac{(s+1)}{(s+10)}$ (HPF, Lead Comp.).

Sol^n:

$$GH(j\omega) = \frac{(1+j\omega)}{(10+j\omega)}$$

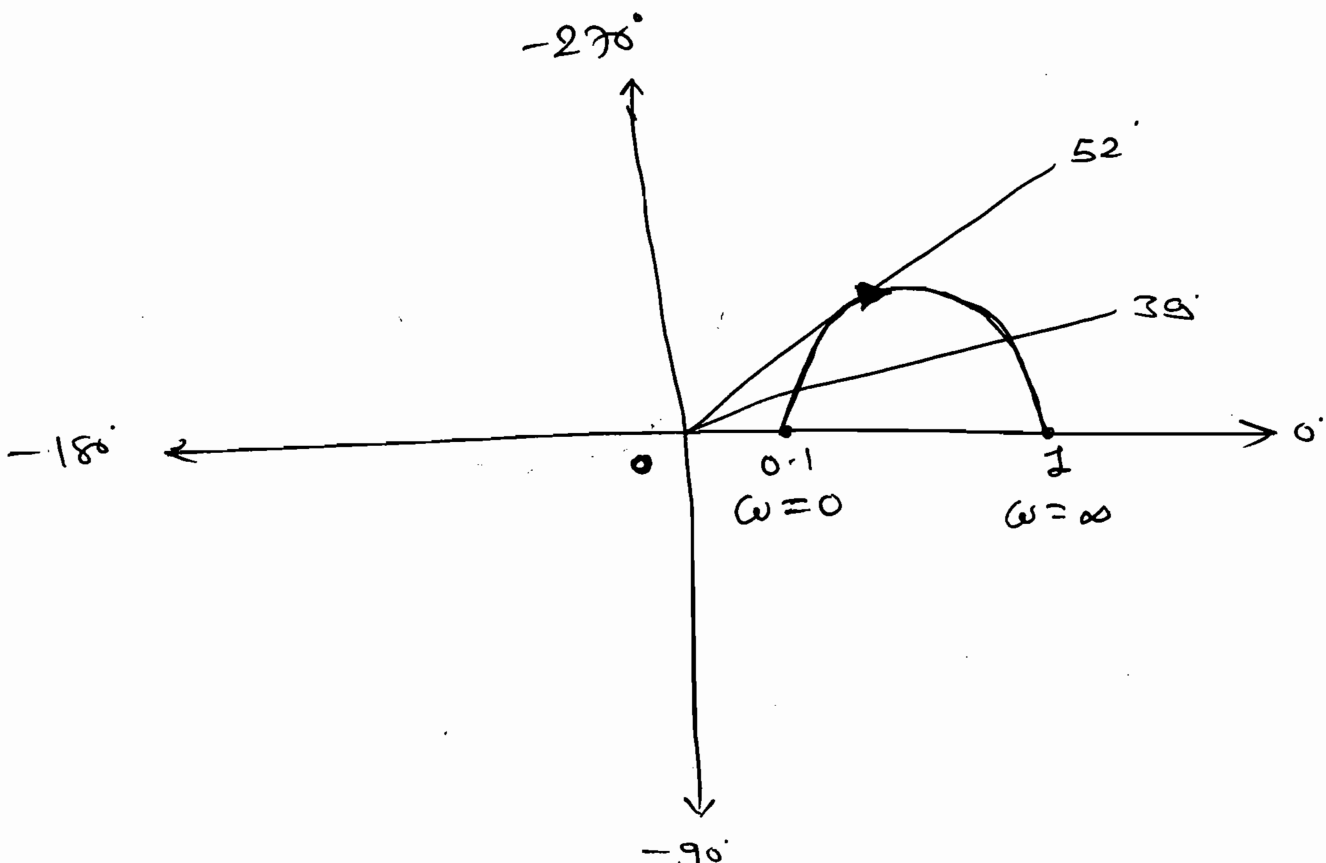
$$\Rightarrow \quad M = \frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2+100}}, \quad \phi = \tan^{-1}(\omega) - \tan^{-1}(\omega/10)$$

ω	M	ϕ
0	0.1	0°
1	0.141	39.29°
2	0.22	52.125°
5	0.456	52.125°
10	0.710	39.29°
⋮		
∞	1	0°

$$\lim_{\omega\to\infty} \frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2+100}}$$

$$= \lim_{\omega\to\infty} \sqrt{\frac{1+1/\omega^2}{1+100/\omega^2}}$$

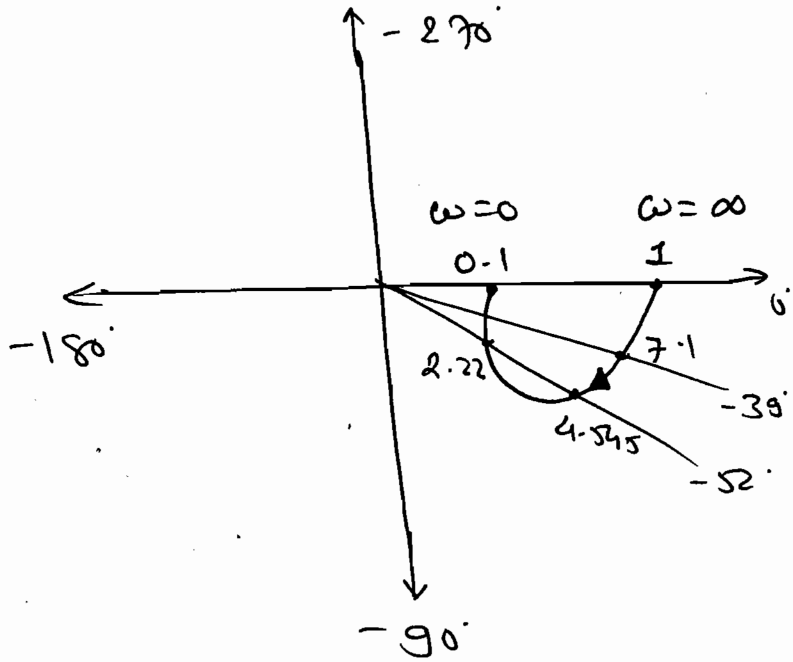
$$= \sqrt{\frac{1+0}{1+0}} = 1.$$



(e) $GH(s) = \dfrac{(s+10)}{(s+1)}$ (LPF Lag Cmp.).

Soln: $M = \dfrac{\sqrt{\omega^2+100}}{\sqrt{\omega^2+1}}$, $\phi = \tan^{-1}(\omega/10) - \tan^{-1}(\omega)$.

ω	M	ϕ
0	10	0°
1	7.1	−39°
2	4.545	−52°
5	2.22	−52°
10	0.14	−39°
⋮	⋮	
∞	1	0



* Procedure to draw the Polar plots:

Step-1:

⇒ Find the M_1 and ϕ_1 at $\omega = 0$.

Step-2:

⇒ Find the M_2 and ϕ_2 at $\omega = \infty$.

Step-3:

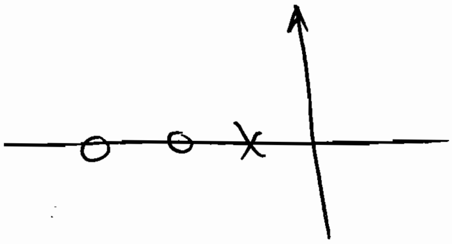
⇒ Ending direction

$\boxed{\phi_1 - \phi_2 = +ve \rightarrow CW. \quad = -ve \rightarrow ACW.}$

Step-4: Starting Direction:

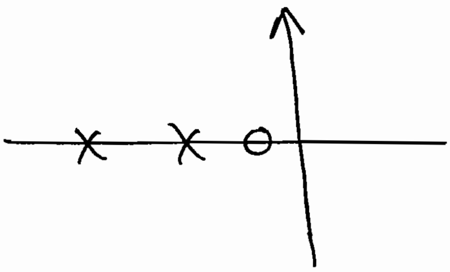
⇒ The starting direction is consider to the TF if it should have only +ve sign terms.

⇒ If the finite Pole is near to the imaginary axis then the S.D. is Cω.



$f_p \rightarrow$ Cω.

⇒ If finite zero near to imaginary then the S.D. is ACω.



$f_z \rightarrow$ ACω.

⇒ The above Procedure is Valid when M at ~~origin~~ i.e. $\omega = 0$ is greater or equal to M at $\omega = \infty$.

$$\boxed{M\big|_{\omega=0} \geq M\big|_{\omega=\infty}}$$

⇒ If the $M\big|_{\omega=0} < M\big|_{\omega=\infty}$ like HPF (when TF consist only zeros (or) zeros at origin and poles = zero) check the Mag.)

⇒ Like HPF draw the polar plot using Standard Procedure.

Q. Draw the Polar Plots for $GH(s) = \frac{1}{s+1}$.

Soln:

$$GH(s) = \frac{1}{s+1}.$$

$s \to j\omega$

$$\Rightarrow GH(j\omega) = \frac{1}{j\omega+1}.$$

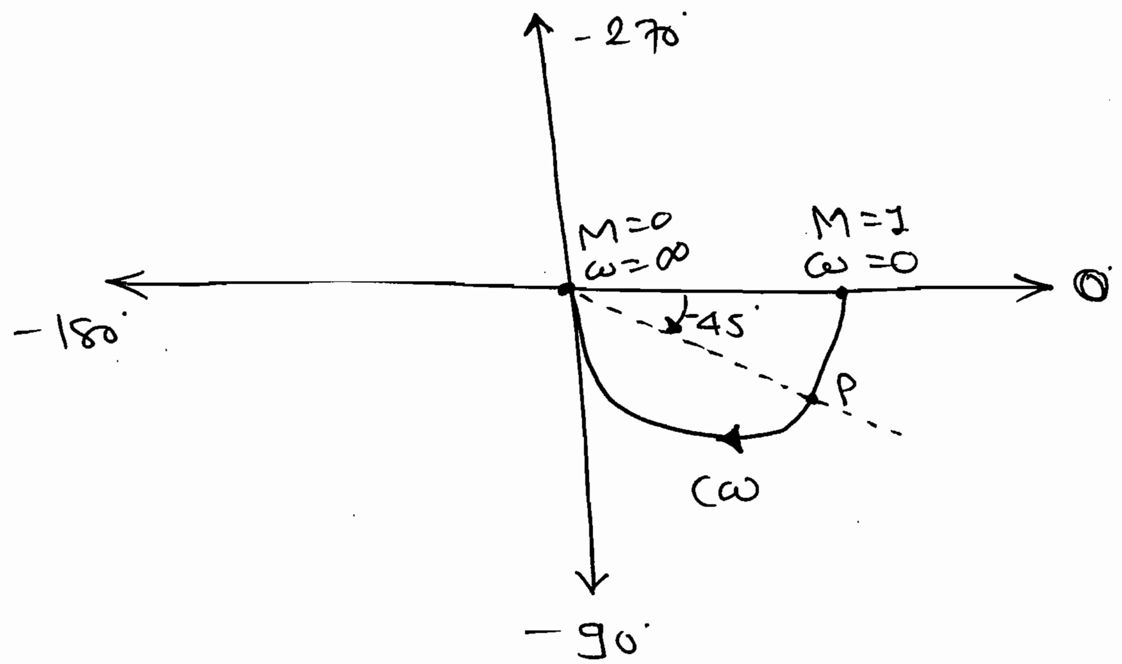
$$M = \frac{1}{\sqrt{\omega^2+1}}, \quad \angle\phi = -\tan^{-1}(\omega).$$

ω	M	ϕ
0	1	0°
∞	0	-90°

$\phi_1 - \phi_2 = 0 - (-90^\circ) = 90^\circ$

$= +ve \Rightarrow$ E.D. $\Rightarrow$ CW.

f-P $\Rightarrow$ S.D. $\Rightarrow$ CW.



$\Rightarrow$ given $\phi = -45^\circ$.

$\therefore -45^\circ = -\tan^{-1}(\omega)$

$\therefore \boxed{\omega = 1 \text{ rad/sec}}$

Rectangular
$\Downarrow$
$\boxed{R\left(\frac{1}{2}, -\frac{1}{2}\right)}$

$$\Rightarrow M\Big|_{\omega=1} = \frac{1}{\sqrt{2}} = 0.707$$

$\Rightarrow$ I.P. $\boxed{P\left(\frac{1}{\sqrt{2}} \angle -45^\circ\right) \Rightarrow \text{polar}}$

$\boxed{a}$ $GH(s) = \dfrac{1}{(s+1)(s+2)}$.

Soln:

$s \rightarrow j\omega$

$GH(j\omega) = \dfrac{1}{(j\omega+1)(j\omega+2)}$.

$M = |GH(j\omega)| = \dfrac{1}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}$.

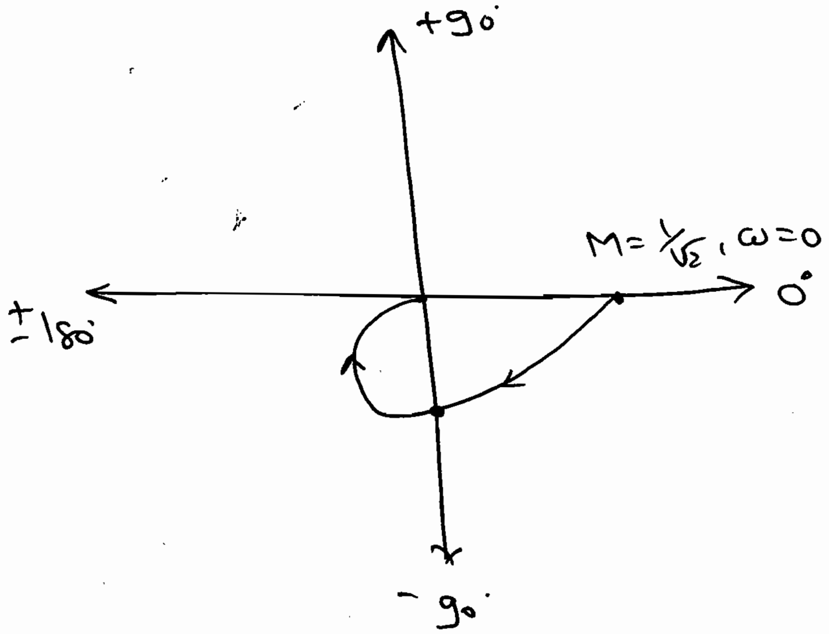
$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2)$.

$\omega = 0 \Rightarrow M = 1/2$. & $\phi_1 = 0^\circ$.

$\omega = \infty \Rightarrow M = 0$ & $\phi_2 = -90^\circ - 90^\circ = -180^\circ$.

E.D. $\Rightarrow$ $\phi_1 - \phi_2 = 0 - (-180^\circ) = +180^\circ \Rightarrow$ CW.

S.D. $\Rightarrow$ FP $\Rightarrow$ CW.



$\Rightarrow$ Intersection Point with -90°.

$\therefore \phi = -90^\circ$

$\Rightarrow -90^\circ = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2)$.

$\Rightarrow 90^\circ = \tan^{-1}(\omega) + \tan^{-1}(\omega/2)$.

$$\Rightarrow \quad 90^\circ = \tan^{-1}\left(\frac{\omega + \omega/2}{1-\omega^2/2}\right).$$

$$\Rightarrow \quad \tan 90^\circ = \frac{\omega + \omega/2}{1-\omega^2/2}.$$

$$\Rightarrow \quad \infty = \frac{3\omega}{2-\omega^2}.$$

$$\therefore \quad 2-\omega^2 = 0$$

$$\Rightarrow \quad \omega^2 = 2 \quad \Rightarrow \quad \boxed{\omega = \sqrt{2} \text{ rad/sec}}$$

$$M\Big|_{\omega=\sqrt{2}} = \frac{1}{\sqrt{2+1}\,\sqrt{4+2}} = \frac{1}{\sqrt{18}}.$$

$\Rightarrow$ Polar $\frac{1}{\sqrt{18}} \angle -90^\circ$

$\Rightarrow$ Rect $\left(0,\ j\frac{1}{\sqrt{18}}\right).$

$\boxed{a}$ $$GH = \frac{1}{(S+1)(S+2)(S+3)}.$$

Sol^n:

$$M = |GH(j\omega)| = \frac{1}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4} \times \sqrt{\omega^2+9}}.$$

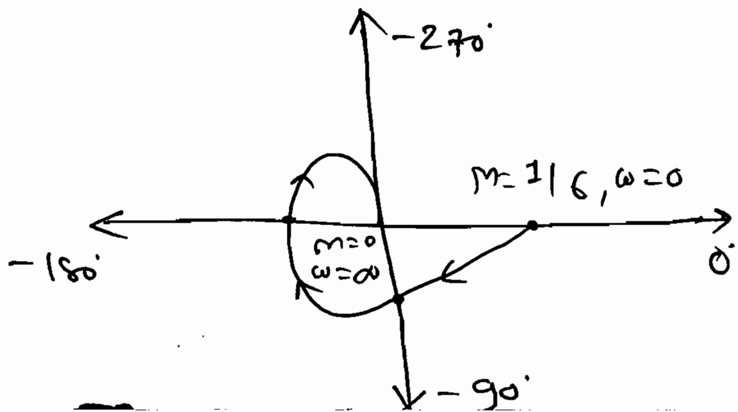
$$\phi = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\omega/3).$$

$\omega = 0 \Rightarrow \quad M = \frac{1}{\sqrt{36}} = 1/6. \quad , \quad \phi = 0^\circ$

$\omega = \infty \Rightarrow \quad M = 0 \quad \& \quad \phi = -90^\circ - 90^\circ - 90^\circ = -270^\circ.$

F.D. $\Rightarrow$ $\phi_1 - \phi_2 = +270^\circ \Rightarrow +ve$
$= CW$

S.D. $\Rightarrow$ finite
pole $\Rightarrow$ CW



$\Rightarrow$ Intersection pt with (-90°).

$$\phi = -90^\circ \Rightarrow -90^\circ = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3).$$

$$\Rightarrow \quad 90^\circ = \tan^{-1}(\omega) + \tan^{-1}\left(\frac{\frac{\omega}{2} + \omega/3}{1 - \omega^2/6}\right).$$

$$\therefore \quad 90^\circ = \tan^{-1}(\omega) + \tan^{-1}\left(\frac{5\omega}{6-\omega^2}\right).$$

$$\therefore \quad 90^\circ = \tan^{-1}\left(\frac{\omega + \frac{5\omega}{6-\omega^2}}{1 - \frac{5\omega^2}{6-\omega^2}}\right).$$

$$\therefore \quad \infty = \frac{\omega + \frac{5\omega}{6-\omega^2}}{1 - 5\omega^2/6-\omega^2}$$

$$\Rightarrow \quad 1 - \frac{5\omega^2}{6-\omega^2} = 0 \Rightarrow \quad 6-\omega^2 - 5\omega^2 = 0$$

$$\boxed{\omega = 1 \text{ rad/sec}}$$

$\Rightarrow$ I.P. with -180°

$$\Rightarrow \quad -180^\circ = -\tan^{-1}\left(\frac{\omega + \frac{5\omega}{6-\omega^2}}{1 - \frac{5\omega^2}{6-\omega^2}}\right).$$

$$\Rightarrow \quad \omega + \frac{5\omega}{6-\omega^2} = 0.$$

$$\therefore \quad 6\omega - \omega^3 + 5\omega = 0$$

$$\therefore \quad \omega^2 = 11$$

$$\boxed{\omega = \sqrt{11} \text{ rad/sec}}$$

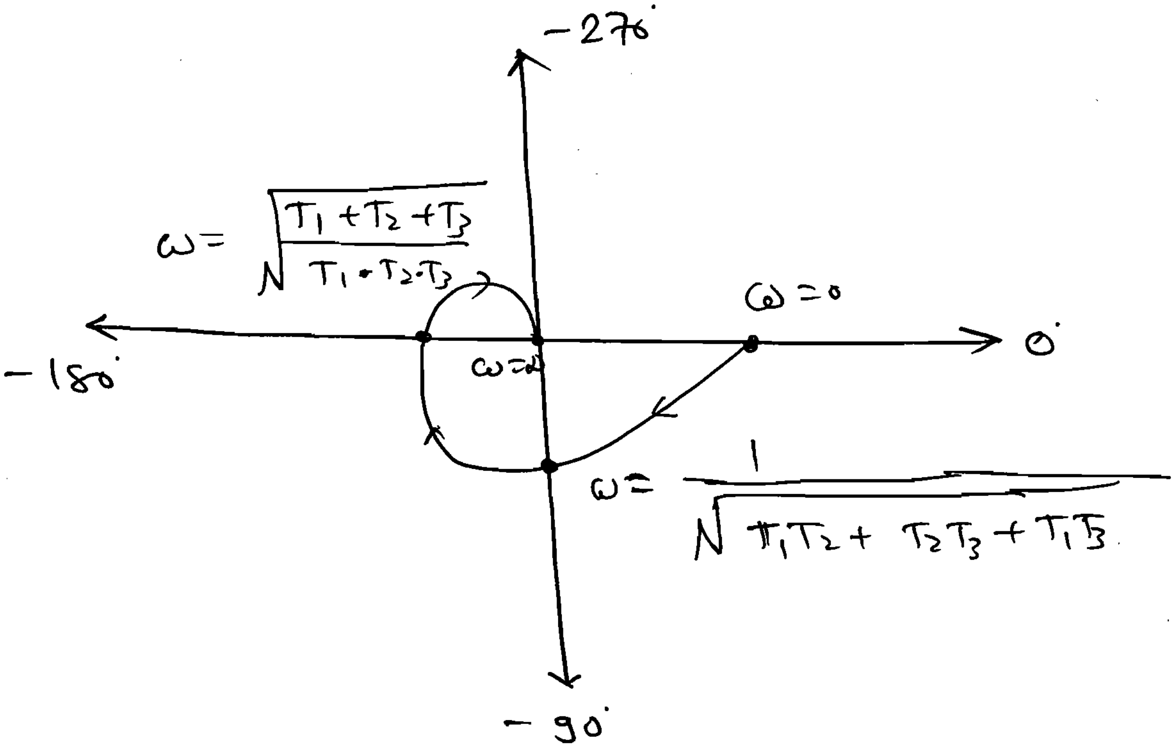
$$\Rightarrow M\Big|_{\omega=1} = \frac{1}{\sqrt{2} \times \sqrt{5}\sqrt{10}} = \frac{1}{\sqrt{100}} = 0.1$$

$\Rightarrow$ I.P. $(0, -j0.1)$.

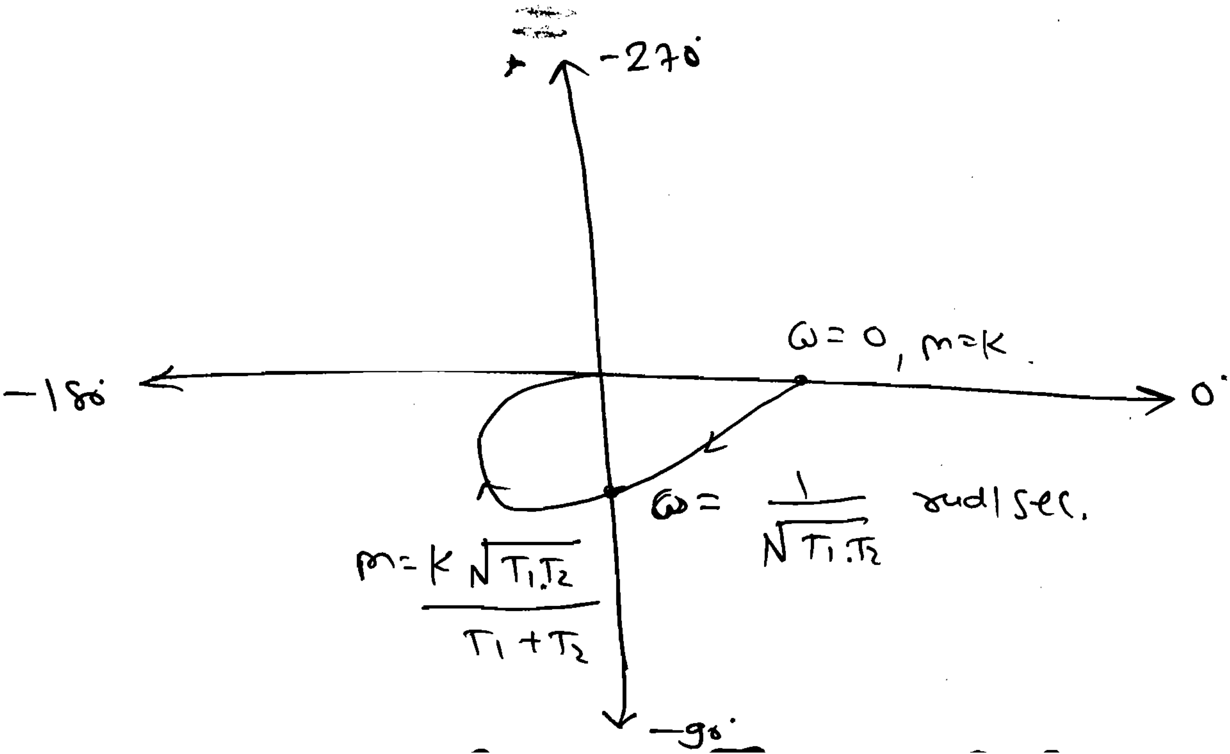
$$\Rightarrow \quad M\Big|_{\omega=\sqrt{11}} = \frac{1}{\sqrt{12}\times\sqrt{16}\times\sqrt{20}} = \frac{1}{60}.$$

I.P. $\left(-\frac{1}{60}, -j0\right)$.

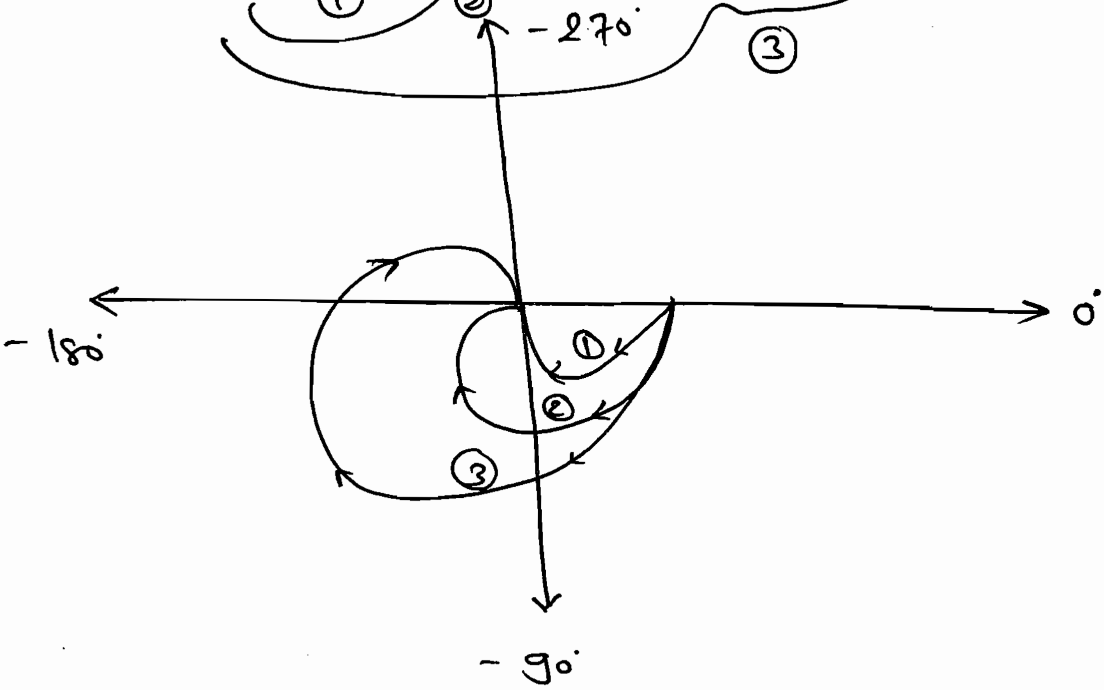
$$* \quad GH = \frac{K}{(sT_1+1)(sT_2+1)(sT_3+1)}$$



$$* \quad GH(s) = \frac{K}{(sT_1+1)(sT_2+1)}$$



* $GH(s) = \dfrac{K}{\underbrace{\underbrace{\underbrace{(sT_1+1)}_{①}(sT_2+1)}_{②}(sT_3+1)}_{③}}$.



Note:

⇒ The addition of each finite pole in the left hand side shift Ending angle by $-90°$, in the clock-wise direction.

$\boxed{a}$ $GH(s) = \dfrac{1}{s(s+1)}$.

Soln:

$$|GH(\omega)| = M = \frac{1}{\omega\sqrt{\omega^2+1}}$$

$$\Rightarrow \quad \phi = -90° - \tan^{-1}(\omega).$$

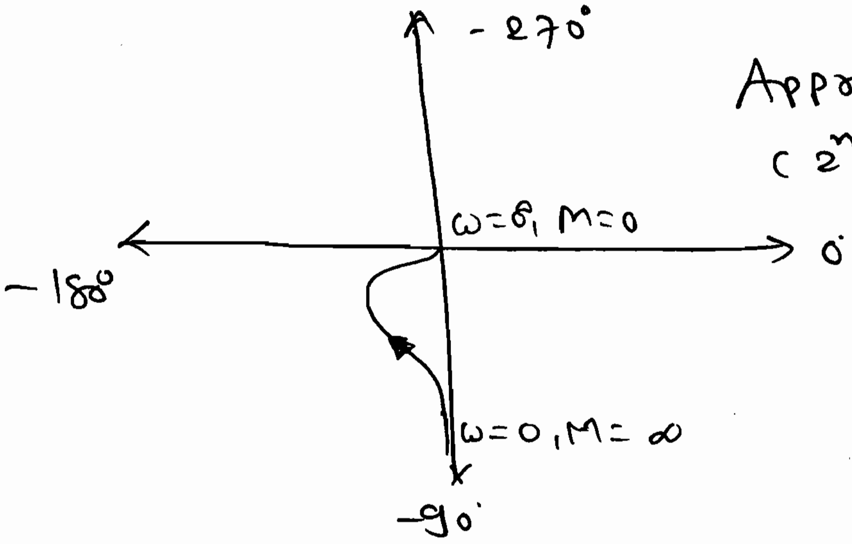
$\omega = 0 \Rightarrow M = \infty$ & $\phi = -90°$

$\omega = \infty \Rightarrow M = 0$ & $\phi = -180°$

E.D. ⇒ $\phi_1 - \phi_2 = -90° + 180° = 90° = CW$

S.D. ⇒ b/p ⇒ CW.

$\Rightarrow$



Approx (2nd priority).

$$\Rightarrow GH = \frac{1}{j\omega(j\omega+1)} \times \frac{1-j\omega}{1-j\omega}$$

$$= \frac{1-j\omega}{j\omega(1+\omega^2)} = \frac{-j(1-j\omega)}{\omega(1+\omega^2)}$$

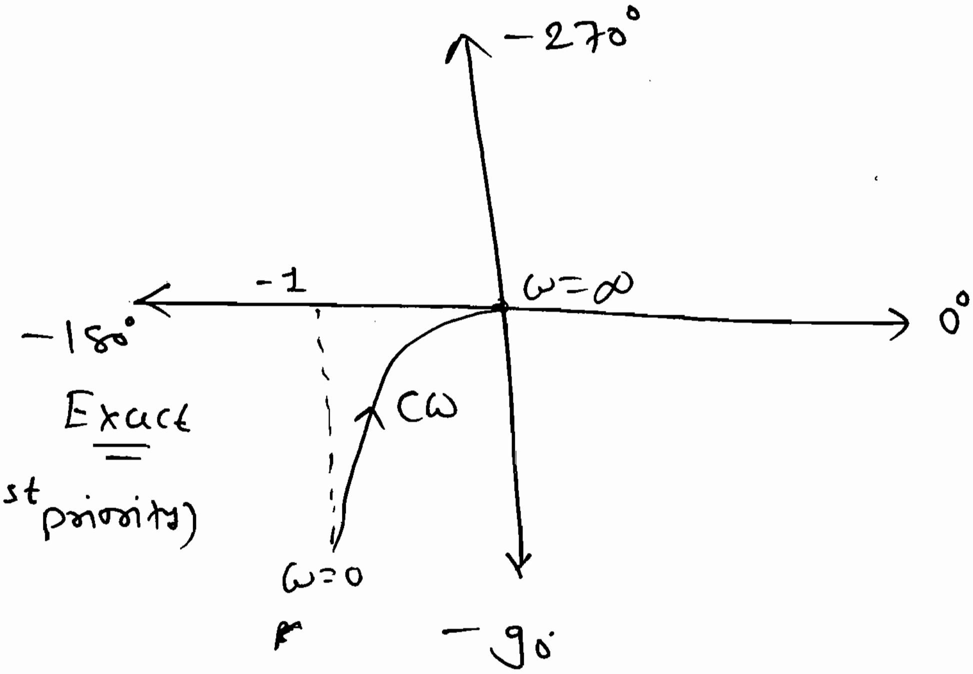
$$= \frac{-\omega - j(1)}{\omega(1+\omega^2)}$$

$$GH = \frac{-\cancel{\omega}}{\cancel{\omega}(1+\omega^2)} - \frac{j}{\omega(1+\omega^2)}$$

$\Rightarrow \omega = 0 \quad GH = -1 - j(\infty).$

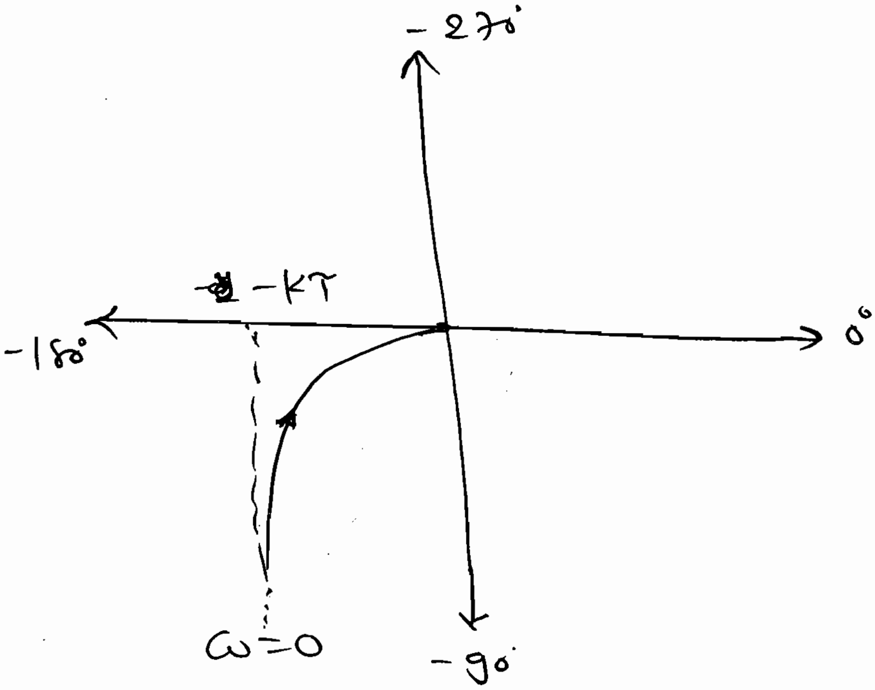
$GH = -1 - j\infty.$

$\Rightarrow$

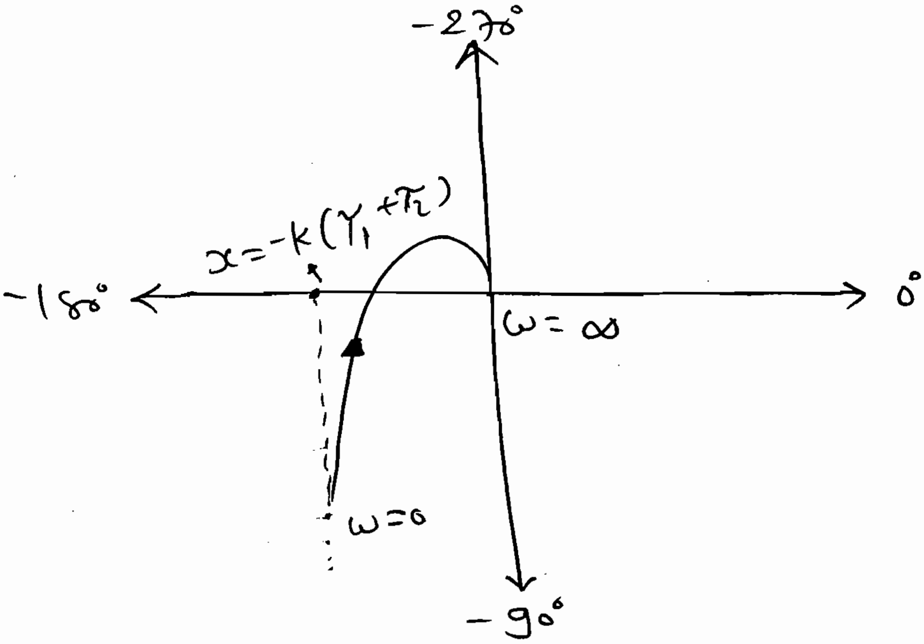


Exact (1st priority)

* $GH(s) = \dfrac{K}{s(s\tau_1+1)}$.



* $GH(s) = \dfrac{K}{s(s\tau_1+1)(s\tau_2+1)}$.



Q $GH = \dfrac{1}{s^2(s+1)}$.

Solⁿ: $M = \dfrac{1}{\omega^2\sqrt{\omega^2+1}}$. & $\phi = -180^\circ - \tan^{-1}(\omega)$.

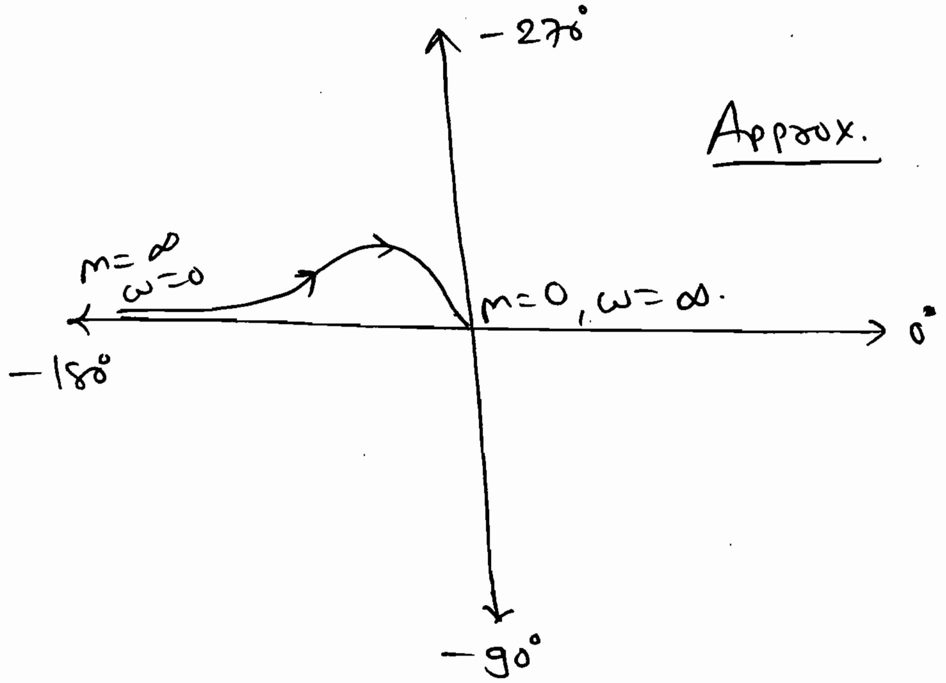
$\omega=0 \Rightarrow M=\infty$ & $\phi = -180^\circ$.

$\omega=\infty \Rightarrow M=0$ & $\phi = -270^\circ$.

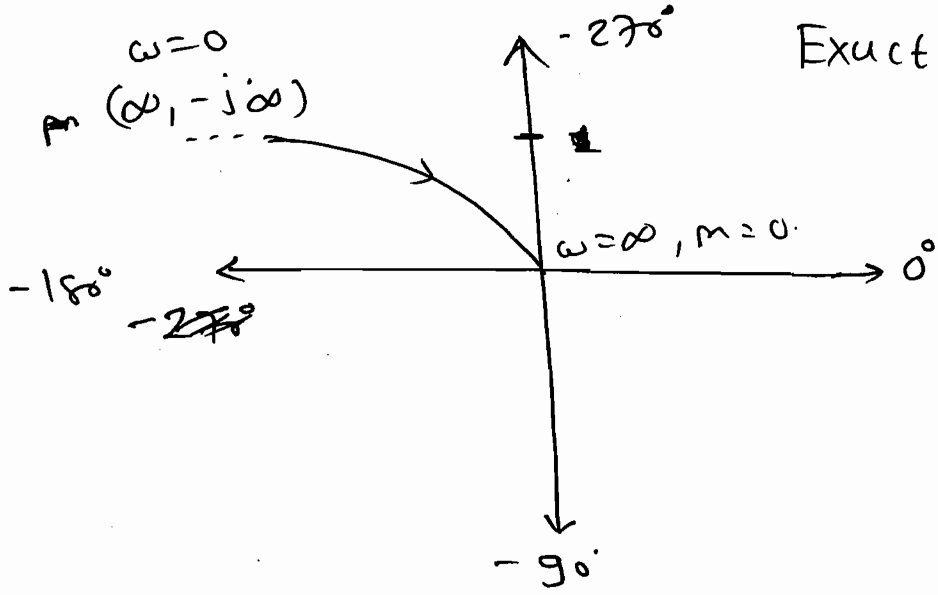
$\Rightarrow$ E.D. $\Rightarrow$ $\phi_1 - \phi_2 = -180^\circ - (-270^\circ) = +90^\circ$ $\Rightarrow$ +ve. $\Rightarrow$ CW

$\Rightarrow$ S.D. $\Rightarrow$ fP $\Rightarrow$ CW.

$\Rightarrow$



$\Rightarrow$



$$\Rightarrow GH(j\omega) = \frac{1}{-\omega^2 (j\omega + 1)} \times \frac{1 - j\omega}{1 - j\omega}.$$

$$= \frac{1 - j\omega}{-\omega^2 (1 + \omega^2)}.$$

$$GH(j\omega) = \frac{-1}{\omega^2 (1+\omega^2)} - \frac{j}{\omega(1+\omega^2)}.$$

$$\Rightarrow GH(j\omega)\Big|_{\omega=0} = -\infty - j\infty = (-\infty, -j\infty).$$

Q $GH = \dfrac{1}{S^3(S+1)}$

Soln:

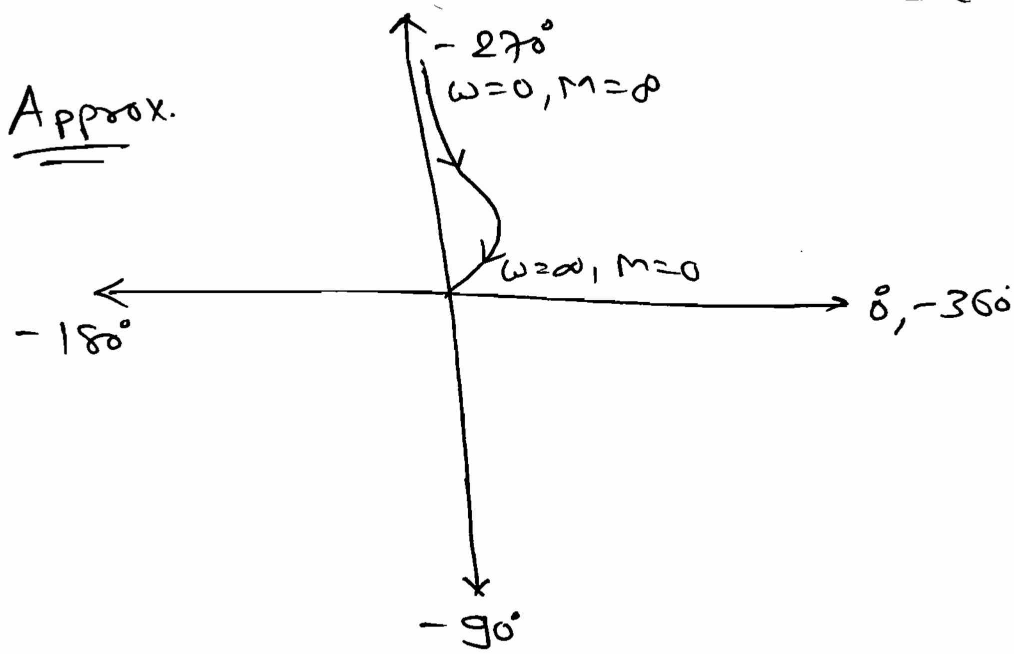
$M = \dfrac{1}{\omega^3\sqrt{\omega^2+1}}$ & $\phi = -270° - \tan^{-1}(\omega)$.

$\omega=0 \Rightarrow M = \infty$ & $\phi = -270°$. E.O. = +ve = cw

$\omega=\infty \Rightarrow M = 0$ & $\phi = -360°$. S.O = finite pole = cw.

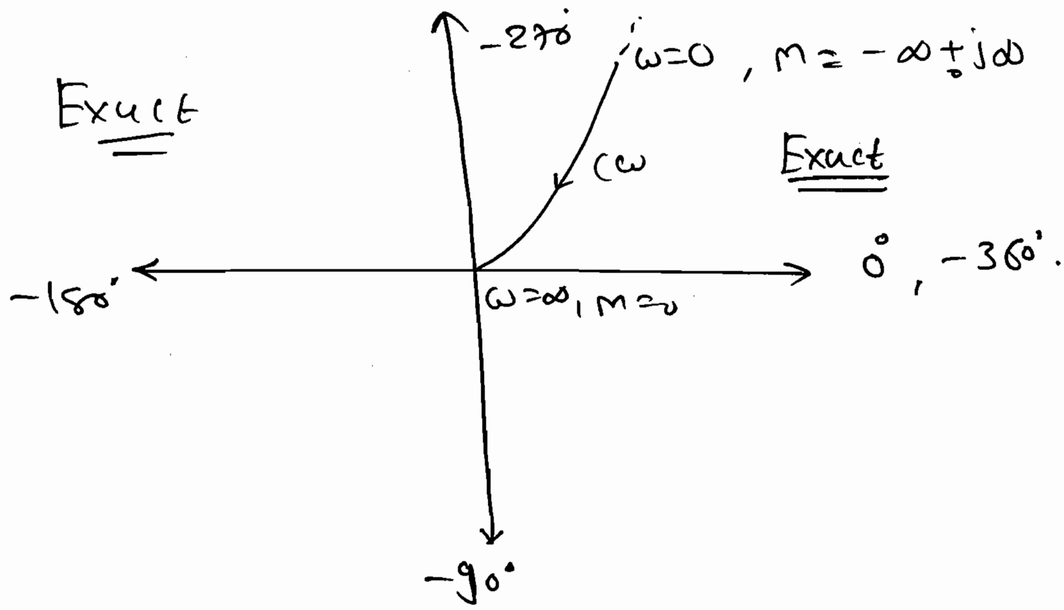
⇒

Approx.



⇒

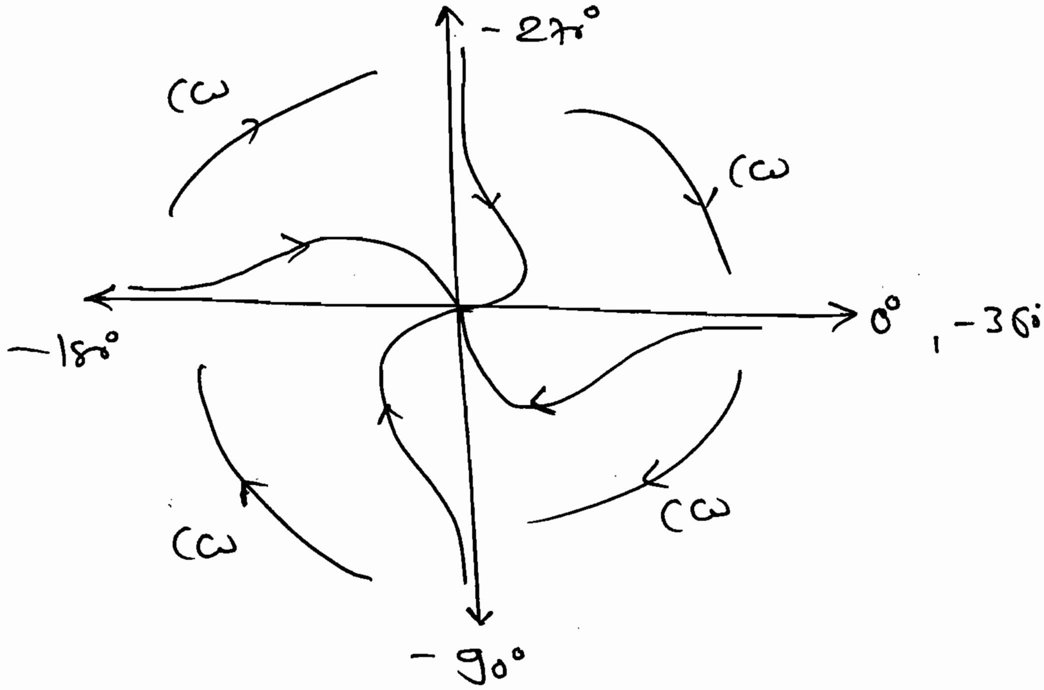
Exact



* Note:

⇒ The addition of each pole at origin shifted the total plot $-90°$ in the clock wise direction.

⇒



$\boxed{a}$ $GH(s) = \dfrac{(s+1)}{s^3}$.

Soln:

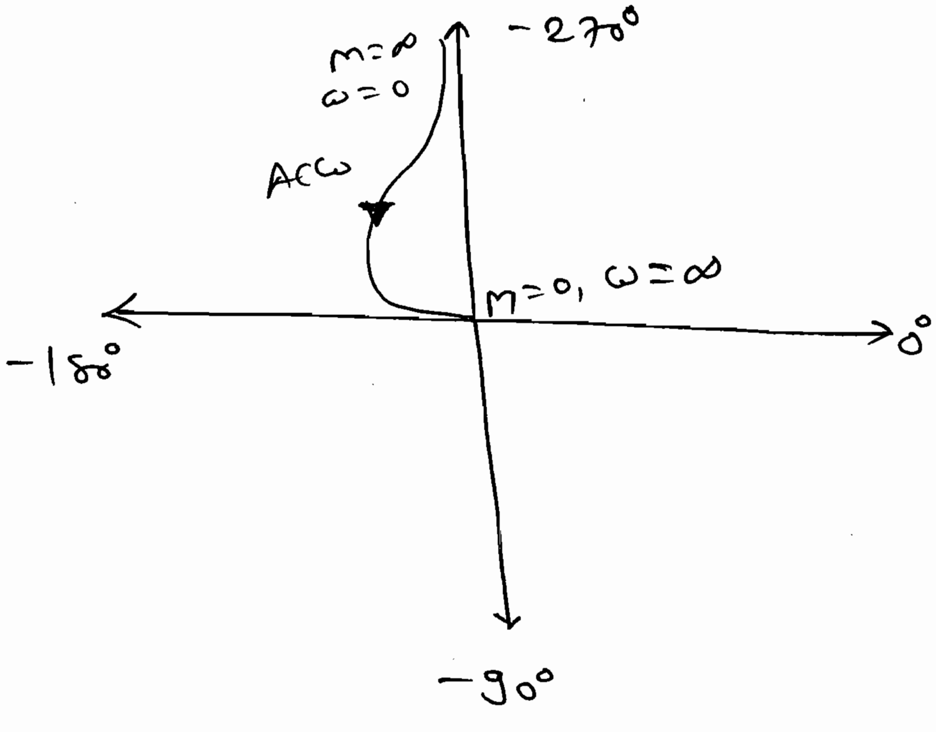
$$M = \frac{\sqrt{\omega^2+1}}{\omega^3} \quad \& \quad \phi = -270^\circ + \tan^{-1}\omega.$$

⇒ $\omega = 0 \Rightarrow M = \infty \quad \& \quad \phi = -270^\circ.$

⇒ $\omega = \infty \Rightarrow M = 0 \quad \& \quad \phi = \cancel{360^\circ}\ -180^\circ$

⇒ E.D. ⇒ $\phi_1 - \phi_2 = -270^\circ + 180^\circ = -ve \Rightarrow$ ACW.

⇒ S.D. ⇒ FZ ⇒ ACW.



$\boxed{Q}$ $GH(s) = \dfrac{(s+1)(s+2)}{s^3}$.

$\Rightarrow$ $M = \dfrac{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}{\omega^3}$. &

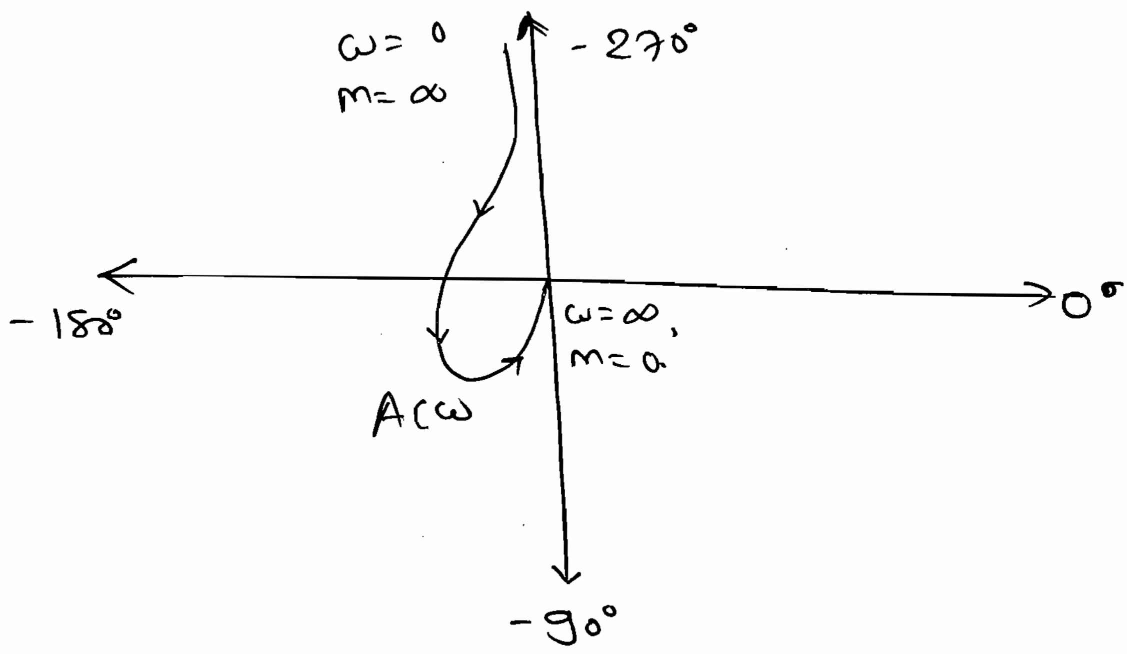
$\phi = -270^\circ + \tan^{-1}\omega + \tan^{-1}(\omega/2)$.

$\Rightarrow$ $\omega = 0$ $\Rightarrow$ $M = \infty$ & $\phi = -270^\circ$.

$\Rightarrow$ $\omega = \infty$ $\Rightarrow$ $M = 0$ & $\phi = -90^\circ$.

$\Rightarrow$ E.P. $\Rightarrow$ $\phi_1 - \phi_2 = -270^\circ + 90^\circ = -ve$ $\Rightarrow$ ACW.

$\Rightarrow$ S.O. $\Rightarrow$ finitzero $\Rightarrow$ ACW.



$\boxed{Q}$ $GH(s) = \dfrac{(s+1)(s+2)(s+3)}{s^3}$

Soln: $M = \dfrac{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4} \times \sqrt{\omega^2+9}}{\omega^3}$.

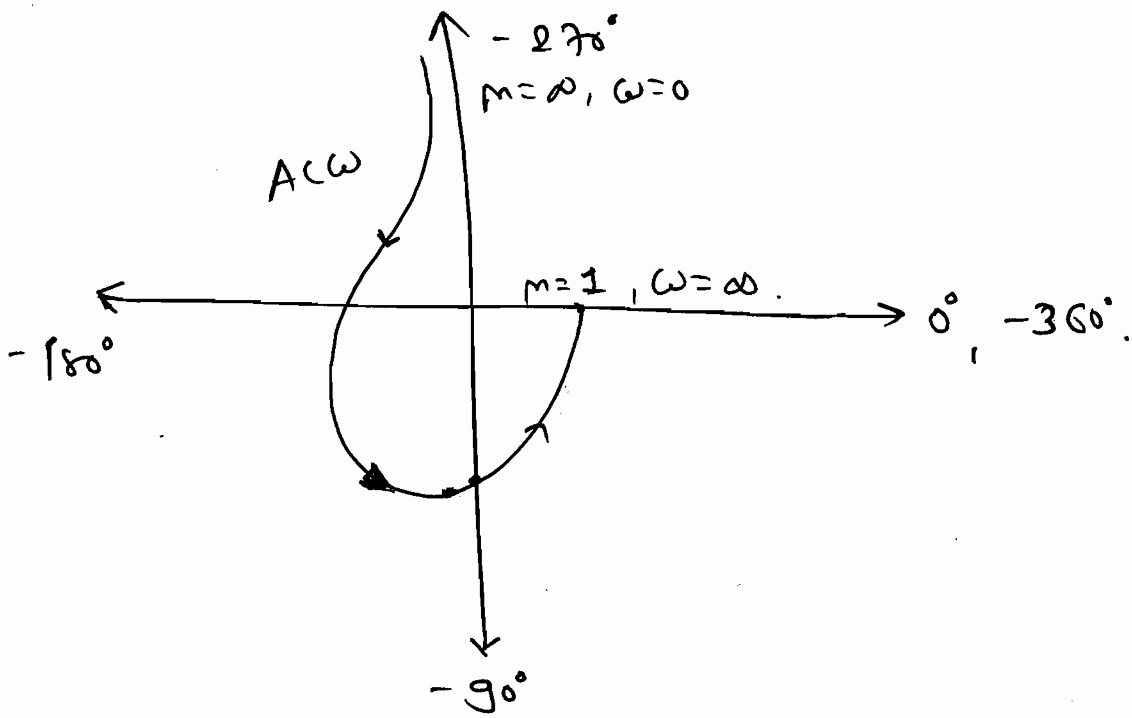
$\Rightarrow$ $\phi = -270^\circ + \tan^{-1}(\omega) + \tan^{-1}(\omega/2) + \tan^{-1}(\omega/3)$.

$\omega = 0$ $\Rightarrow$ $M = \infty$, & $\phi = -270^\circ$.

$\omega = \infty$ $\Rightarrow$ $\boxed{M = \cancel{\infty}\, 1}$ & $\phi = 0^\circ$.

$\Rightarrow$ E.D. $\Rightarrow$ $\phi_1 - \phi_2$ = $-ve$ $\Rightarrow$ ACW

$\Rightarrow$ S.D. $\Rightarrow$ δz = $+ve \Rightarrow$ ACW



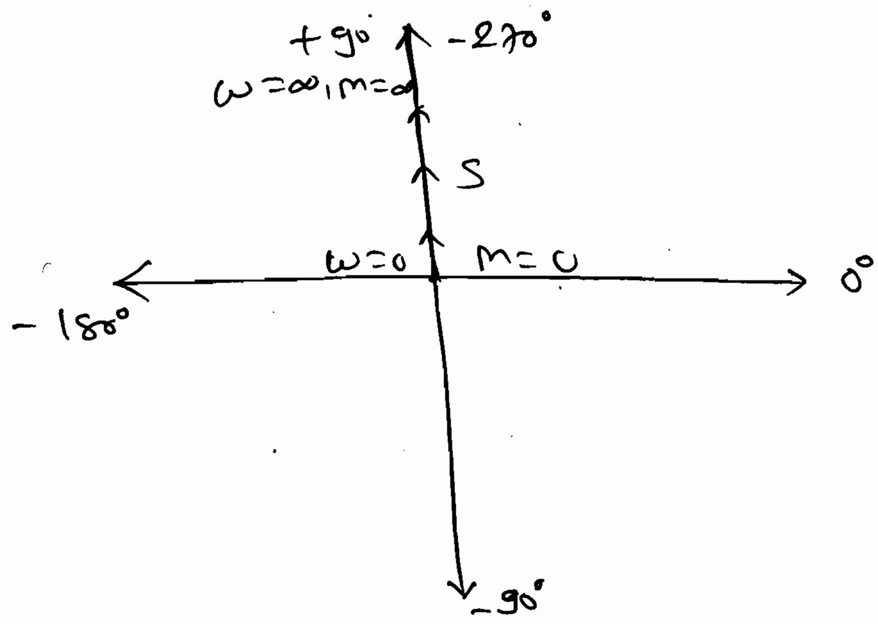
Note: The addition of each ~~time~~ finite zero shift the ending angle +90° in the Anticlockwise direction.

[Q] $GH = S.$

Soln: $M = \omega$ &, $\phi = 90°$

$\omega = 0 \Rightarrow$ $M = 0$ & $\phi = 90°$

$\omega = \infty \Rightarrow$ $M = \infty$ & $\phi = 90°$

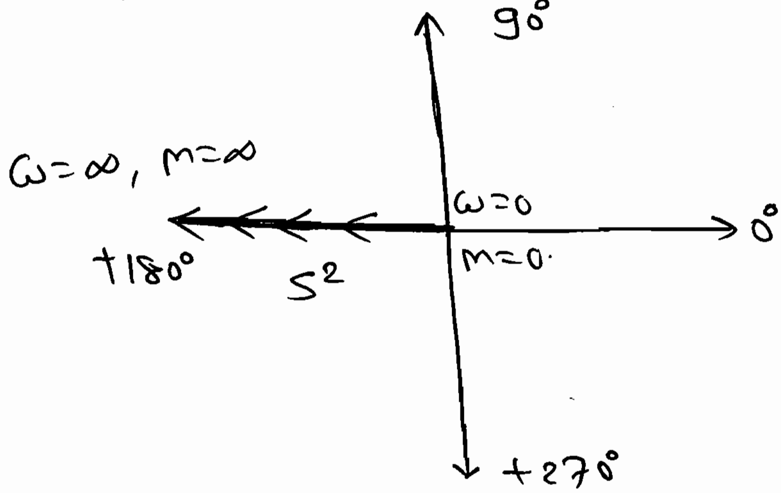


Note:

$\Rightarrow$ Whenever T.F. consist only poles (or) zeros at origin the polar plot is nothing but the angle line.

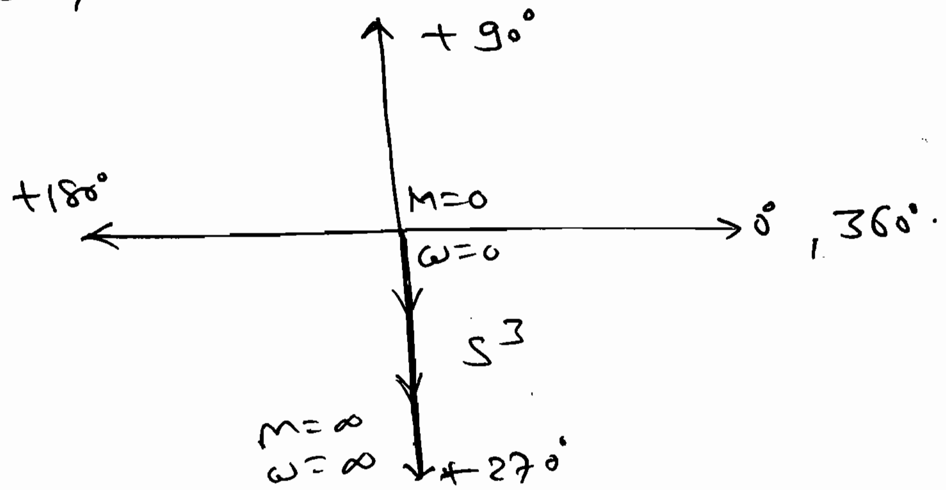
[a] $GH(s) = s^2$

Soln: $M = \omega^2$, $\phi = +180^\circ$



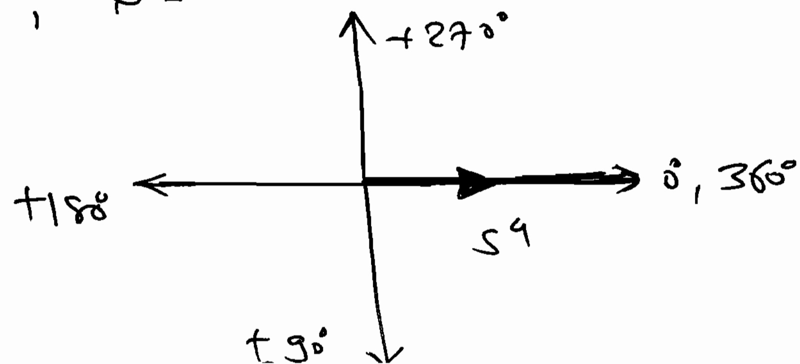
[a] $GH(s) = s^3$.

Soln: $M = \omega^3$, $\phi = +270^\circ$.



[a] $GH(s) = s^4$

Soln: $M = \omega^4$, $\phi = +360^\circ$ (or) 0°



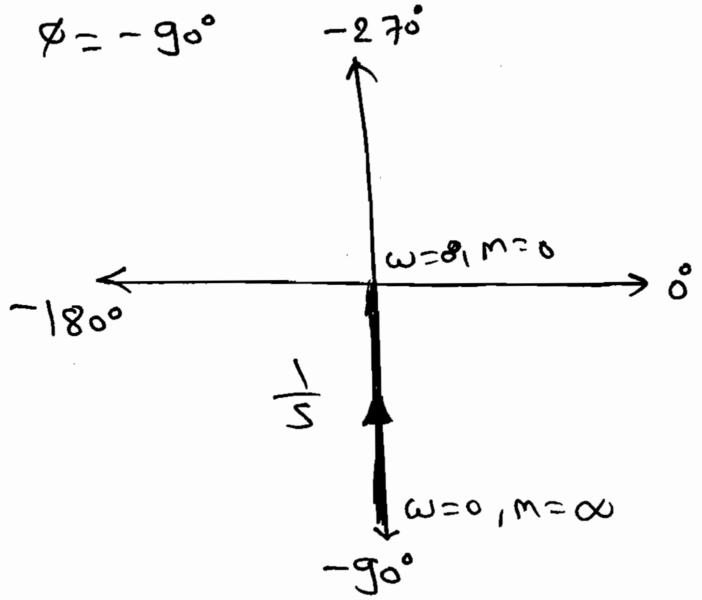
Note:

⇒ Whenever the zero at origin in added the total plot shifted $+90^\circ$ in the A.C.W direction.

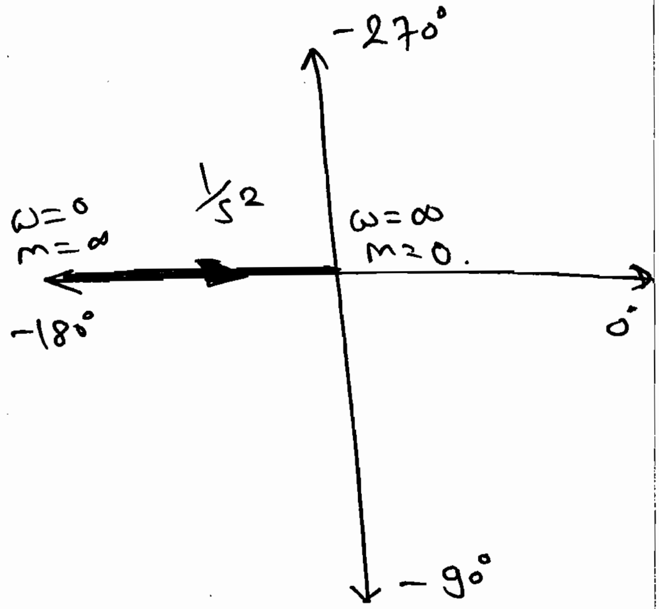
[a] $GH(s) = \frac{1}{s}$

Soln: $M = \frac{1}{\omega}$

$\phi = -90^\circ$

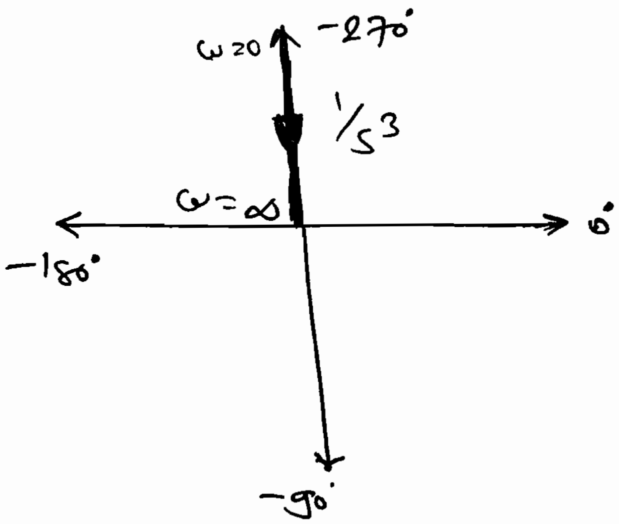


[a] $GH(s) = \frac{1}{s^2}$

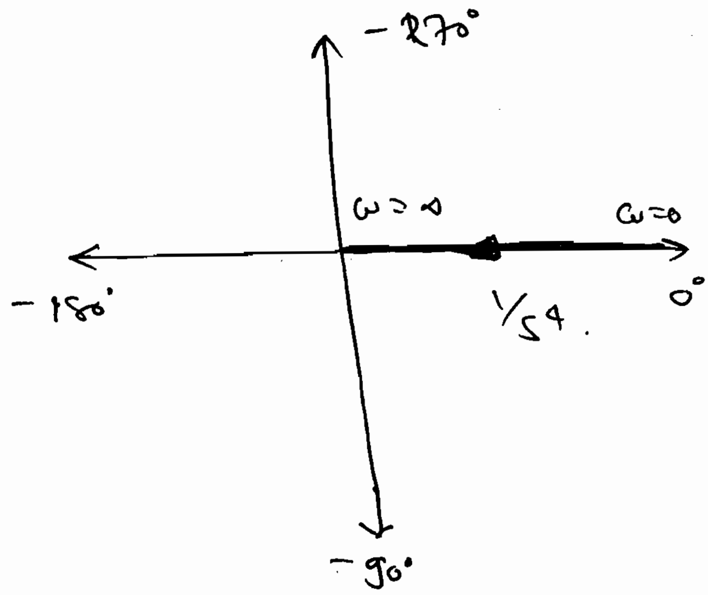


[a] $GH(s) = \frac{1}{s^3}$.

Soln: $M = \frac{1}{\omega^3}$, $\phi = -270^\circ$



[a] $GH(s) = \frac{1}{s^4}$.



Note: Whenever poles are added to the origin the total plot shifted $\mp 90^\circ$ in the the C.W. direction.

$\boxed{Q}$ $GH(s) = \dfrac{(s+1)}{s^3(s+2)}$.

Soln: $GH(j\omega) = \dfrac{(j\omega+1)}{-j\omega^3\,(j\omega+2)}$.

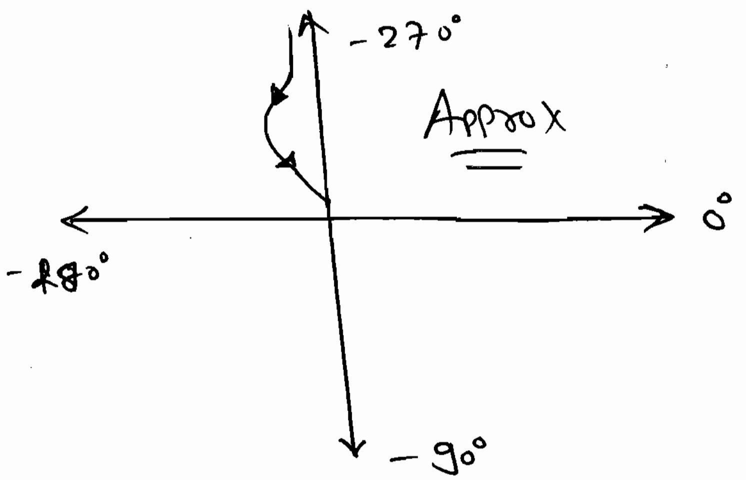
$M = \dfrac{\sqrt{\omega^2+1}}{\omega^3 \times \sqrt{\omega^2+4}}$. $\phi = -270^\circ + \tan^{-1}(\omega) - \tan^{-1}(\omega/2)$.

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -270^\circ$

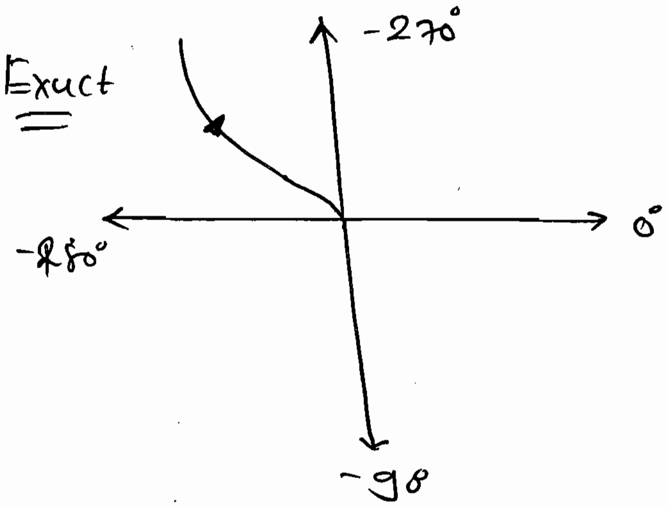
$\omega = \infty \Rightarrow M = 0$ & $\phi = -270^\circ$.

ED $\rightarrow$ X , SD $\rightarrow$ fz $\times$ o -2 -1 $\Rightarrow$ ACCW.

$\Rightarrow$



$\Rightarrow$



$\Rightarrow$ $\boxed{Q}$ $GH(s) = \dfrac{(s+2)}{s^3(s+1)}$.

Soln: $M = \dfrac{\sqrt{\omega^2+4}}{\omega^3\sqrt{\omega^2+1}}$.

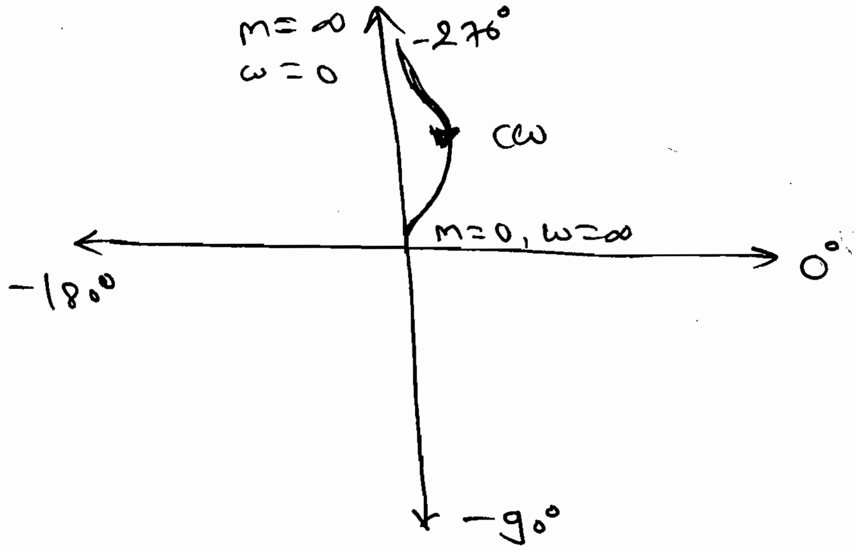
$\Rightarrow \quad \phi = -270^\circ + \tan^{-1}(\omega) - \tan^{-1}(\omega/2).$

$\omega = 0 \Rightarrow \quad M = \infty \quad \& \quad \phi = -270^\circ.$

$\omega = \infty \Rightarrow \quad M = 0 \quad \& \quad \phi = -270^\circ.$

$\Rightarrow \quad$ S.D. $\Rightarrow$ fp $\Rightarrow$ CW.

E.D. $\Rightarrow$ X. $(\because \ \phi_1 - \phi_2 = 0^\circ)$



Q $\quad GH(s) = \dfrac{(s+1)}{s^3(s+2)(s+3)}.$

$\underline{\underline{Sol^n}}$: $\quad M = \dfrac{\sqrt{\omega^2+1}}{\omega^3\sqrt{\omega^2+4}\ \sqrt{\omega^2+9}}.$

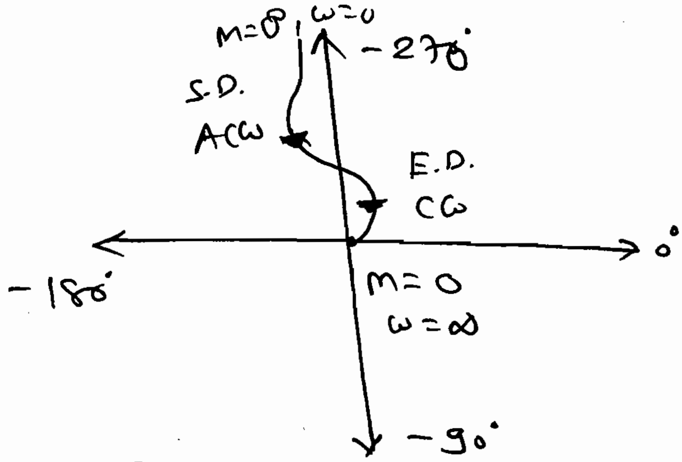
$\phi = -270^\circ + \tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3).$

$\omega = 0 \Rightarrow \quad m = \infty. \quad , \quad \phi = -270^\circ.$

$\omega = \infty \Rightarrow \quad M = 0 \quad , \quad \phi = -360^\circ$

E.D. $\Rightarrow \ \phi_1 - \phi_2 = -270^\circ + 360^\circ = +ve \Rightarrow$ CW.

$\underline{\underline{S.D.}} \Rightarrow$ fizero = ACW.



$\boxed{a}$ $GH(s) = \dfrac{(s+1)}{s^2(s+2)(s+3)}$.

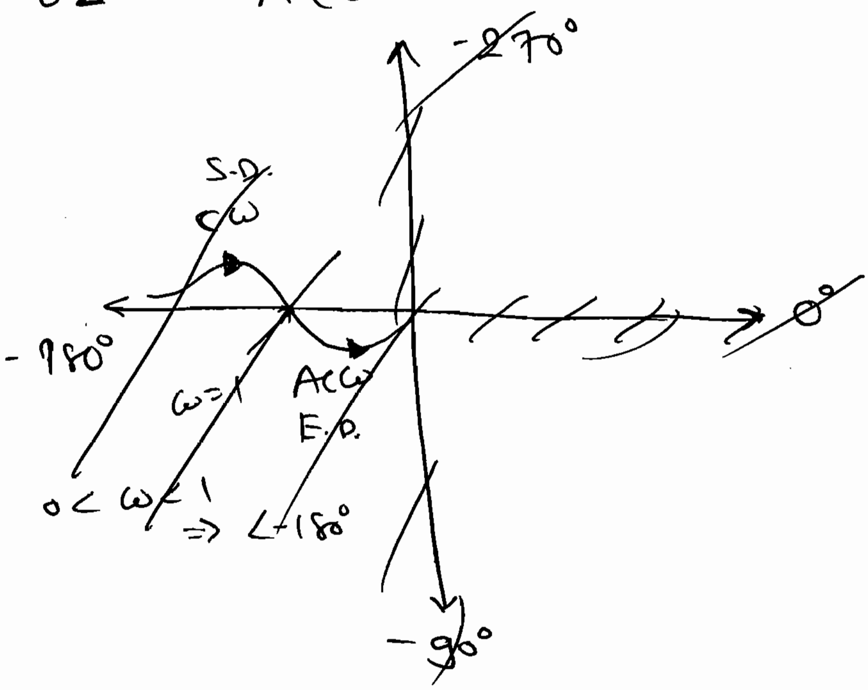
Soln: $M = \dfrac{\sqrt{\omega^2+1}}{\omega^2\sqrt{\omega^2+4}\sqrt{\omega^2+9}}$, $\phi = -180^\circ + \tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3)$.

$\omega = 0 \Rightarrow \quad M = \infty, \quad \phi = -180^\circ$

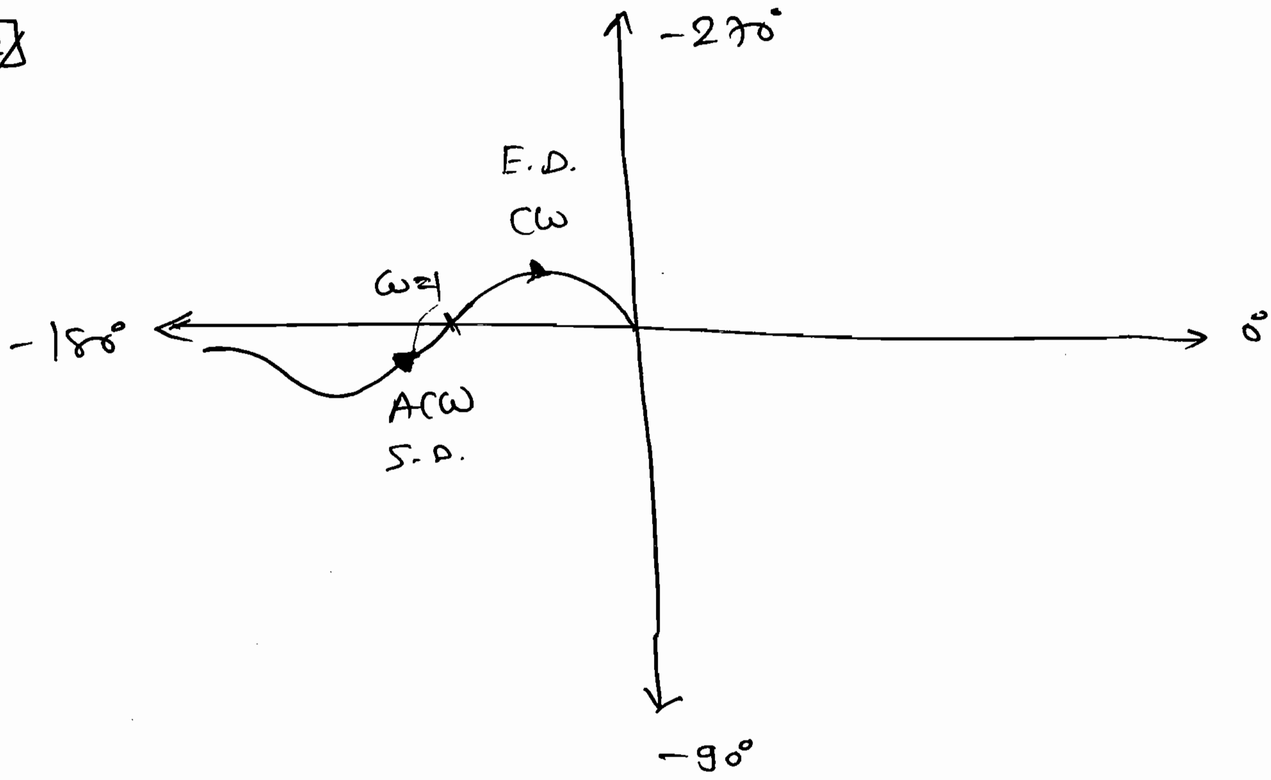
$\omega = \infty \Rightarrow M = 0, \quad \phi = -270^\circ$.

E.D. $\Rightarrow \quad \phi_1 - \phi_2 = -180^\circ + 270^\circ = +ve \Rightarrow$ CW.

S.D. $\Rightarrow$ b2 $\Rightarrow$ ACW.



~~(b)~~



$\Rightarrow$ I.P. with $-180°$.

$\therefore \; -\cancel{180°} = -\cancel{180°} + \tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3)$.

$$\therefore \tan^{-1}(\omega) = \tan^{-1}\left(\frac{\frac{\omega}{2} + \omega/3}{1 - \omega^2/6}\right).$$

$$\therefore \quad \omega = \frac{5\omega}{6 - \omega^2}.$$

$\Rightarrow \quad 6\omega - \omega^3 = 5\omega.$

$6 - \omega^2 = 5$

$\omega^2 = 1 \quad \Rightarrow \quad \boxed{\omega = 1 \text{ rad/sec}}$

$\boxed{a}$ $$GH(s) = \frac{(s+3)}{s^2(s+1)(s+2)}.$$

Sol^n:
$$M = \frac{\sqrt{\omega^2+9}}{\omega^2\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}.$$

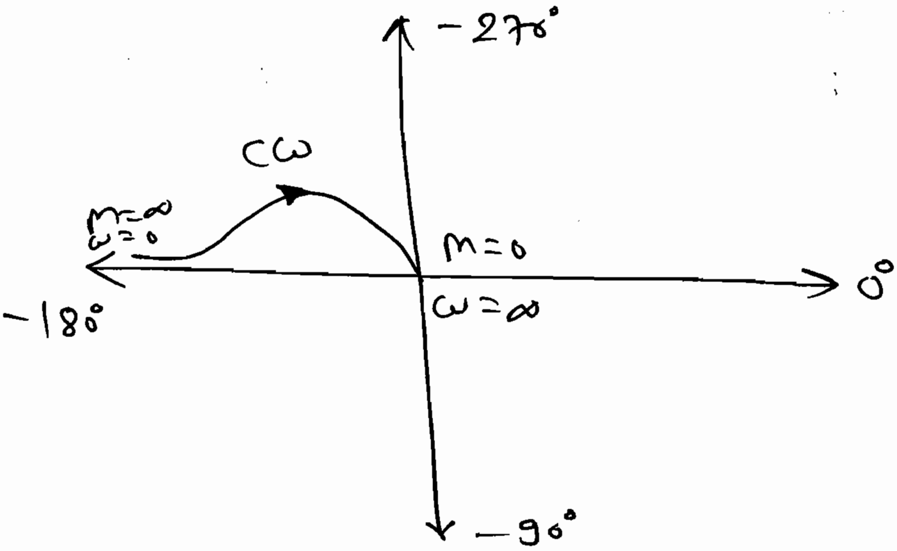
$\Rightarrow \quad \phi = -180° \cancel{+} \tan^{-1}\omega - \tan^{-1}(\omega/2) + \tan^{-1}(\omega/3).$

$\omega = 0 \Rightarrow M = \infty \quad \& \quad \phi = -180°.$

$\omega = \infty \Rightarrow M = 0 \quad \& \quad \phi = -270°.$

$\Rightarrow$ E.D. $\Rightarrow \phi_1 - \phi_2 = -180° + 270° \Rightarrow +ve \Rightarrow$ CW.

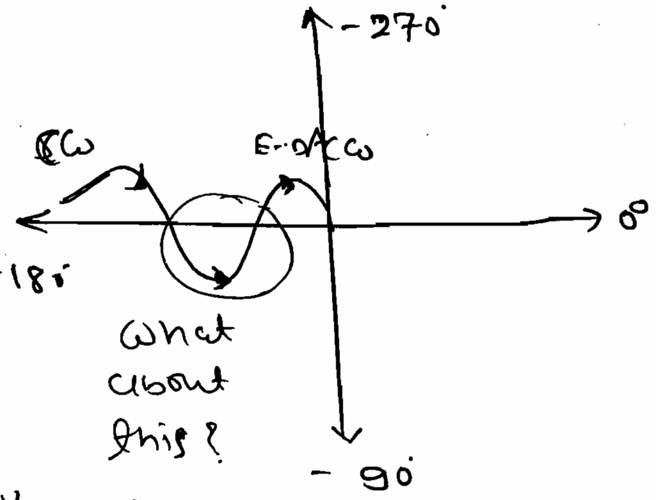
$\Rightarrow$ S.D. $\Rightarrow$ f P $\Rightarrow$ CW.



$\Rightarrow$ I.P. with $-180°$

$\angle GH = -180°$

$\Rightarrow \cancel{-180°} = \cancel{-180°} - \tan^{-1}(\omega) - \tan^{-1}(\omega/2) + \tan^{-1}(\omega/3)$



$\Rightarrow \tan^{-1}(\omega/3) = \tan^{-1}\left(\dfrac{\omega + \omega/2}{1 - \omega^2/2}\right).$

$\Rightarrow \dfrac{\cancel{\omega}}{3} = \dfrac{3\cancel{\omega}}{2-\omega^2}.$

$2-\omega^2 = 9$

$\omega^2 = -7 \quad \Rightarrow \quad \omega = \pm j\sqrt{7}$ X Invalid point.

$\boxed{Q}$ $GH(s) = \dfrac{(s+1)(s+2)}{s^2(s+3)}.$

Sol^n:

$M = \dfrac{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}{\omega^2 \times \sqrt{\omega^2+9}}.$

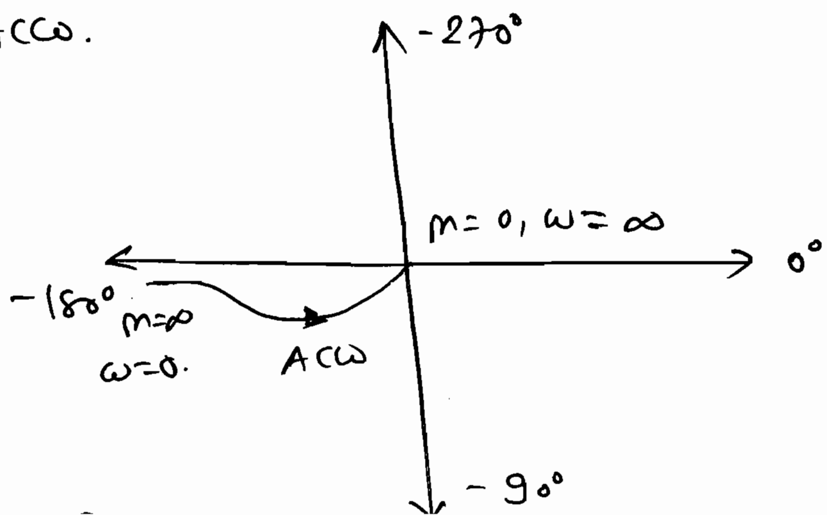
$\Rightarrow \phi = -180° + \tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\omega/3).$

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -180°$

$\omega = \infty \Rightarrow M = 0$ & $\phi = -90°.$

$\Rightarrow$ E.D. $\Rightarrow \phi_1 - \phi_2 = -ve \Rightarrow$ A.C.W.

$\Rightarrow$ S.D. $\Rightarrow$ ~~b2~~ $\Rightarrow$ ACW.



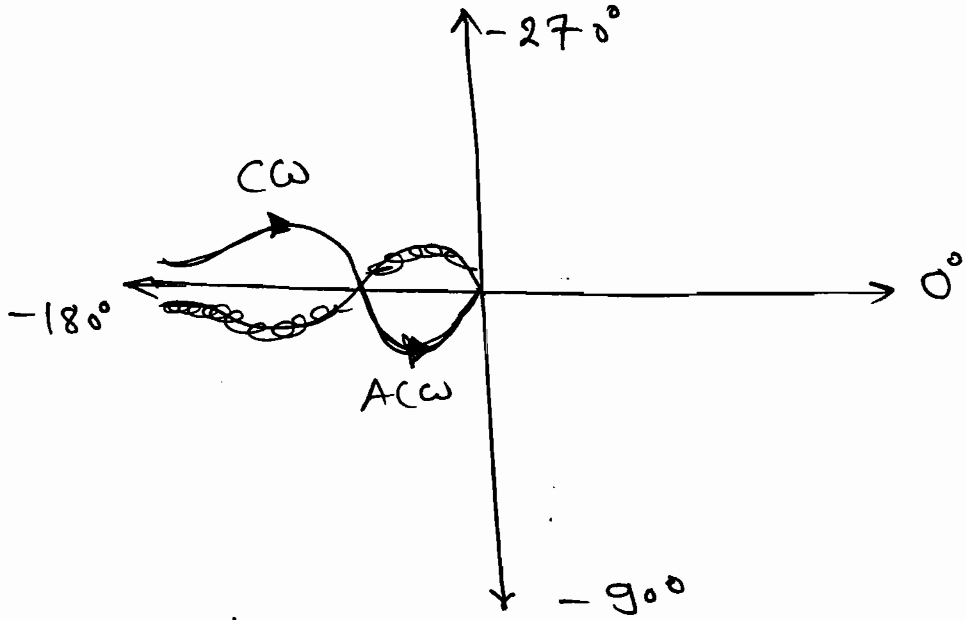
$\boxed{Q}$ $GH(s) = \dfrac{(s+2)(s+3)}{s^2(s+1)}$.

Solⁿ: $M = \dfrac{\sqrt{\omega^2+4} \times \sqrt{\omega^2+9}}{\omega^2 \times \sqrt{\omega^2+1}}$.

$\phi = -180^\circ - \tan^{-1}(\omega) + \tan^{-1}(\omega/2) + \tan^{-1}(\omega/3)$.

$\Rightarrow \omega = 0 \Rightarrow M = \infty, \quad \phi = -180^\circ$. E.D. $\Rightarrow$ ACW

$\Rightarrow \omega = \infty \Rightarrow M = 0, \quad \phi = -90^\circ$. S.D. $\Rightarrow$ FD $\Rightarrow$ CW.



$\boxed{Q}$ $GH(s) = \dfrac{1}{s(s+1)}$.

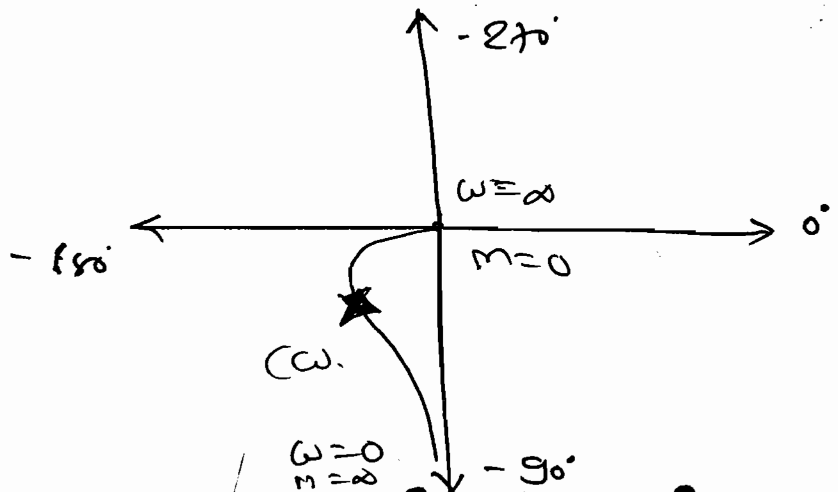
Solⁿ: $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$. & $\phi = -90^\circ - \tan^{-1}(\omega)$.

E.D. $\Rightarrow$ CW

S.D. $\Rightarrow$ CW.

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -90^\circ$

$\omega = \infty \Rightarrow M = 0$ & $\phi = -180^\circ$



$\boxed{Q}$ $GH = \dfrac{1}{s(s-1)}$.

Soln: $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$ & $\phi = -90^\circ - (180^\circ - \tan^{-1}(\omega))$.

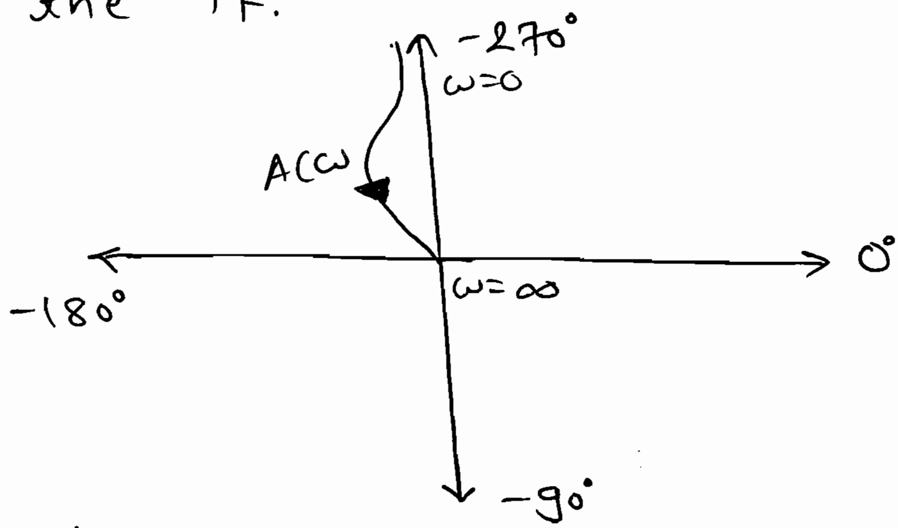
$\phi = -270^\circ + \tan^{-1}(\omega)$.

$\omega = 0 \Rightarrow \phi = -270^\circ$

$\omega = \infty \Rightarrow \phi = -180^\circ$

E.D. $\Rightarrow \phi_1 - \phi_2 = -270^\circ + 180^\circ = -ve \Rightarrow$ ACW.

$\boxed{S.D.}$ X Not required because (-ve) sign in the TF.



$\boxed{Q}$ $GH = \dfrac{1}{s(-s-1)}$.

Soln: $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$ & $\phi = -90^\circ - (180^\circ + \tan^{-1}(\omega))$.

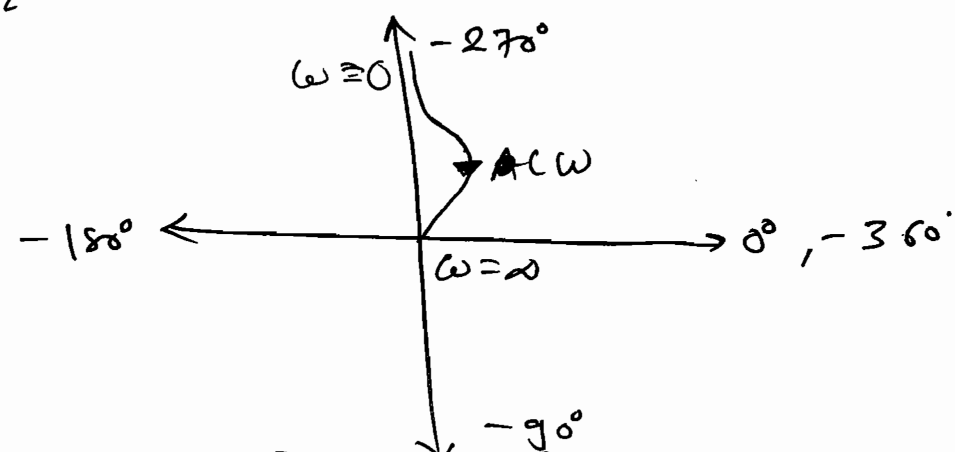
$= -270^\circ - \tan^{-1}\omega$.

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -270^\circ$

$\omega = \infty \Rightarrow M = 0$ & $\phi = -360^\circ$.

E.D. $\Rightarrow \phi_1 - \phi_2 = -270^\circ + 360^\circ = +ve \Rightarrow$ CW.

S.D. X



$\boxed{Q}$ $GH = \dfrac{1}{s(1-s)}$

Sol^n^: $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$ & $\phi = -90^\circ - (-\tan^{-1}(\omega))$

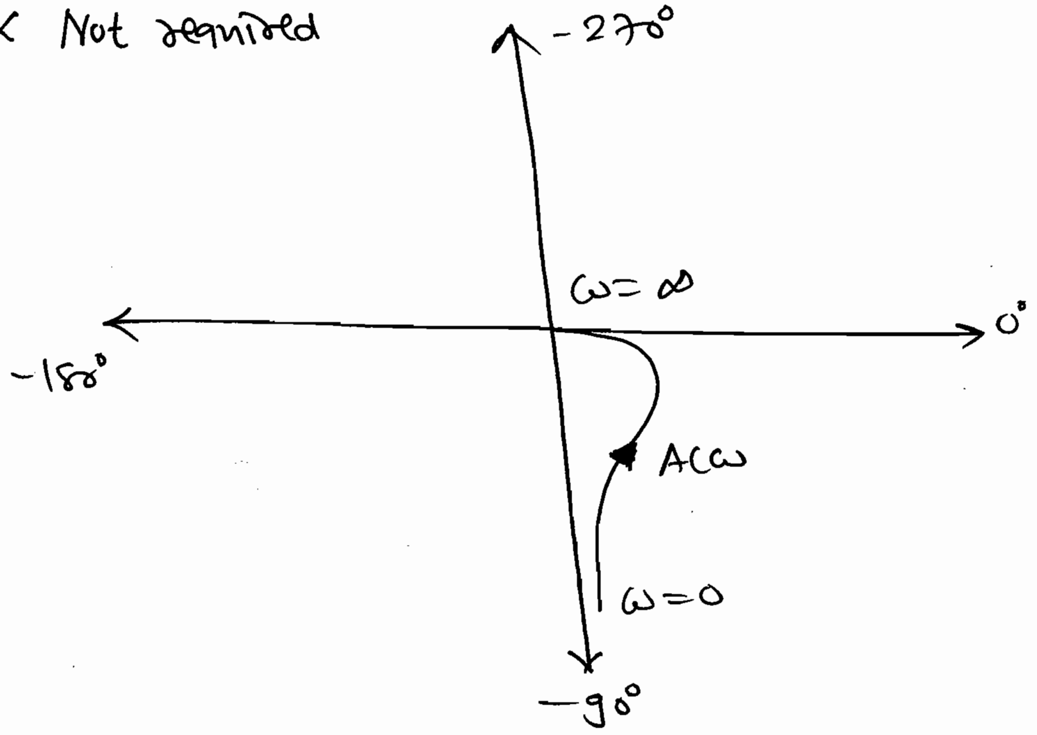
$= -90^\circ + \tan^{-1}\omega$.

$\omega = 0 \Rightarrow M = \infty$ & $\phi = -90^\circ$.

$\omega = \infty \Rightarrow M = 0$ & $\phi = 0^\circ$.

E.D. $\Rightarrow$ $\phi_1 - \phi_2 = -90^\circ = -ve \Rightarrow$ ACW.

S.D. ✗ Not required



Note: The middle intersection pt that means (some freq ~~zone~~ zone C.W & some freq. zone A.C.W).

$\Rightarrow$ The middle intersection point possible when the phase angle having +ve & -ve term ($\tan^{-1}$) ~~of~~ provided that no of finite pole is not equal to no. of finite zero and these should be first two consecutive pole and zero (or) zero and pole otherwise no middle I.P. about that particular point.

Note: E.D. $\phi_1 - \phi_2$ = +ve → CW
= −ve → ACW.

[finite P = finite Z] ⇒ E.D. X

⇒ S.D. fP → CW
fZ → ACW.

if −ve sign in TF. then SD X.

$\boxed{Q}$ $GH = \dfrac{(S+2)}{(S+1).(S-1)}$.

Soln:

$$M = \frac{\sqrt{\omega^2+4}}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+1}}.$$

$$\phi = -\tan^{-1}(\omega) + \tan^{-1}(\omega/2). \mp (180^\circ - \tan^{-1}\omega).$$

$$= \cancel{-\tan^{-1}(\omega)} + \tan^{-1}(\omega/2) \mp 180^\circ + \cancel{\tan^{-1}\omega}$$

$$\phi = -180^\circ + \tan^{-1}(\omega/2).$$

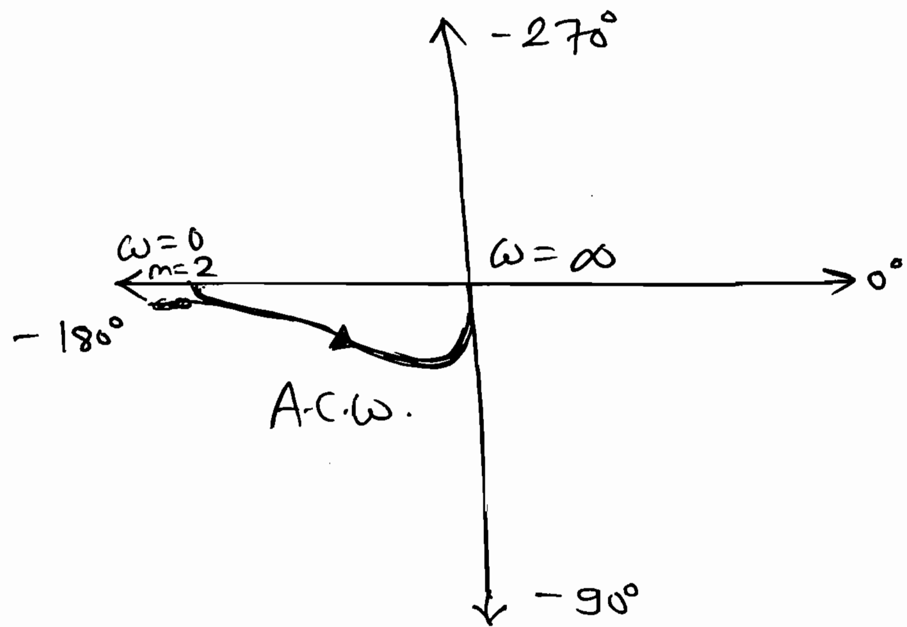
⇒ $\omega = 0$ ⇒ $M = 2$, $\phi = -180^\circ$.

⇒ $\omega = \infty$ ⇒ $M = 0$, $\phi = -90^\circ$.

E.D. $\phi_1 - \phi_2 = -180^\circ + 90^\circ = -90^\circ$ ⇒ ACW.

S.D X

⇒



$\boxed{Q}$ $GH = \frac{(s-3)}{s(s+1)}$

$\underline{\underline{Sol^n:}}$ $M = \frac{\sqrt{\omega^2+9}}{\omega \times \sqrt{\omega^2+1}}$,

$\phi = -90^\circ - \tan^{-1}(\omega) + (180^\circ - \tan^{-1}(\omega/3))$.

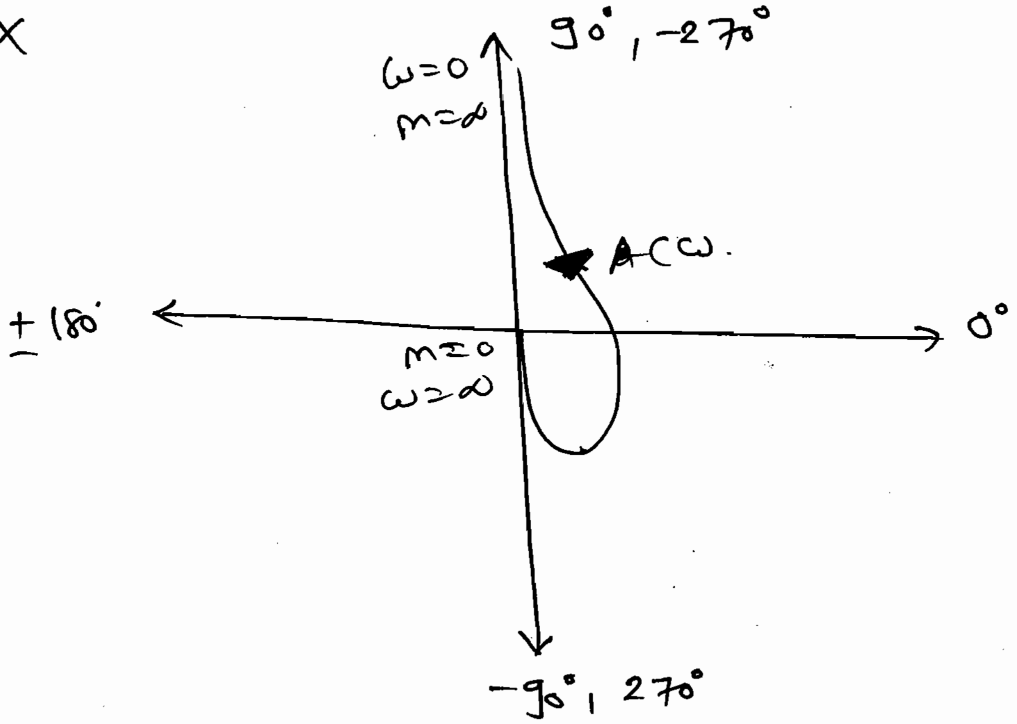
$\phi = +90^\circ - \tan^{-1}(\omega) - \tan^{-1}(\omega/3)$.

$\omega = 0 \Rightarrow M = \infty$, $\phi = +90^\circ$

$\omega = \infty \Rightarrow M = 0$, $\phi = -90^\circ$

$\underline{\underline{E.D.}} \Rightarrow \phi_1 - \phi_2 = +90^\circ - (-90^\circ) = +ve \Rightarrow cw$.

$\underline{\underline{S.D}}$ X



$\boxed{Q}$ $GH(s) = \left(\frac{s+10}{s-10}\right)$.

$\underline{\underline{Sol^n:}}$ $M = \sqrt{\frac{\omega^2+100}{\omega^2+100}} = 1$.

$\phi = \tan^{-1}(\omega/10) - 180^\circ + \tan^{-1}(\omega/10)$.

$\phi = -180^\circ + 2\tan^{-1}(\omega/10)$.

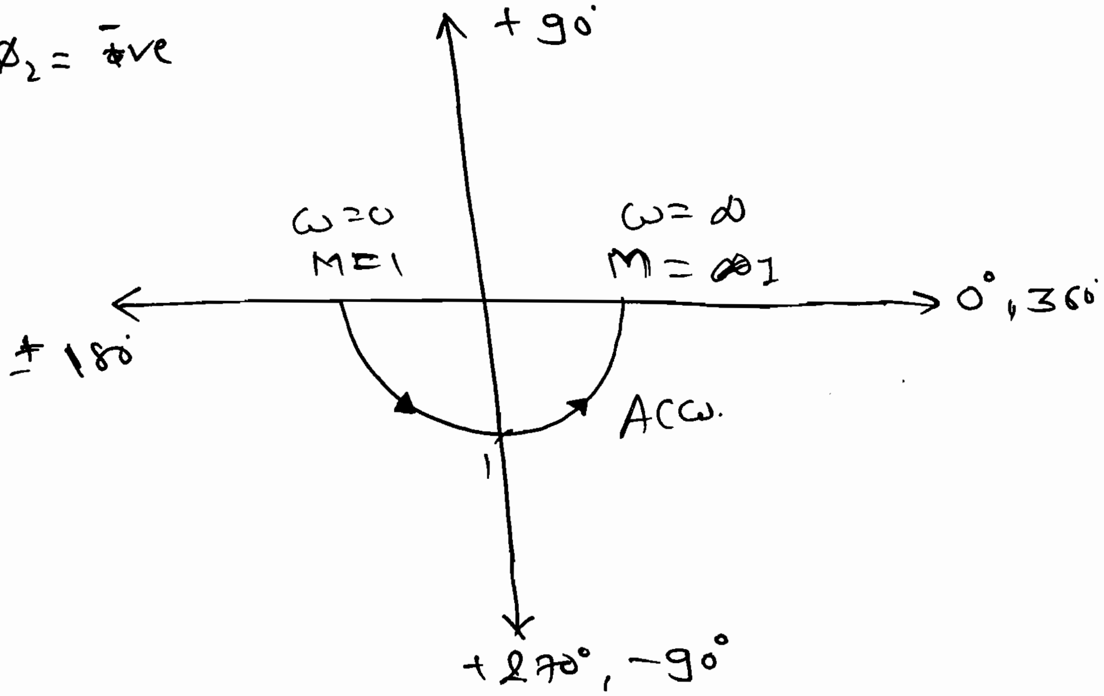
$\therefore \quad \omega = 0 \Rightarrow M = 1 \quad \& \quad \phi = -180^\circ$

$\omega = \infty \Rightarrow M = 1 \quad \& \quad \phi = 0^\circ \cancel{360^\circ}$

E.P. $\phi_1 - \phi_2 = -\text{ve}$

$\Rightarrow$ ACW

S.P X



$\boxed{a}$ $\quad GH = \dfrac{e^{-s}}{s(s+1)}$.

$GH = \dfrac{1-s}{s(s+1)}$ X Don't expand e^{-s} to $(1-s)$.

Soln: $\quad s \to j\omega$

$$GH(j\omega) = \frac{e^{-j\omega}}{j\omega\,(j\omega+1)}.$$

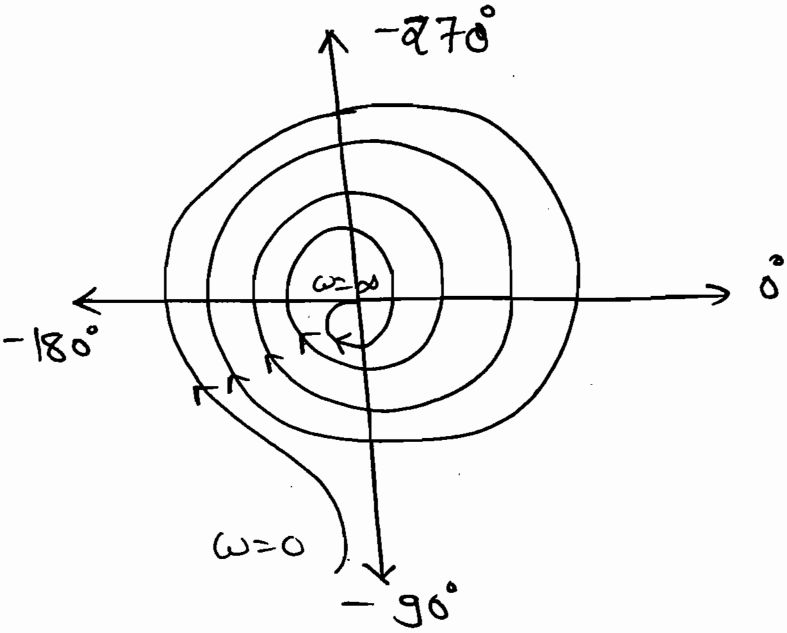
$$\Rightarrow |GH(j\omega)| = M = \frac{1}{\omega\sqrt{\omega^2+1}}.$$

$$\Rightarrow \phi = -90^\circ - \tan^{-1}\omega - \omega\left(\frac{180^\circ}{\pi}\right).$$

$$\therefore \boxed{\begin{aligned} e^{j\theta} &= \cos\theta + i\sin\theta \\ \angle\phi &= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \theta \\ \angle\phi &= \theta \end{aligned}}$$

$$\Rightarrow \angle\phi = -90^\circ - \tan^{-1}\omega - 57.3^\circ\omega.$$

⇒ $\omega\uparrow$	$M\downarrow$	$\angle\phi$
0	∞	-90°
1	0.707	-192°
2	0.22	-267°
3	0.04	-453°
10	0.01	-744°
∞	0	$-\infty$.

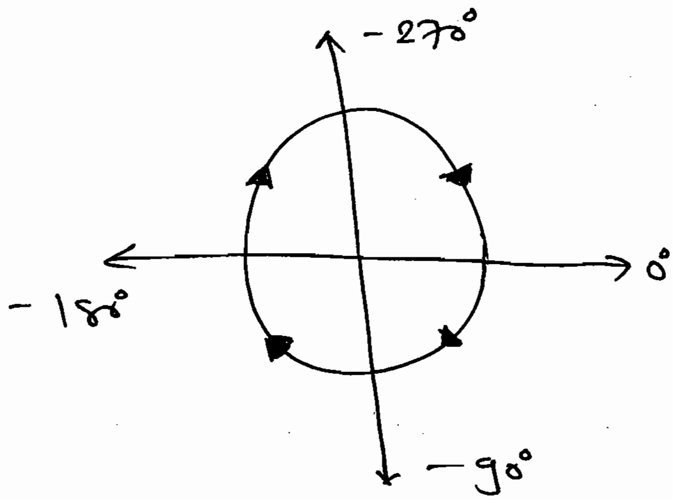


Q $GH = \pi e^{-2s}$

Sol^n: $s \to j\omega \Rightarrow GH(j\omega) = \pi e^{-2j\omega}$

$M = \pi$ & $\phi = -2\omega \times \frac{180^\circ}{\pi}$

ω	M	$\angle\phi$
0	π	0°
$\pi/4$	π	-90°
$\pi/2$	π	-180°
$3\pi/4$	π	-270°
π	π	-360°



* Note: