

B

Section

Algebra & Set Theory

2.1

CHAPTER

Surds, Indices & Logarithms

LAWS OF INDICES

(1) $a^m \times a^n = a^{m+n}$

Example:

$$\rightarrow 2^4 \times 2^{13} = 2^{17}$$

$$\rightarrow 3^8 \times 3^7 = 3^{15}$$

(2) $\frac{a^m}{a^n} = a^{m-n}$

Example:

(i) $\frac{7^5}{7^2} = 7^{5-2} = 7^3$

(ii) $\frac{27^5}{27^2} = 27^3$

(3) $(a^m)^n = a^{mn}$

Example:

(i) $(3^2)^4 = 3^{2 \times 4} = 3^8$

(ii) $(5^3)^9 = 5^{3 \times 9} = 5^{27}$

(4) $(a \times b)^n = a^n \times b^n$

Example:

(i) $(3 \times 5)^2 = 3^2 \times 5^2$

(ii) $(4 \times 5)^3 = 4^3 \times 5^3$

(5) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example:

(i) $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$

(ii) $\left(\frac{5}{8}\right)^3 = \frac{5^3}{8^3} = \frac{125}{512}$

(6) $a^0 = 1$

Here a may be any number

Example:

(i) $5^0 = 1$

(ii) $1000^0 = 1$

(7) $a^x = a^y$

If and only if Condition

Example:

(i) $x = y$ or

(ii) $x = 0, y = 0$

(8) $a^{-m} = \frac{1}{a^m}$

Example:

(i) $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(ii) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

(9) $a^{-1} = \frac{1}{a}$

Example: $10^{-1} = \frac{1}{10}$

Laws of Surds

We write $\sqrt[n]{a} = a^{1/n}$ and it is called a surd of order 'n'

(1) $(\sqrt[n]{a})^n = (a^{1/n})^n = a$

(2) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Example:

(i) $\sqrt[3]{2 \times 5} = \sqrt[3]{2} \times \sqrt[3]{5}$

(ii) $\sqrt[5]{7 \times 13} = \sqrt[5]{7} \times \sqrt[5]{13}$

(3) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example:

(i) $\sqrt[3]{\frac{2}{5}} = \frac{\sqrt[3]{2}}{\sqrt[3]{5}}$

(ii) $\sqrt[5]{\frac{7}{13}} = \frac{\sqrt[5]{7}}{\sqrt[5]{13}}$

(4) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

Example:

(i) $(\sqrt[3]{2})^5 = \sqrt[3]{2^5} = \sqrt[3]{32}$

(ii) $(\sqrt[5]{4})^3 = \sqrt[5]{4^3} = \sqrt[5]{64}$

$$(5) \sqrt[m]{\sqrt[n]{\sqrt[p]{a}}} = \sqrt[mnp]{a} = a^{\frac{1}{mnp}}$$

Example:

$$(i) \sqrt[2]{\sqrt[3]{\sqrt[4]{10}}} = 10^{\frac{1}{2 \times 3 \times 4}} = 10^{\frac{1}{24}}$$

$$(ii) \sqrt[3]{\sqrt[5]{25}} = 25^{\frac{1}{3 \times 5}} = 25^{\frac{1}{15}}$$

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Solved Examples

1. Find the value of $\left(\frac{32}{343}\right)^{-\frac{3}{5}}$

$$\begin{aligned} \text{Sol.: } \left(\frac{32}{343}\right)^{-\frac{3}{5}} &= \left(\frac{2^5}{3^5}\right)^{-\frac{3}{5}} = \left(\frac{2}{3}\right)^{5 \times \frac{-3}{5}} \\ &= \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8} \end{aligned}$$

2. Solve $4^{3x-2} \cdot 8 = 1$

$$\begin{aligned} \text{Sol.: } 4^{3x-2} \times 8 &= (2^2)^{3x-2} \times 2^3 = 1 \\ &= 2^{6x-4+3} = 1 \\ &= 2^{6x-1} = 2^0 \quad [\because 2^0 = 1] \\ &= 2^{6x-1} = 2^0 \Rightarrow x = \frac{1}{6} \end{aligned}$$

3. Find the value of x if $\left(\frac{a}{b}\right)^{3x-5} = \left(\frac{b}{a}\right)^{x+3}$

$$\begin{aligned} \text{Sol.: } \left(\frac{a}{b}\right)^{3x-5} &= \left(\left(\frac{a}{b}\right)^{-1}\right)^{x+3} \\ &= \left(\frac{a}{b}\right)^{3x-5} = \left(\frac{a}{b}\right)^{-x-3} \\ &= 3x-5 = -x-3 \\ &= 4x = 2 \Rightarrow x = \frac{1}{2} \end{aligned}$$

4. Which is greater $\sqrt[4]{4}$ or $\sqrt[5]{5}$

$$\text{Sol.: } \sqrt[4]{4} = 4^{\frac{1}{4}} \text{ and } \sqrt[5]{5} = 5^{\frac{1}{5}}$$

Now the LCM of 4 & 5 is 20, so to compare two surds raise the power by 20.

$$\left(4^{\frac{1}{4}}\right)^{20} = 4^5 = 1024$$

$$\left(5^{\frac{1}{5}}\right)^{20} = 5^4 = 625$$

$$\text{Hence, } \sqrt[4]{4} > \sqrt[5]{5}$$

5. Simplify, $\frac{\sqrt{8} + \sqrt{2}}{\sqrt{8} - \sqrt{2}}$

$$\text{Sol.: } \frac{\sqrt{8} + \sqrt{2}}{\sqrt{8} - \sqrt{2}} \times \frac{\sqrt{8} + \sqrt{2}}{\sqrt{8} + \sqrt{2}}$$

(Multiply numerator & denominator by conjugate of denominator)

$$= \frac{(\sqrt{8} + \sqrt{2})^2}{(\sqrt{8})^2 - (\sqrt{2})^2} = \frac{8 + 2 + 2\sqrt{16}}{8 - 2} = \frac{10 + 8}{6} = 3 \text{ Ans.}$$

6. Simplify $\frac{\sqrt{a^2+1} + \sqrt{a^2-1}}{\sqrt{a^2+1} - \sqrt{a^2-1}}$

Sol.:

$$\frac{\sqrt{a^2+1} + \sqrt{a^2-1}}{\sqrt{a^2+1} - \sqrt{a^2-1}} \times \frac{\sqrt{a^2+1} + \sqrt{a^2-1}}{\sqrt{a^2+1} + \sqrt{a^2-1}}$$

$$= \frac{a^2+1+a^2-1+2(\sqrt{a^2+1})(\sqrt{a^2-1})}{a^2+1-(a^2-1)}$$

$$= \frac{2a^2 + 2\sqrt{a^4-1}}{2} = a^2 + \sqrt{a^4-1}$$

7. Which is greater $\sqrt[3]{4}$ or $\sqrt[4]{5}$

$$\text{Sol.: } \sqrt[3]{4} = 4^{\frac{1}{3}} \text{ and } \sqrt[4]{5} = 5^{\frac{1}{4}}$$

raise the power of both the surds by 12 (LCM of 3 and 4)

$$\left(4^{\frac{1}{3}}\right)^{12} = 4^4 = 256$$

$$\left(5^{\frac{1}{4}}\right)^{12} = 5^3 = 125$$

$$\text{Hence, } \sqrt[3]{4} > \sqrt[4]{5}$$

8. If $2^{2x-1} = \frac{1}{8^{x-3}}$ then find value of x

$$\text{Sol.: } 2^{2x-1} = \frac{1}{8^{x-3}}$$

$$= 2^{2x-1} = \frac{1}{(2^3)^{x-3}} = \frac{1}{2^{3x-9}}$$

$$= 2^{2x-1} = 2^{-3x+9} \left[\because \frac{1}{a^m} = a^{-m} \right]$$

$$= 5x = 10, \quad x = 2$$

9. If $2^x = 3^y = 6^{-z}$ then $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$ is equal to?

$$\text{Sol.: let } 2^x = 3^y = 6^{-z} = k$$

$$\text{So, } 2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 6 = k^{-\frac{1}{z}}$$

We know that $2 \times 3 = 6$ this gives

$$k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}} = \frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$$

$$= \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

10. Arrange $\sqrt{2}$, $\sqrt[3]{4}$ and $\sqrt[4]{6}$ in ascending order

$$\text{Sol.: } \sqrt{2} = 2^{\frac{1}{2}}$$

$$\sqrt[3]{4} = 4^{\frac{1}{3}}, \quad \sqrt[4]{6} = 6^{\frac{1}{4}}$$

Now raise the power of given surds by LCM of 2, 3, 4 that is 12

$$\left(2^{\frac{1}{2}}\right)^{12} = 2^6 = 64$$

$$\left(4^{\frac{1}{3}}\right)^{12} = 4^4 = 256$$

$$\left(6^{\frac{1}{4}}\right)^{12} = 6^3 = 216$$

$$\text{Ans: } \sqrt{2}, \sqrt[4]{6}, \sqrt[3]{4}$$

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Logarithms

Definition

The logarithm of any number of a given base is equal to the index to which the base should be raised to obtain the given number.

For example if

$a^x = c$, then x is called logarithm of a number c to the base a . It is written as $\log_a c = x$

Similarly

$$2^3 = 8 \text{ is similar as } \log_2 8 = 3$$

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16} \text{ is written}$$

$$\text{as } \log_4 \frac{1}{16} = -2$$

$$10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$$

$$3^5 = 243 \Rightarrow \log_3 243 = 5$$

Properties of Logarithms

$$(a) \log_a (m \times n) = \log_a m + \log_a n$$

Example 1.

$$\log_{10} (15) = \log_{10} 3 + \log_{10} 5$$

$$(b) \log_a \left(\frac{m}{n}\right) = \log_a (m) - \log_a (n)$$

Example 2.

$$\log_2 \left(\frac{6}{5}\right) = \log_2 6 - \log_2 5$$

$$(c) \log_a (m^n) = n \log_a m$$

Example 3.

$$\log_5 625 = \log_5 5^4 = 4 \log_5 5$$

$$(d) \log_{a^n} (m) = \frac{1}{n} \log_a (m)$$

Example 4.

$$\log_{2^4} (8) = \frac{1}{4} \log_2 8$$

$$(2) \log_{10^2} 1000 = \frac{1}{2} \log_{10} 1000$$

$$(e) \log_a b = \frac{\log_n b}{\log_n a}$$

[here n may be any natural number]

Example 5.

$$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2}$$

$$\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$$

$$(f) \log_a x = \frac{\log_n x}{\log_n a} = \frac{1}{\log_x a}$$

Example 6.

$$\log_2 5 = \frac{1}{\log_5 2}$$

$$\log_3 8 = \frac{1}{\log_8 3}$$

$$(g) \log_a b \times \log_b a = \frac{\log_n b}{\log_n a} \times \frac{\log_n a}{\log_n b} = 1$$

Remark:

- (1) When base is not mentioned it is taken as 10.
- (2) Logarithms to the base 10 is known as common logarithms.
- (3) $\log_a a = 1$

Example 7.

$$\log_5 5 = 1$$

$$\log_2 2 = 1$$

$$(i) \log_a 1 = 0$$

[a may be any natural number.]

$$(j) a^{\log_a x} = x$$

$$5^{\log_5 3} = 3$$

$$3^{\log_3 7} = 7$$

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Solved Examples

1. Find the logarithm of 32 to the base 2

$$\text{Sol.: } \log_2 32 = \log_2 (2^5)$$

$$= 5 \log_2 2 = 5 \times 1 = 5$$

2. Find the value of $2^{\log_2 5}$

$$\text{Sol.: } a^{\log_a x} = x$$

$$\text{So } 2^{\log_2 5} = 5$$

3. Find the value of $3^{2+\log_3 5}$

$$\text{Sol.: } 3^{2+\log_3 5} = 3^2 \times 3^{\log_3 5}$$

$$= 3^2 \times 5 = 45$$

4. Find the value of $3^{2-\log_3 5}$

$$\text{Sol.: } 3^{2-\log_3 5} = \frac{3^2}{3^{\log_3 5}} = \frac{9}{5}$$

5. Find the value of $\log x^3 + \log x$

$$\text{Sol.: } \log x^3 + \log x = 3 \log x + \log x = 4 \log x$$

6. If $\log_{16} a = \frac{1}{2}$, Find a

$$\text{Sol.: } \log_{16} a = \frac{1}{2}$$

$$a = 16^{1/2}$$

$$a = 4$$

7. $\log_a \sqrt{3} = \frac{1}{6}$, find the value of a

$$\text{Sol.: } \log_a \sqrt{3} = \frac{1}{6}$$

$$= \log_a 3^{1/2} = \frac{1}{6}$$

$$\text{or } a^{1/6} = 3^{1/2}, a = 3^3 = 27$$

8. If $\log_{27} x + \log_9 x + \log_3 x = 11$, find the value of x

$$\text{Sol.: } \log_{27} x + \log_9 x + \log_3 x = 11$$

$$= \frac{1}{3} \log_3 x + \frac{1}{2} \log_3 x + \log_3 x = 11$$

$$= \frac{11}{6} \log_3 x = 11$$

$$= \frac{1}{6} \log_3 x = 1$$

$$= \log_3 x = 6$$

$$= x = 3^6 = 729$$

9. Solve $\log_3 n - \log_3 4 = 2$

$$\text{Sol.: } \log_3 n - \log_3 4 = \log_3 \frac{n}{4} = 2$$

$$\log_3 \frac{n}{4} = 2$$

$$\text{or } \frac{n}{4} = 3^2, n = 3^2 \times 4 = 36$$

10. Find the value of $\log \frac{x^2}{yz} + \log \frac{y^2}{zx} + \log \frac{z^2}{xy}$

$$\text{Sol.: } \log \frac{x^2}{yz} + \log \frac{y^2}{zx} + \log \frac{z^2}{xy}$$

$$= \log \frac{x^2}{yz} \times \frac{y^2}{zx} \times \frac{z^2}{xy}$$

$$= \log \frac{x^2 y^2 z^2}{y^2 z^2 x^2} = \log 1 = 0$$

11. Evaluate $\log_7 \left(\frac{1}{343} \right)$

$$\text{Sol.: } \log_7 \left(\frac{1}{343} \right) = \log_7 \frac{1}{7^3}$$

$$= \log_7 7^{-3} = -3 \log_7 7 = -3$$

12. Evaluate $\log_{100}(0.01)$

Sol.: $\log_{100}(100)^{-1} = -1 \log_{100} 100 = -1$

13. If $\log_{\sqrt{8}} x = 3\frac{1}{3}$, find the value of x

Sol.: $\log_{\sqrt{8}} x = \frac{10}{3}$, $x = \left(2^{\frac{3}{2}}\right)^{\frac{10}{3}}$

$x = 2^{\frac{3}{2} \times \frac{10}{3}} = 2^5 = 32$



Surds, Indices & Logarithms



Practice Exercise: I

1. The value of $(\sqrt{8})^{1/3}$ is:

- (a) 2 (b) 4
(c) $\sqrt{2}$ (d) 8

2. $\left(\frac{1}{216}\right)^{-2/3} + \left(\frac{1}{27}\right)^{-4/3} = ?$

- (a) $\frac{3}{4}$ (b) $\frac{2}{3}$
(c) $\frac{4}{9}$ (d) $\frac{1}{8}$

3. If $\sqrt{2^n} = 64$, then the value of n is:

- (a) 8 (b) 4
(c) 6 (d) 12

4. If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then n equals:

- (a) 0 (b) 2
(c) 3 (d) 4

5. If $(\sqrt{3})^5 \times 9^2 = 3^\alpha \times 3\sqrt{3}$, then α equals:

- (a) 2 (b) 3
(c) 4 (d) 5

6. If x, y, z are real numbers, then the value of:

$\sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x}$ is

(a) xyz (b) $\sqrt{xyz}$

(c) $\frac{1}{xyz}$ (d) 1

7. $\left(\frac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a-b)} \cdot \left(\frac{x^a}{x^b}\right)^{(a+b-c)} = ?$

- (a) x^{abc} (b) x^{a+b+c}
(c) $x^{ab+bc+ca}$ (d) 1

8. If $2^{x+4} - 2^{x+2} = 3$, then x is equal to:

- (a) 0 (b) 2
(c) -1 (d) -2

9. If $2^{2x-1} = \frac{1}{8^{x-3}}$, the value of x is:

- (a) 3 (b) 2
(c) 0 (d) -2

10. If $\sqrt{5 + \sqrt[3]{x}} = 3$, then x is equal to:

- (a) 125 (b) 64
(c) 27 (d) 9

11. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals:

- (a) $\frac{xz}{x+z}$ (b) $\frac{xz}{2(x-z)}$
(c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(x+z)}$

12. If $x = y^a$, $y = z^b$ and $z = x^c$, then the value of abc is:

- (a) 4 (b) 3
(c) 2 (d) 1



Solutions

1. Ans. (c)

$(\sqrt{8})^{1/3} = (8^{1/2})^{1/3} = 8^{\left(\frac{1}{2} \times \frac{1}{3}\right)} = 8^{1/6}$

$= (2^3)^{1/6} = 2^{\left(3 \times \frac{1}{6}\right)} = 2^{1/2} = \sqrt{2}$

2. Ans. (c)

$\left(\frac{1}{216}\right)^{-2/3} + \left(\frac{1}{27}\right)^{-4/3}$

$$= (216)^{2/3} + (27)^{4/3} = (6^3)^{2/3} + (3^3)^{4/3}$$

$$= 6^{\left(3 \times \frac{2}{3}\right)} + 3^{\left(3 \times \frac{4}{3}\right)} = 6^2 + 3^4 = \frac{36}{81} = \frac{4}{9}$$

3. Ans. (d)

$$\sqrt{2^n} = 64 \Rightarrow 2^{n/2} = 64 = 2^6$$

$$\therefore \frac{n}{2} = 6 \text{ or } n = 12$$

4. Ans. (c)

$$\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$$

$$\Rightarrow \frac{3^{2n} \times 3^5 \times (3^3)^3}{3 \times (3^4)^4} = 3^3$$

$$\text{or } \frac{3^{2n} \times 3^5 \times 3^9}{3^1 \times 3^{16}} = 3^3$$

$$\text{or } 3^{2n+5+9} = 3^3 \times 3^1 \times 3^{16}$$

$$\text{or } 3^{2n+14} = 3^{20}$$

$$\therefore 2n + 14 = 20$$

$$\text{or } 2n = 6 \text{ or } n = 3$$

5. Ans. (d)

$$(\sqrt{3})^5 \times 9^2 = 3^\alpha \times 3\sqrt{3}$$

$$\Rightarrow (3^{1/2})^5 \times (3^2)^2 = 3^\alpha \times 3^1 \times 3^{1/2}$$

$$\therefore 3^{5/2} \times 3^4 = 3^\alpha \times 3^1 \times 3^{1/2}$$

$$\text{or } 3^{\left(\frac{5}{2}+4\right)} = 3^{\left(\alpha+1+\frac{1}{2}\right)}$$

$$\text{or } 3^{13/2} = 3^{\alpha+3/2} \quad \text{So, } \alpha + \frac{3}{2} = \frac{13}{2}$$

$$\text{or } \alpha = \left(\frac{13}{2} - \frac{3}{2}\right) = 5$$

6. Ans. (d)

$$\sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x}$$

$$= \sqrt{\frac{y}{x}} \cdot \sqrt{\frac{z}{y}} \cdot \sqrt{\frac{x}{z}} = \frac{\sqrt{y}}{\sqrt{x}} \times \frac{\sqrt{z}}{\sqrt{y}} \times \frac{\sqrt{x}}{\sqrt{z}} = 1$$

7. Ans. (d)

Given Exp.

$$= x^{(b-c)(b+c-a)} \cdot x^{(c-a)(c+a-b)} \cdot x^{(a-b)(a+b-c)}$$

$$= x^{(b^2-c^2)+(c^2-a^2)+(a^2-b^2)} \cdot x^{-a(b-c)-b(c-a)-c(a-b)}$$

$$= x^0 \cdot x^0 = 1$$

8. Ans. (d)

$$2^{x+4} - 2^{x+2} = 3$$

$$\Rightarrow 2^{x+2}(2^2 - 1) = 3$$

$$\Rightarrow 2^{x+2} = 1 = 2^0$$

$$\therefore x + 2 = 0 \text{ or } x = -2$$

9. Ans. (b)

$$2^{2x-1} = \frac{1}{8^{x-3}} \Rightarrow 2^{2x-1} = \frac{1}{(2^3)^{(x-3)}}$$

$$\Rightarrow 2^{2x-1} = \frac{1}{2^{3x-9}} \Rightarrow 2^{2x-1} = 2^{9-3x}$$

$$\therefore 2x - 1 = 9 - 3x \text{ or } 5x - 10 \text{ or } x = 2$$

10. Ans. (b)

On squaring both sides, we get :

$$5 + \sqrt[3]{x} = 9 \text{ or } \sqrt[3]{x} = 4$$

Cubing both sides, we get

$$x = (4 \times 4 \times 4) = 64$$

11. Ans. (d)

$$\text{Let } a^x = b^y = c^z = k$$

$$\text{Then } a = k^{1/x}, b = k^{1/y} \text{ and } c = k^{1/z}$$

$$b^2 = ac \Rightarrow k^{2/y} = k^{1/x} \cdot k^{1/z}$$

$$\Rightarrow k^{2/y} = k^{\left(\frac{1}{x} + \frac{1}{z}\right)}$$

12. Ans. (d)

$$x = y^a = (z^b)^a = z^{ab} = (x^c)^{ab} = x^{abc} \therefore abc = 1$$

Surds, Indices & Logarithms



Practice Exercise: II

- The value of $\log_{343} 7$ is:
 - $\frac{1}{3}$
 - -3
 - $-\frac{1}{3}$
 - 3
- The value of $\log_{10}(.0001)$ is:
 - $\frac{1}{4}$
 - $-\frac{1}{4}$
 - -4
 - 4
- If $\log_8 x = \frac{2}{3}$, then the value of x is:
 - $\frac{3}{4}$
 - $\frac{4}{3}$
 - 4
 - 3
- $\log_{32} x = 0.8$, then x is equal to:
 - 25.6
 - 16
 - 10
 - 12.8
- If $\log_4 x + \log_2 x = 6$, then x is equal to
 - 2
 - 4
 - 8
 - 16
- If $\log 2 = 0.30103$, then the number of digits in 5^{20} is:
 - 14
 - 16
 - 18
 - 25
- The value of $\log_2(\log_5 625)$ is:
 - 2
 - 5
 - 10
 - 15
- $(\log_b a \times \log_c b \times \log_a c)$ is equal to:
 - 0
 - 1
 - abc
 - $a+b+c$
- $\left[\frac{1}{(\log_a bc) + 1} + \frac{1}{(\log_b ca) + 1} + \frac{1}{(\log_c ab) + 1} \right]$ is equal to:
 - 1
 - 2
 - 3
 - $\frac{3}{2}$
- If $\log_2[\log_3(\log_2 x)] = 1$, then x is equal to:
 - 512
 - 128
 - 12
 - 0
- If $\log_{10} 2 = 0.3010$, then $\log_2 10$ is:
 - $.3322$
 - 3.2320
 - 3.3222
 - 5
- If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 5$ is:
 - 0.3241
 - 0.6911
 - 0.6990
 - 0.7525
- The value of $(\log_9 27 + \log_8 32)$ is:
 - 4
 - 7
 - $\frac{7}{2}$
 - $\frac{19}{6}$
- If $\log_5(x^2 + x) - \log_5(x + 1) = 2$, then the value of x is:
 - 5
 - 32
 - 25
 - 10
- If $\log x + \log y = \log(x + y)$, then:
 - $x = y$
 - $xy = 1$
 - $y = \frac{x-1}{x}$
 - $y = \frac{x}{x-1}$
- The value of $\left[\frac{1}{\log_{(p/q)} x} + \frac{1}{\log_{(q/r)} x} + \frac{1}{\log_{(r/p)} x} \right]$ is:
 - 3
 - 2
 - 1
 - 0
- If $\log 2 = 0.3010$ and $\log 3 = 0.4771$ then the value of $\log 4.5$ is:
 - 0.6532
 - 0.7727
 - 0.3266
 - None of these

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Solutions

1. Ans. (a)

Let $\log_{343} 7 = m$.

Then, $(343)^m = 7 \Rightarrow (7^3)^m = 7$

$\Rightarrow 7^{3m} = 7$

$$\Rightarrow 3m = 1 \Rightarrow m = \frac{1}{3}.$$

$$\therefore \log_{343} 7 = \frac{1}{3}.$$

2. Ans. (c)

Let $\log_{10}(.0001) = m$. Then,

$$10^m = .0001 \Rightarrow 10^m = \frac{1}{10000} \Rightarrow 10^m$$

$$= \frac{1}{10^4} \Rightarrow 10^m = 10^{-4} \Rightarrow m = -4$$

$$\therefore \log_{10}(.0001) = -4.$$

3. Ans. (c)

$$\log_8 x = \frac{2}{3} \Rightarrow x = 8^{2/3} = (2^3)^{2/3}$$

$$= 2^{(3 \times \frac{2}{3})} = 2^2 = 4.$$

4. Ans. (b)

$$\log_{32} x = 0.8 \Rightarrow x = (32)^{0.8}$$

$$= (2^5)^{4/5} = 2^4 = 16.$$

5. Ans. (d)

$$\log_4 x + \log_2 x = 6 \Rightarrow \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 6$$

$$\therefore \frac{\log x}{2 \log 2} + \frac{\log x}{\log 2} = 6 \Rightarrow 3 \log x = 12 \log 2$$

$$\text{or } \log x = 4 \log 2 \Rightarrow \log x$$

$$= \log 2^4 \text{ or } x = 2^4 = 16.$$

6. Ans. (a)

$$\log 5^{20} = 20 \log 5 = 20 \times \left[\log \left(\frac{10}{2} \right) \right]$$

$$= 20 \times [\log 10 - \log 2] = 20 \times [1 - 0.3010]$$

$$= 20 \times 0.6990 = 13.9800$$

$$\therefore \text{Characteristic} = 13.$$

$$\therefore \text{Number of digits in } 5^{20} \text{ is } 14.$$

7. Ans. (a)

$$\text{Let } \log_5 625 = x. \text{ Then, } 5^x$$

$$= 625 = 5^4 \text{ or } x = 4$$

$$\text{Let } \log_2(\log_5 625) = y. \text{ Then, } \log_2(4) = y$$

$$\text{or } 2^y = 4 = 2^2. \text{ So, } y = 2.$$

$$\therefore \log_2(\log_5 625) = 2.$$

8. Ans. (b)

$$\text{Given Exp.} = \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a} \right) = 1.$$

9. Ans. (a)

$$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b}$$

$$+ \frac{1}{\log_c ab + \log_c c}$$

$$= \frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)} + \frac{1}{\log_c(abc)}$$

$$= \log_{abc} a + \log_{abc} b + \log_{abc} c$$

$$= \log_{abc}(abc) = 1.$$

10. Ans. (a)

$$\log_2 [\log_3 (\log_2 x)] = 1$$

$$\Rightarrow \log_3 (\log_2 x) = 2$$

$$\Rightarrow \log_2 x = 3^2 = 9$$

$$\Rightarrow x = 2^9 = 512$$

11. Ans. (c)

$$\log_2 10 = \frac{1}{\log_{10} 2} = \frac{1}{0.3010} = 3.3222$$

12. Ans. (c)

$$\log_{10} 5 = \log_{10} \left(\frac{10}{2} \right) = \log_{10} 10 - \log_{10} 2$$

$$= 1 - \log_{10} 2 = (1 - 0.3010) = 0.6990.$$

13. Ans. (d)

$$\log_9 27 + \log_8 32$$

$$\log_{3^2} 3^3 + \log_{2^3} 2^5$$

$$\frac{3}{2} \log_3 3 + \frac{5}{3} \log_2 2$$

$$\text{So, } \frac{3}{2} + \frac{5}{3} = \frac{19}{6}$$

14. Ans. (c)

$$\log_5 (x^2 + x) - \log_5 (x + 1) = 2$$

$$\Rightarrow \log_5 \left(\frac{x^2 + x}{x+1} \right) = 2$$

$$\therefore \log_5 \left[\frac{x(x+1)}{x+1} \right] = 2$$

$$\text{or } \log_5 x = 2 \text{ or } x = 5^2 = 25.$$

15. Ans. (d)

$$\log x + \log y = \log(x+y)$$

$$\Rightarrow \log(x+y) = \log(xy)$$

$$\therefore x+y = xy \text{ or } x = y(x-1)$$

$$\Rightarrow y = \frac{x}{x-1}$$

16. Ans. (d)

Given Expression

$$= \log_x \left(\frac{p}{q} \right) + \log_x \left(\frac{q}{r} \right) + \log_x \left(\frac{r}{p} \right)$$

$$\log_x \left(\frac{p}{q} \times \frac{q}{r} \times \frac{r}{p} \right) = \log 1 = 0.$$

17. Ans. (a)

$$\log 4.5 = \log \left(\frac{9}{2} \right) = \log 9 - \log 2$$

$$= \log(3^2) - \log 2 = 2\log 3 - \log 2$$

$$= (2 \times 0.4771 - 0.3010) = 0.6532.$$

