Electromagnetics Test 2

Number of Questions: 35 **Time: 60 min.**

Directions for questions 1 to** 35**: Select the correct alternative from the given choices.

1. Laplacian of a scalar is
(A) the gradient of the divergence
(B) the divergence of the gradient
(C) only divergence
(D) only gradient

2. Laplace equation in spherical co–ordinates is given by

(A) $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2 V}{\partial\varphi^2}+\frac{\partial^2 V}{\partial Z^2}$

(B) $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\varphi^2}$

(C) $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\varphi^2}$

(D) $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial^2 V}{\partial\varphi^2}$

3. Find $\overline{D}$ at P(5, 7, 0) caused by a uniform surface charge density $\rho_s = 41.7\mu C/m^2$ on the plane at x = 7.
(A) $-20.85\hat{a}_x \mu C/m^2$ (B) $41.7\hat{a}_z \mu C/m^2$
(C) Zero (D) $20.85\hat{a}_y \mu C/m^2$

4. The electric and magnetic field in a medium whose $\varepsilon_r = 5$.

$E = \frac{70}{\rho}\sin\left(10^4 t+\rho z\right)\hat{a}_\varphi$ V/m

$H = \frac{H_0}{\rho}\sin\left(10^4 t+\beta z\right)\hat{a}_\rho$ A/m

Then find H_0.
(A) 14 (B) 75.4
(C) –0.415 (D) 8.3×10^{-3}

5. A uniform plane wave in free space with $H = 2\sin\left(\omega t-3x-4z\right)\hat{a}_y$ A/m is incident on a dielectric slab then find angle of incidence.
(A) 53.13° (B) 36.86°
(C) 40.89° (D) 49°

6. For a given range of frequency, which one of the following system is best for transmission line load matching?
(A) Single stub
(B) Double stub
(C) Single stub with adjustable position
(D) Quarter wave transformer

7. A 50Ω lossless transmission line, transmits a signal of 500KHz to a load of 150Ω. First V_{min} occurs at
(A) 600m from the load
(B) 150m from the source
(C) 150m from the load
(D) 600m from the source

8. A uniform plane wave has power density 1.7W/m^2 has medium constants $\mu_r = 1$, $\varepsilon_r = 4$, then find maximum value of H.
(A) 134.3 mA/m (B) 250.3mA/m
(C) 253.4 mA/m (D) 145.2 mA/m

9. Cavity is a
(A) High pass filter (B) band stop filter
(C) low pass filter (D) band pass filter

10. A TEM wave impinges obliquely on a dielectric – dielectric boundary with $\varepsilon_{r1} = 2$, $\varepsilon_{r2} = 1$. The angle of incidence for total reflection is
(A) 54.7° (B) 45°
(C) 35.26° (D) does not exist

11. Method of images is applicable to
(A) Both electrostatic fields and electrodynamic fields
(B) Neither electrostatic fields nor electrodynamics fields
(C) Electrodynamics fields only
(D) Electrostatic fields only

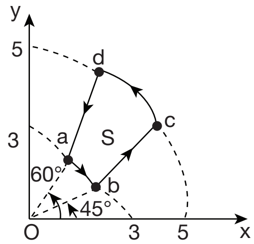
12. For no reflection condition a vertically polarized wave should be incident at the interface between two dielectric having $\varepsilon_1 = 1$ and $\varepsilon_2 = 4$, with an incident angle of
(A) $\tan^{-1}\left(\frac{4}{1}\right)$ (B) $\sin^{-1}(2)$
(C) $\tan^{-1}(2)$ (D) $\sin^{-1}(1/2)$

13. An elliptically polarized wave travelling in positive Z direction in air has X and Y components $E_x = 2\cos(\omega t - \beta z)$ V/m and $E_y = 4\cos(\omega t - \beta z + 80°)$V/m. Then find average power.
(A) 26.52 mW/m^2 (B) 30 mW/m^2
(C) 35 mW/m^2 (D) 47.7mW/m^2

14. The intrinsic impedance of copper at 4GHz, will be – (Assume that $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 6.8 \times 10^5$Mho/m)
(A) $377e^{j\pi/4}$ (B) $120\pi\, e^{j2\pi}$
(C) $0.215e^{j\pi/4}$ (D) $50e^{j2\pi}$

15. The terminating load of an *HF* transmission line with Z = 50Ω working at 250MHz is (50 + *j*30) Ω. Calculate the position of voltage minima nearest of the load.
(A) 1.68m (B) 0.42m
(C) 1.2m (D) 0.6m

16. If $A = \rho \sin\phi \; \hat{a}_\rho + \cos\phi \; \hat{a}_\varphi$ evaluate $\oint A \, d\ell$ around the path shown below figure.



(A) 6 (B) -0.95
(C) 2 (D) Zero

17. Given field $\vec{A} = 3x^2yz\hat{a}_x + x^2z\hat{a}_y + \left(x^2y - 2z\right)\hat{a}_z$ wb/m, it can be said that $\vec{A}$ is
(A) Solenoidal (B) Rotational
(C) Harmonic (D) Conservative

18. A charge of 2 coulomb is placed near conducting ground plate, at a distance of 1*m*. Then force between two charges is
(A) $\frac{1}{2\pi\varepsilon_0}$ (B) $\frac{1}{\pi\varepsilon_0}$
(C) $\frac{2}{\pi\varepsilon_0}$ (D) $\frac{1}{4\pi\varepsilon_0}$

19. A wire of diameter 1mm and conductivity 4.8×10^6S/m has 10^{27}free electrons per cube meter when an electric field of 8mV/m then what is the drift velocity of electrons?
(A) 38.4 mm/sec (B) 2.4 μm/sec
(C) 38.4 μm/sec (D) 61.44 mm/sec

20. In a non magnetic medium $E = 5\cos(2\pi \times 10^8 t - 8x)\; \hat{a}_z$ V/m, then the total power crossing 50cm^2 of plane $3x + y = 2$.
(A) 417.36 mW (B) 542.5 μW
(C) 600 μW (D) 743.μW

21. 50 V peak signal is incident on a 80Ω transmission line and line is transmitted by 250Ω load. The peak value of incident and reflected currents respectively are
(A) 0.357A, 0.625A (B) 0.625A, 0.321A
(C) 0.625A, 0.13A (D) 0.52A, 0.23A

22. A square wave guide operated at 4*G*Hz. If the group velocity is 2×10^8m/sec and wave guide is filled with oil having $\varepsilon = 3.5\varepsilon_0$. Then find largest dimension of the wave guide.
(A) 4.02 cm (B) 2.15 cm
(C) 3.04 cm (D) 2.7 cm

23. An antenna has a gain of 50 dB. Assuming that the main beam of antenna is circular in cross – section, the beam width will be
(A) 29° (B) 50°
(C) 140° (D) 1.1°

24. Consider a region defined by $|x|,|y|,|z| < 1$. Let $\varepsilon = 4\varepsilon_0$, $\mu = 3\mu_0$ and $\sigma = 0$, the displacement current density J_d is equals to $10\sin(10^8 t - bx)\; \hat{a}_y$ mA/m^2. Assume no DC fields are present. Then the magnetic field intensity is given by
(A) $-2.82\cos(10^8 t - bx)\; \hat{a}_y$ A/m
(B) $2.82\, b\sin(10^8 - bx)\; \hat{a}_z$ Am
(C) $-0.74b\cos(10^8 t - bx)\; \hat{a}_z$ A/m
(D) $0.74\cos(10^8 t - bx)\; \hat{a}_y$ A/m

25. Uniform plane wave in a lossy magnetic media has the electric field strength as $E_s = \left(3\hat{a}_x + 5\hat{a}_y\right)e^{-\gamma z}$ where $\gamma = (0.6 + j5)\text{m}^{-1}$. Calculate magnitude of the wave at $z = 3$m and $t = \frac{T}{10}$
(A) 5.83 (B) 0.15
(C) 0.89 (D) 3

26. For an electric field $E = E_0 \cos\omega t$ V/m, what is the phase difference between the conduction current and displacement current?
(A) 45° (B) 90°
(C) 0° (D) 60°

27. A line has 75Ω characteristic impedance and it is terminated by *j*75 ohms. The VSWR on the line is
(A) *j* (B) 1
(C) ∞ (D) Does not exist

28. A 12 cm long vertical dipole radiates into air at 50 MHz with a peak input current of 80mA. Find its efficiency if its loss resistance is 0.2Ω.
(A) 31.6% (B) 61.2%
(C) 16.3% (D) 21.6%

29. In a eighty turn coil, if the flux through each turn is $(4t^3 - 2t)$m wb. The magnitude of the induced emf in the coil at a time of 3sec is
(A) 4.8mV (B) 8.48V
(C) 8.16V (D) 6V

30. A plane wave of 8 *G*Hz is incident normally on a dielectric plate of 4mm thickness. If a phase shift on transmission through the sheet is 90°, then dielectric constant is ________
(A) 4.68 (B) 8
(C) 25 (D) 5.5

31. A dipole antenna with a length of 8cm and carrying a current of 1.5A at a frequency of 10^8Hz, radiates into free space. Find the electric field intensity at a distance of 8 *km* from the antenna.

(A) 0.842mV/m (B) 1.88V/m
(C) 0.97V/m (D) 1.37mV/m

Common data for Questions 32 and 33:
A transmission line has $R = 20\Omega$/km, inductance per Kilometer is 80mH, conductance is zero, capacitance per kilometer is 15μF at $\omega = 2 \times 10^3$ rad/sec.

32. The characteristic impedance of the line is
(A) 50Ω (B) $73.3 \angle{-7.13}°\Omega$
(C) $73.3 \angle{-3.56}°\Omega$ (D) $51.8 \angle 7°\Omega$

33. Find the propagation constant.
(A) $2.2\angle 86°$ (B) $0.153\angle 0°$
(C) $2.19\angle 0°$ (D) $2.19\angle 172°$

Statement for Linked answer Questions 34 and 35:
Given the potential $V = \frac{8}{r}\sin 2\theta \cos\phi$

34. The electric flux density D at $(3, \pi/4, 0)$ is
(A) $8/3\ \hat{a}_r$ (B) $-8/9\ \hat{a}_\phi$
(C) $\frac{-8}{3}\varepsilon_0\ \hat{a}_\theta$ (D) $\frac{8}{9}\varepsilon_0\ \hat{a}_r$

35. Find the work done in moving a 9μ*C* charge from point *A*(1, 15°, 120°) to B(3, 45°, 60°).
(A) 6 μJ (B) 30 μJ
(C) 6 *m*J (D) 30 *m*J

Answer Keys

1. B	**2.** C	**3.** A	**4.** C	**5.** B	**6.** B	**7.** C	**8.** A	**9.** D	**10.** B
11. D	**12.** C	**13.** A	**14.** C	**15.** B	**16.** B	**17.** B	**18.** D	**19.** B	**20.** C
21. B	**22.** C	**23.** D	**24.** C	**25.** C	**26.** B	**27.** C	**28.** B	**29.** B	**30.** D
31. D	**32.** C	**33.** A	**34.** D	**35.** B					

Hints and Explanations

1. Laplacian of a scalar field is the divergence of the gradient Choice (B)

2. Choice (C)

3. $\overline{D} = \varepsilon_0 \overline{E} = \varepsilon_0 \frac{\rho_s}{2\varepsilon_0}\overline{a}_N = \frac{41.7}{2}\overline{a}_x$

[negative sign due to co – ordinate of x is less than Sur face charge density plane. Choice (A)

4. $\frac{E_\varphi}{H_\rho} = -\eta$

$\Rightarrow \frac{-70\sqrt{\varepsilon_r}}{120\pi\sqrt{\mu_r}} = H_0$

$\Rightarrow \frac{-70\sqrt{5}}{120\pi} = -0.415$ Choice (C)

5. The propagation vectors $\tan\theta_I = \frac{3}{4}$

$\theta_I = \tan^{-1}(3/4) = 36.86°$ Choice (B)

6. Single stub is used for a fixed frequency only and double stub is used for a range of frequencies.
Choice (B)

7. Given that $Z_0 = 50\Omega$
$Z_L = 150\Omega$
$\Rightarrow$ $ZL > Z_0$, first V_{max} is located at the load & first V_{min} is located at $\frac{\lambda}{4}$

$\lambda = \frac{C}{f} = \frac{3\times10^8}{5\times10^5} = 600$ m

$\Rightarrow$ V_{min} occurs at 150 m from the load. Choice (C)

8. From pointing vector we know that

$|\vec{P}| = \frac{1}{2}E_m H_m = \frac{\eta}{2}H_m^2$

$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{377}{2} = 188.5\Omega$

$\Rightarrow H_m = \sqrt{\frac{2|\vec{P}|}{\eta}} = \sqrt{\frac{2\times1.7}{188.5}}$

$= 134.3$ mA/m Choice (A)

9. Choice (D)

10. $\frac{\sin\theta_i}{\sin\theta_r} = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$

$\Rightarrow \sin\theta_I = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta_I = 45°$ Choice (B)

11. Choice (D)

12. $\tan\theta_I = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

$\tan\theta_I = 2 \Rightarrow \theta_I = \tan^{-1}(2)$ Choice (C)

13. $E = E_x + E_y = E_x\hat{a}_x + E_y\hat{a}_y$

$|E| = \sqrt{E_x^2 + E_y^2} = \sqrt{16+4} = \sqrt{20}$

$\Rightarrow P = \frac{|E|^2}{2\eta} = 26.52$ mW/m^2 Choice (A)

14. Intrensic impedance (η) = $\sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

$\sigma = 6.8\times10^5$

and $\omega\varepsilon = 4\times10^9\times2\times\pi\times\dfrac{1}{36\pi}\times10^{-9} = 0.2 << \sigma$

So $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma}} = \sqrt{\dfrac{\omega\mu}{\sigma}}e^{j\pi/4}$

$= \sqrt{\dfrac{2\pi\times4\times10^9\times4\pi\times10^{-7}}{6.8\times10^5}}e^{j\pi/4}$

$= 0.215e^{j\pi/4}$ Choice (C)

15. $Z_0 = 50\Omega$

$Z_L = 50 + j30$

$\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{j30}{100 + j30} = \dfrac{30}{104.4}\angle 90° - 16.69$

$= 0.29\angle 73.31°$

Position of voltage minima nearest to load = $\dfrac{\varphi + \pi}{2\beta}$

$[\because \phi = 73.31° = 0.4\pi]$

$= \dfrac{0.4\pi + \pi}{2(2\pi/\lambda)} \left[: \lambda = \dfrac{c}{f} = \dfrac{3\times10^8}{25\times10^7} = \dfrac{6}{5} = 1.2m \right]$

$= \dfrac{(1.4)(1.2)}{4} = 0.42$m from the load

Choice (B)

16. $\oint A.\,dl = \int_S (\nabla\times A)ds$

$\Rightarrow \nabla\times A = \dfrac{1}{\rho}\begin{bmatrix} \hat{a}_\rho & \rho\hat{a}_\varphi & \hat{a}_z \\ \dfrac{\partial}{\partial\rho} & \dfrac{\partial}{\partial\varphi} & \dfrac{\partial}{\partial Z} \\ \rho\sin\varphi & \rho\cos\varphi & 0 \end{bmatrix}$

$= \dfrac{\cos\varphi}{\rho}[1-\rho]$

$\oint A.\,dl = \int_{45°}^{60°}\int_3^5 \dfrac{\cos\varphi}{\rho}(1-\rho)\rho d\rho d\varphi$

$s = (2 - 8)\,[\text{Sin}60° - \text{Sin}45°]$

$= -6\left[\dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}}\right] = -0.95$ Choice (B)

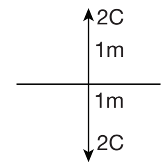
17. $\nabla\times A = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{d}{dx} & \dfrac{d}{dy} & \dfrac{d}{dz} \\ 3x^2yz & x^2z & x^2y - 2z \end{bmatrix}$

$= -\hat{a}_y\left[2x - 3x^2y\right] + \hat{a}_z\left[2xz - 3x^2y\right] \neq 0$

So A is said to be rotational Choice (B)

18. Given that charge is placed near a ground plane so we have to consider method image charge.
So total distance is 2m.

$F = \dfrac{Q_1Q_2}{4\pi\varepsilon_0 r^2} = \dfrac{4}{16\pi\varepsilon_0{}^4} = 1/4\pi\varepsilon_0$



Choice (D)

19. drift velocity = $\dfrac{J}{\rho_v} = \dfrac{\sigma E}{\rho_v}$

$= \dfrac{4.8\times10^6\times8\times10^{-3}}{10^{27}\times1.6\times10^{-19}} = 2.4\mu\text{m/sec}$ Choice (B)

20. On plane $3x + y = 2$

$\hat{a}_n = \dfrac{3\hat{a}_x + \hat{a}_y}{\sqrt{10}}$

$P_{\text{total}} = P_{\text{avg}} \times$ area

$\Rightarrow P_{\text{avg}} = \dfrac{E_0{}^2}{2\eta}$

$\eta = \dfrac{120\pi}{\sqrt{\varepsilon_r}}$

$\beta = \omega\sqrt{\mu\varepsilon_0\varepsilon_r}$

$\Rightarrow \dfrac{\beta C}{\omega} = \sqrt{\varepsilon_r} = \dfrac{8\times3\times10^8}{2\pi\times10^8} = \dfrac{12}{\pi}$

$\Rightarrow P_{\text{avg}} = \dfrac{25\times12}{2\times120\pi\times\pi}$

$\Rightarrow P_{\text{total}} = \left[0.126\times50\times10^{-4}\right]\times\hat{a}_x\left[\dfrac{3\hat{a}_x + \hat{a}_y}{\sqrt{10}}\right]$

$= 600\times10^{-6}\text{W} = 600\ \mu\text{W}$ Choice (C)

21. $I_{\text{incident}} = \dfrac{50V}{80\Omega} = 0.625A$

$I_{\text{reflected}} = \dfrac{V_r}{Z_0} = \dfrac{\Gamma V_i}{Z_0}$ $\left[\because \Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}\right]$

$\Gamma = 0.51$

$I_{\text{reflect}} = \dfrac{0.51\times50}{80} = 0.321$ A Choice (B)

22. We know that $Vp\ Vg = C^2$

$V_P = \dfrac{(3\times10^8)^2}{2\times10^8}$

$= 4.5\times10^8$m/sec

$$\Rightarrow \quad V_P = \frac{C}{\sqrt{\varepsilon_r}\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}}$$

$$\Rightarrow \quad 4.5\times10^8 = \frac{3\times10^8}{\sqrt{3.5}\sqrt{1-\left(\frac{f_c}{4}\right)^2}}$$

$\Rightarrow$ $f_c = 3.73$GHz

For a square wave guide a = b and dominant mode is TM_{11}

$$\Rightarrow \quad f_c = \frac{C}{2\sqrt{\varepsilon_r}}\sqrt{\frac{1}{a^2+\frac{1}{a^2}}}\,;\, f_c = \frac{C}{2\sqrt{\varepsilon_r}}\frac{\sqrt{2}}{a}$$

$$\Rightarrow \quad a = \frac{C}{f_c\sqrt{2\varepsilon_r}} = \frac{3\times10^{10}}{3.73\times10^9\sqrt{7}}$$

= 3.04cm Choice (C)

23. 10log G = 50

$\Rightarrow$ $G = 10^5$

Gain of antema which is circular in cross section is $6\left(\frac{D}{\lambda}\right)^2 = 10^5$

$$\frac{D}{\lambda} = 129$$

$\Rightarrow$ Beam width = $140\frac{\lambda}{D} = 1.08$

≃ 1.1° Choice (D)

24. $J_d = \frac{dD}{dt} \Rightarrow D = \int J_d t + C_1$

[∵ C_1 is zero because no dc fields is present]

$$D = \frac{-10\times10^{-3}}{10^8}\cos\left(10^8 t - bx\right)\hat{a}_y$$

$$E = \frac{D}{\varepsilon} = \frac{-10^{-10}}{\varepsilon}\cos\left(10^8 t - bx\right)\hat{a}_y$$

$= -2.82\cos(10^8 t - bx)\ \hat{a}_y$

$$\Rightarrow \quad \nabla\times E = \frac{\partial E_y}{\partial_x}\hat{a}_z = \frac{-\partial B}{\partial t}$$

$$-2.82b\ \mathrm{Sin}(10^8 t - bx) = -\frac{\partial B}{\partial t}$$

Integrating both sides

$$\frac{-2.82b}{10^8}\cos\left(10^8 t - bx\right) = B = \mu H$$

$$\Rightarrow \quad H = \frac{-2.82b}{10^8\mu}\mathrm{Cos}\left(10^8 t - bx\right)$$

$= 0.74b\cos(10^8 t - bx)\ \hat{a}_z$ A/m Choice (C)

25. $E_S = \left(3\hat{a}_x + 5\hat{a}_y\right)e^{-(0.6+j5)2}$

But $E = R_e\left[E_s e^{j\omega t}\right]$

$$E = \mathrm{Re}\left[\left(3\hat{a}_x + 5\hat{a}_y\right)e^{-(0.6+j5)z}\ e^{j(\omega t-\beta z)}\right]$$

$$\Rightarrow \quad E = \left(3\hat{a}_x + 5\hat{a}_y\right)e^{-0.6z}\cos\left(\omega t - \beta z\right)$$

$$|E| = \sqrt{9+25}e^{-0.6\times3}\cos\left(\frac{2\pi}{T}\times\frac{t}{10} - 15\right)$$

$= 5.8\ e^{-1.8}\cos(21) = 0.89$ Choice (C)

26. $J = \sigma E + \varepsilon\frac{\partial E}{\partial t}$

$= \sigma E_0 \cos\omega t - \varepsilon E_0 \omega \sin\omega t$

$= \sigma E_0 \cos\omega t + \varepsilon E_0 \omega\cos(\omega t + 90°)$

$\Rightarrow$ 90° phase difference between conduction current and displacement current. Choice (B)

27. $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{j-1}{j+1} \Rightarrow |\Gamma| = 1$$

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1}{1-1} = \infty$$ Choice (C)

28. $R_{\text{rad}} = 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2$

$$\Rightarrow \quad \lambda = \frac{C}{f} = \frac{3\times10^{10}}{50\times10^6} = 600\text{cm}$$

$$R_{\text{rad}} = 80\pi^2\left(\frac{12}{600}\right)^2 = 0.316$$

$$\eta = \frac{R_{rad}}{R_{rad}+R_{loss}}\times100$$

$$= \frac{0.316}{0.316+0.2}\times100 = 61.2\%$$ Choice (B)

29. $E_{\text{ind}} = \frac{-Nd\varphi}{dt}$

$$= -80\frac{d\left(4t^3 - 2t\right)\times10^{-3}}{dt}$$

$= -80[12t^2 - 2]\times10^{-3}$ at 3sec.

$= -8.48$ V Choice (B)

30. Phase difference = $\frac{2\pi}{\lambda}$ path difference

$$\pi/2 = \frac{2\pi}{\lambda}\times4\times10^{-3}$$

$\Rightarrow$ λ = 16 mm

$$\Rightarrow \quad V = \frac{C}{\sqrt{\varepsilon_r}} = \lambda f$$

$$\frac{3\times10^8}{\sqrt{\varepsilon_r}} = 16 \times 10^{-3} \times 8 \times 10^9$$

$\Rightarrow$ $\varepsilon_r = 5.49 = 5.5$

Choice (D)

31. length of dipole = dl = 8cm = 0.08m = 8×10^{-2}m

I = 1.5A

$$\lambda = \frac{C}{f} = \frac{3\times10^8}{10^8} = 3\text{m}$$

$$P_{\text{rad}} = 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2 I_{eff}^2$$

$$= 80\pi^2\left(\frac{8\times10^{-2}}{3}\right)^2 \times 1.5^2 = 1.89 \simeq 2$$

$$P_{\text{avg}} = \frac{|E|^2}{2\eta} \text{ and } P_{\text{avg}} \times 4\pi r^2 = P_{\text{rad}}$$

$$\Rightarrow \quad |E|^2 = \frac{2\eta}{4\pi r^2}P_{rad} = \frac{2\times2\times120\pi}{4\pi\times(8\times10^3)^2}$$

$= 1.37$ mV/m

Choice (D)

32. $$Z_0 = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}}$$

$$= \sqrt{\frac{20+j160}{j2\times15\times10^{-3}}}$$

$$= 10^2\sqrt{\frac{1+j8}{j15}} = 73.3 \angle -3.565°$$

Choice (C)

33. $$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(20+j160)\,j30\times10^{-3}}$$

$= 22 \times 10^{-1}\angle 86° = 2.2 \angle 86°$

Choice (A)

34. $D = \varepsilon_0 E$ and $E = -\nabla V$

$$\nabla V = \left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\hat{a}_\varphi\right]$$

$$= \frac{-8}{r^2}\sin 2\theta\,\cos\varphi\hat{a}_r + \frac{16}{r^2}\cos\varphi\, cs2\theta\hat{a}_\theta - \frac{8}{r}\sin 2\theta\sin\varphi\hat{a}_\varphi$$

At (3, π/4, 0) point $E = +\frac{8}{9}\hat{a}_r$

$$D = \varepsilon_0 E = \frac{+8}{9}\varepsilon_0\hat{a}_r$$

Choice (D)

35. $$W = -Q\int_A^B E.dl = QV_{AB} = Q[V_B - V_A]$$

$$= Q\left[\frac{8}{3}\sin 2\times45°\cos 60° - 8\sin 2\times15°\cos 120°\right]$$

$$= 9\times10^{-6}\left[\frac{8}{3}\times1/2 - 8\times\frac{1}{2}(-1/2)\right]$$

$= 9\times10^{-6}[4/3+2] = 30\mu$J

Choice (B)