* $\boxed{\text{Matrices}}$

• matrix :

$$\begin{bmatrix} & \\ & \end{bmatrix}_{m\times n}$$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$

$m \neq n \Rightarrow$ rectangular.
$m = n \Rightarrow$ square.

unit matrix : (square)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

principal diagonal

Null matrix : Or (zero matrix)

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$$

Row matrix :

$$[\quad\quad\quad]_{1\times n}$$

column matrix :

$$\begin{bmatrix} \\ \\ \end{bmatrix}_{n\times 1}$$

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Submatrix :

Any matrix obtained by omitting some rows and columns from a given ($m \times n$) matrix 'A' is called submatrix of 'A'.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & \boxed{5 \quad 6} \\ 7 & \boxed{8 \quad 9} \end{bmatrix}_{3\times3}$$

$$A_1 = \begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}_{2\times2} \quad A_2 = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}_{3\times2}$$

hence, A_1 and A_2 are submatrix of 'A'.

principal Submatrix :

A ~~sequentia~~ square submatrix of a ~~sque~~ square matrix 'A' is called principle submatrix, if its diagonal elements are also the diagonal elements of matrix 'A'

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$$

$$A_1 = \begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}$$

• Addition of matrixes :

Equality :

ⓐ They are of same size.
ⓑ The elements ~~of~~ in the corresponding places are same.

Addition of matrices :
only ~~po~~ possible if the matrix are comfortabl i.e same size.

properties of addition :

① Commutative :
$A+B = B+A.$

② Associative :
$(A+B)+C = A+(B+C)$

③ $A+0 = 0+A = A$

④ $-A+A = 0 = A+(-A)$

⑤ $A-B = A+(-B)$

⑥ $A+B = A+C \Rightarrow B = C$
$B+A = C+A \Rightarrow B = C.$

Scalar multiplication –

if $A = [a_{ij}]_{m\times n}$, then

$$KA = AK = [K a_{ij}]_{m\times n}$$

ex:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$2 \times A = \begin{bmatrix} 1\times 2 & 2\times 2 \\ 3\times 2 & 4\times 2 \end{bmatrix}$$

Theorem – $K(A+B) = KA + KB$

Theorem – $(p+q)A = pA + qA$.

Theorem – $p(qA) = (pq)A$

Theorem – $(-K)A = -(KA) = K(-A)$.

- **Matrix multiplication** –

$$A_{m\times n} \times B_{n\times p} = C_{m\times p}$$

should be equal

ex:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}_{2\times 2}$

$$A\times B = \begin{bmatrix} 2+2 & 1+4 \\ 6+4 & 3+8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 10 & 11 \end{bmatrix}$$

properties of matrix multiplications –

1. associative $(AB)C = A(BC)$
2. $A(B+C) = AB + AC$
3. not commutative.
4. * The equ $AB = 0$ does not necessarily imply that one of the matrices A and B must be a zero matrix.

   – if $AB = 0$, it does not necessarily imply that $BA = 0$.
5. $AI_n = A = I_n A$
6. Associative Law fore multi 'n' matrices –
   $(A_1 A_2)(A_3 A_4) \cdots (\cdot A_n)$

(1) upper triangular matrix :

$$A = \begin{bmatrix} - & - & - & - \\ 0 & - & - & - \\ 0 & 0 & - & - \\ 0 & 0 & 0 & - \end{bmatrix}$$

$\llcorner$ all zero

$A = [a_{ij}]$
if, $a_{ij} = 0$
when, $i > j$

(2) Lower triangular matrix :

$\rightarrow$ all zero

$$A = \begin{bmatrix} - & 0 & 0 & 0 \\ - & - & 0 & 0 \\ - & - & - & 0 \\ - & - & - & - \end{bmatrix}$$

$A = [a_{ij}]$
if, $a_{ij} = 0$
when,
$i < j$

strict triangular matrix,

$A = [a_{ij}]_{m \times n}$
if, $a_{ij} = 0$
~~when, for $i = 1, 2, \dots n$~~

(3) Diagonal matrix : (upper as well as lower tri)

$$A = \begin{bmatrix} - & - & 0's \\ - & - & - \\ 0's & - & - \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\rightarrow$ upper & lower elements 0's

~~Identity~~
(4) Scalar matrix :

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

0's

0's

$\llcorner \rightarrow$ diagonal ele equal.

scalar matrix

sca matrix diagonal val

$$AK = A \times K_1 \times I$$
$$= \cancel{A}\, K_1(A \times I)$$
$$= K_1(A)$$
$$= K_1 A$$

$\therefore$ $AK = K_1 A$

- examples

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -\sqrt{-1}$

$i^4 = 1$

$I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

ex:1

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}_{1\times 3} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix}_{3\times 3} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix}_{3\times 1} = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 3+5x+3 & 2+x+2 \end{bmatrix}_{1\times 3} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix}_{3\times 1} = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 6+5x & 4+x \end{bmatrix}_{1\times 3} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix}_{3\times 1} = 0$$

$$\Rightarrow \begin{bmatrix} 1+6+5x+4x+x^2 \end{bmatrix} = 0$$

$$\Rightarrow x^2+9x+7=0$$

$$x = \frac{-9 \pm \sqrt{81-28}}{2}$$

** $\boxed{x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}}$

(ex: 2)

if $(A+B)^2 = A^2+2AB+B^2$ then what can you say that about A and B ?

$\Rightarrow (A+B)^2 = (A+B)(A+B)$

$= A^2+AB+BA+B^2 = A^2+2AB+B^2$

$\Rightarrow AB+BA = 2AB$

$\boxed{BA = AB}$

(A and B commutative)

(ex: 3) Under what conditions is the matrix equ.

$A^2-B^2 = (A-B)(A+B)$ true ?

$\Rightarrow A^2-B^2 = A^2+AB-BA-B^2$

$AB-BA = 0$

$\Rightarrow AB = BA$ When A and B commute

Idempotent Matrix –

A matrix 'A' such that $A^2 = A$ is called idempotent matrix.

$A \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$A \times A = A$

$A^2 = A$

- Involutory matrices :

A matrix 'A' such that $\boxed{A^2 = I}$ is called involutory.

- Nilpotent Matrices :

A square matrix 'A' is called a nilpotent mtx if there exists a positive integer 'm' such that $A^m = 0$.
If 'm' is the least positive integer such that $A^m = 0$, then 'm' is called the ~~m~~ index of the nilpotent Mxt 'A'.

ex :

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \quad A^3 = 0$$

A
A^2 — $A^2 = 0$ → Index
A^3
A^4
:

$$A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix} \quad A^2 = 0$$

→ Some matrices are Nilpotent Mtx.

Transpose of a matrix:

A matrix obtained by ~~Ching~~ changing its rows into columns and columns into rows is called transpose of a matrix.

properties:

$\rightarrow (A^T)^T = A$

$\rightarrow (A+B)^T = A^T + B^T$

$\rightarrow (kA)^T = kA^T$

$\rightarrow (AB)^T = B^T A^T$

ex:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}.$$

- Symmentrix matrix :

$$A = A^T$$ $$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$a_{ij} = a_{ji}$$

- Skew symmentrix matrix :

$$A = -A^T$$

$$a_{ij} = -a_{ji}$$

→ all the element of principle diagonal should be zero " then it is skew sym mtx.

ex: Show that the matrix "$B^T AB$" is symmetrix or skew symmentric according as 'A' is symmentric or skew symmentric.

⇒ ~~A = A~~

Case a : $A = A^T$

$$(B^T AB)^T = B^T A^T B$$
$$= B^T AB$$

case b : $A = -A^T$

$$(B^T AB)^T = B^T A^T B$$
$$= -B^T AB$$

(ex) if A any square mtx, then show that $A+A^T$ is symmentric and $A-A^T$ is skew symmetric.

$$\Rightarrow (A+A^T)^T = A^T + (A^T)^T$$
$$= A^T + A$$
$$(A+A^T)^T = A + A^T$$ symmentric

(ex)

$$(A-A^T)^T = A^T - A^{T^T}$$
$$= -A + A^T$$
$$= -(A-A^T)$$ skew symmentric

(ex) if A and B are symmentric matrices of order n then show that $AB+BA$ is symmetric and $AB-BA$ is skew-symmentric.

$$A^T = A$$
$$B^T = B$$

$\Rightarrow$

(i) $$(AB+BA)^T = (AB)^T + (BA)^T$$
$$= B^T A^T + A^T B^T$$
$$= BA + AB$$
$$= AB + BA$$

(ii) $$(AB-BA)^T = (AB)^T - (BA)^T$$
$$= B^T A^T - A^T B^T$$
$$= BA - AB$$
$$= -(AB-BA).$$

- Conjugate of number –

$z = x + iy$, x, y = real number and $i = \sqrt{-1}$ is called a complex number.

$\bar{z} = x - iy$, is called the conjugate of the complex number.

- Conjugate of Matrix :

$$A = \begin{bmatrix} 1-2i & 3 \\ 4 & 1+2i \end{bmatrix}_{2\times 2} \xrightarrow{\text{conjugate}} \bar{A} = \begin{bmatrix} 1+2i & 3 \\ 4 & 1-2i \end{bmatrix}$$