Arithmetic Progressions

Exercise 2.1

I. Very Short Answer Type Questions [1 Mark]

1. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

(1) If $a_n = 5n - 4$ is a sequence, then a_{12} is

(a) 48 (b) 52 (c) 56 (d) 62

(2) If $a_n = 3n - 2$, then the value of $a_7 + a_8$ is

(a) 39 (b) 41 (c) 47 (d) 53

(3) The second term of the sequence defined by $a_n = 3n + 2$ is

(a) 2 (b) 4 (c) 6 (d) 8

2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** The arrangement of numbers, *i.e.*, – 4, 16, – 64, 256, – 1024, 4096, ... form a sequence.
Reason (R): An arrangement of numbers which are arranged in a definite order according to some rule, is called a sequence.

(2) **Assertion (A):** Sequence 1, 5, 9, 13, 17, 21, ... is a finite sequence.
Reason (R): A sequence with finite number of terms or numbers is called a finite sequence.

3. Answer the following:

(1) Write down the first six terms of each of the following sequences, whose general terms are:

(a) $a_n = 5n - 3$ (b) $a_n = (-1)^n \cdot 2^{2n}$ (c) $a_n = \frac{2n+1}{n+2}$ (d) $a_n = (-1)^{n-1} \cdot n^2$

(2) Find the 10^{th} term of the sequence defined by $a_n = (-1)^{2n-1} \cdot 5^n$.

(3) Find the difference between the 12^{th} term and 10^{th} term of the sequence whose general term is given by $a_n = 5n - 1$.

Answers

1. (1) (c) 56 (1) (2) (b) 41 (1)
(3) (d) 8 (1)

2. (1) (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(2) (d) Assertion (A) is false but reason (R) is true. (1)

3. (1) (a) 2, 7, 12, 17, 22, 27
(b) –4, 16, –64, 256, –1024, 4096
(c) $1, \frac{5}{4}, \frac{7}{5}, \frac{3}{2}, \frac{11}{7}, \frac{13}{8}$
(d) 1, –4, 9, –16, 25, –36 (1)
(2) –9765625 (1)
(3) 10 (1)

Exercise 2.2

I. Very Short Answer Type Questions [1 Mark]

1. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

(1) In an AP, if $d = -4$, $n = 7$, $a_n = 4$, then a is

(a) 6 (b) 7 (c) 20 (d) 28

(2) The n^{th} term of the AP: a, $3a$, $5a$, ... is

(a) na (b) $(2n - 1)a$ (c) $(2n + 1)a$ (d) $2na$

(3) The first term of an AP is p and the common difference is q, then its 10^{th} term is

(a) $q + 9p$ (b) $p - 9q$ (c) $p + 9q$ (d) $2p + 9p$

(4) If $\frac{4}{5}$, a, 2 are three consecutive terms of an AP, then the value of a is

(a) $\frac{5}{2}$ (b) $\frac{2}{7}$ (c) $\frac{5}{7}$ (d) $\frac{7}{5}$

2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** Common difference of the AP: –5, –1, 3, 7, ... is 4.
Reason (R): Common difference of the AP : a, $a + d$, $a + 2d$, ... is given by $d = 2^{nd}$ term – 1^{st} term.

(2) **Assertion (A):** If n^{th} term of an AP is $7 - 4n$, then its common difference is – 4.
Reason (R): Common difference of an AP is given by $d = a_{n+1} - a_n$.

(3) **Assertion (A):** Common difference of an AP in which $a_{21} - a_7 = 84$ is 14.
Reason (R): n^{th} term of an AP is given by $a_n = a + (n - 1)\,d$.

3. Answer the following:

(1) Write first four terms of the AP, whose first term and the common difference are given as follows: $a = 10, d = 10$

(2) Find the 10^{th} term of the AP: 2, 7, 12, ... [NCERT] **[Imp.]**

(3) In the given AP, find the missing terms: ....., 13, ..., 3. [NCERT]

(4) Find the 6^{th} term from the end of the AP: 17, 14, 11, ..., –40. **[Imp.]**

(5) Which term of the AP: 21, 18, 15, ... is zero? [Delhi 2008 (C)] **[Imp.]**

(6) Write the next term of the AP: $\sqrt{8}, \sqrt{18}, \sqrt{32}$, ... [AI 2008]

(7) Find a, b, and c such that the numbers a, 7, b, 23, c are in AP. [NCERT Exemplar]

(8) Find the 9^{th} term from the end (towards the first term) of the AP: 5, 9, 13, ..., 185. [Delhi 2016]

(9) For what value of k will $k + 9$, $2k – 1$ and $2k + 7$ are the consecutive terms of an AP? [Delhi 2016]

(10) For what value of k will the consecutive terms $2k + 1$, $3k + 3$ and $5k – 1$ form an AP? [Foreign 2016]

(11) Find the eleventh term from the last term of the AP: 27, 23, 19, ..., –65. [CBSE Sample Paper 2018]

(12) If the first three terms of an AP are b, c and $2b$, then find the ratio of b and c. [CBSE Standard SP 2019-20]

(13) Find the value of x so that –6, x, 8 are in AP.

(14) Find the 11^{th} term of the AP: –27, –22, –17, –12, ...

(15) The n^{th} term of an AP is $(7 – 4n)$, then what is its common difference?

(16) Find the common difference of the AP whose first term is 12 and fifth term is 0.

II. Short Answer Type Questions - I [2 Marks]

4. Find how many integers between 200 and 500 are divisible by 8. [AI 2017]

5. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$ is the first negative term? [AI 2017]

6. Is –150 a term of the AP: 17, 12, 7, 2, ...? [Delhi 2011]

7. Find the number of two-digit numbers which are divisible by 6. [AI 2011]

8. Which term of the AP: 3, 14, 25, 36, ... will be 99 more than its 25^{th} term? [AI 2011]

9. Which term of the AP: 3, 15, 27, 39, ... will be 120 more than its 21^{st} term? [Delhi 2019]

10. How many natural numbers are there between 200 and 500, which are divisible by 7? [AI 2011]

11. How many two-digit numbers are divisible by 7? [Foreign 2011]

12. How many two digits numbers are divisible by 3? [Delhi 2019]

13. If $\frac{1}{x+2}, \frac{1}{x+3}$ and $\frac{1}{x+5}$ are in AP, find the value of x. [Foreign 2011]

14. How many three digit numbers are divisible by 11? [AI 2012]

15. In an AP, the first term is 12 and the common difference is 6. If the last term of the AP is 252, find its middle term. [Foreign 2017]

16. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5. [AI 2014]

17. The 4^{th} term of an AP is zero. Prove that the 25^{th} term of the AP is three times its 11^{th} term. [AI 2016]

18. Find the middle term of the AP: 6, 13, 20, ..., 216. [Delhi 2015]

19. The n^{th} term of an AP is $6n + 2$. Find its common difference. [Delhi 2008]

20. Find the 10^{th} term from end of the AP: 4, 9, 14, ..., 254. **[Imp.]**

21. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP. [NCERT Exemplar]

22. Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both. [CBSE Standard SP 2019-20]

III. Short Answer Type Questions - II [3 Marks]

23. Which term of the AP: 115, 110, 105, ... is its first negative term?

24. If the 9^{th} term of an AP is zero, prove that its 29^{th} term is double of its 19^{th} term. [NCERT Exemplar]

25. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle. [NCERT Exemplar]

26. For what value of n, the n^{th} term of two APs: 63, 65, 67, ... and 3, 10, 17, ... are equal. [NCERT]

27. The 8^{th} term of an AP is 37 and its 12^{th} term is 57. Find the AP. **[Imp.]**

28. The p^{th}, q^{th} and r^{th} terms of an AP are a, b and c respectively. Show that $a(q – r) + b(r – p) + c(p – q) = 0$. [Foreign 2016]

29. If the n^{th} terms of two APs: 23, 25, 27, ... and 5, 8, 11, 14, ... are equal, then find the value of n.

IV. Long Answer Type Questions [5 Marks]

30. If m times the m^{th} term of an Arithmetic Progression is equal to n times its n^{th} term and $m \neq n$, show that the $(m+n)^{th}$ term of the AP is zero. [AI2019]

31. The 19^{th} term of an AP is equal to three times its sixth term. If its 9^{th} term is 19, find the AP. [AI 2013]

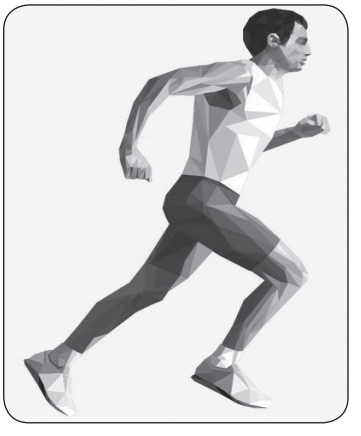
32. The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the AP. **[Imp.]**

33. The eighth term of an AP is half its second term and the eleventh term exceeds one-third of its fourth term by 1. Find the 15^{th} term. **[Imp.]**

34. If 4 times the 4^{th} term of an AP is equal to 18 times the 18^{th} term, then find the 22^{nd} term.

Case Study Based Questions

I. Your friend Veer wants to participate in a 200 m race. Presently, he can run 200 m in 51 seconds and during each day practice it takes him 2 seconds less. He wants to do in 31 seconds.



1. Which of the following terms are in AP for the given situation?
(*a*) 51, 53, 55, ... (*b*) 51, 49, 47, ... (*c*) –51, –53, –55, ... (*d*) 51, 55, 59, …

2. What is the minimum number of days he needs to practice till his goal is achieved?
(*a*) 10 (*b*) 12 (*c*) 11 (*d*) 9

3. Which of the following term is not in the AP of the above given situation?
(*a*) 41 (*b*) 30 (*c*) 37 (*d*) 39

4. If n^{th} term of an AP is given by $a_n = 2n + 3$ then common difference of an AP is
(*a*) 2 (*b*) 3 (*c*) 5 (*d*) 1

5. The value of x, for which $2x$, $x + 10$, $3x + 2$ are three consecutive terms of an AP is
(*a*) 6 (*b*) – 6 (*c*) 18 (*d*) –18

II. India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6^{th} year and 22600 in 9^{th} year.

1. The production during first year is
(*a*) 3000 TV sets (*b*) 5000 TV sets (*c*) 7000 TV sets (*d*) 10000 TV sets

2. The production during 8^{th} year is
(*a*) 10500 (*b*) 11900 (*c*) 12500 (*d*) 20400

3. The production during first 3 years is
(*a*) 12800 (*b*) 19300 (*c*) 21600 (*d*) 25200

4. In which year, the production is 29,200?
(*a*) 10^{th} year (*b*) 12^{th} year (*c*) 15^{th} year (*d*) 18^{th} year

5. The difference of the production during 7^{th} year and 4^{th} year is
(*a*) 6600 (*b*) 6800 (*c*) 5400 (*d*) 7200

Answers and Hints

1. (1) (*d*) 28 (1) (2) (*b*) $(2n-1)a$ (1)
(3) (*c*) $p+9q$ (1) (4) (*a*) $= \frac{7}{5}$ (1)

2. (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(2) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(3) (*d*) Assertion (A) is false but reason (R) is true. (1)

3. (1) 10, 20, 30, 40 (1) (2) 47 (1)
(3) 18, 8 (1) (4) –25 (1)
(5) 8 (1) (6) $\sqrt{50}$ or $5\sqrt{2}$ (1)
(7) $a = -1, b = 15, c = 31$ (1)
(8) Reversing the given AP, we get
185, 181, 174, ..., 9, 5

$$\text{Ninth term } a_9 = a + (9-1)d$$
$$= 185 + 8 \times (-4)$$
$$= 185 - 32$$
$$= 153 \quad (1)$$

(9) Given that $k+9$, $2k-1$ and $2k+7$ are in AP
Then,

$$(2k-1) - (k+9) = (2k+7) - (2k-1)$$
$$\Rightarrow \quad k - 10 = 8$$
$$\Rightarrow \quad k = 18 \quad (1)$$

(10) Given that $2k+1$, $3k+3$ and $5k-1$ are in AP.
So, $(3k+3) - (2k+1) = (5k-1) - (3k+3)$

$$\Rightarrow \quad k + 2 = 2k - 4$$
$$\Rightarrow \quad 2k - k = 2 + 4$$
$$\Rightarrow \quad k = 6 \quad (1)$$

(11) $a_{11} = -25$ (1)
(12) b, c and $2b$ are in AP

$$\Rightarrow \quad c = \frac{3b}{2}$$
$$\therefore \quad b : c = 2 : 3 \quad (1)$$

(13) 1 (1)
(14) 23 (1)
(15)

$$a_n = 7 - 4n$$
$$\Rightarrow \quad a_1 = 7 - 4 \times 1 = 3$$
$$\Rightarrow \quad a_2 = 7 - 4 \times 2 = 7 - 8 = -1$$
$$a_3 = 7 - 4 \times 3 = 7 - 12 = -5$$

Now, $a_2 - a_1 = -1 - 3 = -4$
$a_3 - a_2 = -5 - (-1)$
$= -5 + 1 = -4$
So, the common difference of AP is –4. (1)

(16)

$$A_5 = a_1 + 4d = 0$$
$$12 + 4d = 0$$
$$d = -3 \quad (1)$$

4. AP formed is 208, 216, 224, ..., 496

$$a_n = 496 \quad (1)$$
$$\Rightarrow \quad 208 + (n-1) \times 8 = 496$$
$$\Rightarrow \quad n = 37 \quad (1)$$

5. Here $d = \frac{-3}{4}$ (½)
Let the n^{th} term be first negative term.

$$\therefore \quad 20 + (n-1)\left(\frac{-3}{4}\right) < 0 \quad \Rightarrow \quad 3n > 83 \quad (1)$$
$$\Rightarrow \quad n > 27\frac{2}{3}$$

Hence, 28^{th} term is first negative term. (½)

6. Let

$$a_n = -150$$
$$a + (n-1)d = -150$$
$$\Rightarrow \quad 17 + (n-1)(-5) = -150 \quad (1)$$
$$\Rightarrow \quad (n-1)(-5) = -167$$
$$\Rightarrow \quad n = \frac{167+5}{5} = \frac{172}{5} = 34\frac{2}{5}$$

Here, n is not a natural number.
$\therefore$ –150 is not a term of the given AP. (1)

7. Two-digit numbers which are divisible by 6 are 12, 18, 24, ..., 96
$\because$ Last term,

$$a_n = 96$$
$$\Rightarrow \quad 12 + (n-1)6 = 96 \quad (1)$$
$$\Rightarrow \quad (n-1)6 = 96 - 12 = 84$$
$$\Rightarrow \quad n = 15$$

$\therefore$ There are 15 two-digit numbers divisible by 6. (1)

8. Let a_n be the term which is 99 more than 25^{th} term of given AP.
ATQ, $a_n = a_{25} + 99$

$$\Rightarrow \quad a + (n-1)d = a + 24d + 99 \quad (1)$$
$$\Rightarrow \quad 11(n-1) = 24 \times 11 + 99$$
$$\Rightarrow \quad n = 34$$

Hence, 34^{th} is the required term. (1)

9. AP: 3, 15, 27, 39, ...

$$a = 3, d = 15 - 3 = 12$$
$$a_{21} = a + 20d = 3 + 20 \times 12$$
$$= 3 + 240 = 243$$

120 more than $a_{21} = 243 + 120 = 363$ (1)
Let 363 be n^{th} term.
So, $363 = 3 + (n-1)\,12$

$$\Rightarrow \quad 360 = 12(n-1)$$
$$30 = n - 1 \quad \Rightarrow \quad n = 31$$

Thus, 31^{st} term of the given AP is 120 more than its 21^{st} term. (1)

10. Natural numbers between 200 and 500 which are divisible by 7 are as 203, 210, 217, ..., 497

Let above are n numbers and $a_n = 497$

$a + (n-1)d = 497$ (1)

$\Rightarrow \quad 203 + 7(n-1) = 497$

$\Rightarrow \quad n = 43$

$\therefore$ There are 43 natural numbers between 200 and 500 divisible by 7. (1)

11. Two-digit numbers which are divisible by 7 are 14, 21, 28, ..., 98.

Let $a_n = 98$

$\Rightarrow \quad a + (n-1)d = 98$ (1)

$\Rightarrow \quad 14 + 7(n-1) = 98$

$n = 13$

Hence, there are 13 two-digit numbers which are divisible by 7. (1)

12. 2-digit numbers divisible by 3 are 12, 15, 18, ..., 99 which is in AP.

So, $a_n = 99, d = 15 - 12 = 3$

Now, $a_n = a + (n-1)\,d$ (1)

$\Rightarrow \quad 99 = 12 + (n-1)\,3$

$\Rightarrow \quad 87 = 3\,(n-1)$

$\Rightarrow \quad 29 = n - 1$

$\Rightarrow \quad n = 30$

Thus, 30, 2-digit numbers are divisible by 3. (1)

13. Given term are in AP

So, $\dfrac{2}{x+3} = \dfrac{1}{x+2} + \dfrac{1}{x+5}$

$\Rightarrow \quad \dfrac{2}{x+3} = \dfrac{(x+5)+(x+2)}{(x+2)(x+5)}$ (1)

$\Rightarrow \quad 2x^2 + 14x + 20 = 2x^2 + 13x + 21$

$\therefore \quad x = 1$ (1)

14. Three-digit numbers which are divisible by 11 are 110, 121, 132, ..., 990

Let $a_n = 990$ (1)

$\Rightarrow \quad a + (n-1)d = 990$

$\Rightarrow \quad 110 + 11(n-1) = 990$

$\therefore \quad n = 81$

Hence, there are 81 three-digit numbers which are divisible by 11. (1)

15. Let $a_n = 252 =$ last term

$\Rightarrow \quad a + (n-1)d = 252$

$\Rightarrow \quad 12 + (n-1)6 = 252$

$\Rightarrow \quad n = 41$ (1)

$\therefore$ Since number of terms is odd, so only one middle term.

Now, middle term $= \left(\dfrac{41+1}{2}\right)$

$= 21^{st}$ term

$\therefore \; 21^{st}$ term, $a_{21} = a + 20d$

$= 12 + 20 \times 6$

$= 132$

= middle term value. (1)

16. Numbers between 101 and 999 which are divisible by both 2 and 5 (*i.e.*, by 10) are 110, 120, 130, ... 990.

Now, $a_n = a + (n-1)d$ (1)

$\Rightarrow \quad 990 = 110 + (n-1)10$

$\Rightarrow \quad n = 89$

$\therefore$ Natural numbers which are divisible by 2 and 5 both are 89. (1)

17. $a_4 = a + (4-1)d$

$0 = a + 3d$

$\Rightarrow \quad a = -3d \qquad [\because$ Given, $a_4 = 0]$ (1)

Now $a_{25} = a + (25-1)d = a + 24d$

$= -3d + 24d = 21d = 3 \times 7d$

Hence, $a_{25} = 3 \times a_{11}$

$[\because$ Since $a_{11} = a + (11-1)d = -3d + 10d = 7d]$ (1)

18. Given AP is 6, 13, 20, ..., 216

n^{th} term, $a_n = 216$

$\Rightarrow \quad a + (n-1)d = 216$

$\Rightarrow \quad 6 + 7(n-1) = 216$

$\Rightarrow \quad 7n = 217$

$\Rightarrow \quad n = 31$ (1)

Since, the number of terms in AP are 31, so, the middle most term is 16^{th} term.

$\left[\because \text{ middle term} = \dfrac{(31+1)}{2} = 16^{th} \text{ term}\right]$

$\therefore \quad 16^{th}$ term, $a_{16} = a + 15d = 6 + 15 \times 7 = 111.$ (1)

19. 6 (2) **20.** 209 (2) **21.** $k = 0$ (2)

22. 110, 120, 130, … , 990

$a_n = 990$

$\Rightarrow \quad 110 + (n-1) \times 10 = 990$

$\therefore \quad n = 89$ (2)

23. 25^{th} term (3) **25.** 40°, 60°, 80° (3)

26. 13 (3) **27.** 2, 7, 12, 17, 22, ... (3)

28. Let A and d be the first term and common difference of the given AP, then

$a_p = A + (p-1)d = a$ ...(*i*)

$a_q = A + (q-1)d = b$ ...(*ii*)

$a_r = A + (r-1)d = c$ ...(*iii*)

Now, subtracting (*i*) and (*ii*), we get

$(p-q)d = a - b$

$p - q = \dfrac{a}{d} - \dfrac{b}{d}$ (1)

Multiplying by 'c' on both sides,

$c(p-q) = \dfrac{ca}{d} - \dfrac{cb}{d}$ ...(*iv*)

Now, (*ii*) – (*iii*), we get

$(q-r)d = b - c$

$q - r = \dfrac{b}{d} - \dfrac{c}{d}$

Multiplying by 'a' on both sides,

$a(q-r) = \dfrac{ab}{d} - \dfrac{ac}{d}$ ...(*v*)(1)

Now, $(iii) - (i)$, we get

$$(r-p)d = c - a$$

$$(r-p) = \frac{c}{d} - \frac{a}{d}$$

Multiplying by 'b' on both sides,

$$(r-p)b = \frac{bc}{d} - \frac{ba}{d} \quad \ldots(vi)$$

Adding (iv), (v) and (vi), we get

$a(q-r) + b(r-p) + c(p-q)$

$$= \frac{ab}{d} - \frac{ac}{d} + \frac{bc}{d} - \frac{ba}{d} + \frac{ca}{d} - \frac{cb}{d} = 0 \quad (1)$$

29. $AP_1 = 23, 25, 27, \ldots$

Here, $a_1 = 23$

$d_1 = 25 - 23 = 27 - 25 = 2$

$\therefore$ n^{th} term $= a_1 + (n-1)d_1$

$= 23 + (n-1)2$

$AP_2 = 5, 8, 11, 14, \ldots$ (1)

Here, $a_2 = 5$

$d_2 = 8 - 5 = 11 - 8 = 3$

$\therefore$ n^{th} term $= a_2 + (n-1)d_2$

$= 5 + (n-1)3$ (1)

Now, $23 + (n-1)2 = 5 + (n-1)3$

$\Rightarrow$ $23 + 2n - 2 = 5 + 3n - 3$

$\Rightarrow$ $3n - 2n = 23 - 2 - 5 + 3$

$\Rightarrow$ $n = 26 - 7 = 19$ (1)

30. We know that $a_n = a + (n-1)d$

From the given conditions,

$m[a + (m-1)\,d] = n[a + (n-1)d]$ (1)

$\Rightarrow$ $m[a + (md - d)] = n[a + nd - d]$

$\Rightarrow$ $am + m^2d - md = an + n^2d - nd$ (1)

$\Rightarrow$ $am - an + m^2d - n^2d - md + nd = 0$

$\Rightarrow$ $a(m-n) + d(m^2 - n^2) - d(m-n) = 0$

$\Rightarrow$ $a(m-n) + (m+n)(m-n)d - (m-n)d = 0$ (1)

$\Rightarrow$ $(m-n)\,[a + (m+n)\,d - d] = 0$

$\Rightarrow$ $a + md + nd - d = 0$ (1)

$\Rightarrow$ $a + (m+n-1)d = 0$

Since, $m \neq n$, it is clear that $(m+n)^{th}$ term of the AP is zero. (1)

31. 3, 5, 7, 9, ... (5) **32.** $-13, -8, -3$ (5)

33. 3 (5)

34. Let $a_1, a_2, a_3, \ldots a_n, \ldots$ be the AP with its first term a and common difference d.

It is given that

$$4a_4 = 18a_{18} \quad (1)$$

$\Rightarrow$ $4(a + 3d) = 18(a + 17d)$ (1)

$\Rightarrow$ $4a + 12d = 18a + 306d$ (1)

$\Rightarrow$ $14a + 294d = 0 \Rightarrow 14(a + 21d) = 0$ (1)

$\Rightarrow$ $a + 21d = 0 \Rightarrow a + (22-1)d = 0$

$\Rightarrow$ $a_{22} = 0$

Thus, 22^{nd} term is 0. (1)

Case Study Based Questions

I. 1. (*b*) 51, 49, 47…. **2.** (*c*) 11

3. (*b*) 30 **4.** (*a*) 2

5. (*a*) 6

II.1. (*b*) 5000 TV sets **2.** (*d*) 20400

3. (*c*) 21600 **4.** (*b*) 12^{th} year

5. (*a*) 6600

Exercise 2.3

I. Very Short Answer Type Questions [1 Mark]

1. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

(1) The sum of first five terms of the AP: 3, 7, 11, 15, ... is:

(a) 44 (b) 55 (c) 22 (d) 11

(2) If the first term of an AP is 1 and the common difference is 2, then the sum of first 26 terms is

(a) 484 (b) 576 (c) 676 (d) 625

(3) If the sum to n terms of an AP is $3n^2 + 4n$, then the common difference of the AP is

(a) 7 (b) 5 (c) 8 (d) 6

(4) If a, b, c are in AP then $ab + bc =$

(a) b (b) b^2 (c) $2b^2$ (d) $\frac{1}{b}$

(5) The sum of all natural numbers which are less than 100 and divisible by 6 is

(a) 412 (b) 510 (c) 672 (d) 816

2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation ofassertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** Sum of the first 10 terms of the arithmetic progression –0.5, –1.0, –1.5,... is 27.5.

Reason (R): Sum of first n terms of an AP is given as $S_n = \frac{n}{2}[2a + (n-1)d]$ where a = first term, d = common difference.

(2) **Assertion (A):** The sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Then its n^{th} term, $a_n = 6n - 7$.
Reason (R): n^{th} term of an AP, whose sum of n terms is S_n, is given by $a_n = S_n - S_{n-1}$.

(3) **Assertion (A):** Sum of first hundred even natural numbers divisible by 5 is 500.

Reason (R): Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[a + l]$ where l = last term.

3. Answer the following:

(1) Find the sum of first 10 terms of the AP: 2, 7, 12, ... [NCERT] **[Imp.]**

(2) If the sum of first m terms of an AP is $2m^2 + 3m$, then what is its second term? [Foreign 2010]

(3) Find the sum of first 10 multiples of 6. [AI 2019]

(4) What is the sum of five positive integers divisible by 6? [CBSE Sample Paper 2012]

(5) If the sum of the first q terms of an AP is $2q + 3q^2$, what is its common difference? [AI 2010]

(6) If n^{th} term of an AP is $(2n + 1)$, what is the sum of its first three terms? [CBSE SP 2018-19]

(7) Find the sum of first 100 natural numbers. [CBSE Standard 2020]

II. Short Answer Type Questions - I [2 Marks]

4. Find the sum of first 8 multiples of 3. [CBSE 2018]

5. Find the number of terms of the AP: 54, 51, 48, ... so that their sum is 513. **[Imp.]**

6. In an AP, the first term is –4, the last term is 29 and the sum of all its terms is 150. Find its common difference. [Foreign 2016]

7. Find the sum of all three digit natural numbers, which are multiples of 11. [Delhi 2012]

8. The first and the last terms of an AP are 8 and 65 respectively. If sum of all its terms is 730, find its common difference. [Delhi 2014]

9. The sum of the first n terms of an AP is $4n^2 + 2n$. Find the n^{th} term of this AP. [Foreign 2013]

10. How many terms of the AP: 18, 16, 14, ... be taken so that their sum is zero? [Delhi 2016]

11. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first n terms. [AI 2015]

12. The sum of first n terms of an AP is given by $S_n = 2n^2 + 3n$. Find the sixteenth term of the AP.

III. Short Answer Type Questions - II [3 Marks]

13. How many multiples of 4 lie between 10 and 250? Also find their sum. [AI 2011]

14. Find the sum of first n terms of an AP whose n^{th} term is $5n - 1$. Hence find the sum of first 20 terms. [AI 2011]

15. The sum of first six terms of an AP is 42. The ratio of its 10^{th} term to its 30^{th} term is 1 : 3. Calculate the first and the thirteenth terms of the AP. [AI 2009]

16. Find the sum of all multiples of 7 lying between 500 and 900. [AI 2010]

17. If M, N and T are in AP, prove that $(M + 2N - T)(2N + T - M)(T + M - N) = 4MNT$.

18. In an AP, if the 6th and 13th terms are 35 and 70 respectively, find the sum of its first 20 terms. [Foreign 2011]

19. The sum of the 2^{nd} and the 7^{th} terms of an AP is 30. If its 15^{th} term is 1 less than twice its 8^{th} term, find the AP. [AI 2014]

20. If the ratio of the sum of first n terms of two AP's is $(7n + 1) : (4n + 27)$, find the ratio of their m^{th} terms. [AI 2016]

21. The digits of a positive number of three digits are in AP and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number. [AI 2016]

22. The sums of first n terms of three A.Ps' are S_1, S_2 and S_3. The first term of each AP is 5 and their common differences are 2, 4 and 6 respectively. Prove that $S_1 + S_3 = 2S_2$. **[Imp]**

23. Find the sum of n terms of the series $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots$ [Delhi 2017]

24. Solve the equation: $1 + 4 + 7 + 10 + \ldots + x = 287$.

IV. Long Answer Type Questions [5 Marks]

25. The sum of the first three numbers in an arithmetic progression is 18. If the product of the first and the third terms is 5 times the common difference, find the three numbers. [Al 2019]

26. If m times the m^{th} term of an arithmetic progression is equal to n times its n^{th} term and $m \neq n$, show that the $(m + n)^{th}$ term of the AP is zero. [Al 2019]

27. The first and the last term of an AP are 8 and 350 respectively. If its common difference is 9, how many terms are there and what is their sum? [AI 2011]

28. Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to $\frac{(a + c)(b + c - 2a)}{2(b - a)}$. [NCERT Exemplar][CBSE Standard 2020]

29. If the p^{th} term of an AP is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of the pq terms is $\frac{1}{2}(pq + 1)$. [CBSE 2012]

30. The ratio of the 11^{th} term to the 18^{th} term of an AP is 2 : 3. Find the ratio of the 5^{th} term to the 21^{st} term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms. [NCERT Exemplar]

31. The sum of the first five terms of an AP is 55 and sum of the first ten terms of this AP is 235, find the sum of its first 20 terms. **[Imp.]**

32. The sums of n terms of two APs are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 25^{th} terms. **[Imp.]**

33. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also, find the sum of all numbers on both sides of the middle terms separately. [Foreign 2015]

34. If the ratio of the sum of the first n terms of two APs is $(7n + 1) : (4n + 27)$, then find the ratio of their 9^{th} terms. [AI 2017]

35. If the sum of first 14 terms of an AP is 1050 and its first term is 10, find the 20^{th} term.

36. The first term of an AP is 5, the last term is 45 and sum is 400. Find the number of terms and the common difference.

37. How many terms of the AP: 24, 21, 18, ... must be taken so that their sum is 78?

Case Study Based Questions

I. Pollution—A Major Problem: One of the major serious problems that the world is facing today is the environmental pollution. Common types of pollution include light, noise, water and air pollution.

In a school, students thoughts of planting trees in and around the school to reduce noise pollution and air pollution.

Condition I: It was decided that the number of trees that each section of each class will plant be the same as the class in which they are studying, *e.g.* a section of class I will plant 1 tree a section of class II will plant 2 trees and so on a section of class XII will plant 12 trees.

Condition II: It was decided that the number of trees that each section of each class will plant be the double of the class in which they are studying, *e.g.* a section of class I will plant 2 trees, a section of class II will plant 4 trees and so on a section of class XII will plant 24 trees.

Refer to Condition I

1. The AP formed by sequence *i.e.* number of plants by students is
   (*a*) 0, 1, 2, 3, ..., 12 (*b*) 1, 2, 3, 4, ..., 12 (*c*) 0, 1, 2, 3, ..., 15 (*d*) 1, 2, 3, 4, ..., 15
2. If there are two sections of each class, how many trees will be planted by the students?
   (*a*) 126 (*b*) 152 (*c*) 156 (*d*) 184
3. If there are three sections of each class, how many trees will be planted by the students?
   (*a*) 234 (*b*) 260 (*c*) 310 (*d*) 326

Refer to Condition II

4. If there are two sections of each class, how many trees will be planted by the students?
   (*a*) 422 (*b*) 312 (*c*) 360 (*d*) 540
5. If there are three sections of each class, how many trees will be planted by the students?
   (*a*) 468 (*b*) 590 (*c*) 710 (*d*) 620

II. Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of ₹ 1,18,000 by paying every month starting with the first instalment of ₹ 1000. If he increases the instalment by ₹ 100 every month , answer the following:

1. The amount paid by him in 30^{th} installment is
   (*a*) ₹ 3900 (*b*) ₹ 3500 (*c*) ₹ 3700 (*d*) ₹ 3600
2. The total amount paid by him upto 30 installments is
   (*a*) ₹ 37000 (*b*) ₹ 73500 (*c*) ₹ 75300 (*d*) ₹ 75000
3. What amount does he still have to pay after 30^{th} installment?
   (*a*) ₹ 45500 (*b*) ₹ 49000 (*c*) ₹ 44500 (*d*) ₹ 54000
4. If total installments are 40, then amount paid in the last installment is
   (*a*) ₹ 4900 (*b*) ₹ 3900 (*c*) ₹ 5900 (*d*) ₹ 9400
5. The ratio of the 1^{st} installment to the last installment is
   (*a*) 1 : 49 (*b*) 10 : 49 (*c*) 10 : 39 (*d*) 39 : 10

Answers and Hints

1. (1) (*b*) 55 (1) (2) (*c*) 676 (1)
(3) (*d*) 6 (1) (4) (*c*) $2b^2$ (1)
(5) (*d*) 816 (1)

2. (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(2) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(3) (*d*) Assertion (A) is false but reason (R) is true. (1)

3. (1) 245 (1) (2) 9 (1)
(3) First 10 multiples of 6 are 6, 12, 18, ..., 60.
This is an AP in which $a = 6$, $n = 10$ and $d = 6$.
$\therefore$ Sum of first 10 multiples of 6 = S_{10} (½)

$$\Rightarrow \quad S_{10} = \frac{n}{2}[2a+(n-1)d]$$
$$= \frac{10}{2}[2\times 6+(10-1)6]$$
$$= 5\,(12+54)$$
$$= 5\times 66 = 330 \qquad (½)$$

(4) 90 (1)
(5) Given that,
$S_q = 2q + 3q^2$
$S_1 = 2 + 3 = 5 = T_1$ = First term [put $q = 1$]
$S_2 = 4 + 3(4) = 16$ [put $q = 2$]
$S_3 = 6 + 3(9) = 33$ [put $q = 3$](½)
$\therefore$ 2nd term,
$T_2 = S_2 - S_1 = 16 - 5 = 11$
$\therefore$ 3rd term,
$T_3 = S_3 - S_2 = 33 - 16 = 17$
Common difference
$= T_3 - T_2 = 17 - 11 = 6$ (½)

(6) $a_1 = 3$, $a_3 = 7$, $S_3 = \frac{3}{2}(3+7) = 15$ [½+½]

(7) Natural numbers are 1, 2, 3, 4, ...
The sum of first 100 natural numbers is given by

$$S_n = \frac{n(n+1)}{2} = \frac{100\times(100+1)}{2} \qquad (½)$$
$$= \frac{100\times 101}{2}$$
$$= 50\times 101 = 5050 \qquad (½)$$

4. $S_8 = 3 + 6 + 9 + 12 + ...+ 24$
$= 3(1 + 2 + 3 + ...+ 8)$ (1)
$= 3 \times \frac{8\times 9}{2} = 108$ (1)

5. 18 or 19 (2)

6. $150 = \frac{n}{2}(-4+29) \qquad \left|\because S_n = \frac{n}{2}(a+l)\right|$

$\Rightarrow \quad 300 = 25n \quad \Rightarrow \quad n = 12$ (1)
$\therefore$ Then, $l = a_{12} = 29 = -4 + 11d$
$\Rightarrow \quad 11d = 33 \quad \Rightarrow \quad d = 3$ (1)

7. 3-digit natural numbers which are multiples of 11 are 110, 121, 132, ..., 990
n^{th} term, $990 = 110 + (n-1)11$
$\Rightarrow \quad n = 81$ (1)
$\therefore$ Sum of 'n' terms,

$$S_n = \frac{n}{2}[a+l]$$
$$= \frac{81}{2}[110+990] = 44550$$

$\therefore$ Sum of all three-digit natural numbers, which are multiples of 11 is 44550. (1)

8. $S_n = \frac{n}{2}(a+a_n)$

$$730 = \frac{n}{2}(8+65) \quad \Rightarrow \quad \frac{73n}{2} = 730$$

$\Rightarrow \quad n = 20$ (1)
$\therefore$ Given $a_{20} = 65$, where $a_n = a + (n-1)d$
$\Rightarrow \quad a + 19d = 65 \quad \Rightarrow \quad 8 + 19d = 65$
$\Rightarrow \quad 19d = 57$
Hence, common differences = $d = 3$. (1)

9. Given, $S_n = 4n^2 + 2n$
So, $S_{n-1} = 4(n-1)^2 + 2(n-1)$
$= 4(n^2 - 2n + 1) + 2n - 2$
$= 4n^2 - 8n + 4 + 2n - 2$
$= 4n^2 - 6n + 2$ (1)
$a_n = S_n - S_{n-1} = n^{th}$ term
$= (4n^2 + 2n) - (4n^2 - 6n + 2)$
$= 4n^2 + 2n - 4n^2 + 6n - 2$
$= 8n - 2$ (1)

10. Let the number of terms taken for sum to be zero be n.
Then, sum of n terms
$(S_n) = 0$ (Given)

$$\Rightarrow \quad S_n = \frac{n}{2}[2a+(n-1)d] \qquad (1)$$

$$\Rightarrow \quad 0 = \frac{n}{2}[2\times 18+(n-1)(-2)]$$

$\Rightarrow \quad n = 19$
Hence, sum of 19 terms is 0. (1)

11. $S_5 + S_7 = 167$

$$\Rightarrow \quad \frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d) = 167$$
$$\left\{\because S_n = \frac{n}{2}[2a+(n-1)d]\right\}$$

$\Rightarrow \quad 5a + 10d + 7a + 21d = 167$
$\Rightarrow \quad 12a + 31d = 167$ ...(*i*)
Also, $S_{10} = 235$

$$\Rightarrow \quad \frac{10}{2}(2a+9d) = 235$$

$\Rightarrow \quad 2a + 9d = 47$ ...(*ii*)(1)
Multiplying eq. (*ii*) by 6, we get
$6(2a + 9d) = 6 \times 47$

$\Rightarrow \quad 12a + 54d = 282$ ...(*iii*)

$\therefore$ Subtracting eq. (*i*) from (*iii*), we get

$12a + 54d = 282$

$12a + 31d = 167$

$- \quad - \quad -$

$23d = 115$

$\therefore \quad d = 5$

Putting 'd' in (*ii*) equation, $a = 1$

$\therefore$ Required AP is 1, 6, 11, ... (1)

12. $S_n = 2n^2 + 3n$

$S_1 = 5 = a_1$

$S_2 = a_1 + a_2 = 14 \quad \Rightarrow \quad a_2 = 9$ (1)

$d = a_2 - a_1 = 4$

$a_{16} = a_1 + 15d = 5 + 15(4) = 65$ (1)

13. 60, 7800 (3)

14. Given: $a_n = 5n - 1$

$a_1 = 4$

$\therefore \quad a_2 = 5(2) - 1 = 9$

$d = a_2 - a_1 = 9 - 4 = 5$ (1)

Now, sum of first 'n' terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 4 + 5(n-1)]$$

$$= \frac{n}{2}(8 + 5n - 5) = \frac{n(5n+3)}{2} \quad (1)$$

Now, sum of first 20 terms,

$$S_{20} = \frac{20(5 \times 20 + 3)}{2}$$

$= 10 \times 103 = 1030$ (1)

15. 2 and 26 (3) **16.** 39,900 (3)

18. Given that, $a_6 = 35 \Rightarrow a + 5d = 35$ ...(*i*)

and also $a_{13} = 70 \Rightarrow a + 12d = 70$ ...(*ii*)(1)

On solving the above equations, we get

$a = 10; \quad d = 5$ (1)

Now, sum of first 20 terms,

$$S_{20} = \frac{20}{2}[2 \times 10 + 19 \times 5]$$

$$\left| \because S_n = \frac{n}{2}[2a + (n-1)d] \right|$$

$= 1150$ (1)

19. Given, $a_2 + a_7 = 30$

$\Rightarrow \quad a + d + a + 6d = 30$

$\Rightarrow \quad 2a + 7d = 30$ ...(*i*)(1)

$[\because \ a_n = a + (n-1)d]$

Also, given $a_{15} = 2a_8 - 1$

$\Rightarrow \quad a + 14d = 2(a + 7d) - 1 \quad \Rightarrow \ a = 1$ (1)

Putting the value of a in (*i*), we get

$2 + 7d = 30 \quad \Rightarrow \quad d = 4$

$\therefore \quad a = 1, d = 4$

Hence, AP is 1, 5, 9, 13, 17, ... (1)

20. Let S_n and S'_n be the sum of n terms of two APs. Let a, a' and d, d' be first terms and common differences of two APs. Then

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2a' + (n-1)d']}$$

$$= \frac{a + \left(\frac{n-1}{2}\right)d}{a' + \left(\frac{n-1}{2}\right)d'} = \frac{7n+1}{4n+27} \quad ...(i)(1)$$

Since $\frac{t_m}{t'_m} = \frac{a + (m-1)d}{a' + (m-1)d'}$

[$\because$ Let t_m, t'_m be m^{th} terms of two APs]

So, replacing $\frac{n-1}{2}$ by $m - 1$, *i.e.*, $n = 2m - 1$ in (*i*)

$$\frac{t_m}{t'_m} = \frac{a + (m-1)d}{a' + (m-1)d'} \quad (1)$$

$$= \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Thus, the ratio of their m^{th} terms is

$14m - 6 : 8m + 23.$ (1)

21. Let the required numbers in AP are $a - d$, a, $a + d$ respectively.

Now, $a - d + a + a + d = 15$ [$\because$ Sum of digits = 15]

$\Rightarrow \quad 3a = 15 \ \Rightarrow \ a = 5$ (1)

According to question, number is

$100(a - d) + 10a + a + d$, *i.e.* $111a - 99d$

Number on reversing the digits is

$100(a + d) + 10a + a - d$, *i.e.* $111a + 99d$

Now, as per given condition in question,

$(111a - 99d) - (111a + 99d) = 594$ (1)

$\Rightarrow \quad d = -3$

$\therefore$ Digits of number are $[5 - (-3), 5, (5 + (-3))]$

$= 8, 5, 2.$

$\therefore$ Required number is $111 \times (5) - 99(-3)$

$= 555 + 297 = 852.$ (1)

23. $S_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ...$ upto n terms

$= (4 + 4 + ... + 4) - \frac{1}{n}(1 + 2 + 3 + ... + n)$ (1)

$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$ (1)

$= \frac{7n-1}{2}$ (1)

24. Given equation: $1 + 4 + 7 + 10 + ... + x = 287$

Here, $a = 1$, $d = 4 - 1 = 7 - 4 = 3$

$S_n = 287$

Bur, $S_n = \frac{n}{2}[2a + (n-1)d]$

$287 = \frac{n}{2}[2 \times 1 + (n-1)3]$

$\Rightarrow \quad 287 \times 2 = n(2 + 3n - 3)$

$\Rightarrow \quad 574 = n(3n - 1) = 3n^2 - n$

$\Rightarrow \quad 3n^2 - n - 574 = 0$ (1)

We know that,

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 3 \times (-574)}}{2 \times 3}$$

$$= \frac{1 \pm \sqrt{1 + 6888}}{6} = \frac{1 \pm \sqrt{6889}}{6} = \frac{1 \pm 83}{6} \quad (1)$$

Either $n = \frac{1 \pm 83}{6}$ or $n = \frac{1 - 83}{6}$

$\Rightarrow \quad n = \frac{84}{6}$ or $n = \frac{-82}{6}$

$\Rightarrow \quad n = 14$ or $n = \frac{-41}{3}$

$\therefore \quad n = 14$

Now, $S_n = \frac{n}{2}(a + l) \quad \Rightarrow \quad 287 = \frac{14}{2}(1 + x)$

$\Rightarrow \quad 287 = 7(1 + x) \quad \Rightarrow \quad 287 = 7 + 7x$

$\Rightarrow \quad 7x = 280 \quad \Rightarrow \quad x = \frac{280}{7} = 40$ (1)

25. Let the three numbers in AP are $a - d$, a, $a + d$

Then $a - b + a + a + d = 18$

$\Rightarrow \quad 3a = 18 \quad \Rightarrow \quad a = 6$ (1)

Given: $(a - d)(a + d) = 5d$

$\Rightarrow \quad a^2 - d^2 = 5d \quad \Rightarrow \quad a^2 = 5d + d^2$

$\Rightarrow \quad 36 = 5d + d^2 \quad [\because \ a = 6]$ (1)

$\Rightarrow \quad d^2 + 5d - 36 = 0$

$\Rightarrow \quad d^2 + 9d - 4d - 36 = 0$ (1)

$\Rightarrow \quad d(d + 9) - 4(d + 9) = 0$

$\Rightarrow \quad (d - 4)(d + 9) = 0$

$\Rightarrow \quad d - 4 = 0$ or $d + 9 = 0$

$\Rightarrow \quad d = 4$ or $d = -9$ [Reject]

$\Rightarrow \quad d = 4$ (1)

Thus, three numbers are $a - d$, a, $a + d$

$= 6 - 4, 6, 6 + 4$

$= 2, 6, 10$ (1)

26. We know that $a_n = a + (n - 1)d$

From the given conditions,

$m[a + (m - 1)d] = n[a + (n - 1)d]$

$\Rightarrow \quad m[a + (md - d)] = n[a + nd - d]$ (1)

$\Rightarrow \quad am + m^2d - md = an + n^2d - nd$

$\Rightarrow \quad am - an + m^2d - n^2d - md + nd = 0$ (1)

$\Rightarrow \quad a(m - n) + d(m^2 - n^2) - d(m - n) = 0$

$\Rightarrow \quad a(m - n) + (m + n)(m - n)d - (m - n)d = 0$ (1)

$\Rightarrow \quad (m - n)[a + (m + n)d - d = 0$

$\Rightarrow \quad a + md + nd - d = 0$ (1)

$\Rightarrow \quad a + (m + n - 1)d = 0$

Since, $m \neq n$, it is clear that $(m + n)^{th}$ term of the AP is zero. (1)

27. 39; 6981 (5) **30.** 1 : 3, 5 : 49 (5)

31. 970 (5) **32.** 249 : 447 (5)

33. List of 3-digit number leaving remainder 3 when divided by 4, are 103, 107, 111, ..., 999.

Now, $a_n = 999 \quad \Rightarrow \quad a + (n - 1)d = 999$

$103 + (n - 1)4 = 999 \quad \Rightarrow \quad n = 225$ (1)

Since, number of terms is odd, so there will be only one middle term

Middle term $= \frac{225 + 1}{2} = 113$ (1)

$\therefore \quad a_{113} = a + 112d$

$= 103 + 112 \times 4 = 551$ (1)

There are 112 numbers before 113^{th} term.

$\therefore$ Sum of all terms before middle term

$$S_{112} = \frac{112}{2}[2 \times 103 + 111 \times 4]$$

$= 36400$ (1)

$\therefore$ Sum of all terms $= S_{225} = 123975$

$\therefore$ Sum of terms after middle term

$= S_{225} - (S_{112} + 551)$

$= 87024$ (1)

34. Let the first terms be a and a' and d and d' be their respective common differences.

$$\frac{S_n}{S_n'} = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2a' + (n-1)d']}$$

$$= \frac{7n + 1}{4n + 27} \quad (1)$$

$$\Rightarrow \quad \frac{a + \left(\frac{n-1}{2}\right)d}{a' + \left(\frac{n-1}{2}\right)d'} = \frac{7n + 1}{4n + 27} \quad (1)$$

To get ratio of 9^{th} terms, replacing $\frac{n - 1}{2} = 8$ (1)

$\Rightarrow \quad n = 17$ (1)

Hence, $\frac{t_9}{t_9'} = \frac{a + 8d}{a' + 8d'} = \frac{120}{95}$ or $\frac{24}{19}$ (1)

35. Let common difference be d.

$\Rightarrow \quad \frac{14}{2}[2(10) + (n - 1)d] = 1050$ (2)

$\Rightarrow \quad d = 10$ (1)

$a_{20} = a + 19d$

$= 10 + 19(10) = 200$ (2)

36. $a = 5$

$a_n = 45$

$S_n = 400$

$\Rightarrow \frac{n}{2}(5 + 45) = 400$ (2)

$50n = 800$ (1)

$n = 16$

also $a_n = 45$

$5 + 15d = 45$

$15d = 40$

$d = \frac{8}{3}$ (2)

37. AP is 24, 21, 18, ...

Here, $a = 24$ and $d = 21 - 24 = 18 - 21 = -3$ (1)

Let the sum of n terms of the AP be 78.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow 78 = \frac{n}{2}[2 \times 24 + (n-1)(-3)]$

$\Rightarrow 78 \times 2 = n[48 - 3n + 1]$

$\Rightarrow 156 = n(49 - 3n)$

$\Rightarrow 156 = 49n - 3n^2$

$\Rightarrow 3n^2 - 49n + 156 = 0$ (1)

We know that

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{49 \pm \sqrt{(49)^2 - 4 \times 3 \times 156}}{2 \times 3}$$

$$= \frac{49 \pm \sqrt{2401 - 1872}}{6}$$

$$= \frac{49 \pm \sqrt{529}}{6}$$

$$= \frac{49 \pm 23}{6}$$ (1)

Either $n = \frac{49 + 23}{6}$ or $n = \frac{49 - 23}{6}$

$n = \frac{72}{6}$ or $n = \frac{26}{6} = \frac{13}{3}$

$n = 12$ or $n = 4\frac{1}{3}$

Thus, $n = 12$. (2)

Case Study Based Questions

I. 1. (*b*) 1, 2, 3, 4, ..., 12

2. (*c*) 156 **3.** (*a*) 234

4. (*b*) 312 **5.** (*a*) 468

II. 1. (*a*) ₹ 3900 **2.** (*b*) ₹ 73500

3. (*c*) ₹ 44500 **4.** (*a*) ₹ 4900

5. (*b*) 10 : 49