CHAPTER 13

Probability

INTRODUCTION

Random Experiment :

It is an experiment which if conducted repeatedly under homogeneous condition does not give the same result.

The total number of possible outcomes of an experiment in any trial is known as the **exhaustive number** of events.

For example

(i) In throwing a die, the exhaustive number of cases is 6 since any one of the six faces marked with 1, 2, 3, 4, 5, 6 may come uppermost.

(ii) In tossing a coin, the exhaustive number of cases is 2, since either head or tail may turn over.

(iii) If a pair of dice is thrown, then the exhaustive number of cases is $6 \times 6 = 36$

(iv) In drawing four cards from a well-shuffled pack of cards, the exhaustive number of cases is ${}^{52}C_4$.

Events are said to be **mutually exclusive** if no two or more of them can occur simultaneously in the same trial.

For example,

(i) In tossing of a coin the events head (H) and tail (T) are mutually exclusive.

(ii) In throwing of a die all the six faces are mutually exclusive.

(iii) In throwing of two dice, the events of the face marked 5 appearing on one die and face 5 (or other) appearing on the other are not mutually exclusive.

Outcomes of a trial are **equally likely** if there is no reason for an event to occur in preference to any other event or if the chances of their happening are equal.

For example,

(i) In throwing of an unbiased die, all the six faces are equally likely to occur.

(ii) In drawing a card from a well-shuffled pack of 52 cards, there are 52 equally likely possible outcomes.

The **favourable cases** to an event are the outcomes, which entail the happening of an event.

For example,

(i) In the tossing of a die, the number of cases which are favourable to the " appearance of a multiple of 3" is 2, viz, 3 and 6.

(ii) In drawing two cards from a pack, the number of cases favourable to "drawing 2 aces" is ${}^4C_2$.

(iii) In throwing of two dice, the number of cases favourable to "getting 8 as the sum" is 5, : (2, 6), (6, 2), (4, 4), (3, 5) (5, 3).

Events are said to be **independent if the happening** (or non-happening) of one event is not affected by the happening or non-happening of others.

CLASSICAL DEFINITION OF PROBABILITY

If there are n-mutually exclusive, exhaustive and equally likely outcomes to a random experiment and 'm' of them are favourable to an event A, then the probability of happening of A is denoted by P (A) and is defined by $P(A) = \frac{m}{n}$.

$$P(A) = \frac{\text{No. of elementary events favourable to A}}{\text{Total no. of equally likely elementary events}}$$

Obviously, $0 \le m \le n$, therefore $0 \le \frac{m}{n} \le 1$ so that $0 \le P(A) \le 1$.

P(A) can never be negative.

Since, the number of cases in which the event A will not happen is 'n – m', then the probability P ($\overline{A}$) of not happening of A is given by

$$P(\overline{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$$\Rightarrow \boxed{P(A) + P(\overline{A}) = 1}$$

The **ODDS IN FAVOUR** of occurrence of A are given by $m : (n-m)$ or $P(A) : P(\overline{A})$

The **ODDS AGAINST** the occurrence of A are given by $(n-m) : m$ or $P(\overline{A}) : P(A)$.

EXAMPLE **1. Two dice are thrown simultaneously. The probability of obtaining a total score of seven is**

(a) $\frac{1}{6}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{2}{7}$ **(d)** $\frac{5}{6}$

(e) None of these

Sol.

(a) When two are thrown then there are 6×6 exhaustive cases $\therefore n = 36$. Let A denote the event "total score of 7" when 2 dice are thrown then A = [(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)].

Thus there are 6 favourable cases.

$\therefore m = 6$ By definition $P(A) = \frac{m}{n}$

$$\therefore P(A) = \frac{6}{36} = \frac{1}{6}.$$

EXAMPLE 2. A bag contains 5 green and 7 red balls. Two balls are drawn. The probability that one is green and the other is red is

(a) $\frac{5}{132}$ **(b)** $\frac{7}{132}$ **(c)** $\frac{35}{66}$ **(d)** $\frac{31}{66}$

(e) None of these

Sol.

(c) There are 5 + 7 = 12 balls in the bag and out of these two balls can be drawn in $^{12}C_2$ ways. There are 5 green balls, therefore, one green ball can be drawn in $^5C_1$ ways; similarly, one red ball can be drawn in $^7C_1$ ways so that the number of ways in which we can draw one green ball and the other red is $^5C_1 \times {}^7C_1$.

Hence, P (one green and the other red)

$$= \frac{^5C_1 \times {}^7C_1}{^{12}C_2} = \frac{5}{1} \times \frac{7}{1} \times \frac{1.2}{12.11} = \frac{35}{66}.$$

EXAMPLE 3. A bag contains 5 white and 7 black balls and a man draws 4 balls at random. The odds against these being all black is :

(a) 7 : 92 (b) 92 : 7 (c) 92 : 99 (d) 99 : 92

(e) None of these

Sol.

(b) There are 7 + 5 = 12 balls in the bag and the number of ways in which 4 balls can be drawn is $^{12}C_4$ and the number of ways of drawing 4 black balls (out of seven) is $^7C_4$.

Hence, P (4 black balls)

$$= \frac{^7C_4}{^{12}C_4} = \frac{7.6.5.4}{1.2.3.4} \times \frac{1.2.3.4}{12.11.10.9} = \frac{7}{99}$$

Thus the odds against the event 'all black balls' are

$$(1 - \frac{7}{99}) : \frac{7}{99} : \text{i.e., } \frac{92}{99} : \frac{7}{99} \text{ or } 92 : 7 .$$

EXAMPLE 4. The letters of the word SOCIETY are placed at random in a row. The probability that the three vowels come together is

(a) $\frac{1}{6}$ **(b)** $\frac{1}{7}$ **(c)** $\frac{2}{7}$ **(d)** $\frac{5}{6}$

(e) None of these

Sol.

(b) The word 'SOCIETY' contains seven distinct letters and they can be arranged at random in a row in $^7P_7$ ways, i.e. in 7! = 5040 ways.

Let us now consider those arrangements in which all the three vowels come together. So in this case we have to arrange four letters. S,C,T,Y and a pack of three vowels in a row which can be done in $^5P_5$ i.e. 5! = 120 ways.

Also, the three vowels in their pack can be arranged in $^3P_3$ i.e. 3! = 6 ways.

Hence, the number of arrangements in which the three vowels come together is 120 × 6 = 720

∴ The probability that the vowels come together

$$= \frac{720}{5040} = \frac{1}{7}$$

EXAMPLE 5. There are three events E_1, E_2 and E_3. one of which must, and only one can happen. The odds are 7 to 4 against E_1 and 5 to 3 against E_2. The odds against E_3 is

(a) 4 : 11 (b) 3 : 8 (c) 23 : 88 (d) 65 : 23

(e) None of these

Sol.

(d) Since, one and only one of the three events E_1, E_2 and E_3 can happen, therefore $P(E_1) + P(E_2) + P(E_3) = 1$ .....(1)

∵ Odds against E_1 are 7 : 4

$$\Rightarrow P(E_1) = \frac{4}{4+7} = \frac{4}{11} \quad .....(2)$$

∵ Odds against E_2 are 5 : 3

$$\Rightarrow P(E_2) = \frac{3}{3+5} = \frac{3}{8} \quad ......(3)$$

From (1), (2) and (3), we have, $\frac{4}{11} + \frac{3}{8} + P(E_3) = 1.$

i.e. $P(E_3) = 1 - \frac{4}{11} - \frac{3}{8} = \frac{88-32-33}{88} = \frac{23}{88} = \frac{23}{23+65}$

Hence odds against E_3 is 65 : 23.

ALGEBRA OF EVENTS

Let A and B be two events related to a random experiment. We define

(i) The event "A or B" denoted by "$A \cup B$", which occurs when A or B or both occur. Thus,

$P(A \cup B) =$ Probability that at least one of the events occur

(ii) The event "A and B", denoted by "$A \cap B$", which occurs when A and B both occur. Thus,

$P(A \cap B) =$ Probability of simultaneous occurrence of A and B.

(iii) The event " Not - A" denoted by $\overline{A}$, which occurs when and only when A does not occur. Thus

$P(\overline{A})$ = Probability of non-occurrence of the event A.

(iv) $\overline{A} \cap \overline{B}$ denotes the " non-occurrence of both A and B".

(v) "$A \subset B$" denotes the " occurrence of A implies the occurrence of B".

For example :

Consider a single throw of die and following two events

A = the number is even = {2, 4, 6}

B = the number is a multiple of 3 = {3, 6}

Then $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$, $P(A \cap B) = \frac{1}{6}$

$P(\overline{A}) = \frac{1}{2}$, $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - \frac{2}{3} = \frac{1}{3}$.

ADDITION THEOREM ON PROBABILITY

1. ADDITION THEOREM : If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. ADDITION THEOREM FOR THREE EVENTS : If A, B, C are three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

3. If A and B are **two mutually exclusive events** and the probability of their occurence are P(A) and P(B) respectively, then probability of either A or B occuring is given by
$P(A \text{ or } B) = P(A) + P(B)$
$\Rightarrow P(A + B) = P(A) + P(B)$

EXAMPLE 6. A and B are two events odds against A are 2 to 1. odds in favour of $A \cup B$ are 3 to 1. If $x \leq P(B) \leq y$. then the ordered pair (x, y) is :

(a) $\left(\frac{5}{12}, \frac{3}{4}\right)$ **(b)** $\left(\frac{2}{3}, \frac{3}{4}\right)$ **(c)** $\left(\frac{1}{3}, \frac{3}{4}\right)$ **(d)** $\left(\frac{1}{2}, \frac{3}{7}\right)$

(e) None of these

Sol.

(a) $P(A) = \frac{1}{3}; P(A \cup B) = \frac{3}{4}$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{3}{4}$$

$$\Rightarrow \frac{1}{3} + P(B) - P(A \cap B) = \frac{3}{4}$$

$$\Rightarrow P(B) = \frac{5}{12} + P(A \cap B) \geq \frac{5}{12}$$

$$\text{Also, } P(B) = \frac{5}{12} + P(A \cap B) \leq \frac{5}{12} + \frac{1}{3} = \frac{3}{4}$$

$$\left[\because P(A \cap B) \leq P(A) = \frac{1}{3}\right]$$

Hence, (x, y) is $\left(\frac{5}{12}, \frac{3}{4}\right)$.

EXAMPLE 7. Two cards are drawn from a pack of 52 cards. The probability that either both are red or both are kings is

(a) $\frac{1}{2}$ **(b)** $\frac{1}{321}$ **(c)** $\frac{325}{1326}$ **(d)** $\frac{1}{327}$

(e) None of these

Sol.

(d) 2 cards can be drawn from the pack in $^{52}C_2$ ways. Let A be the event " Two cards are red" and B be the event "Two cards drawn are kings".
The required probability is $P(A \cup B)$.
From addition theorem, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B). \qquad ...(1)$$

Now, P (A) = Probability of drawing two red cards

$$= \frac{^{26}C_2}{^{52}C_2} \quad [\because \text{ There are total 26 red cards}]$$

P (B) = Probability of drawing two king cards

$$= \frac{^{4}C_2}{^{52}C_2} \quad [\because \text{ There are 4 king cards}]$$

$P(A \cap B)$ = Probability of drawing 2 red king cards

$$= \frac{^{2}C_2}{^{52}C_2} \quad [\because \text{ There are just 2 red kings}]$$

Substituting the values in (1), we get

$$P(A \cup B) = \frac{^{26}C_2}{^{52}C_2} + \frac{^{4}C_2}{^{52}C_2} - \frac{^{2}C_2}{^{52}C_2} = \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}$$

$$= \frac{55}{221}.$$

EXAMPLE 8. If A and B are two events, the probability that at most one of these events occurs is :

(a) $P(A') + P(B') - P(A' \cap B')$

(b) $P(A') + P(B') + P(A \cup B) - 1$

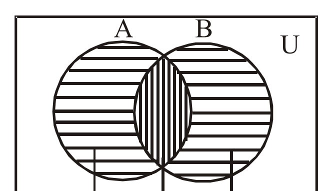
(c) $P(A \cap B') + P(A' \cap B) + P(A' \cap B')$

(d) All above are correct.

(e) None of these

Sol.

(d)



At most one of two events occurs if the event $A' \cup B'$ occurs.

$$P(A' \cup B') = 1 - P(A \cap B)$$

Now, $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$

$$= P(A') + P(B') - [1 - P(A \cup B)]$$

$$= P(A') + P(B') + P(A \cup B) - 1.$$

Finally, since

$$P(A' \cup B') = P[(A')' \cap B'] + P[A' \cap (B')'] + P(A' \cap B')$$

$$= P(A \cap B') + P(A' \cap B) + P(A' \cap B')$$

$$[\because P(A \cup B) = P(A' \cap B) + P(A \cap B') + P(A \cap B)]$$

[See the Venn diagram].

CONDITIONAL PROBABILITY

Let A and B be two events associated with a random experiment.

Then $P\left(\frac{A}{B}\right)$, represents the conditional probability of occurrence of A relative to B.

Also, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

For example :

Suppose a bag contains 5 white and 4 red balls. Two balls are drawn one after the other without replacement. If A denotes the event "drawing a white ball in the first draw" and B denotes the event "drawing a red ball in the second draw".

P (B/A) = Probability of drawing a red ball in second draw when it is known that a white ball has already been drawn in the first draw $= \frac{4}{8} = \frac{1}{2}$

Obviously, P (A/B) is meaning less in this problem.

MULTIPLICATION THEOREM

If A and B are two events, then

$$P(A \cap B) = P(A)\,P(B/A), \text{ if } P(A) > 0$$
$$= P(B)\,P(A/B) \text{ if } P(B) > 0$$

From this theorem we get

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \text{ and } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

For example :

Consider an experiment of throwing a pair of dice. Let A denotes the event " the sum of the point is 8" and B event " there is an even number on first die"

Then A = {(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)},
B = {(2, 1), (2, 2), ..., (2, 6), (4, 1), (4, 2),..... (4, 6), (6, 1), (6, 2),....(6, 6)}

$$P(A) = \frac{5}{36},\ P(B) = \frac{18}{36} = \frac{1}{2},\ P(A \cap B) = \frac{3}{36} = \frac{1}{12}$$

Now, P(A/B) = Prob. of occurrence of A when B has already occurred = prob. of getting 8 as the sum, when there is an even number on the first die

$$= \frac{3}{18} = \frac{1}{6} \text{ and similarly } P(B/A) = \frac{3}{5}.$$

INDEPENDENCE

An event B is said to be independent of an event A if the probability that B occurs is not influenced by whether A has or has not occurred. For two independent events A and B.

$$P(A \cap B) = P(A)\,P(B)$$

Event $A_1, A_2, \ldots\ldots A_n$ are independent if

(i) $P(A_i \cap A_j) = P(A_i)\,P(A_j)$ for all i, j, $i \neq j$, That is, the events are pairwise independent.

(ii) The probability of simultaneous occurrence of (any) finite number of them is equal to the product of their separate probabilities, that is, they are mutually independent.

For example :

Let a pair of fair coin be tossed, here S = {HH, HT, TH, TT}

A = heads on the first coin = {HH, HT}

B = heads on the second coin = {TH, HH}

C = heads on exactly one coin = {HT, TH}

Then P (A) = P(B) = P (C) $= \frac{2}{4} = \frac{1}{2}$ and

$$P(A \cap B) = P(\{HH\}) = \frac{1}{4} = P(A)\,P(B)$$

$$P(B \cap C) = P(\{TH\}) = \frac{1}{4} = P(B)\,P(C)$$

$$P(A \cap C) = P(\{HT\}) = \frac{1}{4} = P(A)\,P(C)$$

Hence the events are pairwise independent.

Also $P(A \cap B \cap C) = P(\phi) = 0 \neq P(A)P(B)P(C)$

Hence, the events A, B, C are not mutually independent.

EXAMPLE **9. The odds against P solving a problem are 8 : 6 and odds in favour of Q solving the same problem are 14 : 10 The probability of the problem being solved, if both of them try it, is**

(a) $\frac{5}{21}$ **(b)** $\frac{16}{21}$ **(c)** $\frac{5}{12}$ **(d)** $\frac{5}{7}$

(e) None of these

Sol.

(b) The odd against P solving a problem = 8 : 6.

$\therefore$ Probability of P not solving the problem $= \frac{8}{14} = \frac{4}{7}$

The odds in favour of Q solving problem = 14 : 10

$\therefore$ Probability of Q not solving the problem $= \frac{10}{24} = \frac{5}{12}$

Hence, the probability of P and Q not solving the problem

$$= \frac{4}{7} \times \frac{5}{12} = \frac{5}{21}$$

$\therefore$ Probability of the problem being solved

= 1 – probability of the problem not being solved

$$= 1 - \frac{5}{21} = \frac{16}{21}.$$

EXAMPLE **10. A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. The probability of occurrence of A is.**

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{5}{6}$ **(d)** $\frac{1}{6}$

(e) None of these

Sol.

(a) Let P (A) = a and P(B) = b Then $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A)P(B) = \frac{1}{6}$, because A and B are independent.

$\therefore a\,b = \frac{1}{6}$ .....(i)

Also $P(\overline{A} \cap \overline{B}) = [1 - P(A)][1 - P(B)]$; .

$$\therefore [1-a][1-b] = \frac{1}{3} \Rightarrow 1 - a - b + ab = \frac{1}{3} \qquad .....(ii)$$

From (i) and (ii) we have $a + b = \frac{5}{6}$ .....(iii)

Solving (i) and (iii) we get, $a = \frac{1}{2}$, $b = \frac{1}{3}$, $\therefore P(A) = \frac{1}{2}$.

EXAMPLE 11. In each of a set of games it is 2 to 1 in favour of the winner of the previous game. The chance that the player who wins the first game shall win three at least of the next four is

(a) $\frac{8}{27}$ (b) $\frac{4}{81}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$

(e) None of these

Sol.

(c) Let W stand for the winning of a game and L for losing it. Then there are 4 mutually exclusive possibilities

(i) W, W, W (ii) W, W, L, W

(iii) W, L, W, W (iv) L, W, W, W.

[Note that case (i) includes both the cases whether he losses or wins the fourth game.]

By the given conditions of the question, the probabilities for (i), (ii), (iii) and (iv) respectively are

$$\frac{2}{3}.\frac{2}{3}.\frac{2}{3};\ \frac{2}{3}.\frac{2}{3}.\frac{1}{3}.\frac{1}{3};\ \frac{2}{3}.\frac{1}{3}.\frac{1}{3}.\frac{2}{3} \text{ and } \frac{1}{3}.\frac{1}{3}.\frac{2}{3}.\frac{2}{3}.$$

Hence the required probability

$$= \frac{8}{27} + \frac{4}{81} + \frac{4}{81} + \frac{4}{81} = \frac{36}{81} = \frac{4}{9}.$$

[$\because$ The probability of winning the game if previous game was also won is $\frac{2}{1+2} = \frac{2}{3}$ and the probability of winning the game if previous game was a loss is $\frac{1}{1+2} = \frac{1}{3}$].

EXAMPLE 12. Three numbers are selected at random without replacement from the set of numbers {1, 2, ..., N}. The conditional probability that the third number lies between the first two, if the first number is known to be smaller than the second, is

(a) 1/6 (b) 1/3 (c) 1/2 (d) 3/4

(e) None of these

Sol.

(b) The number of ways of choosing three numbers out of N is ${}^{N}C_3$. If these numbers are a_1, a_2 and a_3, they must satisfy exactly one of the following inequalities for a successful outcome.

$$a_1 < a_2 < a_3 \qquad a_1 < a_3 < a_2, \qquad a_2 < a_1 < a_3,$$

$$a_2 < a_3 < a_1, \qquad a_3 < a_1 < a_2, \qquad a_3 < a_2 < a_1.$$

Thus the number of ways of arranging the three numbers in a given order is $({}^{N}C_3)$ (6), and there are 3 ways in which the first number is less than the second. Now if A denotes the event : the first number is less than the second number, and B the event : the third number lies between the first and the second, we need to find P(B|A). Since

$$P(B \cap A) = \frac{{}^{N}C_3}{({}^{N}C_3)(6)} = \frac{1}{6} \text{ and } P(A) = \frac{({}^{N}C_3)(3)}{({}^{N}C_3)(6)} = \frac{1}{2},$$

We get $P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{1/6}{1/2} = \frac{1}{3}$.

EXAMPLE 13. Given two bags A and B as follows : Bag A contains 3 red and 2 white balls and bag B contains 2 red and 5 white balls. A bag is selected at random, a ball is drawn and put into the other bag, then a ball is drawn from the second bag. The probability that both balls drawn are of the same colour is

(a) $\frac{187}{1680}$ (b) $\frac{901}{1680}$ (c) $\frac{439}{1680}$ (d) $\frac{437}{1679}$

(e) None of these

Sol.

(b) The whole event consists of the following mutually exclusive ways.

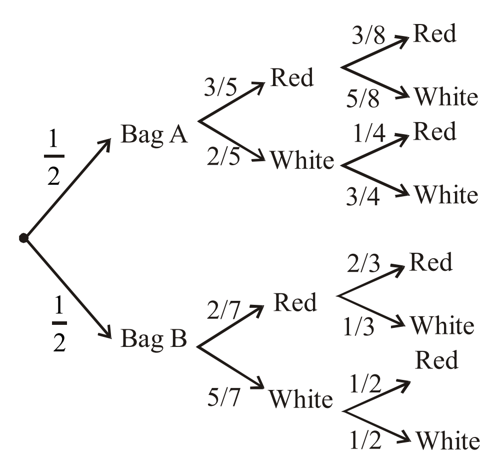
(1) Selecting the bag A, drawing a red ball from A and putting it into bag B and then drawing a red ball from B.

(2) Selecting the bag A, drawing a white ball from A and putting it into bag B and then drawing a white ball from B.

(3) Selecting the bag B, drawing a red ball from B and putting it into A and then drawing a red ball from A.

(4) Selecting the bag B, drawing a white ball from B and putting it into A and then drawing a white ball from A.

The tree diagram of the above processes are shown below, with respective probability of each step



The required probability is

$$= \frac{1}{2} \times \frac{3}{5} \times \frac{3}{8} + \frac{1}{2} \times \frac{2}{5} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{7} \times \frac{2}{3} + \frac{1}{2} \times \frac{5}{7} \times \frac{1}{2}$$

$$= \frac{9}{80} + \frac{3}{20} + \frac{2}{21} + \frac{5}{28} = \frac{901}{1680}$$

EXERCISE

1. In a given race the odds in favour of three horses A, B, C are 1 : 3; 1 : 4; 1 : 5 respectively. Assuming that dead head is impossible the probability that one of them wins is

(a) $\frac{7}{60}$ (b) $\frac{37}{60}$

(c) $\frac{1}{5}$ (d) $\frac{1}{8}$

(e) None of these

2. A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. The probability that only one of them will be selected is

(a) $\frac{6}{7}$ (b) $\frac{4}{35}$

(c) $\frac{6}{35}$ (d) $\frac{2}{7}$

(e) None of these

3. The probability that the 13th day of a randomly chosen month is a Friday, is

(a) $\frac{1}{12}$ (b) $\frac{1}{7}$

(c) $\frac{1}{84}$ (d) $\frac{1}{13}$

(e) None of these

4. If a leap year selected at random, the chance that it will contain 53 Sunday is

(a) $\frac{3}{7}$ (b) $\frac{1}{7}$

(c) $\frac{2}{7}$ (d) $\frac{4}{7}$

(e) None of these

5. A Positive integer N is selected such that $100 < N < 200$. The probability that it is divisible by either 4 or 7 is :

(a) $\frac{38}{99}$ (b) $\frac{24}{99}$

(c) $\frac{34}{99}$ (d) $\frac{14}{99}$

(e) None of these

6. In a single throw with four dice, the probability of throwing seven is

(a) $\frac{4}{6^4}$ (b) $\frac{8}{6^4}$

(c) $\frac{16}{6^4}$ (d) $\frac{20}{6^4}$

(e) None of these

7. If A and B are two independent events with P(A) = 0.6, P(B) = 0.3, then $P(A'\cap B')$ is equal to :

(a) 0.18 (b) 0.28
(c) 0.82 (d) 0.72
(e) None of these

8. If three vertices of a regular hexagon are chosen at random, then the chance that they form an equilateral triangle is :

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$

(c) $\frac{1}{10}$ (d) $\frac{1}{2}$

(e) None of these

9. Six dice are thrown. The probability that different number will turn up is :

(a) $\frac{129}{1296}$ (b) $\frac{1}{54}$

(c) $\frac{5}{324}$ (d) $\frac{5}{54}$

(e) None of these

10. Four balls are drawn at random from a bag containing 5 white, 4 green and 3 black balls. The probability that exactly two of them are white is :

(a) $\frac{14}{33}$ (b) $\frac{7}{16}$

(c) $\frac{18}{33}$ (d) $\frac{9}{16}$

(e) None of these

11. The probability that a person will hit a target in shooting practice is 0.3. If he shoots 10 times, the probability that he hits the target is

(a) 1 (b) $1-(0.7)^{10}$
(c) $(0.7)^{10}$ (d) $(0.3)^{10}$
(e) None of these

12. The probability that at least one of the events A and B occurs is 0.7 and they occur simultaneously with probability 0.2. Then $P(\overline{A})+P(\overline{B}) =$

(a) 1.8 (b) 0.6
(c) 1.1 (d) 0.4
(e) None of these

13. The probability that A can solve a problem is $\frac{2}{3}$ and B can solve it is $\frac{3}{4}$. If both attempt the problem, what is the probability that the problem gets solved?

(a) $\frac{11}{12}$ (b) $\frac{7}{12}$

(c) $\frac{5}{12}$ (d) $\frac{9}{12}$

(e) None of these

14. Three integers are chosen at random from the first 20 integers. The probability that their product is even, is

(a) $\frac{2}{19}$ (b) $\frac{3}{29}$

(c) $\frac{17}{19}$ (d) $\frac{4}{29}$

(e) None of these

15. A die is loaded in such a way that each odd number is twice al likely to occur as each even number. If E is the event of a number greater than or equal to 4 on a single toss of the die, then P(E) is :

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

(e) None of these

16. The probability that two integers chosen at random and their product will have the same last digit is :

(a) $\frac{3}{10}$ (b) $\frac{1}{25}$

(c) $\frac{4}{15}$ (d) $\frac{7}{15}$

(e) None of these

17. Seven people seat themselves indiscriminately at round table. The probability that two distinguished persons will be next to each other is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

(e) None of these

18. Two dice are thrown. The probability that the sum of the numbers coming up on them is 9, if it is known that the number 5 always occurs on the first die, is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

(e) None of these

19. A speaks the truth in 70 percent cases and B in 80 percent. The probability that they will contradict eact. other when describing a single event is

(a) 0.36 (b) 0.38

(c) 0.4 (d) 0.42

(e) None of these

20. One ticket is selected at random from 100 tickets numbered 00, 01, 02,.....99. Suppose S and T are the sum and product of the digits of the number on the ticket, then $P(S = 9 \ / \ T = 0)$ is

(a) $\frac{19}{100}$ (b) $\frac{1}{4}$

(c) $\frac{2}{19}$ (d) $\frac{1}{50}$

(e) None of these

21. A die is loaded such that the probability of throwing the number i is proportional to its reciprocal. The probability that 3 appears in a single throw is :

(a) $\frac{3}{22}$ (b) $\frac{3}{11}$

(c) $\frac{9}{22}$ (d) $\frac{20}{147}$

(e) None of these

22. The probability of getting 10 in a single throw of three fair dice is :

(a) $\frac{1}{6}$ (b) $\frac{1}{8}$

(c) $\frac{1}{9}$ (d) $\frac{1}{5}$

(e) None of these

23. The probability that when 12 balls are distributed among three boxes, the first will contain three balls is,

(a) $\frac{2^9}{3^{12}}$ (b) $\frac{^{12}C_3.2^9}{3^{12}}$

(c) $\frac{^{12}C_3.2^{12}}{3^{12}}$ (d) $\frac{^{12}C_3.2^{11}}{3^{11}}$

(e) None of these

24. A and B toss a fair coin each simultaneously 50 times. The probability that both of them will not get tail at the same toss is

(a) $\left(\frac{3}{4}\right)^{50}$ (b) $\left(\frac{2}{7}\right)^{50}$

(c) $\left(\frac{1}{8}\right)^{50}$ (d) $\left(\frac{7}{8}\right)^{50}$

(e) None of these

25. If n integers taken at random are multiplied together, then the probability that the last digit of the product is 1, 3, 7 or 9 is

(a) $\frac{2^n}{5^n}$ (b) $\frac{4^n - 2^n}{5^n}$

(c) $\frac{4^n}{5^n}$ (d) $\frac{4^n}{7^n}$

(e) None of these

26. A coin is tossed 5 times. What is the probability that head appears an odd number of times?

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{1}{2}$ (d) $\frac{4}{25}$

(e) None of these

27. Atul can hit a target 3 times in 6 shots, Bhola can hit the target 2 times in 6 shots and Chandra can hit the 4 times in 4 shots. What is the probability that at least 2 shots (out of 1 shot taken by each one of them) hit the target ?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{5}{6}$

(e) None of these

28. A bag contain 5 white, 7 red and 8 black balls. If 4 balls are drawn one by one with replacement, what is the probability that all are white ?

(a) $\frac{1}{256}$ (b) $\frac{1}{16}$

(c) $\frac{4}{20}$ (d) $\frac{4}{8}$

(e) None of these

29. A dice is thrown 6 times. If 'getting an odd number' is a 'success', the probability of 5 successes is :

(a) $\frac{1}{10}$ (b) $\frac{3}{32}$

(c) $\frac{5}{6}$ (d) $\frac{25}{26}$

(e) None of these

30. A bag has 4 red and 5 black balls. A second bag has 3 red and 7 black balls. One ball is drawn from the first bag and two from the second. The probability that there are two black balls and a red ball is :

(a) $\frac{14}{45}$ (b) $\frac{11}{45}$

(c) $\frac{7}{15}$ (d) $\frac{9}{54}$

(e) None of these

31. Two dice are tossed. The probability that the total score is a prime number is :

(a) $\frac{1}{6}$ (b) $\frac{5}{12}$

(c) $\frac{1}{2}$ (d) $\frac{7}{9}$

(e) None of these

32. A bag contains 3 white balls and 2 black balls. Another bag contains 2 white balls and 4 black balls. A bag is taken and a ball is picked at random from it. The probability that the ball will be white is:

(a) $\frac{7}{11}$ (b) $\frac{7}{30}$

(c) $\frac{5}{11}$ (d) $\frac{7}{15}$

(e) None of these

33. Suppose six coins are tossed simultaneously. Then the probability of getting at least one tail is :

(a) $\frac{71}{72}$ (b) $\frac{53}{54}$

(c) $\frac{63}{64}$ (d) $\frac{1}{12}$

(e) None of these

34. A bag contains 2 red, 3 green and 2 blue balls. 2 balls are to be drawn randomly. What is probability that the balls drawn contain no blue ball ?

(a) $\frac{5}{7}$ (b) $\frac{10}{21}$

(c) $\frac{2}{7}$ (d) $\frac{11}{21}$

(e) None of these

35. A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals

(a) $\frac{1}{2}$ (b) $\frac{1}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{5}$

(e) None of these

36. The probability that the birth days of six different persons will fall in exactly two calendar months is

(a) $\frac{1}{6}$ (b) ${}^{12}C_2 \times \frac{2^6}{12^6}$

(c) ${}^{12}C_2 \times \frac{2^6 - 1}{12^6}$ (d) $\frac{341}{12^5}$

(e) None of these

37. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

(e) None of these

38. A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(\overline{A}) = \frac{2}{3}$ then $P(\overline{A} \cap B)$ is

(a) $\frac{5}{12}$ (b) $\frac{3}{8}$

(c) $\frac{5}{8}$ (d) $\frac{1}{4}$

(e) None of these

39. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

(e) None of these

40. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$

(c) $\frac{7}{20}$ (d) $\frac{3}{20}$

(e) None of these

41. 2n boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is

(a) $\frac{n}{2n-1}$ (b) $\frac{n-1}{2n-1}$

(c) $\frac{2n-1}{4n^2}$ (d) $\frac{n+1}{2n+1}$

(e) None of these

42. Fifteen persons, among whom are A and B sit down at random at a round table. The probability that there are 4 persons between A and B is

(a) $\frac{9!}{14!}$ (b) $\frac{10!}{14!}$

(c) $\frac{9!}{15!}$ (d) $\frac{1}{7}$

(e) None of these

43. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probbility that they will not win a prize in a single trial is

(a) $\frac{1}{25}$ (b) $\frac{24}{25}$

(c) $\frac{2}{25}$ (d) $\frac{1}{27}$

(e) None of these

ANSWER KEY

1	(b)	7	(b)	13	(a)	19	(b)	25	(a)	31	(b)	37	(a)	43	(b)
2	(d)	8	(c)	14	(c)	20	(c)	26	(c)	32	(d)	38	(a)		
3	(c)	9	(c)	15	(a)	21	(d)	27	(b)	33	(c)	39	(a)		
4	(c)	10	(a)	16	(b)	22	(b)	28	(a)	34	(b)	40	(c)		
5	(c)	11	(b)	17	(a)	23	(b)	29	(b)	35	(a)	41	(a)		
6	(d)	12	(c)	18	(a)	24	(a)	30	(c)	36	(d)	42	(d)		

Hints & Explanations

1. (b) Suppose E_1, E_2 and E_3 are the events of winning the race by the horses A, B and C respectively

$\therefore P(E_1) = \frac{1}{1+3} = \frac{1}{4}$, $P(E_2) = \frac{1}{1+4} = \frac{1}{5}$

$P(E_3) = \frac{1}{1+5} = \frac{1}{6}$

$\therefore$ Probability of winning the race by one of the horses A, B and C

$= P(E_1 \text{ or } E_2 \text{ or } E_3) = P(E_1) + P(E_2) + P(E_3)$

$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{37}{60}$

2. (d) Probability that only husband is selected

$= P(H)P(\overline{W}) = \frac{1}{7}\left(1 - \frac{1}{5}\right) = \frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$

Probability that only wife is selected

$= P(\overline{H})P(W) = \left(1 - \frac{1}{7}\right)\left(\frac{1}{5}\right) = \frac{6}{7} \times \frac{1}{5} = \frac{6}{35}$

$\therefore$ Probability that only one of them is selected

$= \frac{4}{35} + \frac{6}{35} = \frac{10}{35} = \frac{2}{7}$

3. (c) Probability of selecting a month $= \frac{1}{12}$.

13^{th} day of the month is friday if its first day is sunday and the probability of this $= \frac{1}{7}$.

$\therefore$ Required probability $= \frac{1}{12} \cdot \frac{1}{7} = \frac{1}{84}$.

4. (c) A leap-year has 366 days i.e. 52 complete weeks and two days more these two days be two consecutive days of a week. A leap year will have 53 Sundays if out of the two consecutive days of a week selected at random one is a Sunday.

Let S be the sample space and E be the event that out of the two consecutive days of a week one is Sunday, then

S = {(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)}

$\therefore$ n (S) = 7

and E = {(Sunday, Monday), (Saturday, Sunday)

$\therefore$ n (E) = 2

Now, required Probability, $P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$

5. (c) Numbers divisible by 4 are 104, 108.., 196; 24 in number. Numbers divisible by 7 are 105, 112, ...196; 14 in number. Numbers divisible by both, i.e.divisible by 28 are 112, 140, 168, 196; 4 in number. Hence, required probability

$= \frac{24}{99} + \frac{14}{99} - \frac{4}{99} = \frac{34}{99}$

6. (d) Total of seven can be obtained in the following ways

1, 1, 1, 4 in $\frac{4!}{3!} = 4$ ways

[there are four objects, three repeated]

Similarly,

1, 1,2, 3 in $\frac{4!}{2!} = 12$ ways

1, 2,2, 2 in $\frac{4!}{3!} = 4$ ways

Hence, required probability $= \frac{4+12+4}{6^4} = \frac{20}{6^4}$

[$\because$ Exhaustive no. of cases $= 6 \times 6 \times 6 \times 6 = 6^4$]

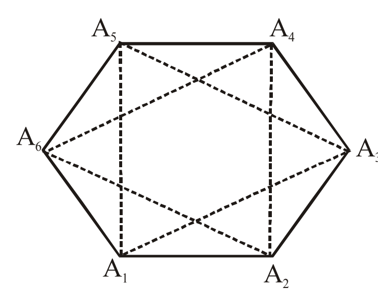
7. (b) Since, A and B are independent events $\therefore$ A' and B' are also independent events

$\Rightarrow P(A' \cap B') = P(A').P(B')$

$= (0.4)(0.7) = 0.28$

$[\because P(A') = 1 - P(A),\ P(B') = 1 - P(B)]$

8. (c) Three vertices can be selected in ${}^6C_3$ ways.



The only equilateral triangles possible are $A_1A_3A_5$ and $A_2A_4A_6$

$p = \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$

9. (c) The number of ways of getting the different number 1, 2, ..., 6 in six dice = 6 !.

Total number of ways $= 6^6$

Hence, required probability $= \frac{6!}{6^6}$

$= \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{6^6} = \frac{5}{324}$

10. (a) Total number of balls = 12

Hence, required probability $= \frac{{}^5C_2 \times {}^7C_2}{{}^{12}C_4} = \frac{14}{33}$

$\because$ No of ways of drawing 2 white balls from 5 white balls $= {}^5C_2$.

Also, No of ways of drawing 2 other from remaining 7 balls $= {}^7C_2$

11. (b) The probality that the person hits the target = 0.3
$\therefore$ The probability that he does not hit the target in a trial $= 1 - 0.3 = 0.7$
$\therefore$ The probability that he does not hit the target in any of the ten trials $= (0.7)^{10}$
$\therefore$ Probability that he hits the target
= Probability that at least one of the trials succeeds
$= 1 - (0.7)^{10}$.

12. (c) We have $P(A \cup B) = 0.7$ and $P(A \cap B) = 0.2$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) = 0.9 \Rightarrow 1 - P(\overline{A}) + 1 - P(\overline{B}) = 0.9$

$\Rightarrow P(\overline{A}) + P(\overline{B}) = 1.1$

13. (a) The probability that A cannot solve the problem $= 1 - \frac{2}{3} = \frac{1}{3}$

The probability that B cannot solve the problem $= 1 - \frac{3}{4} = \frac{1}{4}$

The probability that both A and B cannot solve the problem $= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

$\therefore$ The probability that at least one of A and B can solve the problem $= 1 - \frac{1}{12} = \frac{11}{12}$

$\therefore$ The probability that the problem is solved $= \frac{11}{12}$

14. (c) The total number of ways in which 3 integers can be chosen from first 20 integers is ${}^{20}C_3$.
The product of three integers will be even if at least one of the integers is even. Therfore, the required probability = 1 – Prob. that none of the three integers is even

$$= 1 - \frac{{}^{10}C_3}{{}^{20}C_3} = 1 - \frac{2}{19} = \frac{17}{19}.$$

[Three odd integers can be chosen in ${}^{10}C_3$ ways as there are 10 even and 10 odd integers.

15. (a) If a probability p is assigned to each even number, then 2p is the probability to be assigned to each odd number which gives $2p \times 3 + p \times 3 = 9p = 1$.

$$\Rightarrow p = \frac{1}{9}$$

$\therefore$ P(E) = Probability of getting 4, 5 or 6

$$= \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$$

16. (b) The condition implies that the last digit in both the integers should be 0, 1, 5 or 6 and the probability

$$= 4\left(\frac{1}{10}\right)^2 = \frac{4}{100} = \frac{1}{25}$$

[$\because$ The squares of numbers ending in 0 or 1 or 5 or 6 also 0 or 1 or 5 or 6 respectively]

17. (a) Seven people can seat themselves at a round table in 6! ways. The number of ways in which two distinguished persons will be next to each other = 2 (5) !, Hence, the required probability

$$= \frac{2(5)!}{6!} = \frac{1}{3}$$

18. (a) $\therefore S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

$\therefore n(S) = 36$

& Let $E_1 \equiv$ the event that the sum of the numbers coming up is 9.
& $E_2 \equiv$ the event of occurrence of 5 on the first die.
$E_1 \equiv \{(3,6),(6,3),(4,5),(5,4)\}$

$\therefore n(E_1) = 4$ and

$E_2 = \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$\therefore n(E_2) = 6$

$E_1 \cap E_2 = \{(5,4)\} \quad \therefore n(E_1 \cap E_2) = 1$

Now, $P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{36}$

and $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

$\therefore$ Required Probability

$$= P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$$

19. (b) A and B will contradict each other if one of the events $A \cap B'$ or $A' \cap B$ occurs. The probability of this happening is

$$P[(A \cap B') \cup (A' \cap B) = P(A \cap B') + P(A' \cap B)$$
$$= P(A)P(B') + P(A)P(B),$$

because A and B are independent. Therefore, putting $P(A) = 0.7$ and $P(B) = 0.8$ the required probability is $(0.7)(0.2) + (0.3)(0.8) = 0.38$.

20. (c) We have $(S = 9) = \{09, 18, 27, 36, 45, 54, 63, 72, 81, 90\}$ and $(T = 0) = \{0, 01, ..., 09, 10, 20, ...90\}$
Also $(S = 9) \cap (T = 0) = \{09, 90\}$
Thus $P((S = 9) \cap (T = 0)) = 2/100$
Hence $P(S = 9/T = 0)$

$$= \frac{P((S = 9) \cap (T = 0))}{P(T = 0)} = \frac{2/100}{19/100} = \frac{2}{19}.$$

21. (d) $P(i) = \frac{k}{i}$

$$\Rightarrow 1 = \sum_{i=1}^{6} P(i) = k\sum_{i=1}^{6} \frac{1}{i} = k\frac{49}{20} \Rightarrow k = \frac{20}{49}$$

$$\therefore P(3) = \frac{k}{3} = \frac{20}{49\times 3} = \frac{20}{147}$$

22. (b) Exhaustive no. of cases = 6^3
10 can appear on three dice either as distinct number as following (1, 3, 6) ; (1, 4, 5); (2, 3, 5) and each can occur in 3! ways. Or 10 can appear on three dice as repeated digits as following (2, 2, 6), (2, 4, 4), (3, 3, 4) and each can occur in $\frac{3!}{2!}$ ways.

$\therefore$ No. of favourable cases $= 3\times 3! + 3\times\frac{3!}{2!} = 27$

Hence, the required probability $= \frac{27}{6^3} = \frac{1}{8}$

23. (b) Since each ball can be put into any one of the three boxes. So, the total number of ways in which 12 balls can be put into three boxes is 3^{12}.
Out of 12 balls, 3 balls can be chosen in ${}^{12}C_3$ ways. Now, remaining 9 balls can be put in the remaining 2 boxes in 2^9 ways. So, the total number or ways in which 3 balls are put in the first box and the remaining in other two boxes is ${}^{12}C_3 \times 2^9$.

Hence, required probability $= \frac{{}^{12}C_3.2^9}{3^{12}}$

24. (a) For each toss there are four choices
(i) A gets head, B gets head
(ii) A gets tail, B gets head
(iii) A gets head, B gets tail
(iv) A gets tail, B gets tail
thus, exhaustive number of ways = 4^{50}. Out of the four choices listed above (iv) is not favourable to the required event in a toss. Therefore favourable number of cases is 3^{50}.

Hence, the required probability $= \left(\frac{3}{4}\right)^{50}$

25. (a) In any number the last digit can be one of 0, 1, 2, ... 8, 9. Therefore, the last digit of each number can be chosen in 10 ways. Thus, exhausitive number of ways = 10^n. If the last digit be 1, 3, 7 or 9 none of the numbers can be even or end in 0 or 5. Thus, we have a choice of 4 digits viz. 1, 3, 7 or 9 with which each of n number should end. So, favourable number of ways = 4^n.

Hence, the required probability $= \frac{4^n}{10^n} = \left(\frac{2}{5}\right)_n^n$.

26. (c) Probability of occurence of head in a toss of a coin is 1/2.
Required probability = Prob[Head appears once] + Prob.[Head appears thrice] + Prob.[Head appears five times]

$$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 [5+10+1] = \frac{16}{32} = \frac{1}{2}$$

27. (b) Chandra hits the target 4 times in 4 shots. Hence, he hits the target definitely.
The required probability, therefore, is given by.
P(both Atul and Bhola hit) + P(Atul hits, Bhola does not hit) + P(Atul does not hit, Bhola hits)

$$= \frac{3}{6}\times\frac{2}{6}+\frac{3}{6}\times\frac{4}{6}+\frac{3}{6}\times\frac{2}{6}$$

$$= \frac{1}{6}+\frac{1}{3}+\frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

28. (a) Total number of balls = 5 + 7 + 8 = 20
Probability that the first ball drawn is white

$$= \frac{{}^5C_1}{{}^{20}C_1} = \frac{1}{4}$$

If balls are drawn with replacement, all the four events will have equal probability.
Therefore, required probability

$$= \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{256}$$

29. (b) Let A be the event of getting an odd number.
Here, n (S) = 6 and
n (A) = 3

Probability of getting an odd number $= \frac{3}{6} = \frac{1}{2}$

Hence, probability of not getting an odd number

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Required probability of 5 successes

$$= {}^6C_5 \times\left(\frac{1}{2}\right)^5 \times\frac{1}{2} = \frac{3}{32}$$

30. (c) Required probability
= Probability that ball from bag A is red and both the balls from bag B are black + Probability that ball from bag A is black and one black and one red balls are drawn from bag B

$$= \frac{{}^4C_1}{{}^9C_1}\times\frac{{}^7C_2}{{}^{10}C_2}+\frac{{}^5C_1}{{}^9C_1}\times\frac{{}^3C_1\times{}^7C_1}{{}^{10}C_2}$$

$$= \frac{4}{9}\times\frac{7}{15}+\frac{5}{9}\times\frac{7}{15} = \frac{7}{15}$$

31. (b) Total no. of outcomes when two dice are thrown = n (S) = 36 and the possible cases for the event that the sum of numbers on two dice is a prime number, are (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 1), (5, 6), (6, 1), (6, 5)
Number of outcomes favouring the event = n (A) = 15

Required probability $= \frac{n(A)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

32. (d) The probability of selecting a bag $= \frac{1}{2}$

Now, probability of getting a white ball from the first bag $= \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

and probability of getting a white ball from the second bag $= \frac{1}{2} \times \frac{2}{6} = \frac{1}{6}$

Required Probability = The probability that a white ball is drawn either from the first or the second bag

$= \frac{3}{10} + \frac{1}{6} = \frac{7}{15}$

33. (c) If six coins are tossed, then the total no. of outcomes = $(2)^6 = 64$

Now, probability of getting no tail $= \frac{1}{64}$

Probability of getting at least one tail

$= 1 - \frac{1}{64} = \frac{63}{64}$

34. (b) 2 balls can be drawn in the following ways
1 red and 1 green or 2 red or 2 green

Required probability $= \frac{{}^2C_1 \times {}^3C_1}{{}^7C_2} + \frac{{}^2C_2}{{}^7C_2} + \frac{{}^3C_2}{{}^7C_2}$

$$= \frac{6}{21} + \frac{1}{21} + \frac{3}{21} = \frac{10}{21}$$

35. (a) The event that the fifth toss results a head is independent of the event that the first four tosses results tails.
$\therefore$ Probability of the required event = 1/2

36. (d) Exhaustive number of cases = 12
Favourable cases = ${}^{12}C_2\,(2^6 - 2)$

$\therefore$ Probabiltity $= \frac{{}^{12}C_2(2^6 - 2)}{12^6} = \frac{341}{12^5}$

37. (a) $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{3}$ and $P(E_3) = \frac{1}{4}$;

$$P(E_1 \cup E_2 \cup E_3) = 1 - P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3)$$

$$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$$

38. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;

$$\Rightarrow \frac{3}{4} = 1 - P(\overline{A}) + P(B) - \frac{1}{4}$$

$$\Rightarrow 1 = 1 - \frac{2}{3} + P(B) \Rightarrow P(B) = \frac{2}{3};$$

Now, $P(\overline{A} \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$.

39. (a) $n(S) = {}^5C_2$; $n(E) = {}^2C_1 + {}^2C_1$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^2C_1 + {}^2C_1}{{}^5C_2} = \frac{2}{5}$$

40. (c) A and B will contradict each other if one speaks truth and other false . So, the required

Probability $= \frac{4}{5}\left(1 - \frac{3}{4}\right) + \left(1 - \frac{4}{5}\right)\frac{3}{4}$

$$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}$$

41. (a) Number of ways of forming two groups $= \frac{(2n)!}{n!\,n!}$

We can divide 2n – 2 boys into two groups in $\frac{(2n-2)!}{(n-1)!\,(n-1)!}$. But the two tallest boys can be in any of the groups (each in different). So favourable number of cases $= \frac{2(2n-2)!}{(n-1)!(n-1)!}$

42. (d) Exhaustive number of cases = (15 – 1)! = 14!
Favourable cases
${}^{13}C_4(10-1)! \times 2! \times 4! = 2! \times 13!$
[A, B and four persons are treated as one, A and B can interchange positions]

$\therefore$ Probability $= \frac{2 \times 13!}{14!} = \frac{1}{7}$

43. (b) Total number of possibilities = 25 × 25
Favourable cases for their winning = 25

$\therefore$ P (they win a prize) $= \frac{25}{25 \times 25} = \frac{1}{25}$

$\therefore$ P (they will not win a prize) $= 1 - \frac{1}{25} = \frac{24}{25}$