CBSE Board
Class X Mathematics
Sample Paper 5 (Standard)

Time: 3 hrs **Total Marks: 80**

General Instructions:

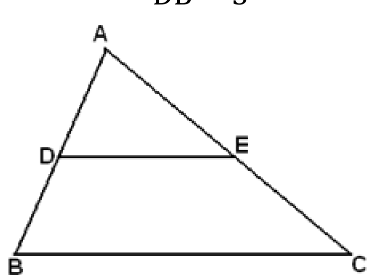
1. This question paper contains **two parts** A and B.
2. Both **Part A** and **Part B** have internal choices.

Part – A:
1. It consists **two sections** - I and II.
2. **Section I** has **16 questions** of **1 mark** each. Internal choice is provided in **5 questions**.
3. **Section II** has **4 questions** on **case study**. Each case study has **5 case-based sub-parts**. An examinee is to attempt any **4 out of 5 sub-parts.** Each subpart carries **1 mark.**

Part – B:
1. It consists **three sections** – III, IV and V
2. **Section III: Question No 21 to 26** are **Very short answer** Type questions of **2 marks** each.
3. **Section IV: Question No 27 to 33** are **Short Answer Type** questions of **3 marks** each.
4. **Section V: Question No 34 to 36** are **Long Answer Type** questions of **5 marks** each.
5. Internal choice is provided in **2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.**

Part A
Section I
Section I has 16 questions of 1 mark each.
(Internal choice is provided in 5 questions)

1. In $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $\frac{AD}{DB} = \frac{2}{3}$ and EC = 4 cm, then find AE.



OR

If the areas of two similar triangles are in the ratio 25: 64, find the ratio of their corresponding sides.

2. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a =?

OR

Find the number of decimal places after which the decimal expansion of the rational number $\frac{23}{2^2 \times 5}$ will terminate.

3. Find: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

OR

If x = a sec θ and y = b tan θ, then find $b^2x^2 - a^2y^2$.

4. If ∆ABC is a right angled at C, then find the value of cos (A + B).

5. What is the distance of A (5, –12) from the origin?

OR

What is the perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0)?

6. Given ∆ABC ~ ∆PQR, if $\frac{AB}{PQ} = \frac{4}{9}$ then $\frac{ar\Delta ABC}{ar\Delta PQR} = \frac{\square}{\square}$

7. What is the sum of the first n natural numbers?

8. Find the probability of getting at most one head when two coins are tossed simultaneously.

9. Which term of the AP 3, 8, 13, 18, … is 88?

OR

Find the 11^{th} term of the AP (5a – x), 6a, (7a + x),…

10. If the arithmetic mean of x, x + 3, x + 6, x + 9 and x + 12 is 10, the find the value of x.

11. What is the name of the quadrilateral, if its diagonals divides each other proportionally?

12. What is the distance between the points (a cos θ + b sin θ, 0) and (0, a sin θ – b cos θ)?

13. Find the length of the hypotenuse of an isosceles right triangle whose one side is $4\sqrt{2}$ cm.

14. Find the sum of the exponents of the prime factors in the prime factorization of 196.

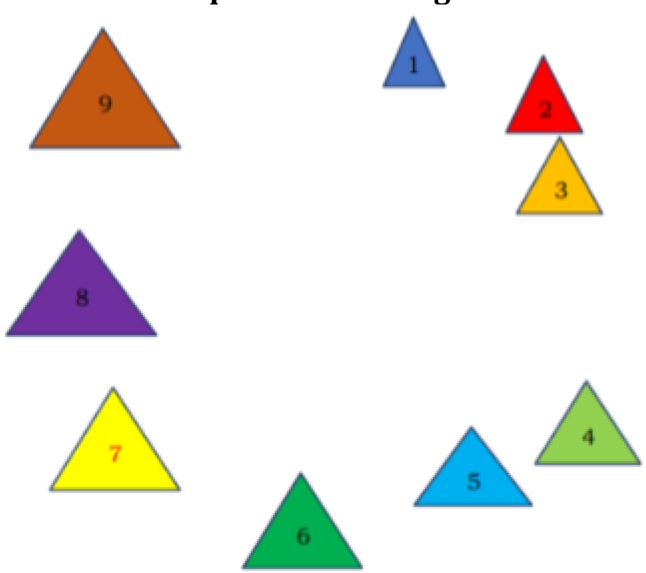
15. Which of the following is a pair of co-primes?
(14, 35), (18, 25), (31, 93), (32, 62)

16. A point P divides the join of A(5, –2) & B(9, 6) in the ratio 3:1. Find the coordinates of P.

Section II
(Q 17 to Q 20 carry 4 marks each)

Case study based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study Based- 1
Numbers and Equilateral Triangles.



Rahul and Sunil were feeling bored during the lockdown. They both created a number game. Sunil prepared nine equilateral triangles and numbered them from 1 to 9. The numbers written on the triangles also represent length of each side of the triangle (in cm).
Sunil arranged them in the form of a circle. He asked Rahul to remove alternate triangles starting from number 1, going clockwise, until only one triangle remained.

(a) The triangle which Rahul removed in the first round are in order, numbered 1, 3, 5, 7, 9. If Rahul continues in the same manner, which numbered triangle will be left in the last?
(i) 4
(ii) 2
(iii) 8
(iv) 6

(b) In the second round, Sunil started counting with triangle numbered 1 and eliminated every third triangle, until only one triangle remained. Which of the following triangle will be left in the end?
Triangle number:
(i) 1
(ii) 3
(iii) 7
(iv) 6

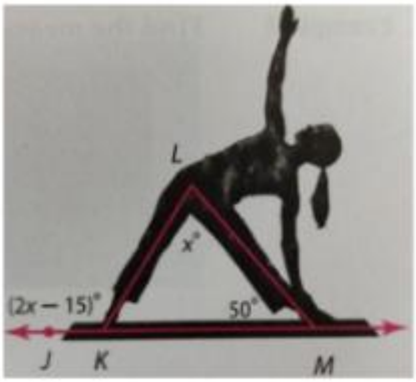
(c) Rahul added two more triangles in the circle and numbered these as triangle 10 and triangle 11. In this round, Rahul started counting with triangle numbered 1, but anticlockwise, and eiminated every fifth triangle, until only one triangle remained. Which triangle will be left in the end?
Triangle number:
(i) 2
(ii) 4
(iii) 5
(iv) 8

(d) If there are 9 triangles, will the perimeters of the triangles follow any pattern? If so, write the pattern?
(i) They are multiple of 3.
(ii) They are multiple of 6.
(iii) They are multiple of 2.
(iv) They are multiple of 4.

(e) Are the areas of the triangles numbered 3, 4 and 6, 8 in proportion? If yes then write down the ratio.
(i) 9: 16
(ii) 3: 4
(iii) 7: 8
(iv) 16: 9

18. Case Study based-2
Types of angles and angle sum property of a triangle

It is 7:00 am!
Shikha rolls out her yoga mat and starts her warm up session with stretching and bending. Anaya her daughter is sitting nearby, observing her mother's daily ritual.
Anaya takes a picture of her mother while she was in a yoga posture and label it as shown.



(a) Angles ∠LKM and ∠JKL are called as?
- (i) Linear Pair of angles
- (ii) Vertically opposite angles
- (iii) Complementary angles
- (iv) Corresponding angles

(b) Find m∠LKM.
- (i) 195° – x
- (ii) 185° – 2x
- (iii) 195° – 2x
- (iv) 185° – x

(c) Find m∠KLM.
- (i) 115°
- (ii) 65°
- (iii) 50°
- (iv) 180°

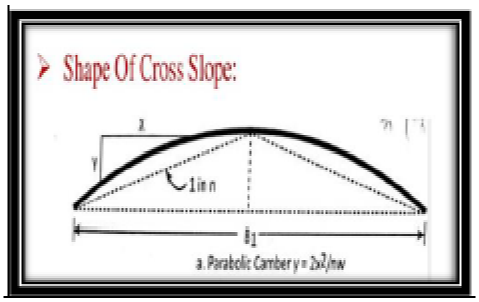
(d) Which of the following is true for △LKM?
- (i) △LKM is an equilateral triangle.
- (ii) △LKM is an isosceles triangle.
- (iii) △LKM is a right angle triangle.
- (iv) All of the above

(e) What is the measurement of the ∠LKJ?
- (i) 115°
- (ii) 65°
- (iii) 50°
- (iv) 180°

19. Case Study Based- 3

Applications of Parabolas-Highway Overpasses/Underpasses

A highway underpass is parabolic in shape.



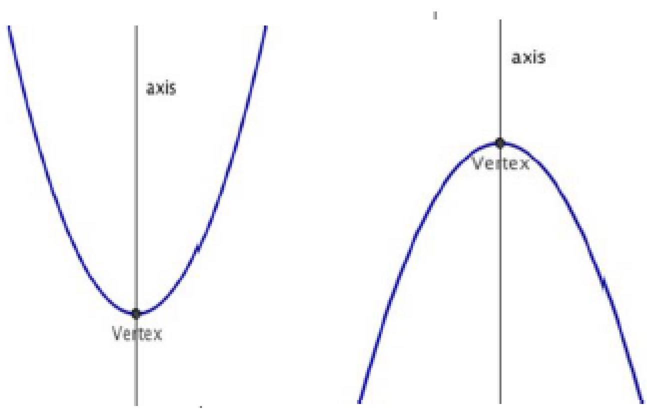
Parabola

A parabola is the graph that results from $p(x)=ax^2+bx+c$

Parabolas are symmetric about a vertical line known as the Axis of Symmetry.

The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the

Vertex



(a) If the highway overpass is represented by $x^2 + 4x + k = 0$ has real and distinct roots then find the condition on k.

(i) $k > 4$

(ii) $k < 4$

(iii) $k \leq 4$

(iv) $k \geq 4$

(b) The quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively: $\frac{1}{4}$ and -1

(i) $k(4x^2 - x - 4)$
(ii) $k(x^2 - 12x + 36)$
(iii) $k(x^2 + 18x + 9)$
(iv) $k(x^2 - 3x + 6)$

(c) Determine the set of values of p for which the quadratic equation $px^2 + 6x + 1 = 0$ has real roots.

(i) $p < 9$
(ii) $p > 9$
(iii) $p \leq 9$
(iv) $p \geq 9$

(d) The representation of Highway Underpass whose zeroes are 9 and –2 is

(i) $x^2 - 9x + 18$
(ii) $x^2 - 12x + 36$
(iii) $x^2 + 18x + 9$
(iv) $x^2 - 7x - 18$

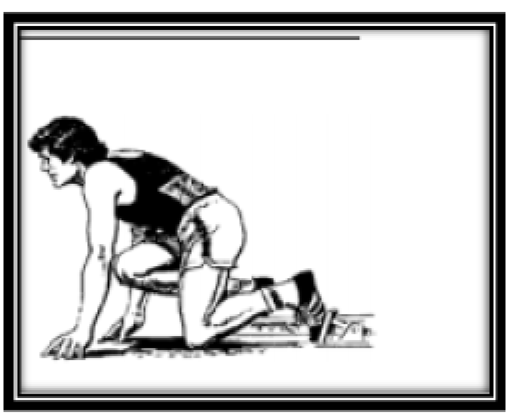
(e) The number of zeroes that polynomial $f(x) = x^2 - 1$ can have is:

(i) 1
(ii) 2
(iii) 0
(iv) 3

20. Case Study Based- 4

110m RACE

A stopwatch was used to find the time that it took a group of students to run 110m.



Time(in sec)	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of students	7	10	15	5	3

(a) Estimate the mean time taken by a student to finish the race.
(i) 54.6
(ii) 63.5
(iii) 43.5
(iv) 50.5

(b) What will be the lower limit of the modal class?
(i) 20
(ii) 40
(iii) 60
(iv) 80

(c) Which of the following are measures of Central Tendency?
(i) Mean
(ii) Median
(iii) Mode
(iv) All of the above

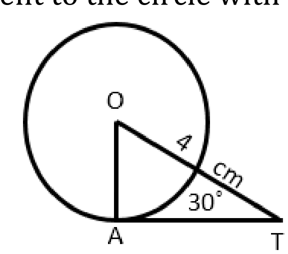
(d) The sum of upper limits of median class and modal class is
(i) 60
(ii) 120
(iii) 80
(iv) 160

(e) How many students finished the race within 1 min?
(i) 18
(ii) 37
(iii) 17
(iv) 8

Part B
All questions are compulsory. In case of internal choices, attempt any one.

Section III
(Q 21 to Q 26 carry 2 marks each)

21. In the given figure, AT is a tangent to the circle with centre O. Find the length of AT.



OR

Show that the tangents at the end points of a diameter of a circle are parallel.

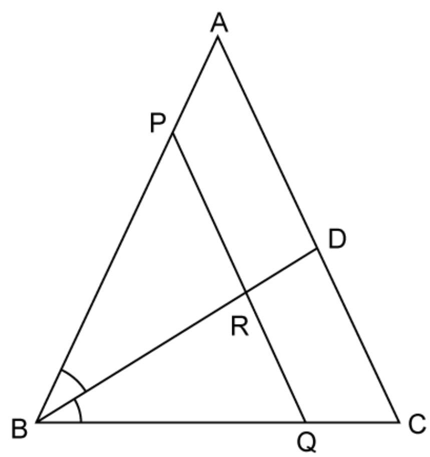
22. Find the zeroes of the quadratic polynomial $x^2 + 7x + 12$ and verify the relationship between the zeroes and its coefficients.

23. A point P is 25 cm away from the centre of a circle, and the length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

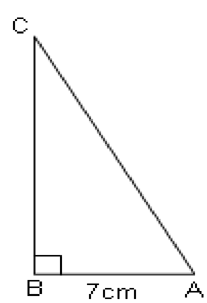
24. The perimeters of two similar triangles ABC and PQR are 32 cm and 24 cm, respectively. If PQ = 12 cm, then find AB.

OR

In ΔABC, the bisector of $\angle B$ meets AC at D. A line PQ || AC meets AB, BC and BD at P, Q and R, respectively. Show that PR $\times$ BQ = QR $\times$ BP.



25. In ΔABC, m$\angle B = 90°$, AB = 7 cm and AC – BC = 1 cm. Determine the values of sin C and cos C.



26. A bag contains 4 white and some red balls. If the probability of drawing a red ball is double that of drawing a white ball, find the number of red balls in the bag.

OR

Cards bearing numbers 1, 3, 5, ..., 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing

(i) A prime number less than 15

(ii) A number divisible by 3 and 5

Section IV

(Q 27 to Q 33 carry 3 marks each)

27. In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108, respectively. Find the minimum number of rooms required, if in each room the same number of participants are to be seated and all of them being in the same subject.

28. The 4th term of an AP is zero. Prove that its 25th term is triple its 11th term.

29. If in a rectangle, the length is increased and breadth is reduced each by 2 metres, then the area is reduced by 28 sq metres. If the length is reduced by 1 metre and breadth is increased by 2 metres, then the area is increased by 33 sq metres. Find the length and breadth of the rectangle.

OR

If three times larger of the two numbers is divided by the smaller one, we get 4 as quotient and 3 as remainder. Also, if seven times the smaller number is divided by the larger one, we get 5 as quotient and 1 as remainder. Find the numbers.

30. If the zeros of the polynomial $f(x) = x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b.

31. The point P divides the join of (2, 1) and (−3, 6) in the ratio 2: 3. Does P lie on the line $x - 5y + 15 = 0$?

OR

Show that a quadrilateral with vertices (0, 0), (5, 0), (8, 4) and (3, 4) is a rhombus. Also find its area.

32. If $\tan\theta = \frac{1}{\sqrt{7}}$, show that $\frac{(\operatorname{cosec}^2\theta - \sec^2\theta)}{(\operatorname{cosec}^2\theta + \sec^2\theta)} = \frac{3}{4}$

33. The cost of fencing a circular field at the rate of Rs. 24 per metre is Rs. 5280. The field is to be ploughed at the rate of Rs. 0.50 per m^2. Find the cost of ploughing the field. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Section V
(Q 34 to Q 36 carry 5 marks each)

34. Draw a line segment AB of length 7 cm. Taking A as the centre, draw a circle of radius 3 cm, and taking B as a centre, draw another circle of radius 2.5 cm. Construct tangents to each circle from the centre of the other circle.

35. A rectangular field is 16 m long and 10 m wide. There is a path of uniform width all around it with an area of 120 m^2. Find the width of the path.

OR

Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, then find the time in which each pipe would fill the cistern.

36. Find the mean, mode and median of the following data:

Class	Frequency
0–10	5
10–20	10
20–30	18
30–40	30
40–50	20
50–60	12
60–70	5

CBSE Board
Class X Mathematics
Sample Paper 5 (Standard) – Solution

Part A
Section I

1.

In ΔABC, DE ∥ BC.

Then, by Basic Proportionality Theorem,

$$\frac{AD}{DB}=\frac{AE}{EC}\Rightarrow\frac{2}{3}=\frac{AE}{4}\Rightarrow AE=\frac{2\times4}{3}=\frac{8}{3}=2.67\text{ cm}$$

OR

Let ΔABC and ΔDEF be similar.

$$\frac{A(\Delta ABC)}{A(\Delta DEF)}=\frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{DF^2}=\frac{25}{64}$$

$$\Rightarrow\frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{DF^2}=\frac{5^2}{8^2}$$

$$\Rightarrow\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=\frac{5}{8}$$

2.

LCM (a, b) × HCF(a, b) = a × b

$\Rightarrow 36\times 2 = a\times 18 \Rightarrow a = \frac{36\times 2}{18} \Rightarrow a = 4$

OR

Here the power of 2 is 2 and the power of 5 is 1.

As 2 > 1

Hence, $\frac{23}{2^2\times5}$ has terminating decimal expansion which terminates after 2 places of decimals.

3.

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}=\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}\qquad\text{..... }(a-b)(a+b)=a^2-b^2$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}=\frac{1+\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}=\sec\theta+\tan\theta$$

OR

$b^2x^2 - a^2y^2 = b^2(a\sec\theta)^2 - a^2(b\tan\theta)^2 = b^2a^2(\sec^2\theta - \tan^2\theta)^2$

$$= b^2a^2\left(\sec^2\theta - \frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$= b^2a^2\sec^2\theta(1 - \sin^2\theta) \quad \ldots\ldots \frac{1}{\cos^2\theta} = \sec^2\theta$$

$$= b^2a^2\sec^2\theta\cos^2\theta \quad \ldots\ldots \cos^2\theta = \frac{1}{\sec^2\theta}$$

$$= a^2b^2$$

4.

ΔABC is a right triangle, right angled at C.

We know that, sum of all the angles of a triangle is 180°

$\angle A + \angle B + \angle C = 180° \Rightarrow \angle A + \angle B = 180° - 90° = 90°$

$\therefore \cos(A + B) = \cos 90° = 0$

5.

The given point is A(5, −12), and let O(0,0) be the origin.

Then, $AO = \sqrt{(5-0)^2 + (-12-0)^2}$

$= \sqrt{5^2 + (-12)^2} = \sqrt{25+144} = \sqrt{169}$

$= 13$ units

OR

Let A(0, 4), B(0, 0) and C(3, 0).

$AB = \sqrt{(0-0)^2 + (0-4)^2} = 4$

$BC = \sqrt{(3-0)^2 + (0-0)^2} = 3$

$AC = \sqrt{(3-0)^2 + (0-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

$\therefore$ Perimeter of a $\Delta ABC = AB + BC + AC = 4 + 3 + 5 = 12$

6.

The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

$$\therefore \frac{ar\Delta ABC}{ar\Delta PQR} = \frac{AB^2}{PQ^2} = \frac{16}{81}$$

7.

Sum of n natural numbers = 1 + 2 + 3 + ... + n

Here, a = 1, d = 2 − 1 = 1

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$\therefore$ Sum of natural numbers

$$= \frac{n}{2}\left[2 \times 1 + (n-1)d\right] = \frac{n}{2}\left[2 + (n-1)\right] = \frac{n(n+1)}{2}$$

8.

When two coins are tossed simultaneously, all possible outcomes are HH, HT, TH, TT.

Total number of possible outcomes = 4

Let E_2 be the event of getting at the most one head.

So, the favourable outcomes are HT, TH, TT.

Number of favourable outcomes = 3

$\therefore$ P (getting at the most 1 head) = P (E_2) = $\frac{3}{4}$

9.

The A.P. is 3, 8, 13, 18, …

⇒ a = 3, d = 5 and an = 88

⇒ a + (n – 1)d = 88

⇒ 3 + (n – 1) × 5 = 88

⇒ 5(n – 1) = 85

⇒ n – 1 = 17

⇒ n = 18

OR

The A.P. is (5a – x), 6a, (7a + x),…

Let A be the first term and D be the difference.

A = 5a – x, D = 6a – 5a + x = a + x, n = 11

an = A + (n – 1)D

⇒a11 = 5a – x + 10(a + x)

⇒a11 = 5a – x + 10a + 10x

⇒a11 = 15a + 9x

10.

x, x + 3, x + 6, x + 9 and x + 12 is 10.

of the numbers = x + x + 3 + x + 6 + x + 9 + x + 12 = 5x + 30 Mean = 10

$\Rightarrow \frac{5x+30}{5} = 10 \Rightarrow 5x + 30 = 50 \Rightarrow 5x = 20 \Rightarrow x = 4$

11.

If the diagonals of a quadrilateral divide each other proportionally, then it is a rectangle.

12.

(a cos θ + b sin θ, 0) and (0, a sin θ − b cos θ)

Distance = $\sqrt{(a\cos\theta + b\sin\theta)^2 + (b\cos\theta - a\sin\theta)^2}$

= $\sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + b^2\cos^2\theta + a^2\sin^2\theta - 2ab\sin\theta\cos\theta}$

= $\sqrt{a^2 + b^2}$

13.

The length of equal sides of an isosceles right triangle is $4\sqrt{2}$ cm.

Hypotenuse = $\sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2} = \sqrt{64} = 8\text{ cm}$

14.

$196 = 2^2 \times 7^2$

The sum of the exponents of the prime factors in the prime factorization of 196 is 4.

15.

(a, b) are co-primes, if HCF of the two numbers is 1.

So, (18, 25) are co-prime.

16.

A point P divides the join of A(5, −2) and B(9, 6) in the ratio 3:1.

The coordinates of P are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right) = \left(\frac{3\times9+1\times5}{3+1}, \frac{3\times6+1\times(-2)}{3+1}\right) = (8,4)$

Section II

17.

(a)

In the first round 1, 3, 5, 7 and 9 numbered triangles are removed.
This means, Rahul is the alternate removing triangles.
In the second round 4 and 8 numbered triangles are removed.
In the third round 6 numbered triangle is removed.
So, 2 numbered triangle will be left in the last.

(b)

Removed triangles numbered in sequence are
3, 6, 9, 4, 8, 5, 2 and 7.
So, 1 numbered triangle will be left in the end.

(c)

Removed triangles numbered in sequence are
8, 3, 9, 2, 6, 10, 11, 7, 4, 1
So, 5 numbered triangle will be left in the end.

(d)

The perimeters of the triangle will follow the below pattern
3, 6, 9, 12, 15, 18, 21, 24 and 27
$\Longrightarrow$ They are multiples of 3.

(e)

We know that, area of an equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2$

The ratio of the areas of first two triangles whose sides are 3 and 4 is 9: 16
The ratio of the areas of two triangles whose sides are 6 and 8 is 36: 64 = 9: 16.

Hence, they are in proportion as their ratio is same and that is 9: 16.

18.

(a) Angles ∠LKM and ∠JKL are called as Linear Pair of angles.

(b) m∠LKM + m∠JKL =180°...... Linear Pair
$\Longrightarrow$ 2x – 15 + m∠LKM=180°
$\Longrightarrow$ m∠LKM= 195° –2x

(c) In △LKM,
m∠LKM + m∠LMK + m∠KLM=180° ...angle sum property of a triangle
$\Longrightarrow$ 195° – 2x + 50 + x = 180°
$\Longrightarrow$ x = 65° = m∠KLM

(d) m∠LKM = 195° –2x = 195 – 2(65) = 195 – 130 = 65°
In △LKM, m∠LKM = m∠KLM = 65°
$\Longrightarrow$△LKM is an isosceles triangle.

(e) m∠LKJ = 2x – 15 = 2(65) – 15 = 130 – 15 = 115°

19.

(a)
$x^2 + 4x + k = 0$
$\Rightarrow$ a = 1, b = 4, c = k
The equation $x^2 + 4x + k = 0$ has real and distinct roots i.e. $b^2 - 4ac > 0$
$\Rightarrow b^2 - 4ac = 4^2 - 4k = 16 - 4k$
$\Rightarrow 16 - 4k > 0$
$\Rightarrow 4k < 16 \Rightarrow k < 4$

(b)

Let the required polynomial be ax^2+bx+c, and let its zeroes be α and β.

$\alpha+\beta=\frac{1}{4}=\frac{-b}{a}$ and $\alpha\beta=-1=\frac{-4}{4}=\frac{c}{a}$

If $a=4k$, then $b=-k$, $c=-4k$

Therefore, the quadratic polynomial is $k(4x^2 - x - 4)$, where k is a real number

(c)

Given equation is $px^2 + 6x + 1 = 0$

Here, a = p, b = 6 and c = 1

The given equation will have real roots, if $b^2 - 4ac \geq 0$.

$\Rightarrow (6)^2 - 4(p)(1) \geq 0$

$\Rightarrow 36 - 4p \geq 0$

$\Rightarrow 36 \geq 4p \Rightarrow p \leq 9$

(d)

A highway underpass is parabolic in shape and a parabola is the graph that results from $p(x) = ax^2 + bx + c$ which has two zeroes (as it is a quadratic polynomial).
Product of zeroes = $9 \times -2 = -18$ and sum of the zeroes = $9 + (-2) = 7$

x^2 – (sum of zeroes)x + product of zeroes
$= x^2 - 7x - 18$

(e)

$f(x) = x^2 - 1 = x^2 + 0x - 1$ is a Quadratic Polynomial.

The number of zeroes that f(x) can have is 2.

20.

(a)

Time (in sec)	No. of students(f)	X	fx
20 – 40	7	30	210
40 – 60	10	50	500
60 – 80	15	70	1050
80 – 100	5	90	450
100 – 120	3	110	330
	$\Sigma f = 40$		$\Sigma fx = 2540$

Mean time taken by a student to finish the race = 2540/40 = 63.5 seconds

(b) The modal class is 60 – 80 as it has the highest frequency i.e 15.
Lower limit of the modal class = 60

(c) Mean, Median and Mode are measures of central tendency.

(d)

Time (in sec)	No. of students(f)	cf
20 – 40	7	7
40 – 60	10	17
60 – 80	15	32
80 – 100	5	37
100 – 120	3	40
	N= Σf = 40	

Here N/2 = 40/2 = 20, Median Class = 60 – 80, Modal Class = 60 – 80
Sum of upper limits of median class and modal class = 80 + 80 = 160

(e) Number of students who finished the race within 1 min = 7 + 10 = 17

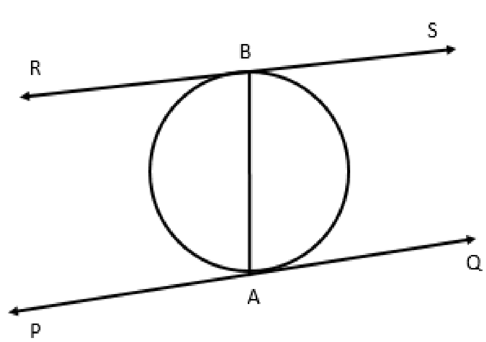
Part B
Section III

21. It is known that radius is perpendicular to the tangent at the point of contact.
Therefore, m$\angle$OAT = 90^o.
In ΔOAT,

$$\cos 30° = \frac{AT}{OT} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{OT} \Rightarrow AT = \frac{\sqrt{3}}{2} \times OT = \frac{\sqrt{3}}{2} \times 4 = 2\sqrt{3} \text{ cm}$$

OR

Let AB be the diameter of the given circle, and let PQ and RS be the tangent lines drawn to the circle at points A and B respectively.
Since tangent at a point to a circle is perpendicular to the radius through the point of contact.
Therefore, AB is perpendicular to both PQ and RS.
$\Rightarrow \angle$PAB = 90° and $\angle$ABS = 90°
$\Rightarrow \angle$PAB = $\angle$ABS
But, these are a pair of alternate interior angles.
Therefore, PQ is parallel to RS.



22.

$x^2 + 7x + 12 = x^2 + 3x + 4x + 12 = x(x + 3) + 4(x + 3) = (x + 3)(x + 4)$

$\therefore$ −3 and −4 are the zeroes of the given polynomial.

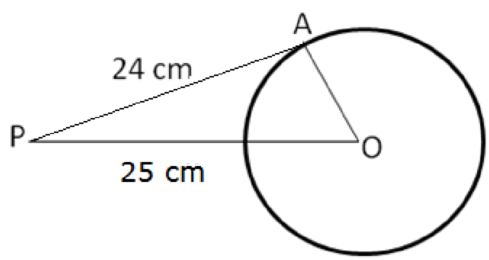
Sum of zeroes = $-3 - 4 = -7 = \frac{-7}{1} = \frac{-\text{coefficient of x}}{\text{coefficient of } x^2}$

Product of zeroes = $(-3)(-4) = 12 = \frac{12}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

23.

PA is the tangent to the circle with centre O and radius, such that PO = 25 cm, PA = 24 cm

In ΔPAO, $\angle A = 90°$………. Since tangent $\perp$ radius,



By Pythagoras' theorem,

$$PO^2 = PA^2 + AO^2 \Rightarrow OA^2 = PO^2 - PA^2 = (25)^2 - (24)^2 = 49$$

$\therefore\ OA = 7cm$

Hence, the radius of the circle is 7 cm.

24.

It is given that ΔABC and ΔPQR are similar triangles, so the corresponding sides of both triangles are proportional.

So, $\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{AB}{PQ}$

Let, $AB = x$ cm

Then, $\frac{x}{12} = \frac{32}{24} \Rightarrow x = \frac{32 \times 12}{24} = 16$ cm

Hence, AB = 16 cm.

OR

Given ΔABC, the bisector of $\angle B$ meets AC at D, line PQ || AC meets AB, BC and BD at P, Q and R, respectively.

To prove: PR $\times$ BQ = QR $\times$ BP

Proof: In ΔBQP,

BR is the bisector of $\angle$B.

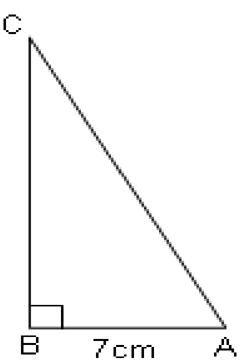
Therefore, by the angle bisector theorem,

$$\frac{BQ}{BP} = \frac{QR}{PR}$$

$\Rightarrow PR \times BQ = QR \times BP$

25.

In ABC, we have



$AC^2 = BC^2 + AB^2$

$(1 + BC)^2 = BC^2 + AB^2$

$\Rightarrow 1 + BC^2 + 2BC = BC^2 + AB^2$

$\Rightarrow 1 + 2BC = 7^2$

$\Rightarrow 2BC = 48$

$\Rightarrow BC = 24$ cm

$\Rightarrow AC = 1 + BC = 1 + 24 = 25$ cm

Hence, $\sin C = \frac{AB}{AC} = \frac{7}{25}$ and $\cos C = \frac{BC}{AC} = \frac{24}{25}$

26.

Let the number of red balls be x. Then,

P (drawing a white ball) $= \frac{4}{4+x}$, and P (drawing a red ball) $= \frac{x}{4+x}$

$$\therefore \frac{x}{4+x} = 2\left(\frac{4}{4+x}\right)$$

$\Rightarrow x(4+x) = 8(4+x) \Rightarrow (x-8)(4+x) = 0 \Rightarrow x = -4$ or $x = 8$

$x \neq -4$........no. of balls can't be negative

$\therefore\ x = 8$

Hence, the number of red balls is 8.

OR

There are 18 cards having numbers 1, 3, 5, …, 35 kept in a bag.

(i) Prime numbers less than 15 are 3, 5, 7, 11, 13.

There are 5 numbers.

$\therefore$ Probability that a card drawn bears a prime number less than 15 = $\frac{5}{18}$

(ii) There is 1 number 15, which is divisible by both 3 and 5.

$\therefore$ Probability of drawing a card bearing a number divisible by both 3 and 5 is $\frac{1}{18}$.

Section IV

27.

To find the minimum number of rooms required, first find the maximum number of participants which can be accommodated in each room such that the number of participants in each room is the same.

This can be determined by finding the HCF of 60, 84 and 108.

$60 = 2^2 \times 3 \times 5,\ 84 = 2^2 \times 3 \times 7,\ 108 = 2^2 \times 3^3$

$\text{H.C.F.} = 2^2 \times 3 = 12$

So, the minimum number of rooms required

$= \frac{\text{Total number of participants}}{12} = \frac{60+84+108}{12} = 21$

28.

Let the first term of the given AP = a and common difference = d

then, $T_n = a + (n-1)d$

$\Rightarrow T_4 = a + (4-1)d,\ T_{25} = a + (25-1)d, \text{ and } T_{11} = a + (11-1)d$

$\Rightarrow T_4 = a + 3d,\ T_{25} = a + 24d, \text{ and } T_{11} = a + 10d$

Now, $T_4 = 0 \Rightarrow a + 3d = 0 \Rightarrow a = -3d$

$\therefore T_{25} = a + 24d = (-3d + 24d) = 21d$

And $T_{11} = a + 10d = -3d + 10d = 7d$

$\therefore T_{25} = 21d = 3 \times (7d) = 3 \times T_{11}$

Hence, the 25^{th} term is triple its 11^{th} term.

29.

Let the length and breadth of the rectangle be x and y respectively.

So the original area of the rectangle = xy

According to the question,

$(x + 2)(y - 2) = xy - 28$

$\Rightarrow xy - 2x + 2y - 4 = xy - 28 \Rightarrow 2x - 2y = 24$ .....(i)

And, $(x - 1)(y + 2) = xy + 33$

$\Rightarrow xy + 2x - y - 2 = xy + 33 \Rightarrow 2x - y = 35$ ....(ii)

Subtracting (i) from (ii), we get y = 11

Substituting this value in (ii), we get

$2x - 11 = 35 \Rightarrow 2x = 46 \Rightarrow x = 23$

Thus, the length and breadth of the rectangle are 23 metres and 11 metres, respectively.

OR

Let the larger number be x and smaller be y. We know that
Dividend = Divisor × Quotient + Remainder
According to the question,
$3x = 4y + 3$
$3x - 4y = 3$........(i)
Also,
$7y = 5x + 1$
$5x - 7y = -1$.......(ii)
Solving (i) and (ii)
we get x = 25 and y = 18
The required numbers are 25 and 18.

30.

Since a – b, a and a + b are the zeros of $f(x) = x^3 - 3x^2 + x + 1$.

$$\therefore (a-b)+a+(a+b) = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\Rightarrow 3a = -\frac{-3}{1} \Rightarrow 3a = 3 \Rightarrow a = 1$$

$$\text{And, } (a-b)\times a\times(a+b) = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

$$\Rightarrow a(a^2-b^2) = -\frac{1}{1} \Rightarrow 1(1-b^2) = -1 \Rightarrow b^2 = 2 \Rightarrow b = \pm\sqrt{2}$$

31.

Given, the point P divides the join of (2, 1) and (−3, 6) in the ratio 2 : 3.

Co-ordinates of the point P $\equiv \left(\frac{2\times(-3)+3\times 2}{2+3}, \frac{2\times 6+3\times 1}{2+3}\right) \equiv \left(\frac{-6+6}{5}, \frac{12+3}{5}\right) \equiv (0,3)$

Now, the given equation is x − 5y + 15 = 0.

Substituting x = 0 and y = 3 in this equation, we have

L.H.S. = 0 − 5(3) + 15 = −15 + 15 = 0 = R.H.S.

Hence, the point P lies on the line x − 5y + 15 = 0.

OR

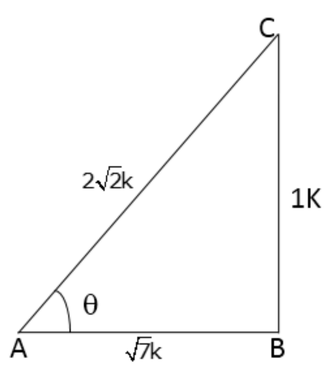
$$\left.\begin{aligned}
AB &= \sqrt{(5-0)^2+(0-0)^2} = \sqrt{25+0} = 5\\
BC &= \sqrt{(8-5)^2+(4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5\\
CD &= \sqrt{(8-3)^2+(4-4)^2} = \sqrt{25+0} = 5\\
DA &= \sqrt{(0-3)^2+(0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5\\
AC &= \sqrt{(8-0)^2+(4-0)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}\\
BD &= \sqrt{(3-5)^2+(4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}
\end{aligned}\right\}$$

Now, AB = BC = CD = DA and AC $\neq$ BD.
Therefore, ABCD is a rhombus.

$$\text{Area} = \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4\sqrt{5} \times 2\sqrt{5} = \frac{1}{2} \times 40 = 20 \text{ sq.units}$$

32.

Let us draw a ΔABC in which ∠B = 90° and ∠A = θ.



$$\text{Given}: \tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{7}}$$

Let $BC = 1k$ and $AB = \sqrt{7}k$, where k is positive

By pythagoras theorem, we have

$$AC^2 = \left(AB^2 + BC^2\right) \Rightarrow AC^2 = \left[\left(\sqrt{7}k\right)^2 + \left(1k\right)^2\right] = 7k^2 + 1k^2 = 8k^2 \Rightarrow AC = 2\sqrt{2}k$$

$$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{2\sqrt{2}k}{1k} = 2\sqrt{2} \text{ and } \sec\theta = \frac{AC}{AB} = \frac{2\sqrt{2}k}{\sqrt{7}k} = \frac{2\sqrt{2}}{\sqrt{7}}$$

$$\Rightarrow \frac{\left(\operatorname{cosec}^2\theta - \sec^2\theta\right)}{\left(\operatorname{cosec}^2\theta + \sec^2\theta\right)} = \left[\frac{\left(2\sqrt{2}\right)^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(2\sqrt{2}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}\right] = \frac{\left(8 - \frac{8}{7}\right)}{\left(8 + \frac{8}{7}\right)} = \frac{\left(\frac{48}{7}\right)}{\left(\frac{64}{7}\right)} = \frac{48}{64} = \frac{3}{4}$$

$$\text{Hence, } \left(\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}\right) = \frac{3}{4}$$

33.

1 m of fencing costs Rs. 24.

Hence for Rs. 5280, the length of fencing = $\frac{5280}{24}$ = 220 m

$\Rightarrow$ Circumference of the field = 220 m

$\therefore 2\pi r = 220$

$\Rightarrow 2 \times \frac{22}{7} \times r = 220 \quad \Rightarrow r = \frac{220 \times 7}{44} = 35$ m

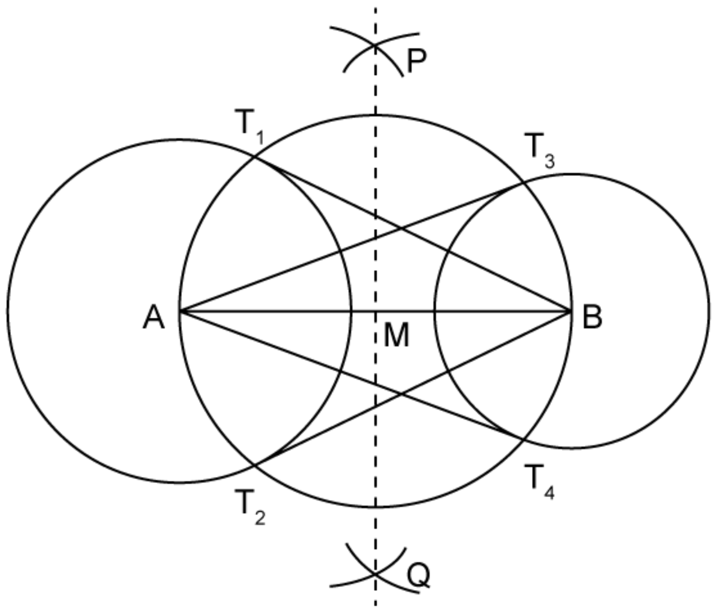
Area of the field = $\pi r^2 = \pi(35)^2 = 1225\pi$ m^2

Cost of ploughing = Rs. 0.50 per m^2

Total cost of ploughing the field = Rs. $1225\,\pi \times 0.50 = \frac{1225 \times 22 \times 1}{7 \times 2}$ = Rs. 1925

Section V

34.



Steps of construction:

- **i.** Draw a line segment AB = 7 cm.
- **ii.** Taking A as the centre and radius 3 cm, a circle is drawn.
- **iii.** With centre B and radius 2.5 cm, another circle is drawn.
- **iv.** With centre A and radius more than $\frac{1}{2}AB$, arcs are drawn on both sides of AB.
- **v.** With centre B and the same radius [as in step (iv)], arcs are drawn on both sides of AB intersecting the previous arcs at P and Q.
- **vi.** Join PQ which meets AB at M.
- **vii.** With centre M and radius AM, a circle is drawn which intersects the circle with centre A at T_1 and T_2 and the circle with centre B at T_3 and T_4
- **viii.** Join AT_3, AT_4, BT_1 and BT_2

Thus, AT_3, AT_4, BT_1 and BT_2 are the required tangents.

35.

Let the width of the path be x metres.

Then, Area of the path = 16 × 10 – (16 – 2x) (10 – 2x) = 120

$$\Rightarrow 16\times10-\left(160-32x-20x+4x^2\right)=120$$

$$\Rightarrow 160-160+32x+20x-4x^2=120$$

$$\Rightarrow -4x^2+52x-120=0$$

$$\Rightarrow 2x^2-26x+60=0$$

$\Rightarrow x^2 - 13x + 30 = 0$

$\Rightarrow x^2 - 10x - 3x + 30 = 0$

$\Rightarrow x(x-10) - 3(x-10) = 0$

$\Rightarrow (x-10)(x-3) = 0$

$\Rightarrow x - 10 = 0 \text{ or } x - 3 = 0$

$\Rightarrow \ x = 10 \text{ or } x = 3$

Hence, the required width is 3 metres as x cannot be 10 m since the width of the path cannot be greater than or equal to the width of the field.

OR

Let the faster pipe take x minutes to fill the cistern.

Then the other pipe takes (x + 3) minutes.

$$\frac{1}{x} + \frac{1}{(x+3)} = \frac{13}{40} \Rightarrow \frac{(x+3)+x}{x(x+3)} = \frac{13}{40}$$

$\Rightarrow 40(2x+3) = 13(x^2 + 3x)$

$\Rightarrow 13x^2 - 41x - 120 = 0$

$\Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x(x-5) + 24(x-5) = 0$

$\Rightarrow (x-5)(13x+24) = 0$

$\Rightarrow x = 5 \quad \text{or} \quad x = \frac{-24}{3}$

$\therefore x = 5 \ \text{(Time cannot be negative)}$

If the faster pipe takes 5 minutes to fill the cistern, then the other pipe takes (5 + 3) minutes = 8 minutes

36.

Let the assumed mean be 35, h = 10. Now, we have

Class	Frequency f_i	Mid-value x_i	$u_i = \left(\frac{x_i - A}{h}\right)$	C.F	$f_i u_i$
0–10	5	5	-3	5	-15
10–20	10	15	-2	15	-20
20–30	18	25	-1	33	-18
30–40	30	35 = A	0	63	0
40–50	20	45	1	83	20
50–60	12	55	2	95	24
60–70	5	65	3	100	15
	N = 100				$\Sigma f_i u_i = 6$

(i) Mean $\bar{x} = A + h\left(\frac{\Sigma f_i u_i}{N}\right) == 35 + 10 \times \left(\frac{6}{100}\right) = 35 + 0.6 = 35.6$

(ii) N = 100, $\frac{N}{2} = 50$

Cumulative frequency just after 50 is 63.

∴ Median class is 30–40.

∴ l = 30, h = 10, N = 100, c = 33, f = 30

$$\therefore \text{Median } M_e = l + h\left(\frac{\frac{N}{2} - c}{f}\right) = 30 + 10\left(\frac{50-33}{30}\right) = 30 + 10\left(\frac{17}{30}\right) = 30 + 5.67 = 35.67$$

(iii) Mode = 3 × median - 2 × mean
= 3 × 35.67 - 2 × 35.6
= 107.01 - 71.2 = 35.81

Thus, mean = 35.6, median = 35.67 and mode = 35.81