SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	2(2)	–	1(3)	–	**3(5)**
2.	Inverse Trigonometric Functions	1(1)	1(2)	–	–	**2(3)**
3.	Matrices	2(2)	1(2)	–	–	**3(4)**
4.	Determinants	1(1)*	–	–	1(5)*	**2(6)**
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	**3(9)**
7.	Integrals	1(1)*	1(2)*	1(3)*	–	**3(6)**
8.	Application of Integrals	1(1)	1(2)	1(3)	–	**3(6)**
9.	Differential Equations	1(1)*	1(2)*	1(3)	–	**3(6)**
10.	Vector Algebra	3(3)#	1(2)*	–	–	**4(5)**
11.	Three Dimensional Geometry	4(4)	–	–	1(5)*	**5(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	1(4)	2(4)	–	–	**3(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : 041

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part -A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Write the cofactor of the element a_{31} in $A = \begin{pmatrix} 3 & 2 & 6 \\ 5 & 0 & 7 \\ 3 & 8 & 5 \end{pmatrix}$.

OR

If A is a square matrix of order 3 and $|2A| = k|A|$, then find the value of k.

2. Evaluate : $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

3. Find the integrating factor of the differential equation $\left\{\frac{e^{-2\sqrt{x}}}{\sqrt{x}} + \frac{y}{\sqrt{x}}\right\}\frac{dx}{dy} = 1 (x \neq 0)$.

OR

Find order and degree of the equation $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} + 4y = \sin x$.

4. If $f(x) = x^2 - 4x + 1$, find $f(A)$, where $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$.

5. Find the unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

OR

Find the projection of the vector $7\hat{i} + \hat{j} - 4\hat{k}$ on $2\hat{i} + 6\hat{j} + 3\hat{k}$.

6. Check whether the function $f(x) = x^3 - 3x^2 - x$ is one-one or not?

7. Evaluate : $\int_1^3 (x-1)(x-2)(x-3)dx$

OR

Evaluate : $\int_{-\pi}^{\pi} x^{10} \sin^7 x \; dx$

8. Check whether the lines having direction ratios $(\sqrt{3}-1, -\sqrt{3}-1, 4)$ and $(-\sqrt{3}+1, \sqrt{3}+1, -4)$ are perpendicular to each other.

9. If the vectors $3\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 4x\hat{j} + y\hat{k}$ are parallel, then the values of x and y.

OR

Find the point which divides the line segment joining the points (–2, 3, 5) and (1, 2, 3) in the ratio 2 : 3 externally.

10. Find vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$.

11. Let $A = \{1, 2, 3, 4\}$. Show that $f = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$ is a bijection from A to A?

12. If $\vec{a}$ and $\vec{b}$ are unit vectors enclosing an angle θ and $|\vec{a}+\vec{b}| < 1$, find the value of θ.

13. Find the area of the region bounded between the line $x = 2$ and the parabola $y^2 = 8x$.

14. Find equation of a line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

15. If $A = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$, such that $A + B + C$ is a zero matrix, then find the matrix C.

16. If the line joining (2, 3, –1) and (3, 5, –3) is perpendicular to the line joining (1, 2, 3) and (3, 5, λ), then find the value of λ.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A teacher arranged a surprise game for students of a classroom having 5 students, namely Amit, Aruna, Eklavya, Yash and Samina. He took a bag containing tickets numbered 1 to 11 and told each student to draw two tickets without replacement.

(i) Probability that both ticket drawn by Amit shows even number, is

(a) 1/11 (b) 2/11 (c) 3/11 (d) 4/11

(ii) Probability that both tickets drawn by Aruna shows odd number, is

(a) 1/11 (b) 2/11 (c) 3/11 (d) 4/11

(iii) When tickets are drawn by Eklavya, find the probability that number on one ticket is a multiple of 4 and on other ticket is a multiple 5.

(a) 4/55 (b) 6/55 (c) 7/55 (d) None of these

(iv) When tickets are drawn by Yash, find the probability that number on one ticket is a prime number and on other ticket is a multiple of 4 .

(a) 3/11 (b) 5/11 (c) 6/11 (d) 2/11

(v) When tickets are drawn by Samina, find the probability that first ticket drawn shows an even number and second ticket drawn shows an odd number.

(a) 2/11 (b) 3/11 (c) 5/11 (d) 8/11

18. An open water tank of aluminium sheet of negligible thickness, with a square base and vertical sides, is to be constructed in a farm for irrigation. It should hold 4000 l of water, that comes out from a tube well.

Based on above information, answer the following questions.

(i) If the length, width and height of the open tank be x, x and y m respectively, then surface area of tank is given by

(a) $S = x^2 + 2xy$ (b) $S = 2x^2 + 4xy$ (c) $S = 2x^2 + 2xy$ (d) $S = 2x^2 + 8xy$

(ii) The relation between x and y is

(a) $x^2y = 4$ (b) $xy^2 = 4$ (c) $x^2y^2 = 4$ (d) $xy = 4$

(iii) The outer surface area of tank will be minimum when depth of tank is equal to

(a) half of its width (b) its width (c) $\left(\frac{1}{4}\right)^{th}$ of its width (d) $\left(\frac{1}{3}\right)^{rd}$ of its width

(iv) The cost of material will be least when width of tank is equal to

(a) half of its depth (b) twice of its depth (c) $\left(\frac{1}{4}\right)^{th}$ of its depth (d) thrice of its depth

(v) If cost of aluminium sheet is ₹ 360/m^2, then the minimum cost for the construction of tank will be
(a) ₹ 2320 (b) ₹ 3320 (c) ₹ 4320 (d) ₹ 5320

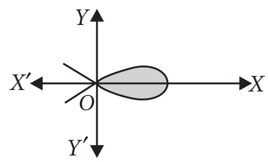
PART - B

Section - III

19. Find the equations of the tangent and the normal to the curve $y = x^3$ at the point $P(1, 1)$.

20. Express $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$, $x \in \left(\frac{-\pi}{2}, \frac{3\pi}{2}\right)$ in the simplest form.

21. Find the area of region bounded by the curve $y^2 = x(1-x)^2$, shown in following figure.



22. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. If an orator is chosen at random, find the probability that a male person is selected. Assume that there are equal number of men and women.

23. Evaluate : $\int \frac{1}{\sqrt{1-\sin x}}\,dx$

OR

Evaluate : $\int \frac{dx}{1-2\sin x\cos x}$

24. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and A adj $A = AA^T$, then find the value of $5a + b$.

25. Find the projection of the vector $2\hat{i}-3\hat{j}-6\hat{k}$ on vector joining the points (5, 6, – 3) and (3, 4, – 2).

OR

If $\vec{a} = 4\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{b} = 3\hat{i}+2\hat{k}$, find $|\vec{b}\times 2\vec{a}|$.

26. Suppose that two cards are drawn at random from a deck of 52 cards. Let X be the number of aces obtained. Then, find the probability distribution of X.

27. If $y = \sin^{-1}x$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

28. Find the solution of the differential equation $\frac{dy}{dx} = \frac{y(1+x)}{x(y-1)}$.

OR

Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x+4y; y=0, x=0$.

Section - IV

29. Show that the curve for which the normal at every point passes through a fixed point is a circle.

30. Find the point on the parabola $y^2 = 2x$ which is closed to the point (1, 4).

31. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, then find $\frac{dy}{dx}$.

OR

Differentiate $\dfrac{2^{x^2+x+1}\cdot\sqrt{4x-1}}{(x^2-1)^{3/2}}$ w.r.t. x.

32. Let $f: A \to B$ be a function defined as $f(x)=\dfrac{2x+3}{x-3}$, where $A = R - \{3\}$ and $B = R - \{2\}$. Is the function f one-one and onto?

33. Find the area of the region bounded by the curve $y = x^2 + x$, the x-axis and the lines $x = 2$, $x = 5$.

34. Evaluate : $\displaystyle\int_0^1 \tan^{-1} x\, dx$

OR

Evaluate : $\displaystyle\int_1^2 \frac{dx}{x(1+x^2)}$

35. If $f(x)$ is continuous at $x = 0$, where

$$f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & \text{for } x > 0 \\ \dfrac{4(1-\sqrt{1-x})}{x}, & \text{for } x < 0 \end{cases}$$, then find $f(0)$.

Section-V

36. Find the vector and cartesian equation of the line through the point $\hat{i}+\hat{j}-3\hat{k}$ and perpendicular to the lines $\vec{r}=2\hat{i}-3\hat{j}+\lambda(2\hat{i}+\hat{j}-3\hat{k})$ and $\vec{r}=3\hat{i}-5\hat{j}+\mu(\hat{i}+\hat{j}+\hat{k})$.

OR

The four points $A(3, 2, -5)$, $B(-1, 4, -3)$, $C(-3, 8, -5)$ and $D(-3, 2, 1)$ are coplanar. Find the equation of the plane containing them.

37. Find the minimum value of $Z = 3x + 4y + 270$ subject to the constraints

$x + y \le 60$
$x + y \ge 30$
$x \le 40,\ y \le 40$
$x \ge 0,\ y \ge 0$

OR

Find the point for which the maximum value of $Z = x + y$ subject to the constraints $2x + 5y \le 100$, $\dfrac{x}{25}+\dfrac{y}{50} \le 1,\ x \ge 0,\ y \ge 0$ is obtained.

38. If $A=\begin{pmatrix} 2 & 3 & 7 \\ 3 & -2 & -1 \\ 1 & 1 & 2 \end{pmatrix}$, find A^{-1}. Using A^{-1} solve the following system of equations :

$2x + 3y + 7z = 12$
$3x - 2y - z = 0$
$x + y + 2z = 4$

OR

If $A=\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$, find (adj A^{-1}).

SOLUTIONS

1. We have, $A = \begin{pmatrix} 3 & 2 & 6 \\ 5 & 0 & 7 \\ 3 & 8 & 5 \end{pmatrix}$

Cofactor of $a_{31} = C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 6 \\ 0 & 7 \end{vmatrix} = 14$

OR

Given, A is a square matrix of order 3

$\therefore \quad |2A| = 2^3|A| = 8|A| = k|A| \quad$ (Given)

$\Rightarrow k = 8$

2. We have, $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right\}$

$= \tan^{-1}\left\{2\cos\left(2 \times \frac{\pi}{6}\right)\right\} \qquad \left[\because \sin^{-1}\frac{1}{2} = \frac{\pi}{6}\right]$

$= \tan^{-1}\left\{2\cos\frac{\pi}{3}\right\} = \tan^{-1}\left[2 \times \frac{1}{2}\right] = \tan^{-1} 1 = \frac{\pi}{4}$

3. We have, $\frac{dy}{dx} - \frac{1}{\sqrt{x}} \cdot y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

This is of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{-1}{\sqrt{x}}$ and $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$.

I.F. $= e^{\int P dx} = e^{\int \frac{-1}{\sqrt{x}} dx} = e^{-2\sqrt{x}}$.

OR

Highest order derivative is $\left(\frac{d^3y}{dx^3}\right)$. So, its order is 3 and degree is 4.

4. We have, $f(x) = x^2 - 4x + 1 \Rightarrow f(A) = A^2 - 4A + I$

$\because \quad A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \quad \therefore \quad A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

$\therefore \quad f(A) = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

5. The unit vector in the direction of a vector $\vec{a}$ is given by $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$. Now, $|\vec{a}| = \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{29}$

Therefore, $\hat{a} = \frac{2\hat{i} + 3\hat{j} + 4\hat{k}}{\sqrt{29}} = \frac{2}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}$

OR

Let $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

Then, $\vec{a} \cdot \vec{b} = (7\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})$

$= 14 + 6 - 12 = 8$

Also, $|\vec{b}| = \sqrt{2^2 + 6^2 + 3^2} = 7$

$\therefore$ Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{7}$

6. We have $f(x) = x^3 - 3x^2 - x$

Clearly, $f(1) = 1 - 3 - 1 = -3$

and $f(-1) = -1 - 3 + 1 = -3$

$\Rightarrow$ Distinct elements have same image, therefore f is not one-one.

7. Let $I = \int_1^3 (x-1)(x-2)(x-3)dx$

$= \int_1^3 (x^3 - 6x^2 + 11x - 6)dx = \left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{11x^2}{2} - 6x\right]_1^3$

$= \left[\frac{81}{4} - \frac{162}{3} + \frac{99}{2} - 18 - \left(\frac{1}{4} - \frac{6}{3} + \frac{11}{2} - 6\right)\right] = 0$

OR

Let $I = \int_{-\pi}^{\pi} x^{10} \sin^7 x \, dx$

Also, let $f(x) = x^{10}\sin^7 x$

Then, $f(-x) = (-x)^{10}[\sin(-x)]^7 = -x^{10}\sin^7 x = -f(x)$

$\Rightarrow f(x)$ is an odd function.

$\therefore \quad I = \int_{-\pi}^{\pi} x^{10}\sin^7 x \, dx = 0$

8. Here, $a_1 = \sqrt{3} + 1,\ b_1 = -\sqrt{3} + 1,\ c_1 = 4$ and

$a_2 = -\sqrt{3} + 1,\ b_2 = \sqrt{3} + 1,\ c_2 = -4$

Since, $\frac{a_1}{a_2} = -1,\ \frac{b_1}{b_2} = -1$ and $\frac{c_1}{c_2} = -1$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow$ Direction ratios of lines are proportional.

Hence, the lines are parallel to each other.

9. Let $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 4x\hat{j} + y\hat{k}$

Since, $\vec{a}$ and $\vec{b}$ are parallel

$\therefore\ \vec{a} = m\vec{b}$, for some $m \in R$

$\Rightarrow 3\hat{i} + 2\hat{j} - \hat{k} = m(6\hat{i} - 4x\hat{j} + y\hat{k})$

$\Rightarrow\ 3 = 6m\ \Rightarrow\ m = \frac{1}{2}$

Also, $-4xm = 2 \Rightarrow \frac{-4x}{2} = 2 \Rightarrow x = -1$

and $ym = -1\ \Rightarrow\ \frac{y}{2} = -1\ \Rightarrow\ y = -2$

OR

Let $C(x, y, z)$ divides the line segment joining the points $A(-2, 3, 5)$ and $B(1, 2, 3)$ in the ratio $2 : 3$ externally.

Now, $\vec{c} = \frac{2\vec{b} - 3\vec{a}}{2 - 3}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are position vectors of A, B and C respectively.

$$= -1\left[2(\hat{i} + 2\hat{j} + 3\hat{k}) - 3(-2\hat{i} + 3\hat{j} + 5\hat{k})\right]$$

$$= -1(8\hat{i} - 5\hat{j} - 9\hat{k}) = -8\hat{i} + 5\hat{j} + 9\hat{k}$$

So, co-ordiantes of $C \equiv (-8, 5, 9)$

10. Let $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Then,

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{4+9+16}} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$$

Hence, the required equation of the plane is

$$\vec{r} \cdot \left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}$$

11. Here $f(1) = 2, f(2) = 3, f(3) = 4, f(4) = 1$

Since no two elements have the same image. So f is one-one. Also, every elements has atleast one pre-image. So, f is onto.

Thus f is bijective.

12. $|\vec{a} + \vec{b}| < 1 \Rightarrow |\vec{a} + \vec{b}|^2 < 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} < 1 \Rightarrow 1 + 1 + 2\vec{a} \cdot \vec{b} < 1$

$\Rightarrow \vec{a} \cdot \vec{b} < -\frac{1}{2} \Rightarrow |\vec{a}|\,|\vec{b}|\cos\theta < -\frac{1}{2}$

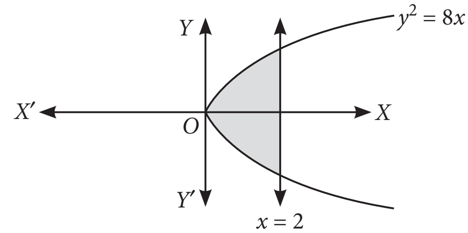
$\Rightarrow 1 \times 1 \times \cos\theta < -\frac{1}{2} \Rightarrow \cos\theta < -\frac{1}{2}$

$\Rightarrow -1 \le \cos\theta < -\frac{1}{2} \Rightarrow \pi \ge \theta > \frac{2\pi}{3}$

13. We have, $y^2 = 8x$ and $x = 2$

$\therefore$ Required area = Area of shaded region

$$= 2 \cdot \int_0^2 \sqrt{8x}\,dx = 4\sqrt{2}\left[\frac{2}{3}x^{3/2}\right]_0^2 = \frac{32}{3} \text{ sq. units.}$$



14. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$.

We know that the line which passes through point $\vec{a}$ and parallel to $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a constant.

$\therefore\ \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$ is the required equation of the line.

15. We have, $A + B + C = O \Rightarrow C = -[A + B]$

$$\Rightarrow C = (-1)\left\{\begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}\right\} = (-1)\begin{bmatrix} 8 & 4 \\ -2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ 2 & -4 \\ -4 & -4 \end{bmatrix}$$

16. D.R.'s of the two lines are 1, 2, –2 and 2, 3, $\lambda - 3$.

Since, lines are perpendicular

$\therefore\ a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow\ 1 \times 2 + 2 \times 3 - 2(\lambda - 3) = 0 \Rightarrow \lambda = 7$

17. (i) (b) : Total number of tickets = 11

Let the event A = First ticket shows even number

and B = Second ticket shows even number

Now, P(Both ticket shows even number) = $P(A) \cdot P(B|A)$

$$= \frac{5}{11} \cdot \frac{4}{10} = \frac{2}{11}$$

(ii) (c) : Let the event A = First ticket shows odd number

and B = Second ticket shows odd number

P(Both ticket shows odd number)

$$= \frac{6}{11} \times \frac{5}{10} = \frac{3}{11}$$

(iii) (a) : Required probability = P(one number is a multiple 4 and other is a multiple 5)

= P(multiple of 5 on first ticket and multiple of 4 on second ticket) + P(multiple of 4 on first ticket and multiple of 5 on second ticket)

$$= \frac{2}{11} \cdot \frac{2}{10} + \frac{2}{11} \times \frac{2}{10}$$

$$= \frac{4}{110} + \frac{4}{110} = \frac{8}{110} = \frac{4}{55}$$

(iv) (d) : Required probability = P(one ticket with prime number and other ticket with a multiple of 4)

$$= \frac{5}{11} \times \frac{2}{10} + \frac{2}{11} \times \frac{5}{10} = \frac{10}{110} + \frac{10}{110} = \frac{2}{11}$$

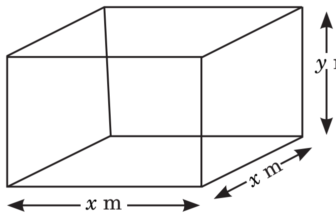
(v) (b) : Let the event A = First ticket shows even number and B = Second ticket shows odd number

Now, P(First ticket shows an even number and second ticket shows an odd number) = $P(A)\cdot P(B|A)$

$$=\frac{5}{11}\times\frac{6}{10}=\frac{30}{110}=\frac{3}{11}$$

18. (i) (d) : Since the tank is open from the top, therefore the total surface area is

$= 2(x \times x + 2\,(xy + yx)) = 2(x^2 + 2(2xy)) = 2x^2 + 8xy$



(ii) (a) : Since, volume of tank should be 4000 l.

$\therefore\ x^2y\ \text{m}^3 = 4000\ l = 4\ \text{m}^3$ [$\because$ 1 litre = 0.001 m^3]

So, $x^2y = 4$

(iii) (a) : Let S be the outer surface area of tank.

Then, $S = x^2 + 4xy$

$\Rightarrow S(x) = x^2 + 4x\cdot\frac{4}{x^2} = x^2+\frac{16}{x}$ [$\because x^2y = 4$]

$\Rightarrow \frac{dS}{dx}=2x-\frac{16}{x^2}$ and $\frac{d^2S}{dx^2}=2+\frac{32}{x^3}$

For maximum or minimum values of S, consider

$\frac{dS}{dx}=0$

$\Rightarrow 2x=\frac{16}{x^2} \Rightarrow x^3 = 8 \Rightarrow x = 2$ m

At $x = 2$, $\frac{d^2S}{dx^2}=2+\frac{32}{2^3}=2+4=6>0$

$\therefore$ S is minimum when $x = 2$

Now as $x^2y = 4$, therefore $y = 1$

Thus, $x = 2y$

(iv) (b) : Since, surface area is minimum when $x = 2y$, therefore cost of material will be least when $x = 2y$.

Thus, cost of material will be least when width is equal to twice of its depth.

(v) (c) : Since, minimum surface area

$= x^2 + 4xy = 2^2 + 4 \times 2 \times 1 = 12\ \text{m}^2$ and

cost per m^2 = ₹ 360

$\therefore$ Minimum cost is = ₹ (12 × 360) = = ₹ 4320

19. The given curve is $y = x^3$.

$\Rightarrow \frac{dy}{dx}=3x^2$

Slope of tangent at (1, 1) is $\left(\frac{dy}{dx}\right)_{(1,1)}=3(1)^2=3$

Equation of tangent at (1, 1) is

$y - 1 = 3(x - 1) \Rightarrow y - 1 = 3x - 3 \Rightarrow 3x - y = 2$

Equation of normal at (1, 1) is

$y-1=\frac{-1}{3}(x-1) \Rightarrow x+3y=4$

20. We write, $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$$=\tan^{-1}\left(\frac{\cos^2\left(\frac{x}{2}\right)-\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)+\sin^2\left(\frac{x}{2}\right)+2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right)$$

$$=\tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right]$$

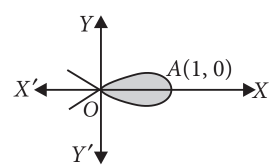
$$=\tan^{-1}\left[\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right]=\tan^{-1}\left[\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right]$$

$$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]=\frac{\pi}{4}-\frac{x}{2}$$

21. Given curve is $y^2 = x(1 - x)^2$

If $y = 0$, then $x(1 - x)^2 = 0 \Rightarrow x = 0, x = 1$

$\therefore$ Required area $=2\int_0^1\sqrt{x}\,(1-x)\,dx$



$$=2\left[\frac{x^{3/2}}{3/2}-\frac{x^{5/2}}{5/2}\right]_0^1$$

$$=2\left[\frac{2}{3}(1)-\left(\frac{2}{5}(1)\right)\right]=\frac{8}{15}\text{ sq. unit}$$

22. Let E_1, E_2 and A denote the events defined as follows :

E_1 = person selected is man

E_2 = person selected is woman

A = person selected is good orator

We have, $P(E_1)=\frac{1}{2}$, $P(E_2)=\frac{1}{2}$

Now, $P(A\,|\,E_1)=\frac{5}{100}$ and $P(A\,|\,E_2)=\frac{25}{1000}$

Required probability is

$$P(E_1\,|\,A)=\frac{P(E_1)\times P(A\,|\,E_1)}{P(E_1)P(A\,|\,E_1)+P(E_2)P(A\,|\,E_2)}$$

$$=\frac{\frac{1}{2}\times\frac{5}{100}}{\frac{1}{2}\times\frac{5}{100}+\frac{1}{2}\times\frac{25}{1000}}=\frac{50}{75}=\frac{2}{3}$$

23. Let $I = \int \frac{1}{\sqrt{1-\sin x}}\,dx = \int \frac{1}{\sqrt{1+\cos\left(\frac{\pi}{2}+x\right)}}\,dx$

$= \int \frac{1}{\sqrt{2\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}}\,dx = \frac{1}{\sqrt{2}}\int \sec\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$

$= \frac{2}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4}+\frac{\pi}{8}+\frac{x}{4}\right)\right|+C$

$= \sqrt{2}\log\left|\tan\left(\frac{3\pi}{8}+\frac{x}{4}\right)\right|+C$

OR

Let $I = \int \frac{dx}{1-\sin 2x}$

Put $\tan x = t \Rightarrow dx = \frac{dt}{1+t^2}$ and $\sin 2x = \frac{2t}{1+t^2}$

$\therefore\ I = \int \frac{dt}{1+t^2-2t} = \int \frac{dt}{(t-1)^2} = \frac{-1}{t-1}+C$

$\Rightarrow I = \frac{-1}{\tan x - 1}+C$

24. We have, $AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$= \begin{bmatrix} 25a^2+b^2 & 15a-2b \\ 15a-2b & 13 \end{bmatrix}$

and $A\cdot(\text{adj}A) = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix}$

$\because A \cdot (\text{adj } A) = AA^T$ is known, so equating the two expressions, we get

$\begin{bmatrix} 25a^2+b^2 & 15a-2b \\ 15a-2b & 13 \end{bmatrix} = \begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix}$

We have, $10a + 3b = 13$ and $15a - 2b = 0$

On solving, we get $a = 2/5$ and $b = 3$

Thus, $5a + b = 2 + 3 = 5$

25. Let $\vec{a} = 2\hat{i}-3\hat{j}-6\hat{k}$, $P \equiv (5, 6, -3)$ and $Q \equiv (3, 4, -2)$

$\therefore\ \overrightarrow{PQ} = (3-5)\hat{i}+(4-6)\hat{j}+(-2+3)\hat{k} = -2\hat{i}-2\hat{j}+\hat{k}$

Now the projection of $\vec{a}$ on $\overrightarrow{PQ}$

$= \frac{\vec{a}\cdot\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{-4+6-6}{\sqrt{4+4+1}} = -\frac{4}{3}$

OR

We have, $\vec{a} = 4\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{b} = 3\hat{i}+2\hat{k}$

$\therefore\ 2\vec{a} = 8\hat{i}+6\hat{j}+4\hat{k}$

Now, $\vec{b}\times 2\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 2 \\ 8 & 6 & 4 \end{vmatrix} = -12\hat{i}+4\hat{j}+18\hat{k}$

$\therefore\ |\vec{b}\times 2\vec{a}| = \sqrt{(-12)^2+4^2+(18)^2} = 22$

26. Total no. of aces = 4

Also, X can take the values 0, 1, 2

$\therefore\ P(X=0) = \frac{^{48}C_2}{^{52}C_2} = \frac{188}{221}$,

$P(X=1) = \frac{^4C_1 \times {}^{48}C_1}{^{52}C_2} = \frac{32}{221}$ and $P(X=2) = \frac{^4C_2}{^{52}C_2} = \frac{1}{221}$

The probability distribution of X is as follows:

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

27. We have, $y = \sin^{-1} x$.

$\therefore\ \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx} = 1$

$\Rightarrow \frac{d}{dx}\left(\sqrt{1-x^2}\cdot\frac{dy}{dx}\right) = 0$

$\Rightarrow \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{d}{dx}\left(\sqrt{1-x^2}\right) = 0$

$\Rightarrow \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} - \frac{dy}{dx}\cdot\frac{2x}{2\sqrt{1-x^2}} = 0$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

28. We have, $\frac{dy}{dx} = \frac{y(1+x)}{x(y-1)}$

$\Rightarrow \left(\frac{y-1}{y}\right)dy = \left(\frac{1+x}{x}\right)dx$

$\Rightarrow \int\left(1-\frac{1}{y}\right)dy = \int\left(\frac{1}{x}+1\right)dx + C_1$

$\Rightarrow y - \log|y| = \log|x| + x + C_1 \Rightarrow x - y + \log|xy| = C$, where $C = -C_1$

OR

Given, $\log\left(\frac{dy}{dx}\right) = 3x+4y \Rightarrow \frac{dy}{dx} = e^{3x+4y}$

$\Rightarrow \frac{dy}{dx} = e^{3x}e^{4y} \Rightarrow \int e^{-4y}dy = \int e^{3x}dx$

$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3}+C$

At $x = 0$, $y = 0$, then $\frac{-1}{4} = \frac{1}{3}+C \Rightarrow C = \frac{-7}{12}$

$\therefore\ \frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}-\frac{7}{12}\Rightarrow 4e^{3x}+3e^{-4y}-7=0$, which is the required solution.

29. Let $P(x, y)$ be an arbitrary point on the given curve. The equation of the normal to the given curve at (x, y) is $Y-y=-\frac{1}{\frac{dy}{dx}}(X-x)$

It is given that the normal at every point passes through a fixed point (α, β) (say).

Therefore, $\beta-y=-\frac{dx}{dy}(\alpha-x)$

$\Rightarrow\ (x-\alpha)dx+(y-\beta)dy=0$

Integrating both sides, we get

$$\int(x-\alpha)dx+\int(y-\beta)dy=C$$

$$\Rightarrow\ \frac{(x-\alpha)^2}{2}+\frac{(y-\beta)^2}{2}=C$$

$\Rightarrow\ (x-\alpha)^2+(y-\beta)^2=r^2$, where $r^2=2C$

Clearly, this equation represents a circle, having centre at (α, β) and radius r.

30. Let $A\ (x, y)$ be the required point which is closest to the point $B(1, 4)$. Then, the distance AB should be minimum and therefore AB^2 should be minimum.

Now, $AB^2=(x-1)^2+(y-4)^2=\left(\frac{y^2}{2}-1\right)^2+(y-4)^2$

$$=\frac{(y^4-32y+68)}{4}$$

Let $f(y)=\frac{y^4-32y+68}{4}$

Then, $f'(y)=\frac{4y^3-32}{4}=y^3-8$ and $f''(y)=3y^2$

Now, $f'(y)=0\Rightarrow y^3-8=0\Rightarrow y=2$

Also, $f''(2)=3\times 4=12>0$

So, $y=2$ is a point of minima.

Now, $y=2\Rightarrow x=\frac{y^2}{2}=\frac{4}{2}=2$.

So, the required point is (2, 2).

31. We have,

$$y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\frac{c}{x-c}+1$$

$$\Rightarrow\ y=\frac{ax^2+bx(x-a)+c(x-a)(x-b)+(x-a)(x-b)(x-c)}{(x-a)(x-b)(x-c)}$$

$$\Rightarrow\ y=\frac{x^3}{(x-a)(x-b)(x-c)}$$

$$\Rightarrow\ \log y=\log\left\{\frac{x^3}{(x-a)(x-b)(x-c)}\right\}$$

$\Rightarrow\ \log y = 3\log x - \{\log(x-a)+\log(x-b)+\log(x-c)\}$

On differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx}=\frac{3}{x}-\left\{\frac{1}{x-a}+\frac{1}{x-b}+\frac{1}{x-c}\right\}$$

$$\Rightarrow\ \frac{dy}{dx}=y\left\{\left(\frac{1}{x}-\frac{1}{x-a}\right)+\left(\frac{1}{x}-\frac{1}{x-b}\right)+\left(\frac{1}{x}-\frac{1}{x-c}\right)\right\}$$

$$\Rightarrow\ \frac{dy}{dx}=y\left\{\frac{-a}{x(x-a)}+\frac{(-b)}{x(x-b)}+\frac{(-c)}{x(x-c)}\right\}$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{y}{x}\left\{\frac{a}{a-x}+\frac{b}{b-x}+\frac{c}{c-x}\right\}$$

OR

Let $y=\frac{2^{x^2+x+1}\cdot\sqrt{4x-1}}{(x^2-1)^{3/2}}$

$$\Rightarrow\ \log y=\log\left[\frac{2^{x^2+x+1}\cdot\sqrt{4x-1}}{(x^2-1)^{3/2}}\right]$$

$$=(x^2+x+1)\log 2+\frac{1}{2}\log(4x-1)-\frac{3}{2}\log(x^2-1)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx}=\log 2\cdot\frac{d}{dx}(x^2+x+1)+\frac{1}{2}\cdot\frac{1}{4x-1}\times\frac{d}{dx}(4x-1)-\frac{3}{2}\cdot\frac{1}{x^2-1}\cdot\frac{d}{dx}(x^2-1)$$

$$=(2x+1)\log 2+\frac{1}{2(4x-1)}\cdot 4-\frac{3}{2(x^2-1)}\cdot 2x$$

$$\Rightarrow\ \frac{dy}{dx}=y\left[(2x+1)\log 2+\frac{2}{4x-1}-\frac{3x}{x^2-1}\right]$$

32. Let $y=f(x)=\frac{2x+3}{x-3}$ ...(i)

Let $x_1, x_2\in A=R-\{3\}$ such that

$f(x_1)=f(x_2)$

$$\Rightarrow\ \frac{2x_1+3}{x_1-3}=\frac{2x_2+3}{x_2-3}$$

$\Rightarrow\ (2x_1+3)(x_2-3)=(2x_2+3)(x_1-3)$

$\Rightarrow\ 2x_1x_2-6x_1+3x_2-9=2x_1x_2-6x_2+3x_1-9$

$\Rightarrow\ -6x_1+3x_2=-6x_2+3x_1$

$\Rightarrow\ 9x_1=9x_2\Rightarrow\ x_1=x_2$

Now, $f(x_1)=f(x_2)\Rightarrow x_1=x_2$

So $f(x)$ is one-one.

For onto, let $y=\frac{2x+3}{x-3}\ \Rightarrow xy-3y=2x+3$

$\Rightarrow\ xy - 2x = 3y + 3 \Rightarrow x(y-2) = 3(y+1)$

$\Rightarrow\ x = \dfrac{3(y+1)}{(y-2)}$ ...(ii)

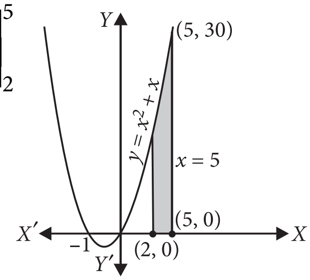
Equation (ii) is defined for all real values of y except 2 which is same as given set $B = R - \{2\}$.

Thus, for every $y \in B$, there exist $x = \dfrac{3(y+1)}{y-2} \in A$ such that $f(x) = y$

Hence, function f is onto.

33. Given curve is $y = x^2 + x$.

Clearly, required area = Area of shaded region



$= \displaystyle\int_2^5 (x^2 + x)\,dx = \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_2^5$

$= \left[\dfrac{125}{3} + \dfrac{25}{2} - \left(\dfrac{8}{3} + 2\right)\right]$

$= \left[\dfrac{250+75}{6} - \dfrac{14}{3}\right]$

$= \dfrac{325}{6} - \dfrac{14}{3} = \dfrac{325-28}{6} = \dfrac{297}{6}$ sq. units

34. Let $I = \displaystyle\int_0^1 \tan^{-1} x\,dx = \int_0^1 \tan^{-1} x \cdot 1\,dx$

$= [\tan^{-1} x \cdot x]_0^1 \displaystyle\int_0^1 \frac{1}{1+x^2} \cdot x\,dx$

$= \left(\dfrac{\pi}{4} - 0\right) - \dfrac{1}{2}\displaystyle\int_0^1 \frac{2x}{1+x^2}dx = \frac{\pi}{4} - \frac{1}{2}I_1$ ...(i)

Consider $I_1 = \displaystyle\int_0^1 \frac{2x}{1+x^2}dx$

Put $1 + x^2 = t \Rightarrow 2x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 2$

$\therefore\ I_1 = \displaystyle\int_1^2 \frac{1}{t}\,dt = [\log t]_1^2 = \log 2 - \log 1 = \log 2$ ...(ii)

$\Rightarrow\ I = \dfrac{\pi}{4} - \log\sqrt{2}$ [From (i) and (ii)]

OR

Let $I = \displaystyle\int_1^2 \frac{dx}{x(1+x^2)}$

Consider, $\dfrac{1}{x(1+x^2)} = \dfrac{A}{x} + \dfrac{Bx+C}{1+x^2}$

$\Rightarrow\ 1 = A(1+x^2) + (Bx+C)\cdot x$

$\Rightarrow\ 1 = x^2(A+B) + Cx + A$

On equating the coefficient of x^2, x and the constant term from both sides, we get $A = 1$, $B = -1$ and $C = 0$

$\therefore\ I = \displaystyle\int_1^2 \frac{1}{x}dx + \int_1^2 \frac{-x}{1+x^2}dx$

$= [\log x]_1^2 - \displaystyle\int_1^2 \frac{x}{1+x^2}dx = \log 2 - \int_1^2 \frac{x\,dx}{1+x^2}$

Put $1 + x^2 = t \Rightarrow 2x\,dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 5$

$\therefore\ I = \log 2 - \dfrac{1}{2}\displaystyle\int_2^5 \frac{1}{t}dt = \log 2 - \frac{1}{2}[\log t]_2^5$

$= \log 2 - \dfrac{1}{2}[\log 5 - \log 2]$

$= \log 2 - \dfrac{1}{2}\log\left(\dfrac{5}{2}\right) = \dfrac{1}{2}\log\left(\dfrac{8}{5}\right)$

35. Since, $f(x)$ is continuous at $x = 0$, therefore

$f(0) = \displaystyle\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$ ...(i)

$\displaystyle\lim_{x\to 0^-} f(x) = \lim_{x\to 0}\left(\frac{4(1-\sqrt{1-x})}{x}\right)$

$= 4\displaystyle\lim_{x\to 0}\left(\frac{1-(1-x)}{x(1+\sqrt{1-x})}\right) = 4\lim_{x\to 0}\left(\frac{1-1+x}{x(1+\sqrt{1-x})}\right)$

$= 4\left(\dfrac{1}{(1+1)}\right) = \dfrac{4}{2} = 2$

From (i), we get $f(0) = 2$.

36. Here we need to find, the equation of the line through the point $(1, 1, -3)$ and perpendicular to the lines

$\dfrac{x-2}{2} = \dfrac{y+3}{1} = \dfrac{z-0}{-3}$ ...(i)

and $\dfrac{x-3}{1} = \dfrac{y+5}{1} = \dfrac{z-0}{1}$ ...(ii)

Let the direction ratios of required line are a, b, c. Then, equations of this line is given by

$\dfrac{x-1}{a} = \dfrac{y-1}{b} = \dfrac{z+3}{c}$ ...(iii)

Direction ratios of line (i) are 2, 1, –3 and line is perpendicular to line (iii) having direction ratios a, b, c

$\therefore\ 2a + b - 3c = 0$ ...(iv)

Similarly $a + b + c = 0$ ...(v)

Solving equation (iv) and (v), we get

$\dfrac{a}{4} = \dfrac{-b}{5} = \dfrac{c}{1}$

$\therefore$ From equation (iii), required equation of the line is $\dfrac{x-1}{4} = \dfrac{y-1}{-5} = \dfrac{z+3}{1}$.

Its vector equation is $\vec{r} = (\hat{i} + \hat{j} - 3\hat{k}) + \lambda(4\hat{i} - 5\hat{j} + \hat{k})$

OR

The equation of the plane passing through the point $A(3, 2, -5)$ is given by

$a(x-3) + b(y-2) + c(z+5) = 0$ ...(i)

If it passes through $B(-1, 4, -3)$ and $C(-3, 8, -5)$, we get

$a(-1-3) + b(4-2) + c(-3+5) = 0$

$\Rightarrow\ -4a + 2b + 2c = 0$

$\Rightarrow\ 2a - b - c = 0$ ...(ii)

Also $a(-3-3) + b(8-2) + c(-5+5) = 0$

$\Rightarrow\ -6a + 6b + 0c = 0$

$\Rightarrow\ a - b - 0c = 0$ ..(iii)

Solving (ii) and (iii) by cross multiplication method, we get

$$\frac{a}{(0-1)} = \frac{b}{(-1-0)} = \frac{c}{(-2+1)}$$

$$\Rightarrow\ \frac{a}{1} = \frac{b}{1} = \frac{c}{1} = k \text{ (say)} \Rightarrow a = k, b = k, c = k.$$

Putting $a = k$, $b = k$ and $c = k$ in (i), we get

$(x-3) + (y-2) + (z+5) = 0$

$\Rightarrow\ x + y + z = 0.$

Thus, the equation of the plane passing through the points $A(3, 2, -5)$, $B(-1, 4, -3)$ and $C(-3, 8, -5)$ is $x + y + z = 0$.

Clearly, the fourth point $D(-3, 2, 1)$ also satisfies $x + y + z = 0$.

Hence, equation of the plane containing the given points is $x + y + z = 0$.

37. Converting inequations into equations, we get

$x + y = 60$ ...(i)

$x = 40$ ...(ii)

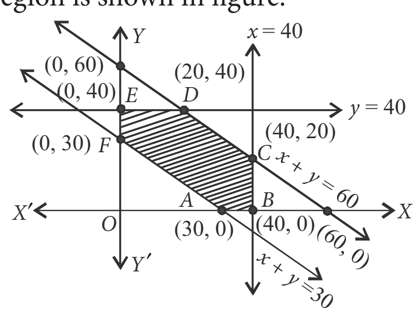
$y = 40$ ...(iii)

$x + y = 30$ ...(iv)

$x = 0$ ...(v)

and $y = 0$ ...(vi)

Let us draw the graph of equations (i) to (vi). The feasible region is shown in figure.



The coordinates of the corner points of the feasible region are $A(30, 0)$, $B(40, 0)$, $C(40, 20)$, $D(20, 40)$, $E(0, 40)$ and $F(0, 30)$.

Let us evaluate Z at these points.

Corner points	Value of $Z = 3x + 4y + 270$
$A(30, 0)$	360 ← Minimum
$B(40, 0)$	390
$C(40, 20)$	470
$D(20, 40)$	490
$E(0, 40)$	430
$F(0, 30)$	390

From the table, the minimum value of Z is 360, which is attained at the point $A(30, 0)$.

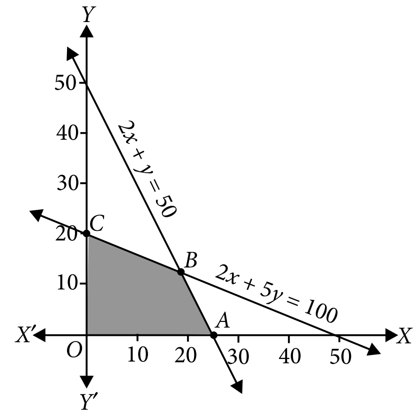
OR

Converting inequations into equations, we get

$2x + 5y = 100$, $2x + y = 50$, $x = 0$ and $y = 0$

i.e., $\frac{x}{50} + \frac{y}{20} = 1$, $\frac{x}{25} + \frac{y}{50} = 1$, $x = 0$ and $y = 0$

Let us draw the graph of above equations.



Clearly, the feasible region is $OABCO$, which is shaded in the figure.

Here, B is the point of intersection of lines $2x + 5y = 100$ and $2x + y = 50$ *i.e.*, $B = \left(\frac{75}{4}, \frac{25}{2}\right)$

We have corner points $A(25, 0)$, $B\left(\frac{75}{4}, \frac{25}{2}\right)$ and $C(0, 20)$.

The values of the objective function $Z = x + y$ at these points are

$Z(A) = 25 + 0 = 25$

$Z(B) = \frac{75}{4} + \frac{25}{2} = 31.25$

$Z(C) = 0 + 20 = 20$

$Z(O) = 0 + 0 = 0$

The maximum value of Z is 31.25, which is attained at $\left(\frac{75}{4}, \frac{25}{2}\right)$.

38. We have, $A = \begin{pmatrix} 2 & 3 & 7 \\ 3 & -2 & -1 \\ 1 & 1 & 2 \end{pmatrix}$

$$\therefore\ |A| = \begin{vmatrix} 2 & 3 & 7 \\ 3 & -2 & -1 \\ 1 & 1 & 2 \end{vmatrix}$$

$= 2(-4 + 1) - 3(6 + 1) + 7(3 + 2) = -6 - 21 + 35$

$= 8 \neq 0$. So A^{-1} exist.

The cofactors of elements of A are

$C_{11} = -3, C_{21} = 1, C_{31} = 11$

$C_{12} = -7, C_{22} = -3, C_{32} = 23$

$C_{13} = 5, C_{23} = 1, C_{33} = -13$

$$\therefore \quad \text{adj} A = \begin{pmatrix} -3 & 1 & 11 \\ -7 & -3 & 23 \\ 5 & 1 & -13 \end{pmatrix}$$

$$\text{and } A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{8}\begin{pmatrix} -3 & 1 & 11 \\ -7 & -3 & 23 \\ 5 & 1 & -13 \end{pmatrix}$$

Given system of equations is

$2x + 3y + 7z = 12$

$3x - 2y - z = 0$

$x + y + 2z = 4$

which can be written in matrix form as

$$\begin{pmatrix} 2 & 3 & 7 \\ 3 & -2 & -1 \\ 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 4 \end{pmatrix}$$

$\Rightarrow \; AX = B \Rightarrow X = A^{-1}B$

$$\Rightarrow \; X = \frac{1}{8}\begin{pmatrix} -3 & 1 & 11 \\ -7 & -3 & 23 \\ 5 & 1 & -13 \end{pmatrix}\begin{pmatrix} 12 \\ 0 \\ 4 \end{pmatrix}$$

$$\text{Now, } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = X = \frac{1}{8}\begin{pmatrix} -36+0+44 \\ -84-0+92 \\ 60+0-52 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Comparing we get $x = 1, y = 1, z = 1$

OR

Here,

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

We know, if A is non-singular matrix,

then $(\text{adj } A^{-1}) = (\text{adj } A)^{-1}$, so we will find $(\text{adj } A)^{-1}$.

The cofactors of elements of A are

$A_{11} = 14 \qquad A_{12} = 11 \qquad A_{13} = -5$

$A_{21} = 11 \qquad A_{22} = 4 \qquad A_{23} = -3$

$A_{31} = -5 \qquad A_{32} = -3 \qquad A_{33} = -1$

$$\therefore \; B = \text{adj } A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}' = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$\therefore \quad |B| = |\text{adj } A| = 14(-4-9) - 11(-11-15) - 5(-33+20)$$

$$= -182 + 286 + 65 = 169 \neq 0$$

Cofactors of B are

$B_{11} = -13 \qquad B_{12} = 26 \qquad B_{13} = -13$

$B_{21} = 26 \qquad B_{22} = -39 \qquad B_{23} = -13$

$B_{31} = -13 \qquad B_{32} = -13 \qquad B_{33} = -65$

$$\therefore \; \text{adj } B = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}' = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$\text{and } B^{-1} = (\text{adj } A)^{-1} = \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

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