# 10

# Area of Triangles and Quadrilaterls

## 10.01 Introduction

We know that the study of Geometry, orginated with the measurement of land in the process of recasting boundaries and distribution of the fields. For example, Kartik distributes his trapezium shaped field by joining the mid points of non-parallel sides between his two daughters. (See Fig 10.01). Is this distribution equal in area? To get answer to this type of problems, there is a need to have a look at area of plane figures.

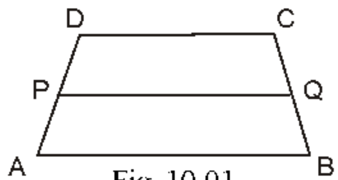



Fig. 10.01

## 10.02 Area

The part of plane enclosed by a simple closed figure, is called the plane region of that figure and magnitude or measure of this plane region is called the area of that field. This magnitude or measure is always expressed with the help of some units, for example 6 square cm ($cm^2$), 9 square metre ($m^2$), 12 hectare etc.

In chapter 7, we have studied about congruent figures. If two plane figures are same in shape and measure, they are called congruent. If they are cut and put on any plane, then both the figures have an equal plane region on that plane that means, two congruent figures are equal in areas. Is its converse also true? Let us try to understand the following activities.

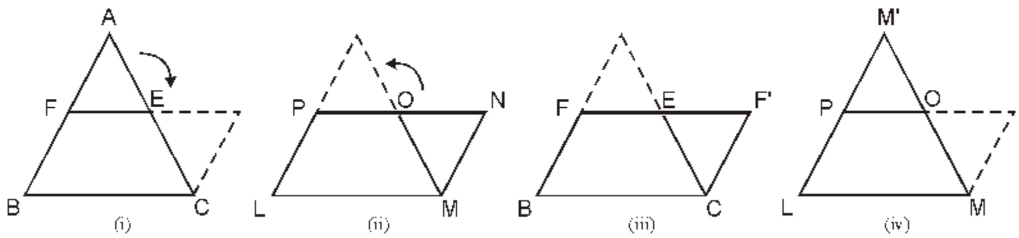



Fig. 10.02

**Step-1**

Prepare a carbon copy of Fig 10.02 (i), (iii) by leaving the dotted part with the help of pencil and scale on a white paper, by using carbon paper putting under this page of book. From prepared carbon copies, cut $\Delta AEF$ of fig 10.02 (i) and according to 10.02 (ii), keeping $E$ on the same point, move $\Delta AEF$ so that $A$ reach to point C and paste. Now we will get a quadrilateral $BCF'F$. Similarly, from Fig. 10.02 (iii), cut a $\Delta MNO$ and at $O$ still, move such that $N$ comes to $P$ and paste as in Fig. 10.02 (iv). In this way a $\Delta LMM'$ will be obtained.

**Step-2**

Now put quadrilateral $BCF'F$ on Fig. 10.02 (iii) and $\Delta LMM'$ on Fig. 10.02 (i). Are they cover each other completely? Yes, they are covering. It means that quadrilateral $BCF'F \cong$ quadrilateral $LMNP$ and $\Delta LMM' \cong \Delta BCA$ . Are these congruent figures equal in area? Definitely they are equal. But converse of it that $\Delta ABC$ and quadrilateral $BCF'F$ and quadrilateral $LMNP$ and $\Delta LMM'$ which are equal in area. Area $\Delta$ ABC, quadrilateral $BCF'F$ and quadrilateral $LMNP$, $\Delta LMM'$ are congruent? Undoubtebly no.

So, we can say that ***congruent figures are equal in area but the figures equal in area need not necessarily congruent.***

If quadrilateral $BCF'F \cong$ quadrilateral $LMNP$ and $\Delta LMM' \cong \Delta BCA$ , then we write it as :

$$ar(BCF'F) = ar(LMNP) \text{ and } ar(LMM') = ar(BCA)$$

Let us see the Fig 10.03. You may observe that plane region figure $T$ is made by adding two figures $X$ and $Y$ in plane region. Now, you can easily see that

Area of figure $T$ = Area of figure $X$ + Area of figure $Y$

OR $\quad ar(T) = ar(X) + ar(Y)$

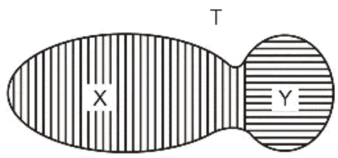



Fig. 10.03

In the previous classes, you have studied the formulae for finding the areas of different figures such as reactangle, square, triangle, etc. In this chapter, you will learn about the relation between area of geometrical figures under the condition when they lie on the same base and between the same pair of parallel lines. By studying this we will try to understand the deep knowledge of these formulae.

## 10.03 Figures Made on Same Base and Between Pair of Same Lines

Look at the following figures 10.04.

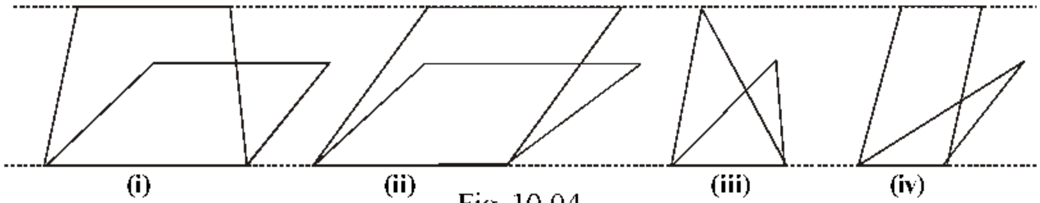



Fig. 10.04

In Fig. 10.04 (i), (ii), (iii) and (iv) in each case two figures are made on the same base but they are not made between same dotted wall of pair of parallel lines.

Now see the figures 10.05 given below.

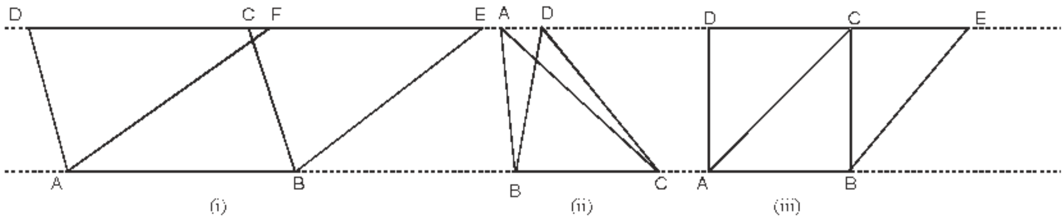



Fig. 10.05

In Fig. 10.05 in each case two figures are between two parallel lines. Here, in Fig. 10.05 (i) parallelogram *ABCD* and *ABEF* are on same base *AB* and between same parallel lines *AB* and *DE*. In Fig. 10.05 (ii), $\Delta ABC$ and $\Delta$ BCD are on same base *BC* and between same parallel lines *BC* and *AD*. Similarly, in Fig. 10.05 (iii) a square *ABCD* and a parallelogram *ABEC* are on same base *AB* and between same parallel lines *AB* and *DE*.

In Fig. 10.05 (i), (ii), (iii) are such figures that are said to be made on same base and between same parallel lines. In all the figures the bases are common in the two figures and the opposite-vertex of common base, is on the line drawn parallel to the base in each figure.

Now in article 10.06, keeping in mind the knowledge gained till now, which group of figures in fig. (i), (ii), (iii) and (iv) is made on the same base and between same parallel lines?

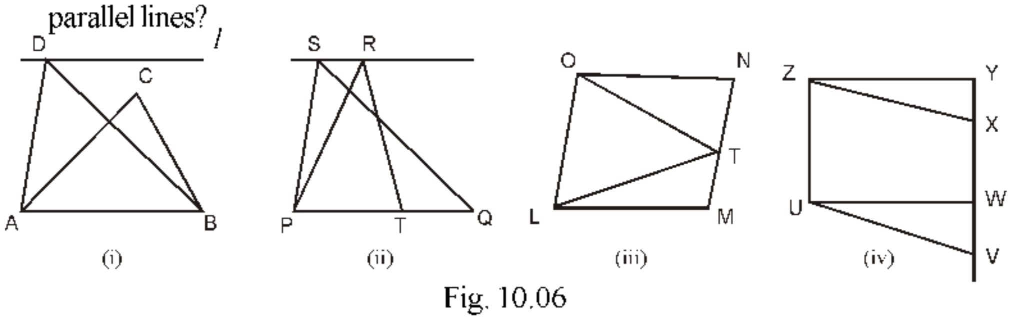



Fig. 10.06

**Let us discuss**

In Fig. 10.06 (i), $\Delta ABC$ and $\Delta ABD$ have common base *AB*, but the vertex C of $\Delta ABC$ does not lie on the line *l* which is parallel to base *AB*.

In Fig. 10.06 (ii) $PT$ is the base of $\Delta PTR$ and $PQ$ is base of $\Delta PQS$ means there is no common base of both the triangles but both the triangles are made in between two parallel lines $PQ$ and $SR$.

In Fig. 10.06 (iii), parallelogram $LMNO$ and $\Delta LTO$ are on the same base $LO$ and between the parallel lines $LO \quad MN$. Similarly, in Fig. 10.06 (iv), parallelogram $UVXZ$ and rectangle $UWYZ$ are made on same base $UZ$ and between a pair of parallel lines $UZ$ and $VY$. In this way Fig. 10.06 (i), (ii) are not in category of the figures, made on same base and between same parallel lines while Fig. 10.06 (iii) and (iv) are said to be in this category.

## EXERCISE 10.1

1. Which of the following figures are lying on the same base and between the same parallel lines? Write common base and pair of parallel lines in such a case.

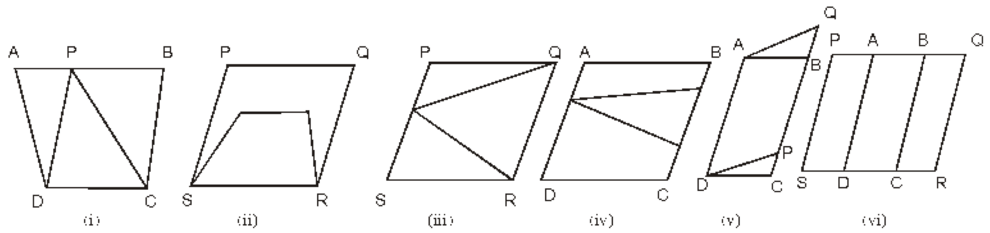



Fig. 10.07

2. Draw the following figures, on the same base and between the same parallel lines-

   (i) An obtuse angled triangle and a trapezium.

   (ii) A parallelogram and an isosceles triangle.

   (iii) A square and a parallelogram.

   (iv) A rectangle and a rhombus.

   (v) A rhombus and a parallelogram.

**Activity 10.2**

**Step-1**

Make two carbon copies of a parallelogram by keeping two carbon papers between three white papers and labell their vertices by $A, B, C, D$. Mark a point $P$ on side CD by pressing such that it also appears on carbon copies.

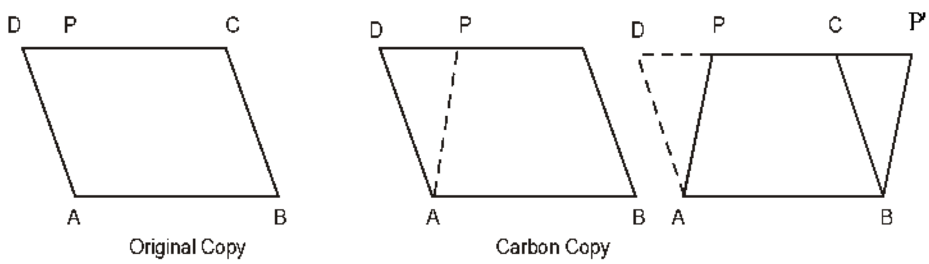



Fig. 10.08

**Step-2**

(i) Cut original copy and paste it on a page of your exercise note book.

(ii) Cut $\Delta APD$ made by joining P to A on carbon copy. Paste $\Delta APD$ on otherside of carbon copy, in such a way that after cutting side $AD$ should coincide with the side $BC$ of trapezium $ABCP$. Keep in mind that $A$ should be on $B$ and $D$ on C.

Thus we are getting two new paralellograms $ABCD$ and $ABP'P$. Paste one of these two quadrilaterals on your exercise note book, on same page as fig. 10.08 (iii).

**Step-3**

Paste another new parallelogram $ABP'P$ on original copy such that side $AB$ of both parallelograms should coincide. (See Fig 10.09).

In a new figure two parallelograms $ABCD$ and $ABP'P$ are made on same base and between a pair of same parallel lines.

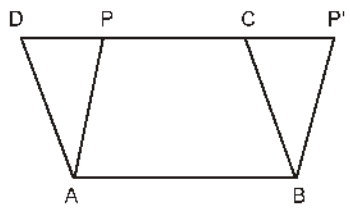



Fig. 10.09

Can you say that parallelogram $ABCD$ and $ABP'P$ are equal in area? Let us see.

$\because$ $\qquad \Delta APD \cong \Delta BP'C$

($\Delta APD$ is pasted after cutting)

$\therefore$ $\qquad ar(APD) = ar(BP'C)$

Adding ar ($ABCP$) on both sides, we get

$$\text{ar }(APD) - \text{ar }(ABCP) = \text{ar }(ABCP) - \text{ar }(BP'C)$$

or $\qquad \text{ar }(ABCD) = \text{ar }(ABP'P)$

$\Rightarrow$ Both parallelograms which are made on same base $AB$ and between the parallel lines ($AB$ | $DP'$), are equal in area.

Let us try to prove this result by any other method.

**Theorem 10.1.** ***Two parallelograms, made on same base and between the same parallelal lines, are equal in areas.***

**Given** : Two parallelograms $ABCD$ and $ABFE$ whose base is $AB$ and are between two parallel lines $AB$ and $DF$

**To Prove** : Area of parallelogram $ABCD$ – Area of parallelogram $ABFE$

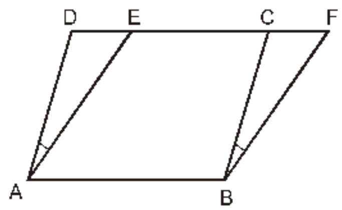



Fig. 10.10

**Proof :** In $\Delta ADE$ and $\Delta BCF$

$AE = BF$ (Opposite sides of parallelogram $ABFE$)

$\angle DAE = \angle CBF$ (Corresponding angles)

$AD = BC$ (Opposite sides of parallelogram $ABCD$)

$\Delta ADE \cong \Delta BCF$ (By SAS congruency rule)

$\therefore$ ar ($\Delta ADE$) = ar ($\Delta BCF$)

Adding ar (ABC) on both sides, we get

ar ($\Delta ADE$) + ar ($ABCE$) = ar (BCF) + ar ($ABCE$)

$\therefore$ ar ($ABCD$) = ar ($ABFE$) **Hence Proved**

**Corollary 1.**

***A parallelogram and a rectangle are lying on same base and between two parallel lines, then their areas are equal and area of parallelogram is equal to the product of its base and distance between two parallel lines.***

**Given** : In Fig. 10.11, $ABCD$ is a parallelogram and $EFCD$ is a rectangle.

Also AL $\perp$ DC

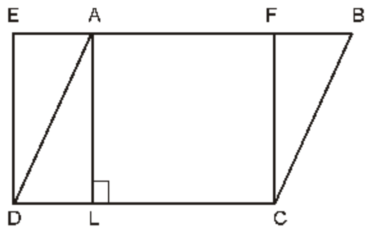



Fig. 10.11

**To Prove** : (i) ar ($ABCD$) = ar ($EFCD$)

(ii) $ar(ABCD) = DC \times AL$

**Proof** : (i) As rectangle is also a parallelogram,

$\therefore$ ar ($ABCD$) = ar ($EFCD$) . . . (i)

(ii) $\because$ Area of rectangle = length $\times$ breadth

$\therefore$ $ar\ (EFCD) = DC \times FC$

$\therefore$ $ar\ (ABCD) = DC \times FC$ [From (i)] ...(ii)

$\because$ $AL \perp DC$ (given)

So, $AFCL$ is also a rectangle.

$\therefore$ $AL = FC$ ... (iii)

Thus, ar ($ABCD$) = $DC \times AL$ [From (ii) and (iii)] **Hence Proved**

**Corollary 2.**

***If a triangle and a parallelogram are made on the same base and between pair of same parallel lines, then area of the triangle is half of the area of parallelogram.***

**Given** : $\Delta ABP$ and parallelogram $ABCD$ are made on same base $AB$ and between same parallel lines $AB$ and $PC$.

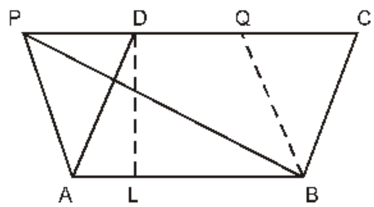



Fig. 10.12

**To Prove** : $ar(PAB) = \frac{1}{2} ar(ABCD)$

**Construction** : Draw $BQ \parallel AP$.

**Proof :**

$\therefore$ $AB \parallel CD$ (given)

$\therefore$ $AB \parallel PQ$

and $AP \parallel BQ$ (By construction)

$\therefore$ $ABQP$ is a parallelogram.

$\therefore$ ar (ABCD) = ar (ABQP) (By theorem 10.1)

And $\Delta ABP \cong \Delta QPB$ (A diagonal $B$ divides a parallelogram in two congruent triangles)

$\therefore$ $ar(ABP) - $ ar $(QPB)$

$$= \frac{1}{2} ar(ABQP)$$

$\Rightarrow$ $ar(ABP) = \frac{1}{2} ar(ABCD)$ **Hence Proved**

**Corollary 3.**

$$\textbf{\textit{Area of triangle}} = \frac{\mathbf{1}}{\mathbf{2}} \times \textbf{\textit{base}} \times \textbf{\textit{height}}$$

From Fig. 10.12, if $DL \perp AB$,

$\therefore$ ar(ABCD) $- AB \times DL$ then by corollary

But $ar(PAB) = \frac{1}{2} ar(ABCD)$ (From corollary 2)

$\therefore$ $ar(PAB) = \frac{1}{2} AB \times DL$

or Area of triangle $= \frac{1}{2} \times$ base $\times$ height **Proved**

## EXERCISE 10.2

1. In Fig 10.13, $ABCD$ is a parallelogram, in which $AE \perp DC$ and $CF \perp AD$. If AB = 16 cm, $AE - 8$ cm and $CF - 10$ cm, then find the value of $AD$.

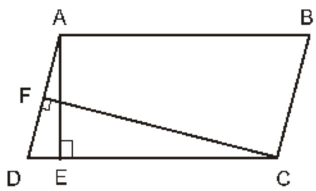



Fig. 10.13

2. If $E, F, G$ and $H$ are respectively the mid-points of sides of a parallelogram. Show that $ar(EFGH) = \frac{1}{2} ar(ABCD)$

3. $P$ and $Q$ are respectively points lying on the side $DC$ and $AD$ respectively of a parallelogram $ABCD$. Show that ar (APB) = ar (BQC).

4. In Fig. 10. 14, $P$ is any point in interior of a parallelogram $ABCD$. Show that :

(i) $ar(APB) + ar(PCD) = \frac{1}{2} ar(ABCD)$

(ii) $ar(APD) + ar(PBC) = ar(APB) + ar(PCD)$

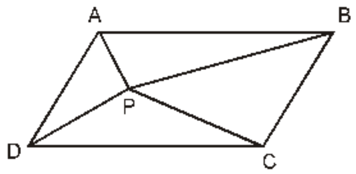



Fig. 10.14

5. In Fig. 10. 15, $PQRS$ and $ABRS$ are parallelograms and X is any point on side $TR$. Show that :

(i) $ar(PQRS) = ar(ABRS)$

(ii) $ar(AXS) = \frac{1}{2} ar(PQRS)$

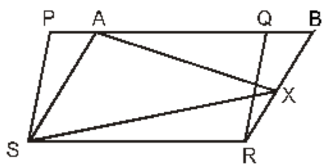



Fig. 10.15

6. A farmer had a field in the form of a parallelogram $PQRS$. He took any point $A$ situated on $RS$ and joined it to points $P$ and $Q$. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portion of the field separately. How should he do it?

## 10.4. Triangles on the same base and between same parallel lines :

**Theorem 10.2. Area *of triangles, on same base and between same parallel lines, are equal.***

**Given** : Δ ABC and Δ DBC are on base *BC* and between parallel lines *BC* and *AF*.

**To Prove** : ar(Δ *ABC*) − *ar*(Δ *DBC*)

**Construction** : From point C draw two lines *CE* and *CF* parallel to *AB* and *BD* respectively.

**Proof** : *ABCE* and *DBCF* are between same parallel lines *BC* and *AF*.

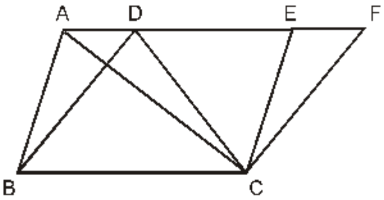



Fig. 10.16

$\therefore$ ar (*ABCE*) = ar (*DBCF*) ...(1)

[From theorem 10.1]

$\because$ *AC is* a diagonal of parallelogram *ABCE*,

$\therefore$ $\text{ar}(\Delta\,\text{ABC}) = \frac{1}{2}\ \text{ar}\ (ABCE)$ ...(2)

Similarly, *DC* is a diagonal of parallelogram *DBCF*,

$\text{ar}\ (DBC) - \frac{1}{2}\ \text{ar}\ (DBCE)$ ... (3)

From (1), (2) and (3), we get

ar (Δ *ABC*) – ar (Δ *DBC*) **Proved**

**Theorem 10.3.** ***If area of two triangles are equal and one side of a triangle is equal to one side of other triangle, then their corresponding altitudes are equal.***

**Given** : In Δ *ABC* and Δ *DEF*

(i) ar (Δ *ABC*) – ar (Δ *DEF*)

(ii) *BC* – *EF*

**To Prove** : Altitude *AN* = Altitude *DM*

**Proof** : In Δ *ABC*, *AN* is the altitude to the corresponding side *BC*.

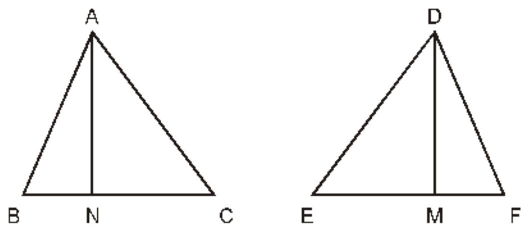



Fig. 10.17

$\therefore\ ar(\Delta\text{ABC}) = \frac{1}{2}\times\text{BC}\times\text{AN}$ [By corollary 3] ...(1)

Similarly $\text{ar}\ (\Delta\,\text{DEF}) = \frac{1}{2}\times\text{EF}\times\text{DM}$ [By corollary 3]] ... (2)

But, given ar (Δ *ABC*) – ar (Δ *DEF*)

$\therefore$ $\frac{1}{2}\times\text{BC}\times\text{AN} = \frac{1}{2}\times\text{EF}\times\text{DM}$

But $BC = EF$ (Given)

$\therefore$ $AN = DM$ **Hence Proved**

## 10.5. Baudhayan Theorem

Baudhayan gave us a very important result on a right angled triangle which is known as Baudhayan Theorem. This theorem is also famous by the name of Pythagorus Theorem. Now we will prove it.

**Theorem 10.4.** ***In a right angled triangle, square made on hypotenuse, is equal to the sum of the squares made on other two sides.***

**Given** : In $\Delta ABC$, $\angle C = 90°$, and squares on sides $AB$, $BC$ and $CA$ are $ADEB, CBFG$ and $ACHK$ respectively.

**To Prove** : Square $ADEB =$ Square $ACHK$ + Square $CBFG$

**Construction** : From point C draw $CM \parallel BE$ which intersects $AB$ at $L$. Join $BK$ and $CD$.

**Proof** : $\angle BAD = \angle CAK = 90°$

Adding $\angle$ CAB both sides

$$\angle BAD \quad \angle CAB = \angle CAK \quad \angle CAB$$

$\Rightarrow$ $\angle CAD = \angle BAK$ ...(1)

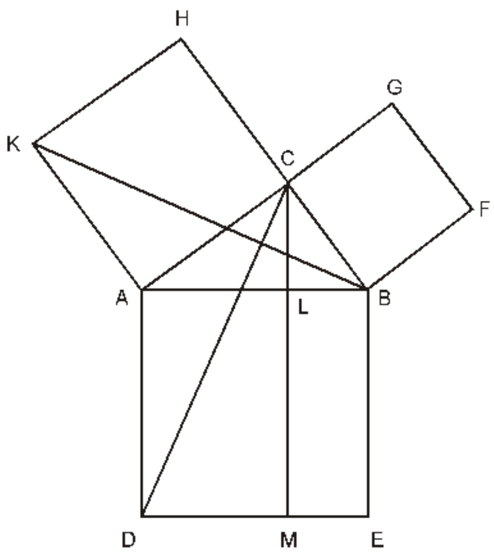



Fig. 10.18

In $\Delta BAK$ and $\Delta DAC$, we get

$AB = AD$

(Sides of a square $ABED$)

$\angle BAK = \angle$ DAC [From (1)]

$AK = AC$

[Sides of square $ACHK$]

$$\Delta BAK \cong \Delta CAD$$

(By SAS congruency rule) ...(2)

But $\angle BCA = \angle ACH = 90°$

$\angle BCA + \angle ACH = 180°$

$\Rightarrow BCH$ is a straight line.

$CH \parallel AK$ (Opposite sides of square $ACHK$)

ΔBAK and square $ACHK$ *are* on same base AK and between same parallel lines $AK$ and $BH$.

$\therefore$ $ar\ (\Delta BAK) = \frac{1}{2}$ ar (square $ACHK$) ...(3)

Similarly, ΔADC and rectangle $ADML$ are on same base $AD$ and between same parallel lines $AD$ and $CM$.

$\therefore$ ar ($\Delta\ CAD$) = $\frac{1}{2}$ ar (rectangle $ADML$) ... (4)

$\therefore$ From (2), (3) and (4), we get

ar ($\Delta\ CAD$) – ar ($\Delta\ BAK$) – $\frac{1}{2}$ ar of square ($ACHK$) – $\frac{1}{2}$ ar (of rectangle $ADML$)

$\therefore$ ar (square $ACHK$) – ar (rectangle $ADML$) ... (5)

Similarly, ar (square $CBFG$) − ar (rectangle $LMEB$) ...(6)

Adding (5) and (6), we get

ar (square $ACHK$) + ar (square $CBFG$) = ar (rectangle $ADML$) − ar (rectangle $LMEB$)

$\therefore$ ar (square ADEB) = ar (square ACHK) + ar (square CBFG) **Proved**

**Theorem 10.5. (Converse of Baudhayan Theorem)**

***In a triangle, if square of a side is equal to the sum of the squares of other two sides, angle opposite to this side, is a right angle.***

**Given :** In $\Delta\ ABC$, $AB^2 \quad BC^2 = AC^2$

To Prove : $\angle B = 90°$

**Construction** : Construct a $\Delta\ PQR$ such that $\angle Q = 90°$, $PQ - AB$ and $QR - BC$

**Proof** : In $\Delta\ PQR$ by Baudhayan theorem

$PR^2 - PQ^2 + QR^2$

But $PQ - AB$ and $QR = BC$.

$\therefore$ $PR^2 - AB^2 + BC^2$ ...(1)

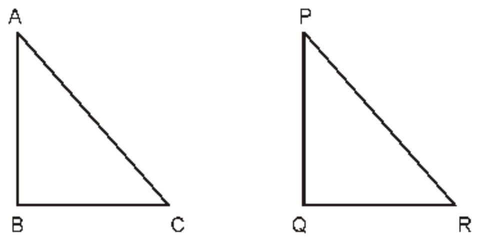



Fig. 10.19

But given that

$$AC^2 = AB^2 + BC^2 \qquad ...(2)$$

From (1) and (2)

$$PR^2 = AC^2 \Rightarrow PR = AC \qquad ...(3)$$

Now in $\Delta ABC$ and $\Delta PQR$, we get

| | |
|---|---|
| $PQ = AB$ | (By construction) |
| $QR = BC$ | (By construction) |
| $PR = AC$ | [From (3)] |
| $\Delta ABC \cong \Delta PQR$ | (By SSS congruency rule) |

$\therefore \quad \angle B = \angle Q = 90^\circ$

But $\quad \angle Q = 90^\circ$

$\therefore \quad \angle B = 90^\circ$ **Hence Proved**

**Illustrative Examples**

**Example 1. *PQRS* is a square. *T* and *U* are the mid-points of PS and *QR* respectively (Fig. 10.20). Find the area of ΔOTS. If PQ = 8 cm and *O* is the point of intersection of *TU* and QS.**

**Solution :** PS = $PQ = 8$ cm and $TU \parallel PQ$

$\therefore \quad ST = \frac{1}{2}PS = \frac{1}{2} \times 8 = 4$ cm

also $\quad PQ = TU = 8$ cm and $PQ \parallel TU$

In ΔPQS, T is the mid point of PS and TO ∥ $PQ$ them

$OT = \frac{1}{2}PQ.$

$\therefore \quad OT = \frac{1}{2}TU = \frac{1}{2} \times 8 = 4$ cm

$\therefore \quad$ ar $(\Delta\, OTS) = \frac{1}{2} \times OT \times TS$

[$\Delta\, OTS$ is a right angle triangle]

$= \frac{1}{2} \times 4 \times 4\,cm^2 = 8\,cm^2$

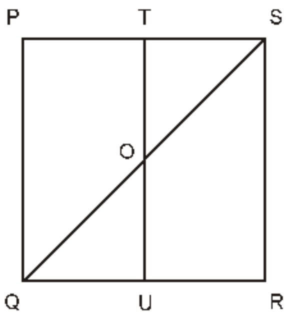


Fig. 10.20

**Exercise 2. *ABCD* is a parallelogram and BC is produced upto *Q* such that AD = *CQ* (Fig 10.21). If *AQ* intersects side *DC* at P. Then show that : ar(ΔBPC) = ar(ΔDPQ)**

**Solution** : ar(ACP) = ar(BCP) ...(1)

[Triangles made on same base PC and between same parallel lines PC and *AB*]

also ar (*ADC*) – ar (*ADQ*) ...(2)

[Triangles made on same base *AD* and between same parallel lines AD and *BQ*]

ar (ADC) - ar (*ADP*) − ar (*ADQ*) - ar (ADP)

ar (APC) = ar (DPQ) ...(3)

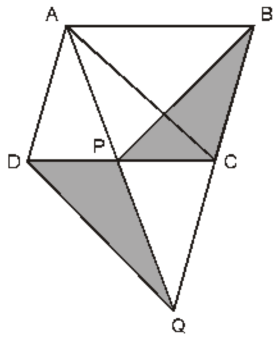


Fig. 10.21

From (1) and (3)

∴ ar (BCP) – ar (DPQ) **Hence Proved**

**Example 3. In Fig. 10.22 *ABCD* is a parallelogram. Point P and *Q* divide side BC in three equal parts. Prove that ar (APQ) = ar (*DPQ*) = $\frac{1}{6}$ ar (ABCD).**

**Solution** : Draw *PR* and *QS* parallel to *AB* from points P and *Q* respectively (Fig. 10.22).

Now, *PQRS* is a parallelogram

whose base $PQ - \frac{1}{3}\ BC$.

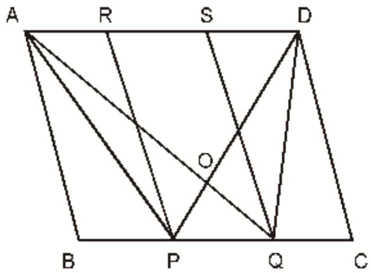


Fig. 10.22

Now ar (APD) = $\frac{1}{2}$ *ar* (*ABCD*)

[triangle and parallelogram *ABCD* lie on same base *AD* and between parallel lines *AD* and *BC*] ...(1)

also $\text{ar}(\text{AQD}) = \frac{1}{2}\text{ar}(\text{ABCD})$ ...(2)

From (1) and (2), we get

ar (APD) = ar (AQD) ...(3)

Subtracting *ar* (*AOD*) from both sides, we get

ar (APD) - ar (AOD) = ar (AQD) - ar (AOD)

∴ ar (APO) = ar (OQD) ...(4)

Adding ar (OPQ) to both sides, we get

ar (APO) + ar (*OPQ*) − ar (*OQD*) + ar (OPQ)

ar (APQ) = ar DPQ

∵ ar (APQ) = $\frac{1}{2}$ ar (PQRS)

and $\qquad ar\,(DPQ) = \frac{1}{2}\, ar(PQRS)$

now $\qquad$ ar (PQRS) = $\frac{1}{3}\, ar\,(ABCD)$

$\therefore \qquad ar(APQ) = ar(DPQ) = \frac{1}{2} ar(PQRS) = \frac{1}{2} \times \frac{1}{3} ar(ABCD)$

$= \frac{1}{6} ar(ABCD)$ **Hence Proved**

**Example 4.** **In Fig. 10.23 $l$, $m$ and n are lines such that $l \parallel m$ and line $n$ intersects line $l$ at P and line $m$ at $Q$. $ABCD$ is a quadrilateral such that vertex $A$ is situated on line $l$, vertices C and D are situated on line $m$ and $AD \parallel n$. Show that :**

**ar (ABCQ) = ar ($ABCDP$)**

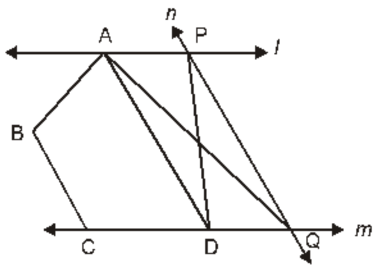


Fig. 10.23

**Solution :** $ar\,(ADP) = ar\,(ADQ)$ ...(1)

[On same base $AD$ and between same parallel lines AD and $PQ$]

Adding ar ($ABCD$) to both sides of (1), we get

ar (ADP) + ar ($ABCD$) = ar ($ADQ$) + ar ($ABCD$)

$ar\,(ABCDP) = ar\,(ABCQ)$ **Hence Proved**

**Example 5.** **In Fig 10.24, $BD \parallel CA$, $E$ is the mid-point of CA and $BD = \frac{1}{2}$ CA.**

**Prove that ar (ABC) = 2 ar ($DBC$).**

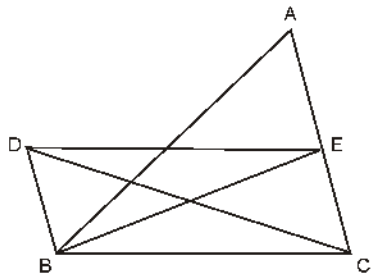


Fig. 10.24

**Solution :** Join $DE$. Here $BCED$ is a parallelogram.

$\because \qquad BD = CE \text{ and } BD \parallel CE$

$\therefore \qquad ar\,(DBC)$ = ar (EBC) ...(1)

(On same base $BC$ and between same parallels $BC$ and $DE$)

$BE$ is median in $\Delta$ ABC.

$\therefore \qquad$ ar (EBC) = $\frac{1}{2}$ ar (ABC)

Now $\quad$ ar (ABC) = ar (EBC) + ar (ABE)

$\therefore \qquad$ ar (ABC) = 2 ar (EBC)

$\therefore \qquad$ ar (ABC) = 2 ar (DBC) $\qquad$ [using (1)] **Hence Proved**

**Example 6. In an acute angled $\Delta$ ABC, $\angle B$ is an acute angle. Therefore, all angles will be less than 90°. $AD$ is perpendicular on BC. Prove that :**

**$AB^2 = AC^2 + BC^2 - 2\ BC \times DC$**

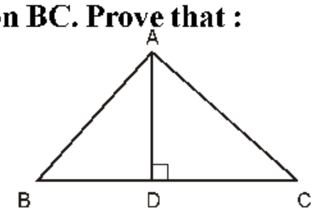



Fig. 10.25

**Solution : Given :** $\Delta ABC$, AD $\perp$ BC

**To Prove :**

$$AB^2 - AC^2 + BC^2 - 2\ BC \times DC$$

**Proof :** In $\Delta$ ABD, $\angle D - 90°$

$\therefore$ $AB^2 = AD^2 \quad BD^2$ (By Budhayan Theorem)

$\Rightarrow$ $AB^2 - AD^2 \quad (BC \quad DC)^2$

$= AD^2 + BC^2 + DC^2 - 2\ BC \times DC$

$= (AD^2 - DC^2) - BC^2 - 2\ BC \times DC$ ...(1)

Also in $\Delta$ ADC, $\angle D = 90^0$

$\therefore$ $AC^2 = AD^2 + DC^2$ ...(2)

From (1) and (2), we get

$AB^2 - AC^2 \quad BC^2 \quad 2\ BC \times DC$ **Hence Proved**

**Example 7. Prove that the sum of the squares of the sides of a rhombus, is equal to the sum of the squares of its diagonals.**

**Solution :**

**Given** : Diagonals $AC$ and $BD$ of a rhombus $ABCD$ interest at point $O$.

**To Prove** : $AB^2 + BC^2 \quad CD^2 + DA^2 - AC^2 \quad BD^2$

**Proof :** We know that diagonals of a rhombus intersect each other at right angles.

Therefore, in $\Delta AOB$, $OA^2 - OB^2 - AB^2$ ...(1)

Similarly, in $\Delta$ BOC, $OB^2 - OC^2 - BC^2$ ...(2)

In $\Delta$ COD, $OC^2 + OD^2 - CD^2$ ...(3)

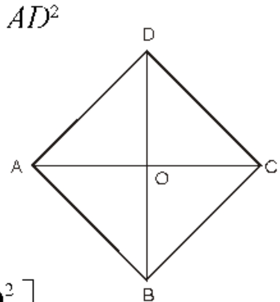
And in $\Delta$ AOD, $OA^2 \quad OD^2 - AD^2$ ...(4)

Adding (1), (2), (3) and (4), we get

$$2(OA^2 + OB^2 - OC^2 - OD^2) - AB^2 + BC^2 + CD^2 + AD^2$$

$\because$ $OA - OC - \frac{1}{2} AC$

and $OB - OD - \frac{1}{2} BD$




$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left[\frac{AC^2}{4} + \frac{AC^2}{4} + \frac{BD^2}{4} + \frac{BD^2}{4}\right]$$

Fig. 10.26

$$\therefore \quad AB^2 + BC^2 + CD^2 + AD^2 = 2\left[\frac{AC^2}{2}+\frac{BD^2}{2}\right]$$

$$= AC^2 + BD^2$$ **Hence Proved**

## EXERCISE 10.3

Write true or false and give reason to your answer :

1. $ABCD$ is a parallelogram and X is the mid-point of AB. If $ar(AXCD) = 24\text{cm}^2$, then ar (ABC) = 24 $\text{cm}^2$.
2. $PQRS$ is a rectangle which is inside a quadrant of a circle of radius 13 cm. A is any point on side $PQ$. If $PS = 5$ cm, then $ar(RAS) = 30\text{cm}^2$.
3. $PQRS$ is a parallelogram whose area is 180 $\text{cm}^2$ and $A$ is any point on diagonal $QS$. Then area of $\Delta ASR$ is 90 $\text{cm}^2$.
4. $ABC$ and $BDE$ are two equilateral triangles such that $D$ *is* the mid-point of side $BC$. Then ar $(BDE) = \frac{1}{4}$ ar $(ABC)$.
5. In Fig 10.27, $ABCD$ and $EFGD$ are two parallelograms and $G$ is the mid-point of side $CD$. Show that :

   ar (DPC) = $\frac{1}{2}$ ar (EFGD)

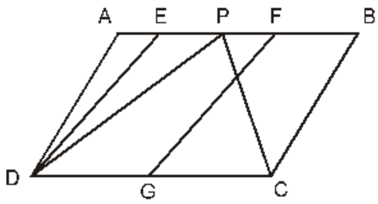



Fig. 10.27

6. In a trapezium $ABCD$, $AB \parallel CD$ and L is the mid-point of side $BC$. A line PQ $\parallel AD$ is drawn through $L$ which meets $AB$ on $P$ and extended $DC$ at $Q$ (Fig. 10.28). Prove that : ar $(ABCD) = ar\ (APQD)$.

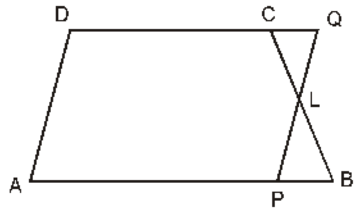



Fig. 10.28

7. If the mid-points of any quadrilateral are joined in a order, then prove that area of

such obtained parallelogram is half of the area of the given quadrilateral (Fig. 10.29). [Hint: Join $BD$ and draw perpendicular from $A$ on $BD$.]

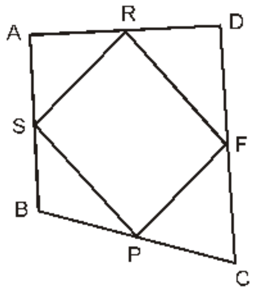



Fig. 10.29

8. A man walks 10 m in east and then 30 m in the north side. Find his distance from initial point.
9. A ladder is placed with a wall such that its lower end at a distance from wall is 7 m. If its other end at reached to the window height of 24 m. Find the length of the ladder.
10. Two poles of height 7 m and 12 m are standing on a plane ground. If distance between their feet is 12 m. Find the distance between upper ends of poles.
11. Find the length of altitude and area of an equilateral triangle whose length of side is a.
12. Find the length of diagonal of a square whose each side is 4 m.
13. If an equilateral triangle $ABC$, $AD$ is perpendicular on $BC$ then prove that $3AB^2 = 4AD^2$
14. $O$ is any point inside a rectangle $ABCD$. Prove that $OB^2 + OD^2 = OA^2 + OC^2$
15. In an obtuse triangle $ABC$, $\angle C$ is an obtuse angle. $AD \perp BC$ meats $BC$ at $D$ on extending farward. Prove that :

$$AB^2 = AC^2 + BC^2 + 2BC \times CD$$

## Important Points

1. If $\Delta ABC \cong \Delta PQR$, then ar ($\Delta ABC$) = ar ($\Delta$ PQR). Total area $R$ of plane figure $ABCD$ is equal to the sum of area of triangular fields $R_1$ and $R_2$ or ar ($R$) = ar ($R_1$) + ar($R_2$) [Figure 10.30]

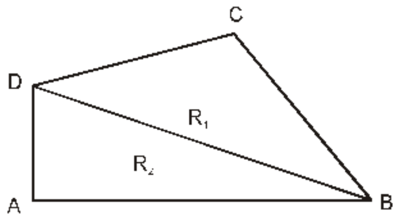



Fig. 10.30

2. Area of two congruent figures are equal but converse of it is not true always.

3. A diagonal of a parallelogram, divides it into two triangles of equal area.

(i) Areas of parallelograms made on same base and between same parallel lines, are equal.

(ii) A parallelogram and a rectangle made on same base and between same parallel lines are equal in areas.

4. Parallelograms made on same base and between same parallel lines, are equal in areas.

5. Triangles made on same base and between same parallel lines are equal in areas.

6. Corresponding altitudes of triangles having equal bases and equal areas, are equal.

7. If a triangle and a parallelogram are made on same base and between same parallel lines, then area of triangle is half of the area of parallelogram.

## Miscellaneous Exercise 10

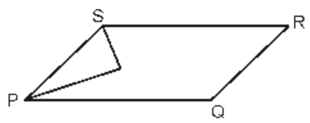
Write correct answer in each of the following :

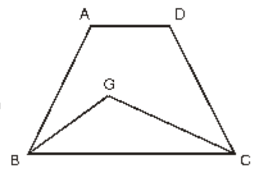
1. Median of a triangle divides it into two :

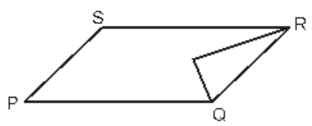
(a) triangles of equal areas (b) congruent triangles

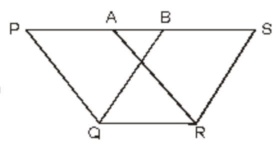
(c) right angled triangles (d) isosceles triangles

2. In which of following figures, you find two polygons made on same base and between same parallel lines :

(a) 


(b) 


(c) 


(d) 


3. The figure, made by joining mid-points of adjecent sides 8 cm and 6 cm of a rectangle is :

(a) a rectangle of area 24 $cm^2$ (b) a square of area 25 $cm^2$

(c) a trapezium of area 24 $cm^2$ (d) a rhombus of area 24 $cm^2$

4. In Fig. 10.31, area of parallelogram ABCD is :

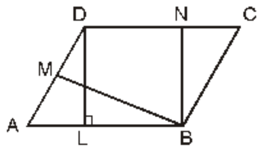



Fig. 10.31

(a) AB × BM (b) BC × BN

(c) DC × DL (d) AD × DL

5. In fig. 10.32, if parallelogram ABCD and rectangle ABEM are of equal areas, then :

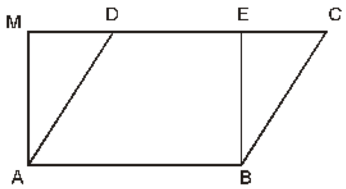



Fig. 10.32

(a) perimeter of ABCD = Perimeter of ABEM

(b) perimeter of ABCD < perimeter of ABEM

(c) perimeter of ABCD > perimeter of ABEM

(d) perimeter of ABCD = $\frac{1}{2}$ (Perimeter of ABEM)

6. Mid points of the sides of a triangle make a simple quadrilateral by taking with any vertex as fourth point, whose area is equal to :

(a) $\frac{1}{2}$ ar (ABC) (b) $\frac{1}{3}$ ar (ABC)

(c) $\frac{1}{4}$ ar (ABC) (d) ar (ABC)

7. Two parallelogram are on same base and between same parallel lines. Ratio of their areas is :

(a) 1 : 2 (b) 1 : 1

(c) 2 : 1 (d) 3 : 1

8. ABCD is a quadrilateral whose diagonal AC divides it into two parts of equal areas, then ABCD :

(a) is a rectangle (b) is always a rhombus

(c) is a parallelogram (d) is all of these

9. A triangle and a parallelogram are on same base and between same parallel lines, then ratio of areas of triangle with area of parallelogram is :

(a) 1 : 3 (b) 1 : 2

(c) 3 : 1 (d) 1 : 4

10. ABCD is a trapezium whose sides AB = a cm and DC = b cm (Fig. 10.33) E and F are mid-points of non-parallel sides. Ratio of ar (ABFE) and ar (EFCD) is :

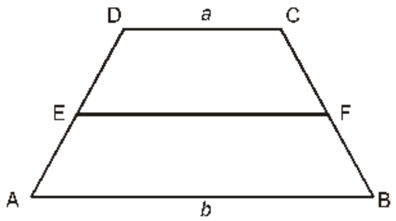



Fig. 10.33

(a) a : b (b) (3a + b) : (a + 3b)

(c) (a + 3b) : (3a + b) (d) (2a + b) : (3a + b)

11. If P is any point on median AD of $\Delta ABC$, then ar (ABP) $\neq$ ar(ACP).

12. If in fig. 10.34, PQRS and EFRS are two parallelogram, then ar (MFR) = $\frac{1}{2}$ ar (PQRS).

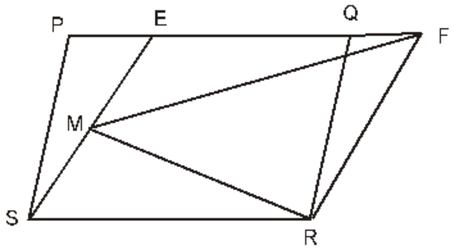



Fig. 10.34

13. In Fig. 10.35 PSDA is a parallelogram. Points Q and R on PS are taken such that PQ = QR = RS and PA || QB || RC.

Prove that ar (PQE) = ar (CFD)

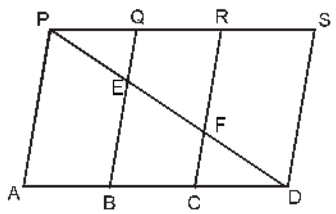



Fig. 10.35

14. X and Y are two points on side LN of $\Delta LMN$ such that LX = XY = YN. Through X a line is drawn parallel to LM, which meets MN at Z. (see Fig. 10.36). Prove that

: ar (LZY) = ar (MZYX)

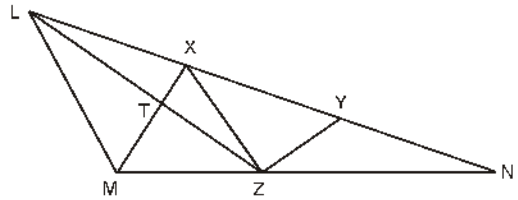



Fig. 10.36

15. Area of parallelogram ABCD is $90\,\text{cm}^2$ [Fig. 10.37]. Find the area of

(i) ar (ABEF) (ii) ar (ABD) (iii) ar (BEF)

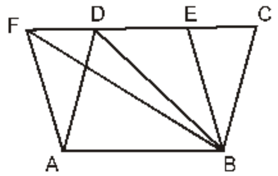



Fig. 10.37

16. In $\Delta ABC, D$ is the mid-point of side AB and P is any point on side BC. If line segment CQ || PD meets side AB at Q (Fig. 10.38), then prove that :

ar (BPQ) = $\frac{1}{2}$ ar (ABC)

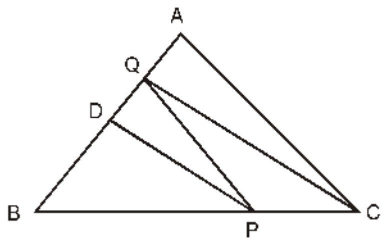



Fig. 10.38

17. ABCD is a square. E and F are respectively mid-points of sides BC and CD. If R is the mid-point of line segment EF (Fig. 10.39), then prove that : $ar(\Delta AER) = ar(\Delta AFR)$

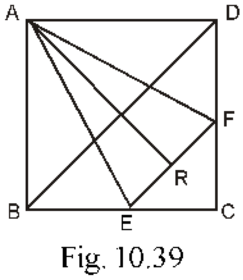



Fig. 10.39

18. O is any point on diagonal PR of a parallelogram PQRS (Fig. 10.40). Prove ar (PSO) = ar (PQO)

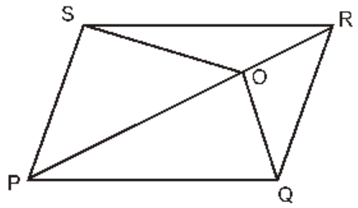


Fig. 10.40

19. ABCD is a parallelogram in which side BC is extended upto point E such that CE = BC (Fig. 10.41). AE intersects side CD at F. If ar (DFB) is 3 $cm^2$, then find the area of parallelogram ABCD.

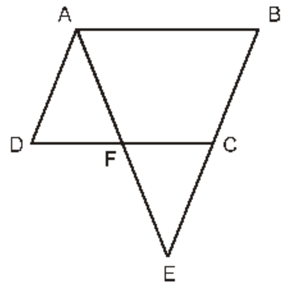


Fig. 10.41

20. Point E is taken on side BC of a parallelogram ABCD. AE and DC are extended so that they meet at F. Prove that ar (ADF) = ar (ABFC).
21. Diagonals of a parallelogram ABCD intersect at O. A line is drawn from O which meets AD at P and BC at Q. Show that PQ divides this parallelogram into two parts of equal areas.
22. Medians BE and CF of a $\Delta ABC$ intersect each other at point G. Prove that area of $\Delta GBC$ is equal to the area of quadrilateral AFGE.
23. In Fig. 10.42 CD || AE and CY || BA. Prove that : ar (CBX) = ar (AXY)

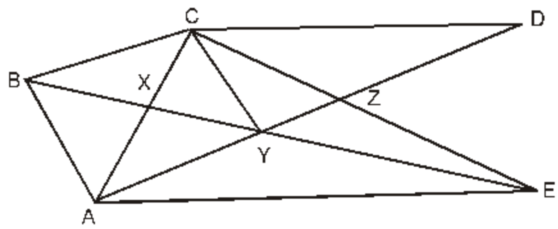


Fig. 10.42

24. ABCD is a trapezium in which AB || CD, CD = 30 cm and AB = 50 cm. If, X and Y are mid-points of AD and BC respectively, then prove that :

ar (DCYX) = $\frac{7}{9}$ ar (XYBA).

25. In $\Delta ABC$, L and M are point on sides AB and AC respectively such that LM || BC. Prove that : ar (LOB) = ar (MOC) is LC and BM intersect at O.

26. In Fig. 10.43 ABCDE is a pentagon. BP drawn parallel to AC, meets extended DC at P and EQ drawn parallel to AD meets extended CD at Q.
Prove that :
ar (ABCDE) = ar (APQ).

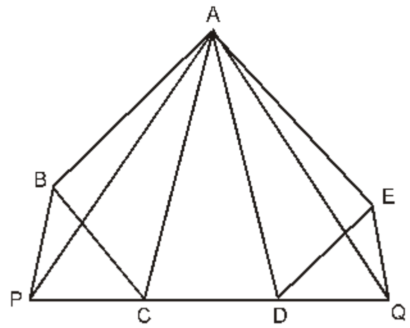



Fig. 10.43

27. If medians of a triangle ABC meet at point G, then prove that :

ar (AGB) = ar (AGC) = ar (BGC) = $\frac{1}{3}$ ar (ABC).

28. In figure 10.44 X and Y are respectively the mid points of sides AC and AB .QP || BC and CYQ and BXP are striaght lines. Show that : ar (ABP) = ar (ACQ)

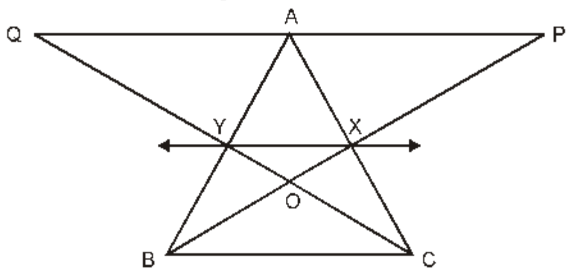



Fig. 10.44

29. In Fig. 10.45, ABCD and AEFD are two parallelogram. Prove that : ar (PEA) = ar (QFD).
[Hint : Join PD]

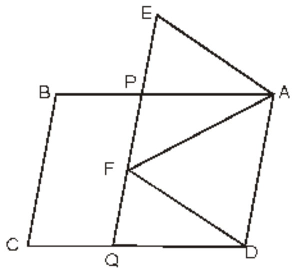


Fig. 10.44

# Answers

## EXERCISE 10.1

**1.** (i) DC and DC || AB ; (iii) QR, QR || PS ; (v) AD, AD || QC

## EXERCISE 10.2

**1.** 12.8 cm

## EXERCISE 10.3

**1.** Flase $ar(A \times CD) = ar(ABCD) - ar(BC \times) = 48 - 12 = 36\,\text{cm}^2$

**2.** True $SR = \sqrt{13^2 - 5^2} = 12 ar(PAS) = \frac{1}{2} ar(PQRS) = 30\,\text{cm}^2$

**3.** Flase area of $\Delta QRS = 90\,\text{cm}^2$ and $ar(ASR) < ar(QRS)$

**4.** True $\frac{ar(BDE)}{ar(ABC)} = \frac{\sqrt{3}(BD)^2}{\sqrt{3}(BC)^2} = \frac{1}{4}$

4

**5.** Flase $ar(DPC) = \frac{1}{2}(ABCD) = ar(EFGD)$

**6.** $10\sqrt{10}$ m

**9.** 25 m

**10.** 13 m

**11.** $\frac{\sqrt{3}}{a}a, \frac{\sqrt{3}}{4}a^2$

**12.** $4\sqrt{2}$

## Miscellaneous Exercise 10

| | | | |
|---|---|---|---|
| **1.** A | **2.** D | **3.** D | **4.** C |
| **5.** C | **6.** A | **7.** D | **8.** D |
| **9.** B | **10.** B | | |

**11.** Flase: $ar(ABD) = (ACD)$ and $ar(PBD) = ar(PCD)$ $\therefore$ $ar(ABP) = ar(ACP)$

**12.** True: $ar(PQRS) = ar(EFRS) = 2ar(MFR)$

**15.** (i) 90 $cm^2$ ; (ii) 45 $cm^2$ ; (iii) 45 $cm^2$

**19.** 13 $cm^3$