6. Linear programming

Exercise: 6.1

1. A manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry and Then sent to machine shop for finishing. The number of Man hours of labor required in each shop for production of A and B and the number of man hours available for the
Firm are as follows:

Gadgets	Foundry	Machine shop
A	10	5
B	6	4
Time available (In hour)	60	35

Profit on the sale of A is ₹ 30 and B is₹ 20 per unit. Formulate the LPP to have maximum profit.

Solution: Let the number of gadgets A produced by the firm be x and the number of gadgets B produced by the firm be y. The profit on the sale of A is ₹ 30 per unit and on the sale 0f B is ₹20 per unit. The total
∴ profit is $z = 30x + 20y$.
This is a linear function which is to be maximized.
Hence, it is the objective function. The constraints are as per the following table:

	Gadget A (x)	Gadget B (y)	Total available Time (in hour)
Foundry	10	6	60
Machine shop	5	4	35

From the above table total man-hours of labor required for x units of gadget A and y units of gadget B in foundry is $(10x + 6y)$ hours and total man-hours of labor required in machine shop is $(5x + 4y)$ hours.
Since, maximum time available in foundry and machine.

Shops are 60 hours and 35 hours respectively, therefore, the constraints are $10x + 6y < 60$, $5x + 4y < 35$ Since, x and y cannot be negative, we have x2 0, y 2 0. Hence, the given LPP can be formulated as. Maximize $z = 30x + 20y$, subject to $10x + 6y \leq 60$, $5x + 4y \leq 35$, $x \geq 0$, $y \geq 0$.

2. In a cattle breeding firm, it is prescribed that the food ration for One animal must contain 14, 22 and 1 unit of nutrients A, B and C respectively. Two different kinds of fodder are available. Each unit weight of These two contains the following amounts of these three nutrients:

Fodder → Nutrients ↓	Fodder 1	Fodder 2
Nutrients A	2	1
Nutrients B	2	3
Nutrients c	1	1

The cost of fodder 1 is ₹ 3 per unit and that of fodder 2 is ₹ 2 per unit.
Formulate the LPP to minimize the cost.
Solution: Let x units of fodder 1 and y units of fodder 2 be prescribed.
The cost of fodder 1 is ₹ 3 per unit and cost of fodder 2 is ₹ 2 per unit. '
The total cost is $z = 3x + 2y$.
This is the linear function which is to be minimized.
Hence, it is the objective function. The constraints are as per the following table:

Fodder → Nutrients ↓	Fodder 1	Fodder 2	Minimum requirements
Nutrients A	2	1	14
Nutrients B	2	3	22
Nutrients c	1	1	1

From the above table fodder contains $(2x + y)$ units 0f nutrients A, $(2x + 3y)$ Units of nutrients B and $(x + y)$ units of nutrients C. The minimum Requirements of these nutrients are 14 units, 22 units and 1 unit respectively.
Therefore, the constraints are:
$2x + y \geq 14$, $2x + 3y \geq 22$, $x + y \geq 1$
Since, number of units (i.e. x and y) cannot be negative,
we have, $x \geq 0$, $y \geq 0$.
Hence, the given LPP can be formulated as
Minimize $z = 3x + 2y$, subject to $2x + y \geq 14$
$2x + 3y \geq 22$, $x + y \geq 1$, $x \geq 0$, $y \geq 0$

3. A company manufactures two types of chemicals A and B. Each chemical requires two types of raw material P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B.

Chemical → Raw material ↓	A	B	Availability
p	3	2	120
Q	2	5	160

The company gets profits of ₹ 350 and ₹ 400 by selling one unit of A and one unit of B respectively. Formulate the problem as LPP to maximize profit.

Solution: Let the company manufactures x units Of Chemical A and 3/ units of chemical B. Then the total profit to the company is p = ₹ (350x + 400y).
This is a linear function which is to be maximized.
Hence, it is the objective function.
The constraints are as per the following table:

Chemical → Raw material ↓	A (x)	B (y)	Availability
p	3	2	120
Q	2	5	160

The raw material P required for x units of chemical A and y units of Chemical B is 3x + 2y. Since, the maximum Availability of P is 120, we have the first constraint as 3x + 2y< 120.
Similarly, considering the raw material Q we have 2x + 5y< 160.
Since, x and y cannot be negative, we have, x 2 0, y 2 0.
Hence, the given LPP can be formulated as:
Maximize p: 350x + 400y, subject to $3x + 2y \leq 120$, $2x + 5y \leq 160$, $x \geq 0$, $y \geq 0$.

**4. A printing company prints two types of magazines A and B.
The company earns ₹ 10 and ₹ 15 on mega zines A and B per copy.
These are processed on three Machines I, II, III. Magazine
A requires 2 hours on Machine I, 5 hours on Machine II, and
2 hours on machine III. Magazine B requires 3 hours.
On machine I, 2 hours on machine II, and 6 hours on Machine III.
Machines I, II, III are available for 36, 50 and 60 hours per week
Respectively. Formulate the Linear programming problem to maximize the profit.**

Solution: Let the company prints x magazine of type A and 3/ magazine of type B.

Profit on sale of magazine A is ₹ 10 per copy and magazine B is ₹ 15 per copy.
Therefore, the total earning 2 of the company is
z = ₹ (10x + 15y). This is a linear function which is to be maximized
Hence, it is the objective function.
The constraints are as per the following table:

Magazine Type / Machine type	Time required per unit		Available time per week (in hours)
	Magazine A (X)	Magazine B (y)	
Machine I	2	3	36
Machine II	5	2	50
Machine III	2	6	60

From the above table, the total time required for machine I
Is (2x + 3y) hours, for machine II is (5X + 2y) hours and for machine III
Is (2x + 6y) hours. The machine I, II, III are available for 36, 50 and 60 hours
Per week.
Therefore, the constraints are $2x + 3y \leq 36$, $5x + 2y \leq 50$,
$2x + 6y \leq 60$. Since, x and y cannot be negative, we have,
$X \geq 0$, $y \geq 0$. Hence, the given LPP can be formulated as
Maximize $z = 10x + 15y$, subject to $2x + 3y \leq 36$ $5x + 2y \leq 50$, $2x + 6y \leq 60$, $x \geq 0$, $y \geq 0$.

5. A manufacturer produces bulbs and tubes. Each of these must be processed through two machines M_1 and M_2. A package of bulbs require 1 hour of work on Machine M_1 and 3 hours of work on Machine M_2. A package of tubes require 2 hour's on Machine M_1 and 4 hours on Machine M_2. He earns a profit of ₹ 13.5 per package of bulbs and ₹ 55 per package of tubes. Formulate the LLP to maximize the profit.

Solution: Let the number of packages of bulbs produced
By manufacturer be x and packages of tubes be y.
The manufacturer earns a profit of ₹ 13.5 per package of
Bulbs and ₹ 55 per package of tubes.
Therefore, his total profit is p = ₹ (13.5x + 55y)
This is a linear function which is to be maximized.
Hence, it is the objective function.
The constraints are as per the following table:

	Bulbs (x)	Tubes (y)	Available time
Machine M_1	1	2	10
Machine M_2	3	4	12

From the above table, the total time required for Machine M_1 is $(x+2y)$ hours and for Machine M_2 is $(3x + 4y)$ hours.
Given Machine M_1 and M_2 are available for at most 10 Hours and 12 hours a day respectively.
Therefore, the constraints are $x + 2y \le 10$, $3x + 4y \le 12$
Since, x and y cannot be negative, we have, $x \ge 0, y \ge, 0$.
Hence, the given LPP can be formulated as:
Maximize $p = 13.5x + 55y$, subject to $x + 2y \le 10$ $3x + 4y \le 12$, $x \ge 0$, $y \ge 0$.

6. A company manufactures two types of fertilizers F_1 and F_2. Each type of fertilizer requires two raw materials A and B. The number of units of A and B required to manufacture one Unit of fertilizer F_1 and F_2 and availability of the raw materials A and B per day are given in the following table:

Fertilizers → Raw material ↓	F_1	F_2	Availability
A	2	3	40
B	1	4	70

By selling one unit of F_1 and one unit of F_2, company gets a profit Of ₹ 500 and ₹ 750 respectively. Formulate the problem as LPP to maximize the profit.

Solution: Refer to the solution of Q. 3.

Ans. Maximize $z = 500x + 750y$, subject to $2x + 3y \le 40$, $x + 4y \le 70$, $x \ge 0$, $y \ge 0$.

7. A doctor has prescribed two different kinds of feeds A and B to form a weekly diet for a sick person. The minimum requirement of fats, carbohydrates and proteins are 18, 28, 14 units respectively. One unit of food A has 4 units of fat, 14 units of carbohydrates and 8 units of protein. One unit of food B has 6 units

of fat, 12 units of carbohydrates and 8 units of protein.
The price of food A is ₹ 4.5 per unit and that of food B is ₹ 3.5 per unit.
Form the LPP, so that the sick person's diet meets the requirements
At a minimum cost.

Solution: Let the diet of sick person include x Units of Food A
and y units of food B.
Then x ≥ 0, y ≥ 0.
He prices of food A and B is ₹ 45 and ₹ 35 per unit sportively:
Therefore, the total cost is z = ₹ (4.5x + 35y).
This is the linear function which is to be minimized.
Hence, it is objective function.
The constraints are as per the following table:

Magazine Type / Machine type	Food A (X)	Food B (y)	Minimum requirements
Fat	4	6	18
Carbohydrates	14	12	28
Proteins	8	8	14

From the above table, the sick person's diet will include (4x + 6y) units of fats, (14x + 12y) units of carbohydrates And (8x + 8y) units of proteins. The minimum requirements of these ingredients are 18 units, 28 units and 14 units respectively.
Therefore, the constraints are 4x+6y ≥ 18, 14x + 12y ≥ 28, 8x + 8y ≥ 14,
Hence, the given LPP can be formulated as Minimize z = 4.5x + 3.5y, subject to 4x+ 6y ≥ 18,
14x + 12y ≥ 28, 8x + 8y ≥ 14, x ≥ 0, y≥ o.

8. If John drives a car at a speed of 60 km/hour, he has To spend 5 per km on petrol. If he drives at a faster Speed of 90 km/hour, the cost of petrol increases to ₹ 8 per km. He has? 600 to spend on petrol and Wishes to travel the maximum distance within an Hour. Formulate the above problem as LPP.

Solution: Let John travel x1 km at a Speed of 60 km/hour and
x2 km at a speed of 90 km /hour.
Therefore, time required to travel a distance of x1 km is
$\frac{x_1}{60}$ $60\ hours$
and the time required to travel a distance of
$x\ 2km\ is\ \frac{x2}{90}\ hours.$

Then total time required to travel is
$\left(\frac{x1}{60} + \frac{x2}{90}\right)$ *hours*
Since, he wishes to travel the maximum distance within an hour.
$\frac{x1}{60} + \frac{x2}{90} \leq 1$
He has to spend ₹ 5 per km on petrol at a speed of
60 km/hour and f 8 per km at a speed of 90 km/hour.
∴ the total cost of travelling is ₹ (5x1 + 8x2)
Since, he has ₹ 600 to spend on petrol,
5x1 + 8x2 ≤ 600
Since, distance 15 never negative, x1 ≥ 0, x2 ≥ 0
Total distance travelled by John is z = (x1 + x2) km.
This is the linear function which is to be maximized.
Hence, it is objective function.
Hence, the given LPP can be formulated as:
Maximize z : = x1 + x2 , subjected to
$\frac{x1}{60} + \frac{x2}{90} \leq 1$
5x1 + 8x2 ≤ 600, x1 ≥ 0, x2 ≥ 0.

9. The company makes concrete bricks made up of cement and sand. The weight of a concrete brick has to be at least 5 kg. Cement costs ₹ 20 per kg and sand costs ₹ 6 per kg. Strength consideration dictate that a Concrete brick should contain minimum 4 kg of cement And not more than 2 kg of sand. Formulate the LPP for the cost to be minimum.

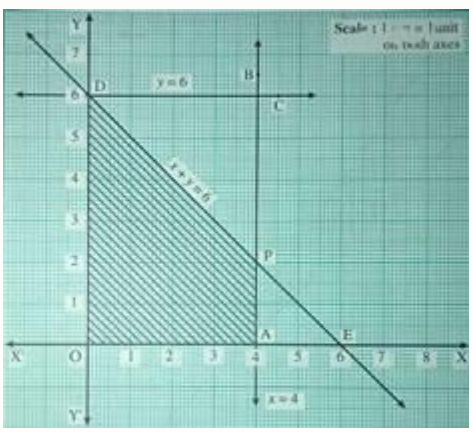
Solution: Let the company use x1 kg of cement and x2 kg of sand to make concrete bricks.
Cement costs ₹ 20 per kg and sand costs ₹ 6 per kg. the total cost c = ₹ (20x1 + 6x2)
This is a linear function which is to be minimized.
Hence, it is the objective function.
Total weight of brick = (x 1 + x2) kg
Since, the weight of concrete brick has to be at least 5 kg
∴ x1 + x2 ≥ 5
Since, concrete brick should contain minimum 4 kg of
Cement and not more than 2 kg of sand,
x1 ≥ 4, and 0 ≤ x2 ≤ 2
Hence, the given LPP can be formulated as:
Minimize c = 20x1 + 6x2, subject to x1 + x2 ≥ 5, x1 ≥ 4,
0 ≤ x2 ≤2, x1 ≥ 0, x2 ≥ 0.

Exercise 6.2

Solve the following LPP by graphical method:
1. Maximize z = 11x + 8y
Subject to x ≤ 4, y ≤6, x + y ≤ 6, x ≥ 0, y ≥ 0.
Solution: First we draw the lines AB, CD and ED whose
Equation are x = 4, y = 6 and x + y = 6 respectively.

Line	equation	Point on the x -axis	Point on the y - axis	sign	Region
AB	X = 4	A(4 ,0)	-	≤	Origin side Of the line AB
CD	Y = 6	-	D (0 ,6)	≤	Origin side Of the line CD
ED	X + y = 6	E (6 ,0)	D (0 ,6)	≤	Origin side Of the line ED

The feasible region is shaded portion OAPDO in the figure.



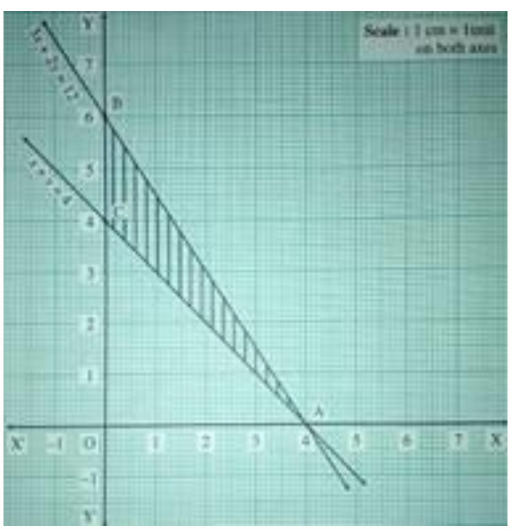
The vertices of the feasible region are o(0,0) A(4,0)
P and D (0,6)
P is the point of intersection of the lines x + y = 6 and
X = 4. Substitute x =4 in x+ y = 6 we get
4 + y = 6
∴ y =2
∴ P is (4,2)
∴ the corner points of feasible region are o(0,0) A(4,0)

P and D (0,6)
The values of the objects function z = 11x + 8y at these Vertices are
Z(0) = 11(0)+8(0) = 0 + 0 = 0
Z(A) = 11(4) + 8(0) = 44 + 0 = 44
Z(P) = 11(4) + 8(2) = 44 + 16 = 60
Z(D) = 11(0) + 8(2) = 0 + 16 = 16
∴ z has maximum value 60, when x = 4 and y = 2.

2. Maximize z = 4x + 6y
Subject to 3x + 2y ≤ 12, x + y ≥ 4, x , y ≥ 0

Solution:
First we draw the lines AB and AC whose
Equation are 3x + 2y = 12 and x + y = 4 respectively.

Line	equation	Point on the x - axis	Point on the y -axis	sign	Region
AB	3x + 2y = 12	A(4,0)	B(0,6)	≤	Origin side Of the line AB
CD	x + y = 4	A(4,0)	C(0,4)	≤	Non-Origin side Of the line AC



The Feasible region is the Δ ABC which is shaded in the Figure.
The vertices of the feasible region (i.e corner points) are A(4,0) B(0,6) and C(0,4)
The value of the objective function z = 4x + 6y at these Vertices are

$Z(A) = 4(4) + 8(0) = 6(0) = 16 + 0 = 16$
$Z(B) = 4(0) + 6(6) = 0 + 36 = 36$
$Z(C) = 4(0) + 6(4) = 0 + 24 = 24$
$\therefore$ z has maximum value 36, when $x = 0$, and $y = 6$.

3. Maximize $z = 7x + 11y$,
Subject to $3x + 5y \leq 26$, $5x + 3y \leq 30$, $x \geq 0$, $y \geq 0$.

Solution:
First we draw the lines AB and CD whose
Equation are $3x + 5y = 26$ and $5x + 3y = 30$ respectively.

Line	equation	Point on the x - axis	Point on the y - axis	sign	Region
AB	$3x + 5y = 26$	$A\left(\frac{25}{3},0\right)$	$B\left(0,\frac{26}{5}\right)$	$\leq$	Origin side Of the line AB
CD	$5x + 3y = 30$	C(6,0)	D(0,10)	$\leq$	Origin side Of the line CD

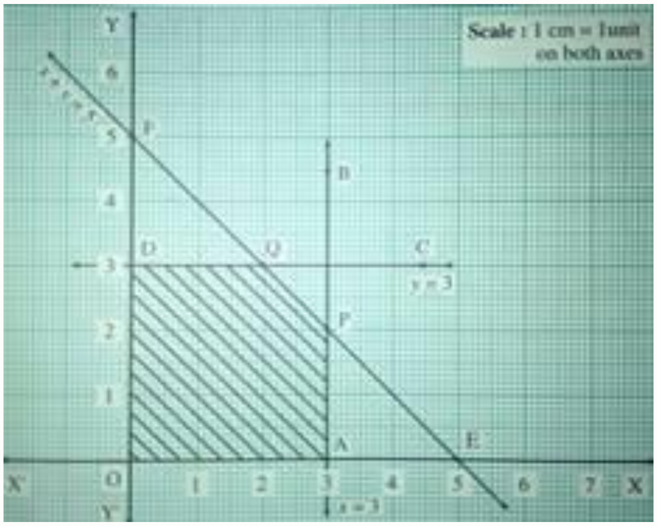
The Feasible region is OCPBO which is shaded in the Figure.
The vertices of the feasible region are O (0,0), C(6,0), $P \text{ and } B\left(0\frac{26}{5}\right)$.
The vertex p is the point of intersection of the lines
$3x + 5y = 26$ ... (1)

25x + 15y = 150 … (2)
On subtracting, we get
$16x = 72$
$\therefore x = \frac{72}{16} = \frac{9}{2} = 4.5$
Substituting x = 4.5 in equation (2), we get
5(5.4) + 3y = 30
22.5 + 3y = 30
∴ 3y = 7.5 ∴ y = 2.5
P is (4.5, 2.5)
The value of the objective function z = 7x + 11y at these
Corner points are:
Z(0) = 7(0) + 11(0) = 0 + 0 = 0
Z(C) = 7(6) + 11(0) = 42 + 0 = 42
Z(P) = 7(4.5) + 11(2.5) = 31.5 + 27.5 = 59.0 = 59
$Z\ (B) = 7(0) + 11\left(\frac{26}{5}\right)$
$= \frac{286}{5} = 57.2$
∴ z has maximum value 59, when x = 4.5, and y = 2.5.

4. Maximize z = 10x + 25y,
Subject to 0 ≤ x ≤ 3, 0 ≤ y ≤ 3, x + y ≤ 5.

Solution:
First we draw the lines AB, CD and EF whose
Equation are x =3, y =3 and x + y = 5 respectively.

Line	equation	Point on the x -axis	Point on the y - axis	sign	Region
AB	x = 3	A (3 , 0)	-	≤	Origin side Of the line AB
CD	y = 3	-	D(0,3)	≤	Origin side Of the line CD
EF	X + y = 5	E (5,0)	F(0,5)		Origin side Of the line EF



The feasible region is OAPQDO which is shaded in the Figure.

The vertices of the feasible region are O(0,0), A(3,0) P, Q and D (0,3).

P is the point of intersection of the line $x + y = 5$ and $x = 3$.

Substituting $x = 3$ in $x + y = 5$, we get

$3 + y = 5 \quad \therefore y = 2$

$\therefore$ P is (3,2)

Q is the point of intersection of the line $x + y = 5$ and $Y = 3$

Substituting $y = 3$ in $x + y = 5$, we get

$X + 3 = 5, \quad \therefore x = 2, \quad \therefore$ Q is (2, 3)

The value of the objective function $z = 10x + 25y$ at these Vertices are

$Z(O) = 10(0) + 25(0) = 0 + 0 = 0$

$Z(A) = 10(3) + 25(0) = 30 + 0 = 30$

$Z(P) = 10(3) + 25(2) = 30 + 50 = 80$

$Z(Q) = 10(2) + 25(3) = 20 + 75 = 95$

$Z(D) = 10(0) + 25(3) = 0 + 75 = 75$

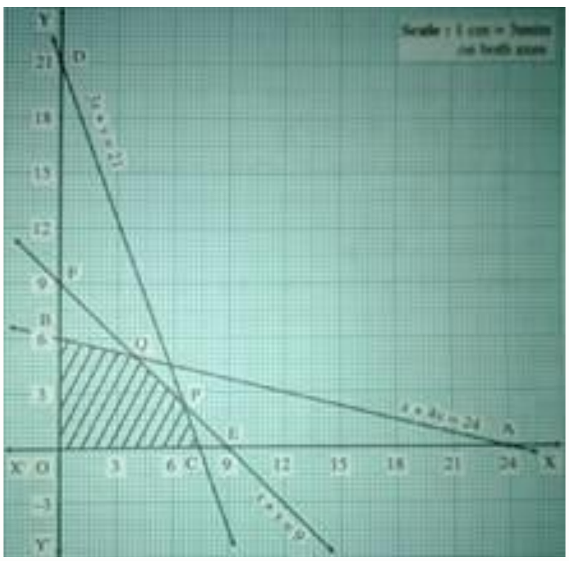
$\therefore$ z has maximum value 95, when $x = 2$, and $y = 3$.

5. Maximize $z = 3x + 5y$,
Subject to $x + 4y \leq 24$, $3x + y \leq 21$, $x + y \leq 9$, $x \geq 0$, $y \geq 0$.

Solution:

First we draw the lines AB, CD and EF whose Equation are $x + 4y = 24$, $3x + y = 21$, $x + y = 9$ Respectively.

Line	equation	Point on the x - axis	Point on the y - axis	sign	Region
AB	X+4y = 3	A (24 , 0)	B(0,6)	≤	Origin side Of the line AB
CD	3x + y = 21	C(7,0)	D(0,21)	≤	Origin side Of the line CD
EF	X + y = 9	E (9,0)	F(0,9)	≤	Origin side Of the line EF



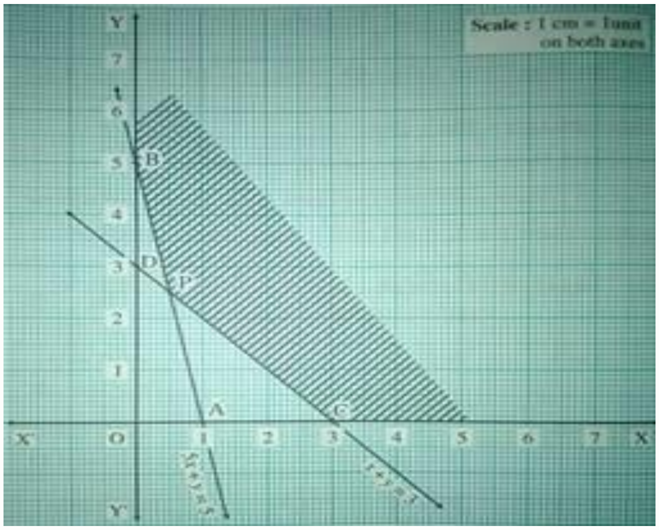
The feasible region is OCPQBO which is shaded in
The figure.
The vertices of the feasible region are O(0,0) C(7,0)
P, Q and B(0,6).
$3x + y = 21$ ... (1)
And $x + y = 9$ ... (2)
On subtracting we get
$2x = 12 \quad \therefore x = 6$
Substituting $x = 6$ in equation (2), we get
$6 + y = 9 \quad \therefore y = 3$
$\therefore p \equiv (6,3)$
Q is the point of intersection of lines
$X + 4y = 24$ ... (3)
And $x + y = 9$ ...(2)
On subtracting we get,

$3y = 15 \therefore y = 5$
Substituting $y = 5$ in equation (2), we get
$X + 5 = 9 \quad \therefore x = 4$
$\therefore Q \equiv (4, 5)$
$\therefore$ the corner points of the feasible region are
O(0,0), C(7,0), P(6,3), Q(4,5) and B(0,6)
The values of the objective function $z = 3x + 5y$ at these
Corner points are
$Z(O) = 3(0) + 5(0) = 0 + 0 = 0$
$Z(A) = 3(7) + 5(0) = 21 + 0 = 21$
$Z(P) = 3(6) + 5(3) = 18 + 15 = 33$
$Z(Q) = 3(4) + 5(5) = 12 + 25 = 37$
$Z(D) = 3(0) + 5(6) = 0 + 30 = 30$
$\therefore$ z has maximum value 37, when $x = 4$, and $y = 5$.

6. Maximize $z = 7x + y$,
Subject to $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$,

Solution:
First we draw the lines AB, CD whose
Equation are $5x + y = 5$, and $x + y = 3$
Respectively.

Line	equation	Point on the x -axis	Point on the y -axis	sign	Region
AB	$5x + y = 5$	A(1,0)	B (0,5)	$\geq$	Non- Origin side Of the line AB
CD	$X + y = 3$	C(3,0)	D(0,3)	$\geq$	Non-Origin side Of the line CD



The feasible region is XCPBY which is shaded in the figure.
The vertices of the feasible region are C (3,0) P, And B(0,5).
P is the points of the intersection of the lines
5x + y = 5 and x + y =3
On subtracting, we get

$4x = 2 \quad \therefore x = \frac{1}{2}$

$Substituting\ x = \frac{1}{2}\ in\ x + y = 3,$

We get

$\frac{1}{2} + y = 3$

$\therefore y = \frac{5}{2} \quad \therefore P = \left(\frac{1}{2}, \frac{5}{2}\right)$

The value of the objective function z = 7x + y at these
Vertices are
Z(C) = 7(3) + 0 = 21

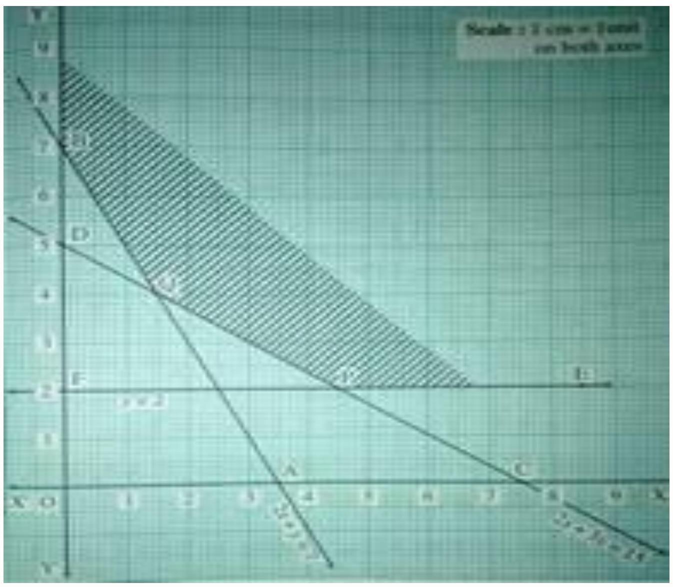
$Z(P) = 7\left(\frac{1}{2}\right) + \frac{5}{2}$

$\rightarrow = \frac{7}{2} + \frac{5}{2} = 6$

Z(B) = 7(0) + 5 = 5
∴ z has maximum value 5, when x = 0, and y = 5.

7. Maximize z = 8x + 10y,
Subject to 2x + y ≥7, 2x + 3y ≥15, y ≥ 2, x ≥ 0, y ≥ 0.
Solution:
First we draw the lines AB, CD and EF whose
Equation are 2x + y =7, 2x + 3y = 15 and y =2
Respectively.

Line	equation	Point on the x -axis	Point on the y -axis	sign	Region
AB	2x+y = 7	A (3.5,0)	B(0,7)	≥	Non-Origin side Of the line AB
CD	2x + 3y = 15	C(7.5,0)	D(0,5)	≥	Non-Origin side Of the line CD
EF	y = 2	-	F(0,2)	≥	Non-Origin side Of the line EF



The feasible region is EPQBY which is shaded in the figure.
The vertices of the feasible region are P, Q and B (0,7)
P is the points of intersection of the lines 2x + 3y = 15 and
Y =2.
Substituting y =2 in 2x + 3y =15, we get
2x + 3(2) =15
∴ 2x =9 ∴ x =4.5 ∴ P = (4.5,2)
Q is the point of intersection of the lines
2x + 3y =15 … (1)
And 2x + y = 7 … (2)
On subtracting, we get
2y = 8 ∴ y =4
∴ from (2), 2x + 4 = 7
∴ 2x = 3 ∴ x = 1.5
∴ Q = (1.5,4)

The value of the objective function $z = 8x + 10y$ at these Vertices are

$Z(P) = 8(4.5) + 10(2) = 36 + 20 = 56$

$Z(Q) = 8(1.5) + 10(4) = 12 + 40 = 52$

$Z(B) = 8(0) + 10(7) = 70$

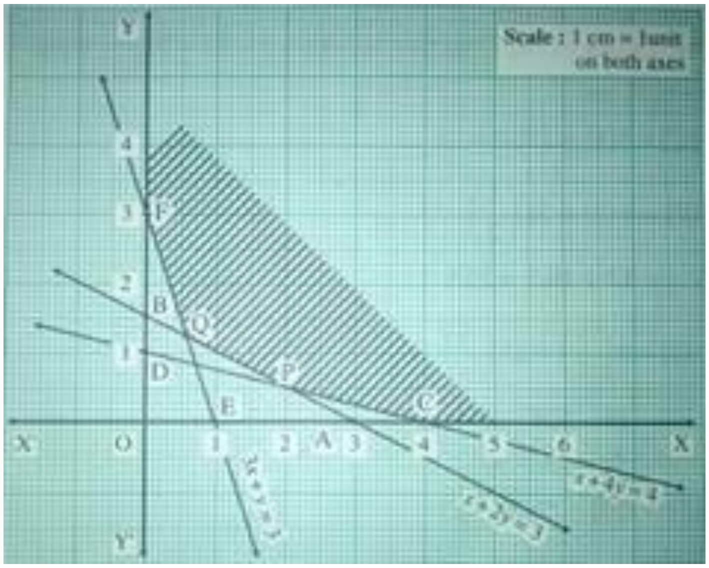
$\therefore$ z has maximum value 52, when $x = 1.5$, and $y = 3$.

8. Maximize $z = 6x + 21y$,
Subject to $x + 2y \geq 3$, $x + 4y \geq 4$, $3x + y \geq 3$, $x \geq 0$, $y \geq 0$

Solution:

First we draw the lines AB, CD and EF whose Equation are $x + 2y = 3$, $x + 4y = 4$ and $3x + y = 3$ Respectively.

Line	equation	Point on the x -axis	Point on the y -axis	sign	Region
AB	$X + 2y = 3$	A (3.,0)	$B\left(0,\frac{3}{2}\right)$	$\geq$	Non-Origin side Of the line AB
CD	$X + 4y = 4$	C(4,0)	D(0,1)	$\geq$	Non-Origin side Of the line CD
EF	$3x + y = 3$	E (1,0)	F(0,3)	$\geq$	Non-Origin side Of the line EF



The feasible region is XCPQFY which is shaded in the figure.
The vertices of the feasible region are C(4,0) P, Q and F(0,3). P is the points of intersection of the lines

X+ 4y = 4

And x + 2y = 3

On subtracting, we get

$2y = 1 \quad \therefore y = \frac{1}{2}$

Substituting

$y = \frac{1}{2}\ in\ x + 2y$

$= 3, we\ get$

$x + 2\left(\frac{1}{2}\right) = 3$

$\therefore$ x = 2

$\therefore p \ \equiv \left(2, \frac{1}{2}\right)$

Q is the point of intersection of the lines

X + 2y =3 ... (1)

And 3x + y = 3 ... (2)

Multiplying equation (1) by 3 we get

3x + 6y = 9

Subtracting equation (2) from this equation, we get

5y = 6

$\therefore y = \frac{6}{5}$

$\therefore from\ (1), x + 2\left(\frac{6}{5}\right) = 3$

$\therefore x = 3 - \frac{12}{5} = \frac{3}{5}$

$\therefore Q \equiv \left(\frac{3}{5}, \frac{6}{5}\right)$

The values of the objective function z = 6x + 21y at these Vertices are

Z(C) = 6(4) + 21(0) = 24

$z(P) = 6(2) + 21\left(\frac{1}{2}\right)$

= 12 + 10.5 = 22.5

$Z(Q) = 6\left(\frac{3}{5}\right) + 21\left(\frac{6}{5}\right)$

$= \frac{18}{5} + \frac{126}{5}$

$= \frac{144}{5} = 28.8$

Z(F) = 6(0) + 21(3) = 63.

∴ z has maximum value 22.5, when x = 2, and

$y = \frac{1}{2}$