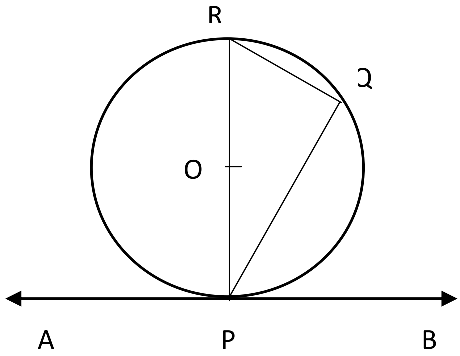
Circles

92) In the figure, AB is a tangent to a circle with center O. Prove that $\angle BPQ = \angle PRQ$.

If $\angle BPQ = 60^\circ$, find $\angle RPQ$.



2014/2015 (3 marks)

$\angle PQR = 90^\circ$ (Angle in semi circle)

$\angle RPB = 90^\circ$ (Radius is perpendicular to tangent)

So, $\angle RPQ + \angle BRQ = 90^\circ$ .....(1)

In

$\angle PQR + \angle RPQ + \angle PRQ = 180^\circ$

So, $+\angle RPQ + \angle PRQ = 180^\circ$

Also, $\angle RPQ + \angle PRQ = 90^\circ$ ......(2)

From eq. (1) and (2) we have

$\angle BPQ = \angle PRQ$

From eq. (1) ,$\angle RPQ = 90^\circ - \angle BPQ$

$90^\circ - 60^\circ = 30^\circ$

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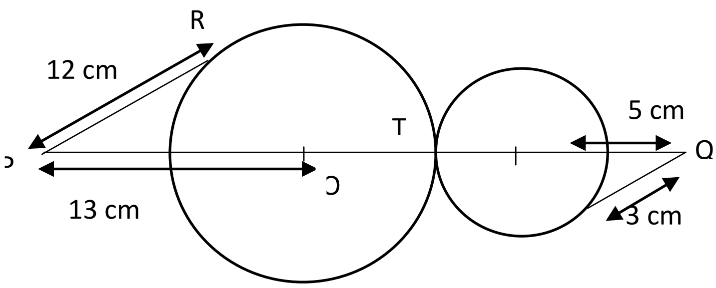
93) Two circles with centers at O and O' touch each other externally at T as shown:



If PR=12 CM, PO=13 cm, O'Q =5 cm and SQ=3 cm, find the length of line segment PQ.

2014/2015 [4 marks]

Join OR and O'S

In $\triangle ORP$, $\angle ORP = 90^\circ$ (Radius is perpendicular to the tangent)

$\therefore$ $OR^2 = OP^2 - PR^2$

$= (13)^2 - (12)^2$

$=169-144$

$=25$

So, $OR=\sqrt{25}=5$ cm.

$\rightarrow$ OT= 5 cm (Radii of the same circle)

Similarly, in $\triangle O'QS$,

$\angle O'SQ=90^\circ$

$\therefore$ $O'S^2= O'Q^2-QS^2$

So, $O'S=$ $\overline{\quad}$ $= 4$ cm.

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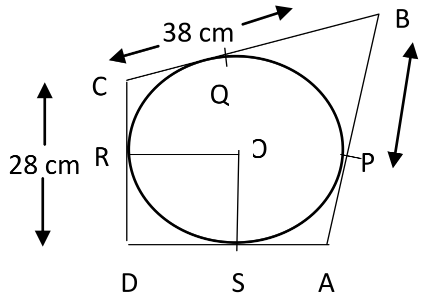
94) In the given figure, ABCD is a quadrilateral, in which $\angle ADC = 90^\circ$, BC = 38 cm, CD =28 cm and BP = 25 cm. Find the radius of the circle with center O.



2014/2015 (2 marks)

13)

AS=AP (Tangents from external point are equal)

BP=QB =25 cm (Tangents from external point are equal)

QC = CR = (38-25)cm = 13 cm. (Tangents from external point are equal)

RD=DS=(28-13)cm = 15 cm. (Tangents from external point are equal)

So. Radius of the circle =OS=RD= 15 cm. (OSDR is a square)

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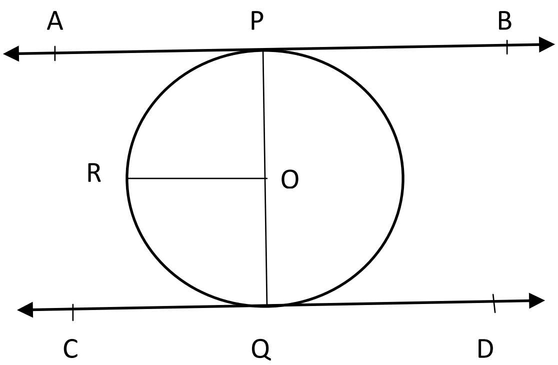
95) Prove that line segment joining the points of contact of two parallel tangents to a circle is diameter of the circle.



2011/2012/2014/2015 [2 marks]

To prove : POQ is a diameter.

Construction: Through O, draw OR||BA or OR||CD as AB and CD are parallel tangents.

Proof: $\angle OPA=90^\circ$ (Radius is always perpendicular to tangent)

OR||BA (by construction)

$\therefore \angle OPA+\angle POR=180^\circ$.

$\rightarrow \angle POR=180^\circ-90^\circ=90^\circ$

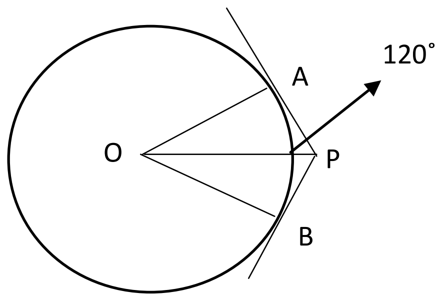
Similarly, $\angle QOR=90^\circ$

$\therefore \angle POR+\angle QOR=180^\circ$.

→POQ is straight line through the center O. So, PQ is a diameter.

---

96) Two tangents PA and PB are drawn to a circle with center O, such that $\angle APB = 120^\circ$. Prove that OP =AP+BP =2AP.



2011/2012/2014/2015 (2 marks)

Let PA and PB be two tangents to the circle with center O (see figure)

Join OA and OB.

$\triangle OAP \cong \triangle OBP$ (RHS)

$\angle APO = \angle BPO$ (CPCT)

$$= \frac{1}{2}\angle APB = \frac{1}{2}\times 120^\circ = 60^\circ.$$

In right, $\triangle OAP$,

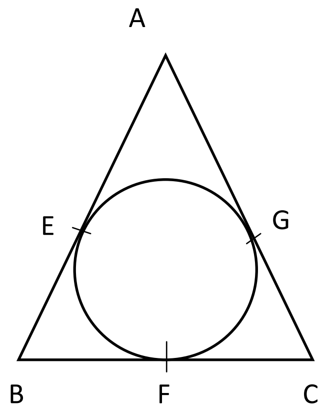
$$\frac{AP}{OP} = \cos 60^\circ = \frac{1}{2}$$

OP =2AP

=AP+BP(AP=BP).

---

97) If the isosceles triangle ABC of the figure given below, AB =AC, show that BF=FC.



2011/2012/2014/2015 (2 marks)

From the figure, AB = AC (Given)

Also, AE=AG (Tangents from the external points are equal)

So, AB - AE = AC – AG

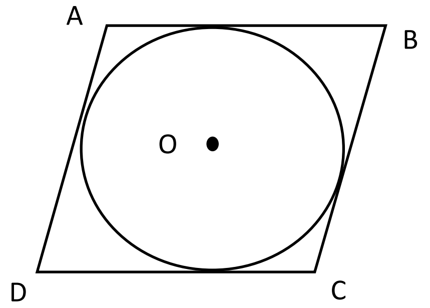
→ BE=CG .....(1)

But BE = BF and CG =CF (Tangents from external points are equal)

So, from eq. (1), BF = CF.

---

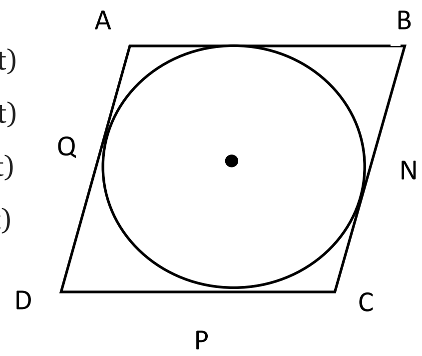
98) Prove that the parallelogram circumscribing a circle is rhombus.



2014/2015 (2 marks)

: ABCD is a parallelogram touching the circle at M, N, P and Q. (see figure)

To prove: ABCD is rhombus.



Proof: AQ = AM (Tangents from external point)

DQ=DP (Tangents from external point)

BN=MB (Tangents from external point)

NC=PC (Tangents from external point)

Adding the above, we get

AD + BC = AB + CD.

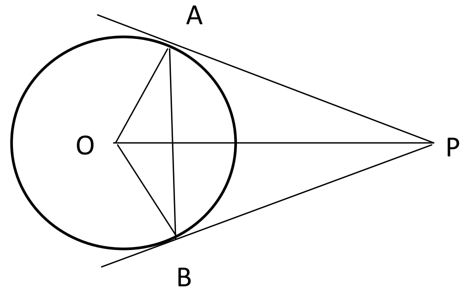
But AD=BC and AB =CD. (Opposite sides of ||gm)

→ AD=AB=BC=CD

→ It is a rhombus.

---

99) In the given figure, OP is equal to the diameter of the circle. Prove that ABP is an equilateral triangle.



2011/2012/2015 (3 marks)

let radius (OA) = r.

OP=2r.

Also, ∠OAP=90° (Tangent is ⊥ to radius through the point of contact).

In right, △OAP,

$$\text{Sin}(\angle OPA)=\frac{OA}{OP}=\frac{r}{2r}=\frac{1}{2}.$$

→∠OPA=30°

Similarly, from △OPB.

∠OPB=30°

∠APB=30°+30°=60°.

Since PA =PB (lengths of tangents from an external point are equal), therefore

∠PAB=∠PBA.

In △APB,

∠APB+∠PAB+∠PBA=180° (Angle sum property of triangle)

→ 60°+2∠PAB=180°

→∠PAB=60°

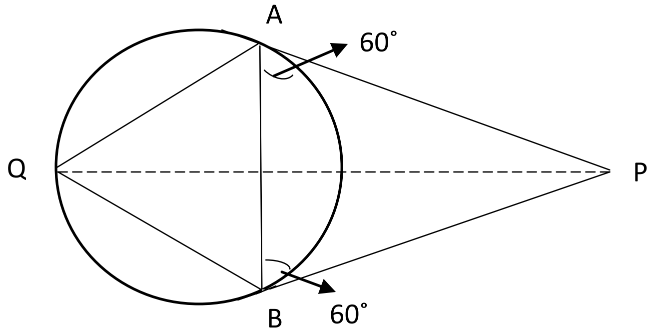
→∠PBA=60°

Since all angles are 60°, therefore △ABP is equilateral.

---

100) PA and PB are the tangents of a circle which circumscribes an equilateral △ABQ.

If ∠PAB=60°, as shown in the figure, prove that QP bisects AB at right angles.



2015 (4 marks)

$\left.\begin{array}{l}\angle QAB = 60° \\ \angle QBA = 60°\end{array}\right\}$ (△ABQ is equilateral)

So, $\angle PAQ = \angle PAB + \angle QAB = 60^\circ + 60^\circ = 120^\circ$

Similarly $\angle PBQ = 120^\circ$ ....(1)

Now, in $\triangle PAQ$ and $\triangle PBQ$,

$PA = PB$ (Tangents from external point)

$AQ = BQ$ ($\triangle ABQ$ equilateral)

$\angle PAQ = \angle PBQ$ (Each=120°, shown above)

$\triangle PAQ \cong \triangle PBQ$ (by SAS)

$\angle APQ = \angle BPQ$(CPCT).....(2)

Let QP intersects AB at M.

Now, in $\triangle PAM$ and $\triangle PBM$,

$\angle APM = \angle BPM$ [From (2)]

$PA = PB$

$PM = PM$

So, $\triangle PAM \cong \triangle PBM$ (by SAS)

$\rightarrow$ $AM = BM$ (CPCT).....(3)

and $\angle AMP = \angle BMP$ (CPCT)

But $\angle AMP + \angle BMP = 180^\circ$

$\rightarrow \angle AMP + \angle AMP = 180^\circ$

$\rightarrow$ $2\angle AMP = 180^\circ$

$\rightarrow \angle AMP = 90^\circ$ ......(4)

From (3) and (4) we get that QP bisects AB at right angles.

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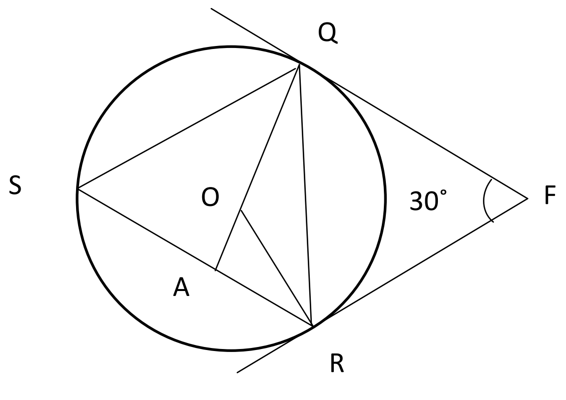
101) Tangents PQ and PR are drawn to circle such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.



2015 (4 marks)

Draw $QA \perp PQ$ intersecting RS at A.

So, $\angle QAS = 90^\circ$, because RS||PQ.

Also, QA will pass through center O of the circle. Join OR.

So, $\angle ROQ + \angle RPQ = 180^\circ$

$\rightarrow \angle ROQ + 30^\circ = 180^\circ$

$\rightarrow \angle ROQ = 150^\circ$

But $\angle RSQ = \frac{1}{2}\angle ROQ$.

So, $\angle RSQ = \frac{1}{2} \times 150^\circ = 75^\circ$.

Therefore, from $\triangle QSA$,

$\angle SQA = 180^\circ - 90^\circ - 75^\circ = 15^\circ$.

Also we have:

$\angle PQR + \angle PRQ + \angle QPR = 180^\circ$

$\rightarrow \angle PQR + \angle PRQ + 30^\circ = 180^\circ$ ($\angle PQR = \angle PRQ$ because $PQ = PR$)

$\rightarrow 2\angle PQR = 150^\circ$

$\rightarrow \angle PQR = \frac{150^\circ}{2} = 75^\circ$

But $\angle APQ = 90^\circ$ (Angle between tangent and radii)

So, $\angle AQR = \angle AQP - \angle PQR$.

$= 90^\circ - 75^\circ - 15^\circ$.

So, $\angle RQS = \angle SQA + \angle AQR$

$= 15^\circ + 15^\circ = 30^\circ$