CLASS : XII CHAPTER : 5 CONTINUITY & DIFFERENTIABILITY

EXERCISE : 5.4 (NCERT)

Differentiate the following w.r.t. x.

Q No 1 $\frac{d}{dx}\left(\frac{e^x}{\sin x}\right)$

$$= \frac{\sin x \cdot \frac{d}{dx}(e^x) - e^x \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2} \quad \Big| \ \frac{u}{v} \text{ form.}$$

$$= \frac{\sin x \cdot e^x - e^x \cos x}{\sin^2 x} = \frac{e^x(\sin x - \cos x)}{\sin^2 x}.$$

Q No. 2 $\frac{d}{dx}\left(e^{\sin^{-1}x}\right)$

$$= e^{\sin^{-1}x} \cdot \frac{d}{dx}(\sin^{-1}x) = e^{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}} = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}.$$

Q No. 3 $\frac{d}{dx}\left(e^{x^3}\right)$

$$= e^{x^3} \cdot \frac{d}{dx}(x^3) = e^{x^3} \cdot 3 \cdot x^2 = 3x^2 e^{x^3}$$

Q No. 4 $\frac{d}{dx}\left(\sin(\tan^{-1}e^{-x})\right)$

$$= \cos(\tan^{-1}e^{-x}) \cdot \frac{d}{dx}(\tan^{-1}e^{-x})$$

$$= \cos(\tan^{-1}e^{-x}) \times \frac{1}{1+(e^{-x})^2} \times \frac{d}{dx}(e^{-x})$$

$$= \cos(\tan^{-1}e^{-x}) \cdot \frac{1}{1+e^{-2x}} \cdot e^{-x} \cdot \frac{d}{dx}(-x)$$

$$= \cos(\tan^{-1}e^{-x}) \cdot \frac{1}{1+e^{-2x}} \cdot e^{-x}(-1)$$

$$= \frac{1}{\sqrt{1+(e^{-x})^2}}\left(\frac{-e^{-x}}{1+e^{-2x}}\right) \quad \left[\because \cos(\tan^{-1}t) = \frac{1}{\sqrt{1+t^2}}\right.$$

Q No. 5 $\frac{d}{dx}\left[\log(\cos e^x)\right]$

$$= \frac{1}{\cos e^x} \cdot \frac{d}{dx}(\cos e^x) = \frac{1}{\cos e^x} \cdot -\sin e^x \cdot \frac{d}{dx}(e^x)$$

$$= -\frac{\sin e^x}{\cos e^x} \cdot e^x = -e^x \cdot \tan e^x.$$

Q No. 6 $\frac{d}{dx}\left(e^x + e^{x^2} + \cdots e^{x^5}\right)$

$$= \frac{d}{dx}(e^x) + \frac{d}{dx}(e^{x^2}) + \frac{d}{dx}(e^{x^3}) + \frac{d}{dx}(e^{x^4}) + \frac{d}{dx}(e^{x^5})$$

$$= e^x + e^{x^2}\cdot\frac{d}{dx}(x^2) + e^{x^3}\cdot\frac{d}{dx}(x^3) + e^{x^4}\cdot\frac{d}{dx}(x^4) + e^{x^5}\cdot\frac{d}{dx}(x^5)$$

$$= e^x + e^{x^2}\cdot 2x + e^{x^3}\cdot 3x^2 + e^{x^4}\cdot 4x^3 + e^{x^5}\cdot 5x^4$$

$$= e^x + 2x\, e^{x^2} + 3x^2 e^{x^3} + 4x^3 e^{x^4} + 5x^4 e^{x^5}$$

Q No 7. $\frac{d}{dx}\left(\sqrt{e^{\sqrt{x}}}\right) =$

$$= \frac{d}{dx}\left(e^{\sqrt{x}}\right)^{1/2} = \frac{1}{2}\left(e^{\sqrt{x}}\right)^{1/2-1}\cdot\frac{d}{dx}e^{\sqrt{x}}$$

$$= \frac{1}{2}\left(e^{\sqrt{x}}\right)^{-1/2}\cdot e^{\sqrt{x}}\cdot\frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}}\cdot e^{\sqrt{x}}\cdot\frac{1}{2}x^{-1/2} = \frac{e^{\sqrt{x}}}{2\sqrt{e^{\sqrt{x}}}}\cdot\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{4\sqrt{x}}\cdot\left(e^{\sqrt{x}}\right)^{1-1/2} = \frac{1}{4\sqrt{x}}\left(e^{\sqrt{x}}\right)^{1/2} = \frac{1}{4\sqrt{x}}e^{\frac{\sqrt{x}}{2}}$$

Q No 8 $\frac{d}{dx}\left[\log(\log x)\right]$; $x > 1$

$$= \frac{1}{\log x}\cdot\frac{d}{dx}(\log x) = \frac{1}{\log x}\cdot\frac{1}{x} = \frac{1}{x\log x}.$$

Q No. 9: $\frac{d}{dx}\left[\frac{\cos x}{\log x}\right]$

$$= \frac{\log x\,\frac{d}{dx}(\cos x) - \cos x\,\frac{d}{dx}(\log x)}{(\log x)^2} = \frac{\log x\cdot(-\sin x) - \cos x\cdot\frac{1}{x}}{(\log x)^2}$$

$$= \frac{-(x\sin x\cdot\log x + \cos x)}{x(\log x)^2};\quad x > 0$$

Q No 10 $\frac{d}{dx}\left(\cos(\log x + e^x)\right) = -\sin(\log x + e^x)\cdot\frac{d}{dx}(\log x + e^x)$

$$= -\sin(\log x + e^x)\left(\frac{1}{x} + e^x\right) = -\frac{\sin(\log x + e^x)(1 + xe^x)}{x}.$$