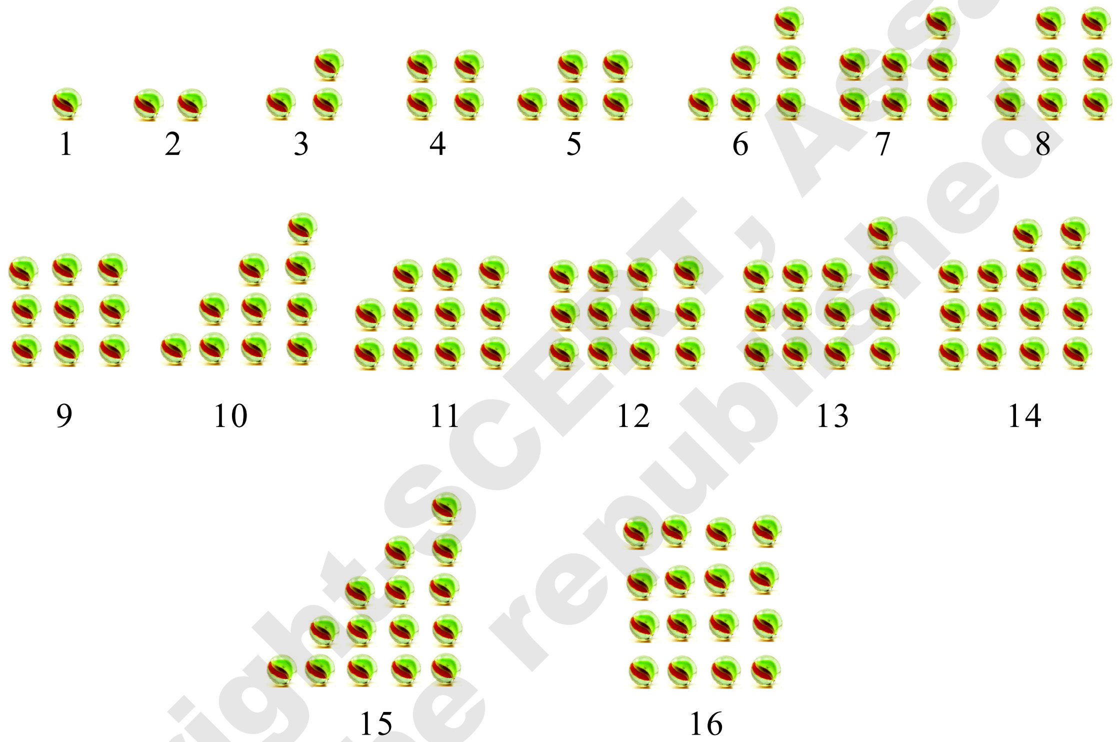
Chapter-6

SQUARES AND SQUARE ROOTS

6.1 Observe the following patterns made of marbles.



Here, observe the patterns of 4, 9 and 16. The number of rows and number of columns of each of them are equal. Also the number of marbles in each row and each column are equal. In this way you can arrange 25 or 36 marbles.

Let us see the total number of marbles in these patterns.

$4 = 2 \times 2$	$9 = 3 \times 3$	$16 = 4 \times 4$	$25 = 5 \times 5$
$= 2^2$	$= 3^2$	$= 4^2$	$= 5^2$

What have you seen? We can express each of 1, 4, 9 etc. as the square of a number. These numbers are square numbers.

That means, **the number which can be expressed as a square of some number is known as Square number**.

Of the above patterns, except 1, 4, 9 and 16, the other numbers cannot be arranged in this form. The numbers of this type are not square numbers.

Activity : Find the square number between 10 and 100.

Let us see a technique :

$1 \times 1 = 1^2 = 1$

$2 \times 2 = 2^2 = 4$

$3 \times 3 = 3^2 = 9$

$4 \times 4 = 4^2 = 16$ etc.

1, 4, 9, 16, 25, 36, 49, ... etc. are known as *Perfect Squares.*

If x is a natural number then $x \times x$ i.e x^2 is a square number. It is applicable for integer also.

Observe : $\frac{1}{4} = \left(\frac{1}{2}\right)^2$, Thus, $\frac{1}{4}$ is also a square number. But it is not a perfect square of an integer.

In this lesson, when we discuss about square numbers, we shall consider the perfect square of integer only. The diagramatic representation given above is aplicable only for perfect square numbers.

You observe the square size table given here. After observing the table you will see that all the square numbers are placed at the diagonal. Now you find out the perfect square numbers in between 1 and 1000. Check the perfect squares you have obtained to see if any perfect square has been left out in between any two of these perfect squares. For example, see if there is any perfect square in between two perfect square 81 and 100 or not.

×	1	2	3	4	5	6	7	8	9	10
1	1									
2		4								
3			9							
4				16						
5					25					
6						36				
7							49			
8								64		
9									81	
10										100

You have seen that $9 \times 9 = 9^2 = 81$ and $10 \times 10 = 10^2 = 100$. Now for a perfect square number to lie in between 81 and 100, the number must be a square of natural number between 9 and 10. But there is no any natural number between 9 and 10. Therefore, there are no any perfect square between 81 and 100.

Now, let us see what happens in case of integers. We know that just like 9 and 10 are integers, the numbers – 9 and – 10 are also integers.

$$9^2 = 81, \quad (-9)^2 = 81, \quad 10^2 = 100, \quad (-10)^2 = 100$$

Just as there is no any integer between 9 and 10, so is the case for – 9 and –10.

Notice that we can square any number. Consider a rational number. Let the number be $\frac{19}{2}(=9.5)$. Now, $9.5 \times 9.5 = (9.5)^2 = 90.25$

Thus squaring 9.5 we get 90.25. However, remember that 90.25 is square of a rational numbers, but it is not a perfect square number. You can check the squares of different numbers in this way.

Now, we shall discuss different properties of square numbers. By square numbers, you have to consider the perfect squares of integers.

Observe carefully the perfect square numbers that you obtained in between 1 and 1000 (including 1 and 1000).

(i) Observe the square numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169 etc. What are the digits in the unit places of these numbers? In a square number, the digit in the unit place can be any one among the numbers 0, 1, 4, 5, 6 and 9. But this digit cannot be any of 2, 3, 7 or 8.

Conversely, if a number has any one of these digit 0, 1, 4, 5, 6 and 9 at its unit place, can we say that it a square number? For example, 10, 11, 14 etc. have 0, 1, 4 at their unit places respectively. But they are not square numbers.

Therefore, if a number has 0, 1, 4, 5, 6 and 9 at its unit place it does not mean that it is a perfect square. We can just guess only. But if digits like 2, 3, 7 or 8 lie at the unit place, then we can be sure that the number is not a perfect square number.

(ii) Let us find out squares of some odd numbers.

$1^2 = 1$, $3^2 = 9$, $5^2 = 25$, $17^2 = 289$, $97^2 = 9409$

Observe the squares. Every square is an odd number. Is not it? You also try to find out the squares of some other odd numbers. Therefore we can conclude that *square of an odd number is an odd number*.

(iii) Now let us find out squares of some even numbers.

$2^2 = 4$, $4^2 = 16$, $12^2 = 144$, $34^2 = 1156$, $96^2 = 9216$

Here, every square is an even number. Isn't it?

You also try to find out squares of some even numbers. Squares of an even number is also an even number, isn't it? So, we can conclude that *squares of even numbers are even*.

6.2 Some interesting patterns of square numbers.

6.2.1 Consider the sum of consecutive odd natural numbers of the following.

$$1 + 3 = 4,\ 1 + 3 + 5 = 9,\ 1 + 3 + 5 + 7 = 16 \text{ etc.}$$

Here 4, 9, 16, ... etc. are square numbers. These can be expressed as follows.

$1 = 1^2 = 1$

$1 + 3 = 4 = 2^2$ (Sum of first two odd numbers)

$1 + 3 + 5 = 9 = 3^2$ (Sum of first three odd numbers)

$1 + 3 + 5 + 7 = 16 = 4^2$ (Sum of first four odd numbers)

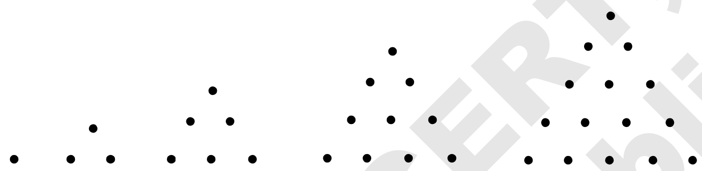
Similarly, we have,

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$

$1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2$

Observe the above pattern. There is a relation between the sum of the consecutive odd numbers with the square numbers. The relation is *the sum of first **n** consecutive odd natural numbers is $\boldsymbol{n^2}$.*

6.2.2 Observe the following patterns. These are arranged in triangular pattern.



1 3 6 10 15 etc.

(Triangular number pattern)

Let us know

For a natural number n, $\frac{n(n+1)}{2}$ is a triangular number. For example,

If $n = 1$, triangular number is $\frac{1(1+1)}{2} = 1$

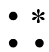
$n = 2$, triangular number is $\frac{2(2+1)}{2} = 3$

$n = 3$, triangular number is $\frac{3(3+1)}{2} = 6$ etc.

6.2.3 Relation between square numbers with triangular numbers.

Let us add two consecutive triangular numbers.

$1 + 3 = 4 = 2^2$



Let us demonstrate it with the help of points.

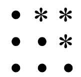
* (one) + • (three)

$= 1 + 3 = 4$

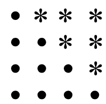
$3 + 6 = 9 = 3^2$

$6 + 10 = 16 = 4^2$

$10 + 15 = 25 = 5^2$ etc.



* (three) + • (six)
$= 3 + 6 = 9$



* (six) + • (ten)
$= 6 + 10 = 16$

Therefore, we can say that sum of two consecutive triangular numbers is a square number.

Exercise 6.1

1. How many perfect square numbers are there in between 1 and 200 ?
2. What will be the digit in the unit place of each of the squares of the following numbers?
   (i) 51 (ii) 99 (iii) 205 (iv) 3400 (v) 1987
3. Write the reasons why the following numbers are not square numbers.
   (i) 4347 (ii) 24832 (iii) 35493 (iv) 403388 (v) 182000
4. (i) Write five numbers whose squares are even.
   (ii) Write five numbers whose squares are odd.
5. Without adding directly, find the sum (using relevant properties).
   (i) $1 + 3 + 5 + 7$
   (ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$
   (iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25$
6. Can 36 be expressed as the sum of 6 consecutive odd numbers?
7. Write 5 consecutive triangular numbers greater than 15. From these numbers consider two consecutive numbers and examine whether their sums are square numbers or not.

6.3 An easy method to find the square of a number

Let us discuss an easy method to find the square of a number having 5 in the unit place. Let us see how to find the squares of the numbers 15, 25, 35, 45, ...etc. easily.

Directly squaring them you will get,

$15^2 = 225$ $25^2 = 625$ $35^2 = 1225$ $45^2 = 2025$

Now let them arrange in following ways.

$15^2 = 225 = (1 \times 2)$ hundred $+ 25$,	(1×2) hundred is 2 hundreds $= 200$
$25^2 = 625 = (2 \times 3)$ hundred $+ 25$,	(2×3) hundred is 6 hundreds $= 600$
$35^2 = 1225 = (3 \times 4)$ hundred $+ 25$	
$45^2 = 2025 = (4 \times 5)$ hundred $+ 25$	

Look at the technique – add 1 with 2 (digit in the tenth place) to make it 3 and multiply by the number 2.

$$\begin{array}{r|l} 2 & 5 \\ \times\ 2 & 5 \\ \hline \rightarrow 6 & 25 \leftarrow \end{array}$$

Product of two 5's at unit place.

Again observe,

Increase the tenth place digit 6 by 1 to make it 7 and multiply by 6.

$$\begin{array}{r|l} 6 & 5 \\ 6 & 5 \\ \hline \rightarrow 42 & 25 \leftarrow \end{array}$$

Product of two 5's at unit place.

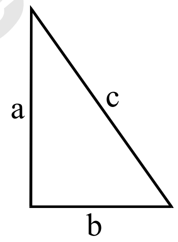
Observe once again.

$$\begin{array}{r|l} 8 & 5 \\ \times\ 8 & 5 \\ \hline 9 \times 8 \rightarrow\ 72 & 25 \ \leftarrow 5 \times 5 \end{array}$$

6.4 Pythagorean Triplets

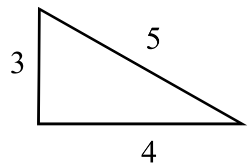
One special property of a right angled triangle is,

Square of the hypotenuse of a right angled triangle is equal to the sum of the squares of the other two sides.



That means in the adjoining right angled triangle, $a^2 + b^2 = c^2$ where, *a, b* and *c* represent the measures of the sides of the right angled triangle.

Now you consider another example. If the length of the sides containing a right angle are 4 units and 3 units, then what will be the length of the hypotenuse?



Here, $3^2 + 4^2 = 9 + 16 = 25$

We know that $5^2 = 25$, therefore length of the hypotenuse is 5 unit.

That means $3^2 + 4^2 = 5^2$

Thus, *the triplets of such three numbers where sum of squares of two numbers is equal to the square of the third one are called Pythagorean Triplet* .e.g. (3, 4, 5) is a Pythagorean triplet.

If you try , you will also get three natural numbers a, b and c such that $a^2 + b^2 = c^2$.

You may check whether 6, 8 and 10 are pythagorean triplet or not.

Since $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

Therefore , 6, 8 and 10 are pythagorean triplet.

Try yourself

Which of the following are pythagorean triplet?

(i) 15, 20, 25 (ii) 7, 24, 25 (iii) 5, 10, 13

For any natural number p (p>1) we know that

$(2p)^2 + (p^2 - 1)^2 = (p^2 + 1)^2$

Have you noticed that left hand side is sum of two squares and right hand side is also a square. That means it is in the form of $a^2 + b^2 = c^2$. i.e. $2p$, $p^2 - 1$ and $p^2 + 1$ form a pythogorean triplet.

Let us form some pythagorean triplets for different values of p.

Let $p = 4$

then $2p = 8$, $p^2 - 1 = 15$, $p^2 + 1 = 17$

Now $(2p)^2 + (p^2 - 1)^2$

$= 8^2 + 15^2$

$= 64 + 225$

$= 289$

$= 17^2$ $[\because 17 \times 17 = 289]$

Thus, $8^2 + 15^2 = 17^2$ and so 8, 15, 17 are Pythagorean triplet.

Using $(2p)^2 + (p^2 - 1)^2 = (p^2 + 1)^2$, we can have some other pythagorean triplets. But this relation is not true for all the pythagorean triplets. For example (5, 12, 13) does not satisfy this relation.

Notice that whatever may be the value of p (where p>2) among $2p$, $p^2 - 1$ and $p^2 + 1$, $2p$ is the smallest and $p^2 + 1$ is largest. i.e. $2p < p^2 - 1 < p^2 + 1$.

Exercise 6.2

1. Find the squares of following numbers.

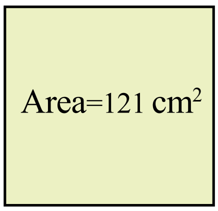
   (i) 35 (ii) 55 (iii) 95

2. Write three pythagorean triplets.
3. Find a pythagorean triplet whose smallest term is10.

6.5 Square roots

Let us discuss about square root with the help of an example.

Suppose, area of a square is121 sq. cm. From this data can you say the length of the side of the square?



We know that area of a square = (side)^2

Therefore we need a number whose square is 121. Here whatever length of the side we get, it is the square root of 121.

We know that $11^2 = 121$

Therefore, the length of the side of the square is 11 cm whose area is 121 sq. cm.

Here 11 is square root of 121.

We know that

$1^2 = 1$, therefore square root of 1 is 1

$2^2 = 4$, therefore square root of 4 is 2

$3^2 = 9$, therefore square root of 9 is 3

$14^2 = 196$, therefore square root of 196 is 14

Observe

$(-1)^2 = 1$ and $1^2 = 1$. Therefore we can say square roots of 1 are 1 and –1.

$(-2)^2 = 4$ and $2^2 = 4$. Therefore we can say square roots of 4 are 2 and –2.

A perfect square has two square roots. In this lesson, we shall consider only the positive square root of a number. The positive square root of a number is denoted by the symbol '$\sqrt{\ }$'.

For example, $\sqrt{9} = 3$ (not –3); $\sqrt{196} = 14$ (not –14) etc.

Statements	Conclusions
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$

Statements	Conclusions
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$

6.6 Method of finding the square root of a number

6.6.1 Finding square root by prime factorisation method

We know that $25 = 5 \times 5 = 5^2$

$\therefore \sqrt{25} = 5$

Again, $169 = 13 \times 13 = 13^2$

$\therefore \sqrt{169} = 13$

Therefore, it is seen that if a square number is factorised in two same factors, then one factor is square root of that number.

Let us consider another examples.

(i) Square root of 225

Expressing 225 into prime factors we get,

$225 = \underline{3 \times 3} \times \underline{5 \times 5} = 3^2 \times 5^2 = (3\times5)^2$

$\therefore \sqrt{225} = 3 \times 5 = 15$

$$\begin{array}{r|l} 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

(ii) Square root of 900

Expressing 900 into prime factors we get,

$900 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5} = 2^2 \times 3^2 \times 5^2$

$= (2 \times 3 \times 5)^2$

$\therefore \quad \sqrt{900} = 2 \times 3 \times 5 = 30$

$$\begin{array}{r|l} 2 & 900 \\ \hline 2 & 450 \\ \hline 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

(iii) Square root of 7056

Expressing 7056 into prime factors we get,

$7056 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7} = 2^2 \times 2^2 \times 3^2 \times 7^2$

$= (2 \times 2 \times 3 \times 7)^2$

$\therefore \quad \sqrt{7056} = 2 \times 2 \times 3 \times 7$

$= 84$

$$\begin{array}{r|l} 2 & 7056 \\ \hline 2 & 3528 \\ \hline 2 & 1764 \\ \hline 2 & 882 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

What have you learnt from above examples? For a large number first we divide it into small numbers as factors. Out of them equal factors are paired. Selecting one factor, from each pair and multiplying them we get the square root of the given number.

To factorise, it will be convenient to use prime factors.

Try yourself Find square roots of following numbers –

(i) 256 (ii) 2304 (iii) 74529

Example : If square of a number is 5184, what is the number?

Solutions : Here, given that square of a number is 5184. To find the number i.e. to find the square root of 5184.

Expressing 5184 into prime factors we get,

$5184 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$\therefore \quad \sqrt{5184} = 2 \times 2 \times 2 \times 3 \times 3 = 72$

or, $72^2 = 5184$

Therefore, squaring 72 we get 5184.

$$\begin{array}{r|l} 2 & 5184 \\ \hline 2 & 2592 \\ \hline 2 & 1296 \\ \hline 2 & 648 \\ \hline 2 & 324 \\ \hline 2 & 162 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Example : By what smallest number 180 must be multiplied so that the product becomes a perfect square? What is the square root of the number thus obtained?

Solutions : $180 = 9 \times 4 \times 5 = 3^2 \times 2^2 \times 5$

In the right hand side, 5 is alone. Therefore, 5 must be multiplied by another 5 to make it a square. Therefore both sides must be multiplied by 5.

$\therefore \quad 180 \times 5 = 3^2 \times 2^2 \times 5 \times 5 = 3^2 \times 2^2 \times 5^2$

$\therefore \quad 180 \times 5$ i.e. 900 is a perfect square number.

$\therefore$ Multiplying 180 by 5, we have a perfect square number and the number is 900.

Square root of 900 is $3 \times 2 \times 5 = 30$

Example : By what smallest number 2645 must be divided so that the quotient becomes a perfect square? What is the square root of the number so obtained?

Solutions : $2645 = 5 \times 23 \times 23$

Here it is seen that the number 5 is lying single. Therefore dividing both sides by 5 we get,

$$(2645) \div 5 = (5 \times 23 \times 23) \div 5$$

$$\text{i.e.} \quad 529 = 23 \times 23$$

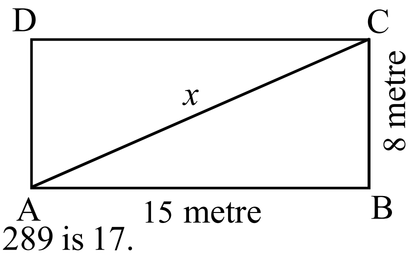
Thus, dividing 2645 by 5, a square number 529 is obtained and square root of 529 is 23.

Example : The length and breadth of a small rectangular park are 15 metre and 8 metre respectively. Find the diagonal of that rectangular park.

Solutions : For the park ABCD, let length of diagonal AC be x metres.

In the figure, ABC is a right angled tringle.

Now, using pythagorus theorem, we get,



$AC^2 = AB^2 + BC^2$

or, $x^2 = 15^2 + 8^2$

or, $x^2 = 225 + 64 = 289$

We know that $17^2 = 289$ and $(-17)^2 = 289$

But length can't be negative. Therefore, square root of 289 is 17.

$$\therefore \ x = 17$$

$\therefore$ Length of diagonal is 17 metre.

Example : Find the smallest square number that is divisible by each of the numbers 8, 12 and 18.

Solutions : Notice that we are to find the smallest number divisible by 8, 12 and 18 which is also a perfect square number.

First find the LCM of 8, 12, 18

LCM = $2 \times 2 \times 3 \times 3 \times 2 = 72$

$$\begin{array}{r|l} 2 & 8,\ 12,\ 18 \\ \hline 2 & 4,\ 6,\ 9 \\ \hline 3 & 2,\ 3,\ 9 \\ \hline & 2,\ 1,\ 3 \end{array}$$

Here, 72 is divisible by 8, 12 and 18 and it is smallest of all. But it is not a perfect square number. When 72 is factorised into primes, 2 will occur in single. If we multiply by another 2 then it becomes

$$72 \times 2 = 2 \times 2 \times 3 \times 3 \times 2 \times 2 = 2^2 \times 3^2 \times 2^2$$

Therefore right hand side is squared now. So 72×2 i.e. 144 is the smallest perfect square number divisible by 8, 12 and 18.

Example : Find the square root of the following –

(i) 2^8 (ii) 9×36

Solutions :

(i) We know that $2^8 = (2^4)^2$

$$\therefore \sqrt{2^8} = 2^4$$

(ii) We know that $9 \times 36 = 3^2 \times 6^2$

$$\therefore \sqrt{9 \times 36} = 3 \times 6 = 18$$

Exercise 6.3

1. Answer whether the following are true or false.
   (i) The square root of an even perfect square number is even.
   (ii) Sum of $1 + 3 + 5 + 7 + 9 + 11 + 13$ is a square number.
   (iii) A number has 8 at its unit place, therefore the number may be a square number.
   (iv) A square number has 1 at the unit place. Therefore its square root may have 1 or 9 at its unit place.
2. What will be the possible digits in the unit places of the square roots of the following numbers.
   (i) 8281 (ii) 5476 (iii) 172225 (iv) 12100
3. Find the square roots of the following numbers by prime factorisation method.
   (i) 256 (ii) 729 (iii) 1764 (iv) 5184
   (v) 7744 (vi) 5929 (vii) 8836 (viii) 4225
4. What is the smallest number by which each of the following numbers must be multiplied to make it a perfect square?
   (i) 15 (ii) 45 (iii) 150 (iv) 175
5. (i) Find the smallest perfect square number divisible by 8, 15 and 20.
   (ii) Find the least perfect square number divisible by 12, 20 and 25.
6. (i) By what least number 4032 is to be divided to get a perfect square number. Find the square root of the quotient.

(ii) By what smallest number 14112 is to be multiplied to get a perfect square number. Find the square root of the product.

7. In a school, there are all total 1024 students. In the morning prayer they are asked to make rows such that number of rows and number of students in each row are same. Find the number of rows and number of students in each row.

8. In a tea garden, number of rows of tea plants and number of tea plants in each row are same. 835 plants are supplied to the garden. But it is seen that more plants are required to satisfy the above condition. How many additional tea plants are required?

6.6.2 Finding square root by division method.

Before explaining the method let us observe the following results.

$4^2 = 16$	$10^2 = 100$	$32^2 = 1024$	$100^2 = 10000$	$388^2 = 150544$
$9^2 = 81$	$31^2 = 961$	$99^2 = 9801$	$103^2 = 10609$	$1234^2 = 1522756$

Squaring different numbers you will see that

(i) If the square number has two digits, then its square root has one digit.

(ii) If the square number has three or four digits, then its square root has two digits.

(iii) If the square number has five or six digits, then its square root has three digits. Now we shall discuss how to find square root using division method.

Rule of finding square roots by division method :

Step 1 : Place a bar over every pair of digits starting from the digit at unit's place of the number whose square root to be found. If a single digit is left at the extreme left, put a bar on it also.

Step 2 : Find the largest number whose square is less than or equal to the number of the extreme left group. This number will be divisor and quotient both.

Step 3 : Find the remainder by subtracting the product of divisor and quotient from the first group and immediately bring down the next group right to the remainder. This number will be the new dividend.

Step 4 : To get the new divisor, make the quotient double and enter it with a blank on its right. To fill up this place consider the largest possible digit which will be the new digit in the quotient. Notice that the product of divisor with the new digit of quotient is less or equal to the dividend.

Step 5 : Repeat steps 2, 3, 4 till all the groups have been taken up. When remainder becomes zero, then quotient is the required square root and if the remainder is not zero then the number is not a perfect square number.

Let us try to explain with an example.

Example : Find the square root of 74529

Solutions : In the first step, placing bars over every pair of digits starting from the unit place of the number 74529, there will be three groups. That is $\overline{7}\ \overline{45}\ \overline{29}$.

In the second step, we are to find the largest number whose square is less or equal to the first group i.e. 7 and such a number is 2.

Dividend = 7

Divisor = 2

Quotient = 2

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{7}\ \overline{4\,5}\ \overline{2\,9} \\ & -4 \\ \hline & 3 \end{array}$$

Now subtracting 4, product of divisor and quotient from 7 we get 3.

Now, bring down the second group right to the remainder. Thus we have 345. Therefore dividend = 345. Now to get the new divisor make the quotient twice and enter it with a blank on its right.

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{7}\ \overline{4\,5}\ \overline{2\,9} \\ & -4 \\ \hline 4\_ & 3\,4\,5 \end{array}$$

Now we are to put a number right to 4 such that the product of this number and 4_ is near or equal to 345. Let us find such a number.

$4\boldsymbol{1} \times \boldsymbol{1} = 41$
$4\boldsymbol{2} \times \boldsymbol{2} = 84$
$4\boldsymbol{3} \times \boldsymbol{3} = 129$
$4\boldsymbol{4} \times \boldsymbol{4} = 176$
$4\boldsymbol{5} \times \boldsymbol{5} = 225$
$4\boldsymbol{6} \times \boldsymbol{6} = 276$
$4\boldsymbol{7} \times \boldsymbol{7} = 329$
$4\boldsymbol{8} \times \boldsymbol{8} = 384$

$$\begin{array}{r|l} & 2\ \ 7 \\ \hline 2 & \overline{7}\ \overline{4\,5}\ \overline{2\,9} \\ & -4 \\ \hline 4\,\underline{7} & 3\,4\,5 \\ & -3\,2\,9 \\ \hline & \ \ 1\,6 \end{array}$$

It is seen that putting 7 in the blank place, we get the nearest smaller number 329 to 345. On subtraction we get 16.

Similarly, this time 29 will come down and new dividend will be 1629.

So, 2 × 27=54 is written.

Now in similar way we should put a number right to 54 such that product of this number and 54_ is less them or equal to 1629.

$$\begin{array}{r|l} & 2\ \ 7 \\ \hline 2 & \overline{7}\ \overline{4\,5}\ \overline{2\,9} \\ & -4 \\ \hline 4\,\underline{7} & 3\,4\,5 \\ & -3\,2\,9 \\ \hline 5\,4\,\_ & \ \ 1\,6\,2\,9 \end{array}$$

Proceeding in the same way, putting 3 right to 54, we get $543 \times 3 = 1629$.

```
          2   7   3
    2  | 7  45  29
       | - 4
   47  |  3 45
       | - 3 29
  543  |    16 29
       |  - 16 29
       |        0
```

It is seen that remainder = 0

∴ square root of 74529 is 273

Example : Find square root. (i) 961 (ii) 8649 (iii) 416200801

Solutions :

(i)

```
        3   1
   3 |  9  61
     |  9
  61 |     61
     |     61
```

∴ $\sqrt{961} = 31$

(ii)

```
         9   3
    9 | 86  49
      | 81
  183 |  5  49
      |  5  49
```

∴ $\sqrt{8649} = 93$

(iii)

```
           2  0  4  0  1
      2 |  4 16 20 08 01
        |  4
     40 |    16
        |    00
    404 |    16 20
        |    16 16
   4080 |        4 08
        |        0 00
  40801 |        4 08 01
        |        4 08 01
```

∴ $\sqrt{416200801} = 20401$

6.6.3 Square roots of decimal numbers :

Step 1 : Decimal numbers have two parts – Integral part and Decimal part. In integral part put bars on each pairs from unit's place as usual. Place bar even if the left most digit is alone by itself.

Step 2 : For the right hand part of decimal i.e. decimal part, place bars on every pair of digits right to the decimal, that means from begining with the first decimal place. If the last digit is all by itself you can put a zero at the end. (Why ? Think.)

Observe that placing of bars to the numbers on both sides of decimal point are different.

Step 1 : Now to find the square root use division method as mentioned earlier. Put decimal point (.) immediately after finding the square root of integral part.

Step 1 : Proceeding this way when remainder is zero (0), quotient is the required square root.

Let us try to understand with examples.

Example : Find square root.

(i) 1.69 (ii) 151.29 (iii) 990.3609 (iv) 0.0018931201

Solutions :

(i)

	1. 3
1	$\overline{1}.\overline{69}$
	1
23	69
	69

$\therefore \sqrt{1.69} = 1.3$

(ii)

	1 2. 3
1	$\overline{1}\,\overline{51}.\overline{29}$
	1
22	51
	44
243	729
	729

$\therefore \sqrt{151.29} = 12.3$

(iii)

	3 1. 4 7
3	$\overline{9}\,\overline{90}.\overline{36}\,\overline{09}$
	9
61	90
	61
624	29 36
	24 96
6287	4 40 09
	4 40 09

$\therefore \sqrt{990.3609} = 31.47$

(iv)

	0. 0 4 3 5 1
4	$0.\overline{00}\,\overline{18}\,\overline{93}\,\overline{12}\,\overline{01}$
	16
83	293
	249
865	44 12
	43 25
8701	87 01
	87 01

$\therefore \sqrt{0.0018931201} = 0.04351$

Exercise 6.4

1. Without finding square root, find how many digits are there in the square roots of the following numbers.

   (i) 100 (ii) 21904 (iii) 17850625

2. Using division method, find the square root of each of the following numbers.

   (i) 676 (ii) 841 (iii) 1156

   (iv) 2025 (v) 2704 (vi) 4489

   (vii) 8100 (viii) 14641 (ix) 15129 (x) 21904

3. Find the square roots of the following decimal numbers.

   (i) 51.84 (ii) 79.21 (iii) 98.01

   (iv) 1.44 (v) 6.25 (vi) 973.44

4. For a rectangular field the length is 35 metres and the breadth is 12 metres. What will be the length of the hypotenuse?

5. In a school there are 1089 students. On the first day of the annual sports of the school at the time of flag hoisting, they are asked to stand in such a way that whatever is the number of rows, number of students in each row must be same (i.e. number of rows and number of columns are same). How many rows can be formed with these students?

6. Find the smallest number to be added with each of the following numbers to get perfect square.

   (i) 1220 (ii) 1750 (iii) 5451 (iv) 1015

7. Find the smallest number to be subtracted from each of the following numbers to get a perfect square. Find the square root of the number so obtained.

   (i) 825 (ii) 1450 (iii) 3250 (iv) 6262

8. What is the nearest perfect square number of 4612?

9. Give 5 examples of each of the following –

   (i) Square number whose unit place digit is 4.

   (ii) Square number whose unit place digit is 9.

   (iii) Square number whose unit place digit is 0.

What we have learnt

1. The number which can be expressed as the square of a number is known as square number.
2. Every square number have 0, 1, 4, 5, 6 or 9 at the unit's place.
3. If a number has 2, 3, 7 or 8 at the unit's place we can surely say that the number is not a perfect square number.
4. Square of even numbers are even and square of odd numbers are odd.
5. Square root is the inverse operation of squaring.
6. If a square number has one or two digits then its square root have one digit. Again, if the square number has three or four digits then its square root have two digits, etc.

❑❑❑