Nuclei

basic concepts

1. Composition of Nucleus

The atom consists of central nucleus, containing entire positive charge and almost entire mass. According to accepted model ***the nucleus is composed of protons and neutrons.*** The *proton* was discovered by Rutherford by bombardment of α-particles on nitrogen in accordance with the following equation:

$$\underset{\text{(Nitrogen)}}{{}^{14}_{7}\text{N}} + \underset{(\alpha\text{-particle})}{{}^{4}_{2}\text{He}} \longrightarrow \underset{\text{Oxygen}}{{}^{17}_{8}\text{O}} + \underset{\text{Proton}}{{}^{1}_{1}\text{H}}$$

The superscripts (on the top) denote the *mass number* and subscripts (in the base) denote the *atomic number.* Symbolically a nuclide is written as ${}^{A}_{Z}X$ or ${}_{Z}X^{A}$, where A is the mass number and Z is the atomic number.

The *neutron* was discovered by J. Chadwick by the bombardment of α-particles on beryllium in accordance with

$$\underset{\text{(Beryllium)}}{{}^{9}_{4}\text{Be}} + \underset{(\alpha\text{-particle})}{{}^{4}_{2}\text{He}} \longrightarrow \underset{\text{(Carbon)}}{{}^{13}_{6}\text{C}} + \underset{\text{(Neutron)}}{{}^{1}_{0}n}$$

A neutron is neutral (zero charge) particle and its mass number is 1.

The number of protons in a nucleus is called atomic number (Z) while the number of nucleons (*i.e.*, protons + neutrons) is called the *mass number* (A). In general mass number>atomic number (except for hydrogen nucleus where $A = Z$).-

Since neutron is neutral, it is used for artificial disintegration.

2. Size of Nucleus

According to experimental observations, the radius of the nucleus of an atom of mass number A is

$$R = R_0 A^{1/3} \qquad \text{where} \qquad R_0 = 1.2\times10^{-15}\ \text{m} = 1.2\ \text{fm}$$

3. Atomic Masses

The masses of atoms, nuclei, etc., are expressed in terms of atomic mass unit represented by amu or 'u'. For this mass of C-12 is taken as standard.

$$1\ \text{u} = \frac{\text{mass of carbon - 12 atom}}{12}$$

$$= 1.660565\times10^{-27}\ \text{kg}$$

mass of proton (m_p) = 1.007276 u

mass of neutron (m_n) = 1.008665 u

mass of electron (m_e) = 0.000549 u

4. Isotopes, Isobars and Isotones

The nuclides having the same atomic number (Z) but different mass number (A) are called isotopes. The nuclides having the same mass number (A), but different atomic number (Z) are called isobars. The nuclides having the same number of neutrons (A–Z) are called **isotones.**

5. Nuclear Instability: Radioactivity

Becquerel discovered that some heavy nuclei ($A > 180$ like radium) are unstable and spontaneously decay into other elements by the emission of certain radiations: α, β and γ-radiations. This phenomenon is called **radioactivity.**

6. Properties of α,β and γ-Radiations

α-particles: (*i*) α-particles are helium nuclei, so they have positive charge $+2e$ and mass nearly four times the mass of proton.

(*ii*) On account of positive charge, α-particles are deflected by electric and magnetic fields.

(*iii*) α-particles have strong ionizing power.

(*iv*) α-particles have small penetrating power.

(*v*) α-particles are scattered by metallic foils (*eg.*, gold foils).

(*vi*) α-particles produce fluorescence in some substances like zinc sulphide.

(*vii*) α-particles affect photographic plate feebly.

β-particles: (*i*) β-particles are fast moving electrons.

(*ii*) The speed of β-particles is very high ranging from 0.3 c to 0.98 c (c = speed of light in vacuum).

(*iii*) β-particles carry negative charge equal to $-e = -1.6\times10^{-19}$ C; so they are deflected by electric and magnetic fields opposite to the direction of deflection of α-particles.

(*iv*) β-particles have small ionising power (100 times smaller than α-particles).

(*v*) β-particles have large penetrating power (100 times larger than α-particles).

(*vi*) β-particles cause fluorescence.

(*vii*) β-rays are similar to cathode rays.

γ-Rays: (*i*) γ-rays are electromagnetic radiations, of wavelength 0·01 Å.

(*ii*) γ-rays are neutral, so they are not affected by electric and magnetic fields.

(*iii*) γ-rays travel in vacuum with the speed of light.

(*iv*) γ-rays have the highest penetrating power.

(*v*) γ-rays have the least ionising power.

(*vi*) γ-rays are similar to X-rays

7. Radioactive Decay Laws

Rutherford-Soddy law

(*i*) Radioactivity is a nuclear phenomenon. It is independent of all physical and chemical conditions.

(*ii*) The disintegration is random and spontaneous. It is a matter of chance for any atom to disintegrate first.

(*iii*) The radioactive substances emit α or β-particles along with γ-rays. These rays originate from the nuclei of disintegrating atom and form fresh radioactive products.

(*iv*) The rate of decay of atoms is proportional to the number of undecayed radioactive atoms present at any instant. If N is the number of undecayed atoms in a radioactive substance at any time t, dN the number of atoms disintegrating in time dt, the rate of decay is $\frac{dN}{dt}$ so that

$$-\frac{dN}{dt} \propto N \text{ or } \frac{dN}{dt} = -\lambda N \qquad ...(i)$$

where λ is a constant of proportionality called the **decay (or disintegration) constant**.

Equation (*i*) results

$$N = N_0\, e^{-\lambda t} \qquad ...(ii)$$

where N_0 initial number of undecayed radioactive atoms.

8. Radioactive Displacement Laws

(*i*) When a nuclide emits an α-particle, its mass number is reduced by four and atomic number by two,

i.e., $_{Z}^{A}X \longrightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He + \text{Energy}$

(*ii*) When a nuclide emits a β-particle, its mass number remains unchanged but atomic number increases by one,

i.e., $_{Z}^{A}X \longrightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}\beta + \bar{\nu} + \text{Energy}$,

where $\bar{\nu}$ is the antineutrino.

The β-particles are not present initially in the nucleus but are produced due to the disintegration of neutron into a proton,

i.e., $_{0}^{1}n \longrightarrow {}_{1}^{1}H + {}_{-1}^{0}\beta + \bar{\nu}$ (antineutrion)

When a proton is converted into a neutron, positive β-particle or positron is emitted.

$$_{1}^{1}H \longrightarrow {}_{0}^{1}n + {}_{1}^{0}\beta + \nu \text{ (neutrino)}$$

(*iii*) When a nuclide emits a gamma photon, neither the atomic number nor the mass number changes.

9. Half-life and Mean life

The half-life period of a radioactive substance is defined as the time in which one-half of the radioactive substance is disintegrated. If N_0 is the initial number of radioactive atoms present, then in a half life time T, the number of undecayed radioactive atoms will be $N_0 / 2$ and in next half $N_0 / 4$ and so on.

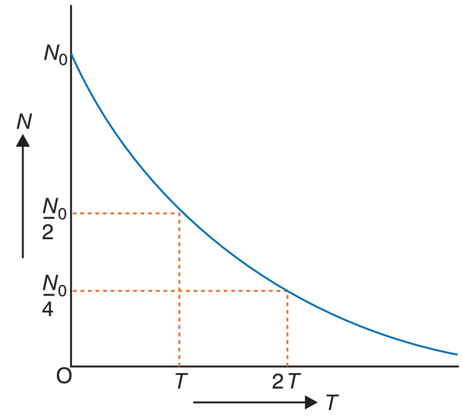
That is $t = T$ (half-life), $N = \frac{N_0}{2}$

$\therefore$ From relation $N = N_0 e^{-\lambda T}$ ...(*i*)

we get, $\frac{N_0}{2} = N_0 e^{-\lambda T}$ or $e^{-\lambda T} = \frac{1}{2}$ ...(*ii*)

From equations (*i*) and (*ii*), we get

$$\frac{N}{N_0} = e^{-\lambda t} = \left(\frac{1}{2}\right)^{t/T} \quad ...(iii)$$



Equation (*iii*) is the basic equation for the solution of half-life problems of radioactive elements.

The half-life T and disintegration constant λ are related as

$$T = \frac{0.6931}{\lambda} \quad ...(iv)$$

The mean life of a radioactive substance is equal to the sum of life time of all atoms divided by the number of all atoms,

i.e., Mean life,

$$\tau = \frac{\text{sum of life time of all atoms}}{\text{total number of atoms}} = \frac{1}{\lambda} \quad ...(v)$$

From equations (*iv*) and (*v*), we get

$$T = 0.6931\,\tau \text{ i.e., } T < \tau \quad ...(vi)$$

10. Activity of Radioactive Substance

The activity of a radioactive substance means the rate of decay (or the number of disintegrations/sec). This is denoted by

$$A = \left|\frac{dN}{dt}\right| = \left|\frac{d}{dt}(N_0 e^{-\lambda t})\right| = \lambda N \quad ...(vii)$$

If A_0 is the activity at time $t = 0$, then,

$$A_0 = \lambda N_0.$$

$$\therefore \quad \frac{A}{A_0} = \frac{N}{N_0} = e^{-\lambda t}$$

i.e., $A = A_0 e^{-\lambda t}$ ...(*viii*)

11. Units of Radioactivity

(1) Curie: It is defined as the activity of radioactive substance which gives 3.7×10^{10} disintegration/sec which is also equal to the radioactivity of 1 g of pure radium.

(2) Rutherford: It is defined as the activity of radioactive substance which gives rise to 10^6 disintegrations per second.

(3) Becquerel: In SI system the unit of radioactivity is becquerel.

$$1 \text{ becquerel} = 1 \text{ disintegration/second}$$

12. Simple Explanation of α-decay, β-decay and γ-decay

α-emission: A proton in nucleus has a binding energy of nearly 8 MeV; so to come out of a nucleus, it requires an energy of 8 MeV; but such amount of energy is not available to a proton; hence proton as such cannot come out of nucleus on its own. On the other hand, mass of α-particle is subsequently less than the total mass of 2 protons + 2 neutrons. According to Einstein's mass energy equivalence relation, sufficient energy is released in the formation of an α-particle within the nucleus. This energy appears in the form of kinetic energy of α-particle. With this kinetic energy, α-particle hits the wall of nucleus again and again and finally escapes out. The process may be represented as

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + \underset{(\alpha\text{-particle})}{{}^{4}_{2}He}$$

β-emission: β-particles are not the constituents of nucleus, then question is why and how they are emitted by radioactive nucleus. Pauli, in 1932, suggested that at the time of emission of a β -particle, a neutron in nucleus is converted into a proton, a β-particle and an antineutrino. This may be expressed as

$$^{1}_{0}n \longrightarrow {}^{1}_{1}H + {}^{0}_{-1}\beta + \bar{\nu}$$

In general $$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \bar{\nu}$$

Antineutrino is a massless and chargeless particle. The energy of the above process is shared by β-particle and antineutrino; that is why the energy of β-particle ranges from 0 to certain maximum value.

γ-emission: When α or β-particle is emitted from a nucleus, the residual nucleus is left in an excited state. The excited nucleus returns to its ground state by the emission of a γ-photon.

Thus γ-photon is emitted either with α-emission or with β-emission.

13. Mass Energy Equivalence Relation

According to Einstein, the mass and energy are equivalent *i.e.*, mass can be converted into energy and vice-versa. The mass energy equivalence relation is $E = mc^2$.

Accordingly, 1 kg mass is equivalent to energy

$$= 1 \times (3 \times 10^8)^2 = 9 \times 10^{16} \text{ joules}$$

and $$1 \text{ amu} = \frac{1}{6.02 \times 10^{26}} \text{kg mass}$$

is equivalent to energy 931 MeV.

14. Mass Defect

It is observed that the mass of a nucleus is always less than the mass of constituent nucleons (*i.e.,* protons + neutrons). This difference of *mass* is called the mass defect. Let (Z, A) be the mass of nucleus, m_p = the mass of proton and m_n = mass of neutron, then the mass defect

$$\Delta m = \text{Mass of nucleons} - \text{Mass of nucleus}$$
$$= Zm_p + (A - Z)m_n - M_{\text{nucleus}}$$

15. Binding Energy per Nucleon

This **mass defect** is in the form of binding energy of nucleus, which is responsible for binding the nucleons into a small nucleus.

$\therefore$ Binding energy of nucleus = $(\Delta m)\, c^2$

and $$\text{Binding energy per nucleon} = \frac{(\Delta m)c^2}{A}$$

16. Nature of Nuclear Forces

The protons and neutrons inside the nucleus are held together by strong attractive forces. These attractive forces cannot be gravitational since forces on repulsion between protons > > attractive gravitational force between protons. These forces are short range attractive forces called *nuclear forces*. The nuclear forces are strongest in nature, short range and charge independent, therefore the force between proton-proton is the same as the force between neutron-neutron or proton-neutron.

Yukawa tried to explain the existence of these forces, accordingly the proton and neutron do not have independent existence between nucleus. The proton and neutron are interconvertible through negative and positive π-mesons, *i.e.*,

$$\text{Proton} \underset{\pi^+}{\overset{\pi^-}{\rightleftarrows}} \text{Neutron and Neutron} \xrightarrow{\pi^\circ} \text{Neutron}$$

The existence of meson gives rise to meson field which gives rise to attractive nuclear forces.

The mass of π-meson = 273 × mass of electron.

17. Nuclear Reaction

When a beam of monoenergetic particles (*e.g.*, α-rays, neutrons etc.) collides with a stable nucleus, the original nucleus is converted into a nucleus of new element. This process is called a nuclear reaction. A typical nuclear reaction is

$$a + \mathrm{X} \rightarrow \mathrm{Y} + b$$

where a is incident energetic particle, X is target nucleus, Y is residual nucleus and b is outgoing particle. This reaction in compact form is expressed as

$$X\,(a, b)\,Y$$

In a nuclear reaction mass number, electric charge, linear momentum, angular momentum and total energy are always conserved. The energy of reaction is

$$\mathrm{Q} = (M_a + M_X)\,c^2 - (M_b + M_Y)c^2$$

18. Nuclear Fission

The splitting of heavy nucleus into two or more fragments of comparable masses, with an enormous release of energy is called *nuclear fission*. For example, when slow neutrons are bombarded on ${}_{92}\mathrm{U}^{235}$, the fission takes place according to reaction

$${}^{235}_{92}\mathrm{U} + \underset{\text{(slow neutron)}}{{}^{1}_{0}n} \longrightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3({}^{1}_{0}\mathrm{n}) + 200\,\mathrm{MeV}$$

In nuclear fission the sum of masses before reaction is greater than the sum of masses after reaction, the difference in mass being released in the form of fission energy.

Remarks:

1. It may be pointed out that it is not necessary that in each fission of uranium, the two fragments Ba^{141} and Kr^{92} are formed but they may be any stable isotopes of middle weight atoms. The most probable division is into two fragments containing about 40% and 60% of the original nucleus with the emission of 2 or 3 neutrons per fission.
2. The fission of U^{238} takes place by fast neutrons.

19. Nuclear Fusion

The phenomenon of combination of two or more light nuclei to form a heavy nucleus with release of enormous amount of energy is called *nuclear fusion*. The sum of masses before fusion is greater than the sum of masses after fusion, the difference in mass appearing as fusion energy.

For example, the fusion of two deuterium nuclei into helium is expressed as

$${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \longrightarrow {}^{4}_{2}\mathrm{He} + 21.6\,\mathrm{MeV}.$$

Thus, fusion process occurs at an extremely high temperature and high pressure as in sun where temperature is 10^7 K.

Remarks:

1. For the fusion to take place, the component nuclei must be brought within a distance of 10^{-14} m. For this they must be imparted high energies to overcome the repulsive force between nuclei. This is possible when temperature is enormously high.
2. The principle of hydrogen bomb is also based in nuclear fusion.
3. The source of energy of sun and other star is *nuclear fusion*. There are two possible cycles:

(a) Proton-proton cycle:

$$ {}_1^1H + {}_1^1H \longrightarrow {}_1^2H + {}_1^0\beta + \nu. \text{ (Neutrino)} + \text{Energy} $$

$$ {}_1^2H + {}_1^1H \longrightarrow {}_2^3He + {}_0^1n + \text{Energy} $$

$$ {}_2^3He + {}_2^3He \longrightarrow {}_2^4He + {}_1^1H + {}_1^1H + \text{Energy} $$

Net result is $ {}_1^1H + {}_1^1H + {}_1^1H + {}_1^1H \longrightarrow {}_2^4He + 2{}_1^0\beta + 2\nu + \text{Energy (26.7 MeV)}$

(b) Carbon-nitrogen cycle:

$$ {}_1^1H + {}_6^{12}C \longrightarrow {}_7^{13}N + \text{Energy} $$

$$ {}_7^{13}N \longrightarrow {}_6^{13}C + {}_1^0\beta + \nu \text{ (neutrino)} $$

$$ {}_6^{13}C + {}_1^1H \longrightarrow {}_7^{14}N + \text{(Energy)} $$

$$ {}_7^{14}N + {}_1^1H \longrightarrow {}_8^{15}O + \text{Energy} $$

$$ {}_8^{15}O \longrightarrow {}_7^{15}N + {}_1\beta^0 + \nu \text{ (neutrino)} $$

$$ {}_7^{15}N + {}_1^1H \longrightarrow {}_6^{12}C + {}_2^4He + \text{Energy} $$

Net result is $ {}_1^1H + {}_1^1H + {}_1^1H + {}_1^1H \longrightarrow {}_2^4He + 2{}_1^0\beta + 2\nu + \text{Energy (26.7 MeV)}$

The proton-proton cycle occurs at a relatively lower temperature as compared to carbon-nitrogen cycle which has a greater efficiency at higher temperature.

At the sun whose interior temperature is about 2×10^6 K, the proton-proton cycle has more chances for occurrence.

Selected NCERT Textbook Questions

Composition of nucleus and Radioactivity

Q. 1. Two stable isotopes of lithium ${}_3^6Li$ and ${}_3^7Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u respectively. Find the atomic weight of lithium.

Ans. Masses of isotopes are $m_1 = 6.01512$ u, $m_2 = 7.01600$ u

Percentage of isotopes are $P_1 = 7.5\%$, $P_2 = 92.5\%$

$$ \text{Average atomic mass } = \frac{P_1 m_1 + P_2 m_2}{P_1 + P_2} $$

$$ = \frac{7.5 \times 6.01512 + 92.5 \times 7.01600}{7.5 + 92.5} = \mathbf{6.941\,u} $$

Q. 2. Find the nuclear reactions for

(i) α-decay of ${}_{88}^{226}Ra$

(ii) α-decay of ${}_{94}^{242}Pu$

(iii) β^--decay of ${}_{15}^{32}P$

(iv) β^--decay of ${}_{83}^{210}Bi$

(v) β^+-decay of ${}_6^{11}C$

(vi) β^+-decay of ${}_{43}^{97}Tc$

(vii) Electron capture of ${}_{54}^{120}Xe$

Ans. (*i*) $^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\text{He}$ (*ii*) $^{242}_{94}\text{Pu} \rightarrow ^{238}_{92}\text{U} + ^{4}_{2}\text{He}$

(*iii*) $^{32}_{15}\text{P} \rightarrow ^{32}_{16}\text{S} + _{-1}e^{0} + \bar{\nu}$ (*iv*) $^{210}_{83}\text{Bi} \rightarrow ^{210}_{84}\text{Po} + _{-1}e^{0} + \bar{\nu}$

(*v*) $^{11}_{6}\text{C} \rightarrow ^{11}_{5}\text{B} + _{+1}e^{0} + \nu$ (*vi*) $^{97}_{43}\text{Tc} \rightarrow ^{97}_{42}\text{Mo} + _{+1}e^{0} + \nu$

(*vii*) $^{120}_{54}\text{Xe} + _{+1}e^{0} \rightarrow ^{120}_{53}\text{I} + \upsilon$

Q. 3. A radioactive isotope has a half-life of *T* years. How long will it take the activity to reduce (*a*) 3.125% (*b*) 1% of its original value?

Ans. $$\frac{R}{R_0} = \left(\frac{1}{2}\right)^n \quad \ldots(i)$$

(*a*) $$\frac{R}{R_0} = \frac{3.125}{100} = \frac{1}{32} = \left(\frac{1}{2}\right)^5$$

From (*i*), $$\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^n$$

$\Rightarrow$ n = (5 half lives), or $\frac{t}{T} = 5$

$\therefore$ $\boldsymbol{t = 5\,T}$

(*b*) $$\frac{R}{R_0} = \frac{1}{100} = \left(\frac{1}{2}\right)^n$$

$\therefore$ $\log 100 = n \log 2$

$$n = \frac{\log_{10} 100}{\log_{10} 2} = \frac{2}{0.3010} = 6.64$$

$\boldsymbol{t = 6.64\,T}$

Q. 4. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of Indus-Valley civilisation. Given living plant gives about 15 decays per minute and half-life of carbon 14=5730 years.

Ans. Activity, $R = \lambda N$

Initial activity, $R_0 = \lambda N_0$

$\therefore$ $$\frac{N}{N_0} = \frac{R}{R_0}$$

Given $\frac{R}{R_0} = \frac{9}{15}$ $\Rightarrow$ $\frac{N}{N_0} = \frac{9}{15}$

From relation $N = N_0 e^{-\lambda t}$, we have $\frac{N}{N_0} = e^{-\lambda t}$

or $-\lambda t = \log_e \frac{N}{N_0}$ or $t = \frac{\log_e \frac{N_0}{N}}{\lambda} = \frac{2.303 \times \log_{10} \frac{15}{9}}{0.693/T}$

$$= \frac{2.303 \times (\log_{10} 1.6667)\,T}{0.693} = \frac{2.303 \times 0.2218 \times 5730}{0.693}$$

$\approx$ **4224 years**

Q. 5. Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi (millicurie) strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

Avogadro Number = 6.02×10^{23} per g-atom.

Ans. We have $R = \lambda N$ ...(*i*)

Given $R = 8.0 \text{ mCi} = 8.0\times10^{-3}$ Ci

$= 8.0\times10^{-3}\times3.7\times10^{10} \text{ s}^{-1} = 29.6\times10^{7} \text{ s}^{-1}$

$$\lambda = \frac{0.6931}{T} = \frac{0.6931}{5.3\text{years}} = \frac{0.6931}{5.3\times365\times24\times60\times60}\text{s}^{-1} = 4.15\times10^{-9}\text{s}^{-1}$$

From equation (*i*)

Number of undecayed nuclei, $N = \dfrac{R}{\lambda} = \dfrac{29.6\times10^{7}}{4.15\times10^{-9}} = 7.13\times10^{16}$ atoms

The mass of 6.02×10^{23} atoms is 60 grams, so mass of $N = 7.13\times10^{16}$ atoms is

$$= 7.13\times10^{16}\left(\frac{60}{6.02\times10^{23}}\right)\text{g}$$

Required mass of Co, $\dfrac{7.13\times10^{16}\times60}{6.02\times10^{23}}\text{g} = \mathbf{7.1\times10^{-6}\,g}$

Q. 6. The half life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Ans. We have $\dfrac{dN}{dt} = \lambda N$ ...(*i*)

Disintegration constant $\lambda = \dfrac{0.6931}{T}$ ...(*ii*)

Here T=28 years = $28\times3.154\times10^{7}$ seconds

90 g of ^{90}Sr contain 6.023×10^{23} atoms

∴ Number of Sr-90 atoms in 15 mg $(=15\times10^{-3}\text{g})$

$$N = \frac{15\times10^{-3}}{90}\times6.023\times10^{23} = 1.00\times10^{20}$$

Disintegration rate, $\dfrac{dN}{dt} = \left(\dfrac{0.6931}{28\times3.154\times10^{7}}\right)\times1.00\times10^{20}$

$$= \mathbf{7.85\times10^{10}\ Bq}$$

Q. 7. A source contains two phosphorus radionuclides $^{32}_{15}P$ ($T_{1/2}$ = 14.3 days) and $^{33}_{15}P$ ($T_{1/2}$ = 25.3 days). Initially 10% of the decay comes from, $^{33}_{15}P$ ($T_{1/2}$ = 25.3 days) how long one must wait until 90% do so?

Ans. Let radionuclides be represented as P_1 ($T_{1/2}$ = 14.3 days) and P_2 ($T_{1/2}$ = 25.3 days).

Initial decay is 90% from P_1 and 10% from P_2. With the passage of time amount of P_1 will decrease faster than that of P_2.

As rate of disintegration $\propto N$ or mass M, initial ratio of P_1 to P_2 is 9. Let mass of P_1 be $9x$ and that of P_2 be x.

Let after t days mass of P_1 become y and that of P_2 become $9y$.

Using half-life formula $\dfrac{M}{M_0} = \left(\dfrac{1}{2}\right)^{n}$ where n is number of half lives, $n = \dfrac{t}{T}$.

$$\frac{y}{9x} = \left(\frac{1}{2}\right)^{n_1} \quad ...(i) \quad \text{where} \quad n_1 = \frac{t}{T_1}$$

$$\frac{9y}{x} = \left(\frac{1}{2}\right)^{n_2} \quad ...(ii) \quad \text{where} \quad n_2 = \frac{t}{T_2}$$

Dividing (*i*) by (*ii*), we get

$$\frac{1}{81} = \left(\frac{1}{2}\right)^{n_1 - n_2} \Rightarrow \frac{1}{81} = \left(\frac{1}{2}\right)^{t\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

$\Rightarrow$ Taking log, $\quad \log 1 - \log 81 = t\left(\frac{1}{T_1} - \frac{1}{T_2}\right)(\log 1 - \log 2)$

As $\log 1 = 0$ $\quad t = \dfrac{(\log 81)}{(\log 2)\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} = \dfrac{\log 81}{\log 2}\left(\dfrac{T_1 T_2}{T_2 - T_1}\right) = \dfrac{1.9084}{0.3010} \times \dfrac{14.3 \times 25.3}{(25.3 - 14.3)} =$ **208.5 days**

Nuclear Energy: Fission and Fusion

Q. 8. Obtain the binding energy of a nitrogen nucleus ($^{14}_{7}N$) from the following data in MeV.

m_H = 1.00783 u

m_n = 1.00867 u

m_N = 14.00307 u

Ans. $_7N^{14}$ nucleus contains 7 protons and 7 neutrons.

Mass of 7-protons = $7m_H$ = 7 × 1.00783 u = 7.05481 u

Mass of 7-neutrons = $7m_n$ = 7 × 1.00867 u = 7.06069 u

$\therefore$ Mass of nucleons in $^{14}_{7}N = 7.05481 + 7.06069 = 14.11550$ u

Mass of nucleus $^{14}_{7}N = m_N = 14.00307$ u

$\therefore$ Mass defect = mass of nucleons – mass of nucleus

= 14.11550 – 14.00307 = 0.11243 u

Total Binding energy = 0.11243 × 931 MeV = **104.67 MeV**

Binding energy per nucleon = $\dfrac{104.67}{14}$ = **7.47 MeV/nucleon**

Q. 9. Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data. m_H = 1.007825 u, m_n=1.008665 u, $m(^{56}_{26}Fe)$ = 55.934939 u, $m(^{209}_{83}Bi)$ = 208.980388 u, 1 u =931.5 MeV. Which nucleus has greater binding energy per nucleon?

Ans. Mass defect in $\left(^{56}_{26}Fe\right)$ atom

$= 26\, m_H + (56–26)\, m_n - m(^{56}_{26}Fe)$

= 26 × 1.007825 + 30 × 1.008665 – 55.934939

= 26.203450 + 30.259950 – 55.934939 = 0.528461 u

Total binding energy of $^{56}_{26}Fe$ = 0.528461 × 931.5

= 492.26 MeV.

Binding energy per nucleon, $B_n = \dfrac{492.26}{56}$ =**8.79 MeV/ nucleon**

Mass defect of ($^{209}_{83}Bi$) atom is = $83\, m_H + (209 - 83)m_n - m(^{209}_{83}Bi)$

$= 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$= 83.649475 + 127.091790 - 208.980388$

$= 210.741265 - 208980388 = 1.760877$ u

Total binding energy of $(^{209}_{83}\text{Bi}) = 1.760877 \times 931.5$ MeV $= 1640.26$ MeV

Binding energy per nucleon $B_n = \frac{E}{A}$

$= \frac{1640.26}{209} =$ **7.848 MeV/nucleon**

Obviously $^{56}_{26}\text{Fe}$ has greater binding energy per nucleon.

Q. 10. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}\text{Cu}$ atoms (of mass 62.92960 u). The masses of proton and neutrons are 1.00783 u and 1.00867 u respectively.

Ans. Masses of protons and neutrons in 63 u of Cu

$= Zm_p + (A-Z)m_n = 29m_p + (63-29)m_n$

$= 29 \times 1.00783 + (34 \times 1.00867) = 29.22707 + 34.29478 = 63.52185$ u

Mass of $^{63}_{29}\text{Cu}$ atom $= 62.92960$ u

Mass defect $= 63.52185 - 62.92960 = 0.59225$ u

Energy released in $^{63}_{29}\text{Cu}$ atom $= 0.59225 \times 931$ MeV $= 551.385$ MeV

Number of atoms in 3 g of copper $= \frac{6.02 \times 10^{23}}{63} \times 3 = 2.87 \times 10^{22}$

$\therefore$ Energy required to separate all nucleons (neutrons and protons) from each other

$= 2.87 \times 10^{22} \times 551.385$ MeV $=$ **1.6×10^{25} MeV**

Q. 11. The radionuclide $^{11}_{6}\text{C}$ decays according to

$$^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + \underset{\text{(postitron)}}{e^+} + \nu + Q, \quad T_{1/2} = 20.3 \text{ min}$$

The maximum energy of the emitted positron is 0.960 MeV. Given the mass values

$m(^{11}_{6}\text{C}) = 11.011434$ u and $m(^{11}_{5}\text{B}) = 11.009305$ u

Calculate Q and compare it with the maximum energy of the positron emitted.

[*CBSE Panchkula 2015*]

Ans. Mass difference $\Delta m = m_N(^{11}_{6}\text{C}) - \{m_N(^{11}_{5}\text{B}) + m_e\}$

where m_N denotes that masses are of atomic nuclei.

If we take the masses of atoms, then we have to subtract $6m_e$ from ^{11}C and $5m_e$ from ^{11}B, then

mass difference $= m(^{11}_{6}\text{C} - 6m_e) - \{m(^{11}_{5}\text{B} - 5m_e + m_e\} = \{m(^{11}_{6}\text{C}) - m(^{11}_{5}\text{B}) - 2m_e\}$

$= 11.01143 - 11.009305 - 2 \times 0.000548 = 0.001033$ u

$Q = 0.001033 \times 931.5$ MeV $=$ **0.962 MeV**

This energy is nearly the same as energy carried by positron (0.960 MeV). The reason is that the daughter nucleus is too heavy as compared to e^+ and ν, so it carries negligible kinetic energy.

Total kinetic energy is shared by positron and neutrino; here energy carried by neutrino (E_ν) is minimum, so that energy carried by positron (E_e) is maximum (practically $E_e \approx Q$).

Q. 12. The Q-value of a nuclear reaction

$$A + B \longrightarrow C + D$$

is defined by Q = $(m_A + m_B - m_C - m_D)\,c^2$

where the masses refer to the nuclear rest masses. Determine from the given data whether the following reactions are exothermic or endothermic.

(*i*) $^1_1H + ^3_1H \to ^2_1H + ^2_1H$ **(*ii*) $^{12}_6C + ^{12}_6C \to ^{20}_{10}Ne + ^4_2He$**

Atomic masses are given to be:

$m(^1_1H) = 1.007825$ u $m(^2_1H) = 2.014102$ u

$m(^3_1H) = 3.016049$ u $m(^{12}_6C) = 12.00000$ u

$m(^{20}_{10}Ne) = 19.992439$ u $m(^4_2He) = 4.002603$ u

Take 1 u = 931 MeV

Ans. (*i*) Nuclear reaction is

$$^1_1H + ^3_1H \to ^2_1H + ^2_1H + Q$$

Mass of LHS = $m(^1_1H) + m(^3_1H)$ = 1.007825 + 3.016049 = 4.023874 u

Mass of RHS = $m(^2_1H) + m(^2_1H)$ = 2.014102 + 2.014102 = 4.028204 u

$$Q = [(m_A + m_B - m_C - m_D) \text{ in kg}] \times c^2 \text{ joule}$$
$$= [(m_A + m_B - m_C - m_D)\text{u}] \times 931 \text{ MeV}$$
$$= \left[\{m(^1_1H) + m(^3_1H)\} - \{m(^2_1H) + m(^2_1H)\}\right] \times 931 \text{ MeV}$$
$$= [4.023874 - 4.028204] \times 931 \text{ MeV}$$
$$= -0.00433 \times 931 \text{ MeV} = \mathbf{-4.031 \text{ MeV}}$$

As Q is negative, energy must be supplied for the reaction; hence the reaction is **endothermic**.

(*ii*) Nuclear reaction is $^{12}_6C + ^{12}_6C = ^{20}_{10}Ne + ^4_2He$

$$Q = [\{m(^{12}_6C) + m(^{12}_6C)\} - \{m(^{20}_{10}Ne) + m(^4_2He)\}] \times c^2 \text{ joule}$$
$$= [(12.000000 + 12.000000) - (19.992439 + 4.002603)] \times c^2 \text{ joule}$$
$$= (24.000000 - 23.995042) \times 931 \text{ MeV} = 0.004958 \times 931 \text{ MeV} = \mathbf{4.616 \text{ MeV}}$$

As Q is positive, the energy will be liberated in the reaction, hence the reaction is **exothermic**.

Q. 13. A 1000 MW fission reactor consumes half of its fuel in 5 years. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time and that all energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process. Energy generated per fission of $^{235}_{92}U$ is 200 MeV. **[*HOTS*]**

Ans. Number of U-235 atoms in 1 gram $= \dfrac{1}{235} \times 6 \times 10^{23}$

Energy generated per gram of $^{235}_{92}U = \dfrac{1}{235} \times 6 \times 10^{23} \times 200 \times 1.6 \times 10^{-13}$ Jg^{-1}

P = 1000 MW = 1000×10^6 W

$t = 5 \times 365 \times 24 \times 60 \times 60 = 5 \times 3.154 \times 10^7$ s

Total energy generated in 5 years with 80% time on

$$Q = Pt = \left(1000 \times 10^6 \times \frac{80}{100} \times 5 \times 3.154 \times 10^7\right) \text{J}$$

Amount of $^{235}_{92}U$ consumed in 5 years.

$$m=\frac{\text{Total energy}}{\text{Energy consumed per gram}}=\frac{1000\times10^6\times0.8\times5\times3.154\times10^7}{\left(\frac{1}{235}\right)\times6\times10^{23}\times200\times1.6\times10^{-13}}\text{ gram}$$

$$=\frac{4\times3.154\times235}{6\times3.2}\times10^4\text{ gram}=1.544\times10^6\text{ g}=1544\text{ kg}$$

Initial amount of fuel = 2 × 1544 = **3088 kg**

Q. 14. How long an electric lamp of 100 W can be kept glowing by fusion of 2.0 kg of deuterium? The fusion reaction can be taken as: **[*HOTS*]**

$$\mathbf{{}^2_1H+{}^2_1H\longrightarrow{}^3_2He+n+3.2\text{ MeV}}$$

Ans. Number of deuterium atoms in 2 g = 6.02×10^{23}

Number of deuterium atoms in 2.0 kg is = 6.02×10^{26}

Number of reactions $=\frac{6.02\times10^{26}}{2}=3.01\times10^{26}$

Energy released in one reaction =3.2 MeV

Total energy released, $W=3.01\times10^{26}\times3.2\text{ MeV}=9.632\times10^{26}\text{ MeV}$

$$=9.632\times10^{26}\times1.6\times10^{-13}\text{ J}=15.4\times10^{13}\text{ J}$$

If t second is the required time during which the bulb glows, then $W=Pt$ gives

$$t=\frac{W}{P}=\frac{15.4\times10^{13}}{100}=15.4\times10^{11}\text{ s}$$

$$=\frac{15.4\times10^{11}}{3.15\times10^7}\text{ years}=\mathbf{4.9\times10^4\text{ years.}}$$

Q. 15. For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from inner orbit, say, the K-shell is captured by the nucleus and a neutrino is emitted.

$$e^-+{}^A_ZX\longrightarrow{}_{Z-1}^{\ A}Y+\nu$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

Ans. Consider the two competing processes

Positron emission: $\quad {}^A_ZX\longrightarrow{}_{Z-1}^{\ A}Y+e^++\nu+Q_1$ and

Electron capture, $\quad e^-+{}^A_ZX\longrightarrow{}_{Z-1}^{\ A}Y+\nu+Q_2$

$$Q_1=[m_N({}^A_ZX)-m_N({}_{Z-1}^{\ A}Y)-m_e]c^2$$

Converting nuclear masses into atomic masses

$$Q_1=[m({}^A_ZX)-Zm_e-\{m({}_{Z-1}^{\ A}Y)+(Z-1)m_e\}-m_e]c^2$$
$$=[m({}^A_ZX)-m({}_{Z-1}^{\ A}Y)-2m_e]c^2$$
$$Q_2=[m_N({}^A_ZX)+m_e-m_N({}_{Z-1}^{\ A}Y)]c^2$$
$$=[m({}^A_ZX)-Zm_e+m_e-\{m({}_{Z-1}^{\ A}Y)+(Z-1)m_e\}]c^2$$
$$=[m({}^A_ZX)-m({}_{Z-1}^{\ A}Y)]c^2$$

This means that $Q_1>0$ implies $Q_2>0$; but $Q_2>0$ does not necessarily imply $Q_1>0$. Thus if β^+ emission is energetically allowed, electron capture is necessarily allowed, but not vice-versa.

Multiple Choice Questions [1 mark]

Choose and write the correct option(s) in the following questions.

1. Suppose we consider a large number of containers each containing initially 10000 atoms of a radioactive material with a half life of 1 year. After 1 year, [*NCERT Exemplar*]

(*a*) all the containers will have 5000 atoms of the material.

(*b*) all the containers will contain the same number of atoms of the material but that number will only be approximately 5000.

(*c*) the containers will in general have different numbers of the atoms of the material but their average will be close to 5000.

(*d*) none of the containers can have more than 5000 atoms.

2. The gravitational force between a H-atom and another particle of mass m will be given by Newton's law: [*NCERT Exemplar*]

$F = G\frac{M.m}{r^2}$**, where r is in km and**

(*a*) $M = m_{proton} + m_{electron}$

(*b*) $M = m_{proton} + m_{electron} - \frac{B}{c^2}$ $(B = 13.6 \text{ eV})$

(*c*) M is not related to the mass of the hydrogen atom.

(*d*) $M = m_{proton} + m_{electron} - \frac{|V|}{c^2}$ ($|V|$ = magnitude of the potential energy of electron in the H-atom).

3. If radius of the $^{27}_{13}Al$ nucleus is taken to be R_{Al}, then the radius of $^{125}_{53}Te$ nucleus is nearly

(*a*) $\frac{3}{5}R_{Al}$ (*b*) $\left(\frac{13}{53}\right)^{1/3} R_{Al}$ (*c*) $\left(\frac{53}{13}\right)^{1/3} R_{Al}$ (*d*) $\frac{5}{3}R_{Al}$

4. The equation $_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$ represents

(*a*) β-decay (*b*) γ-decay (*c*) fusion (*d*) fission

5. During a mean life of a radioactive element the fraction that disintegrates is:

(*a*) e (*b*) $\frac{1}{e}$ (*c*) $\frac{e-1}{e}$ (*d*) $\frac{e}{e-1}$

6. How much energy will approximately be released if all the atoms of 1 kg of deuterium could undergo fusion? [Assume energy released per deuterium nucleus is 2 MeV]

(*a*) 2×10^7 kWh (*b*) 9×10^{13} J (*c*) 6×10^{27} calorie (*d*) 9×10^{13} MeV

7. A nuclear reaction is given below. The masses in amu of reactant and product nuclei are given in brackets:

$$\underset{(1.002)}{A} + \underset{(1.004)}{B} \longrightarrow \underset{(1.001)}{C} + \underset{(1.003)}{D} + Q \text{ Mev}$$

The value of energy Q is

(*a*) 1.234 MeV (*b*) 0.91 MeV (*c*) 0.465 MeV (*d*) 1.862 MeV

8. The binding energies per nucleon of deuteron ($_1H^2$) and helium ($_2He^4$) nuclei are 1.1 MeV and 7 MeV respectively. If two deuterons fuse together to form a helium nucleus, then energy produced is:

(*a*) 5.9 MeV (*b*) 23.6 MeV (*c*) 26.9 MeV (*d*) 32.4 MeV

9. When a nucleus in an atom undergoes a radioactive decay, the electronic energy levels of the atom [*NCERT Exemplar*]

(*a*) do not change for any type of radioactivity.

(*b*) change for α and β radioactivity but not for γ-radioactivity.

(*c*) change for α-radioactivity but not for others.

(*d*) change for β-radioactivity but not for others.

10. M_x and M_y denote the atomic masses of the parent and the daughter nuclei respectively in a radioactive decay. The Q-value for a β^- decay is Q_1 and that for a β^+ decay is Q_2. If m_e denotes the mass of an electron, then which of the following statements is correct? [*NCERT Exemplar*]

(*a*) $Q_1 = (M_x - M_y)\, c^2$ and $Q_2 = (M_x - M_y - 2m_e)c^2$

(*b*) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = (M_x - M_y)c^2$

(*c*) $Q_1 = (M_x - M_y - 2\, m_e)\, c^2$ and $Q_2 = (M_x - M_y + 2\, m_e)c^2$

(*d*) $Q_1 = (M_x - M_y + 2\, m_e)\, c^2$ and $Q_2 = (M_x - M_y + 2\, m_e)c^2$

11. When boron $\left({}^{10}_{5}B\right)$ is bombarded by neutron, alpha-particles is emitted. The resulting nucleus has the mass number

(*a*) 11 (*b*) 7 (*c*) 6 (*d*) 15

12. The half life of ${}^{215}At$ is 100 μs. The time taken for the activity of the sample of ${}^{215}At$ to decay to $\frac{1}{16}$ th of its initial value is

(*a*) 400 μs (*b*) 300 μs (*c*) 40 μs (*d*) 6.3 μs

13. For a radioactive material, half-life is 10 minutes. If initially there are 600 number of nuclei, the time taken (in minutes) for the disintegration of 450 nuclei is

(*a*) 20 (*b*) 10 (*c*) 30 (*d*) 15

14. When an α-particle of mass m moving with velocity v bombards on a heavy nucleus of charge Ze, its distance of closest approach from the nucleus depends on m as

(*a*) $\frac{1}{m^2}$ (*b*) m (*c*) $\frac{1}{m}$ (*d*) $\frac{1}{\sqrt{m}}$

15. Tritium is an isotope of hydrogen whose nucleus Triton contains 2 neutrons and 1 proton. Free neutrons decay into $p + \bar{e} + \bar{\nu}$. If one of the neutrons in Triton decays, it would transform into He^3 nucleus. This does not happen. This is because [*NCERT Exemplar*]

(*a*) Triton energy is less than that of a He^3 nucleus.

(*b*) the electron created in the beta decay process cannot remain in the nucleus.

(*c*) both the neutrons in triton have to decay simultaneously resulting in a nucleus with 3 protons, which is not a He^3 nucleus.

(*d*) because free neutrons decay due to external perturbations which is absent in a triton nucleus.

16. Heavy stable nuclei have more neutrons than protons. This is because of the fact that [*NCERT Exemplar*]

(*a*) neutrons are heavier than protons.

(*b*) electrostatic force between protons are repulsive.

(*c*) neutrons decay into protons through beta decay.

(*d*) nuclear forces between neutrons are weaker than that between protons.

17. In a nuclear reactor, moderators slow down the neutrons which come out in a fission process. The moderator used have light nuclei. Heavy nuclei will not serve the purpose because [*NCERT Exemplar*]

(*a*) they will break up.

(*b*) elastic collision of neutrons with heavy nuclei will not slow them down.

(*c*) the net weight of the reactor would be unbearably high.

(*d*) substances with heavy nuclei do not occur in liquid or gaseous state at room temperature.

18. Samples of two radioactive nuclides A and B are taken. λ_A and λ_B are the disintegration constants of A and B respectively. In which of the following cases, the two samples can simultaneously have the same decay rate at any time? [*NCERT Exemplar*]

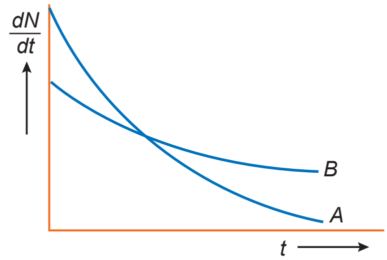
(*a*) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A = \lambda_B$.

(*b*) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$.

(*c*) Initial rate of decay of B is twice the initial rate of decay of A and $\lambda_A > \lambda_B$.

(*d*) Initial rate of decay of B is same as the rate of decay of A at $t = 2h$ and $\lambda_B < \lambda_A$.

19. The variation of decay rate of two radioactive samples *A* and *B* with time is shown in figure. Which of the following statements are true? [*NCERT Exemplar*]



(*a*) Decay constant of A is greater than that of B, hence A always decays faster than B.

(*b*) Decay constant of B is greater than that of A but its decay rate is always smaller than that of A.

(*c*) Decay constant of A is greater than that of B but it does not always decay faster than B.

(*d*) Decay constant of B is smaller than that of A but still its decay rate becomes equal to that of A at a later instant.

20. The binding energy per nucleon in $^{7}_{3}Li$ and $^{4}_{2}He$ are 7.06 MeV and 5.60 MeV respectively, then in the reaction: $p + ^{7}_{3}Li \rightarrow 2\left(^{4}_{2}He\right)$ the energy of proton must be: [*NCERT Exemplar*]

(*a*) 28.24 MeV (*b*) 17.28 MeV (*c*) 1.46 MeV (*d*) 39.2 MeV

Answers

1. (*c*)	**2.** (*b*)	**3.** (*b*)	**4.** (*a*)	**5.** (*c*)	**6.** (*b*)	**7.** (*d*)
8. (*b*)	**9.** (*b*)	**10.** (*a*)	**11.** (*b*)	**12.** (*a*)	**13.** (*a*)	**14.** (*c*)
15. (*a*)	**16.** (*b*)	**17.** (*b*)	**18.** (*b*), (*d*)	**19.** (*c*), (*d*)	**20.** (*b*)	

Fill in the Blanks [1 mark]

1. The rest mass of a nucleus is ________________ than the sum of the rest masses of its constituent nucleons.
2. Complete the equation $^{m}_{n}X \xrightarrow[\alpha\ decay]{}$ ________________.
3. A radioactive isotope of silver has half life of 20 minutes. The fraction of the original activity that remain after one hour is ________________.
4. One atomic mass unit is defined as ________________ of mass of an atom of $_6C^{12}$.
5. Isotopes of an element are the atoms of an element which have ________________ but different atomic weights.
6. Isobars are the atoms of different element which have same ________________ but different atomic number.
7. Isotones are the nuclides which contains ________________.
8. The process responsible for energy production in the sun is ________________.
9. In both the processes of nuclear fission an nuclear fusion, a certain mass disappears. This is called ________________.
10. The Apsara reactor at the Bhabha Atomic Research Centre (BARC), Mumbai, uses ________________ as moderator.

Answers

1. less **2.** $^{m-4}_{n-2}Y$ **3.** $\frac{1}{8}$ **4.** 1/12th

5. same atomic number **6.** atomic weights **7.** same number of neutrons

8. nuclear fusion **9.** mass defect **10.** water

Very Short Answer Questions [1 mark]

Q. 1. Write the relationship between the size of a nucleus and its mass number (*A*). [*CBSE (F) 2012*]

Ans. The relationship is $R = R_0 A^{1/3}$

where R = Radius of nucleus and A = Mass number.

Q. 2. How is the mean life of a radioactive sample related to its half life? [*CBSE (F) 2011*]

Ans. Mean life (τ) and half life ($T_{1/2}$) are related as:

$$\tau = \frac{T_{1/2}}{0.6931}$$

Q. 3. Write two characteristic features of nuclear force which distinguish it from Coulomb's force. [*CBSE (AI) 2011*]

Ans. Characteristic Features of Nuclear Force

(*i*) Nuclear forces are short range **attractive forces** (range 2 to 3 fm) while Coulomb's forces have range upto infinity and may be attractive or repulsive.

(*ii*) Nuclear forces are charge independent forces; while Coulomb's force acts only between charged particles.

Q. 4. Why is it found experimentally difficult to detect neutrinos in nuclear β-decay? [*CBSE (AI) 2014*]

Ans. Neutrinos are chargeless (neutral) and almost massless particles that hardly interact with matter.

Q. 5. In both β^- decay processes, the mass number of a nucleus remains same whereas the atomic number *Z* increases by one in β^- decay and decreases by one in β^+ decay. Explain, giving reason. [*CBSE (F) 2014*]

Ans. In both processes, the conversion of neutron to proton or proton to neutron inside the nucleus.

$${}^{A}_{Z}X \longrightarrow \beta^- + {}_{Z+1}^{A}Y + \overline{\nu}$$

$${}^{A}_{Z}X \longrightarrow \beta^+ + {}_{Z-1}^{A}Y + \overline{\nu}$$

Q. 6. The radioactive isotope *D*-decays according to the sequence.

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2$$

If the mass number and atomic number of D_2 are 176 and 71 respectively, what is the (*i*) mass number, (*ii*) atomic number of *D*? [*CBSE Delhi 2010*]

Ans. The sequence is represented as ${}^{A}_{Z}D \xrightarrow{\alpha} {}^{A-4}_{Z-2}D_1 \xrightarrow{\beta^-} {}^{A-4}_{Z-1}D_2$

(*i*) Given $A - 4 = 176 \Rightarrow$ Mass number of D, A = **180**

(*ii*) $Z - 1 = 71 \Rightarrow$ Atomic number of D, Z = **72**

Q. 7. Two nuclei have mass numbers in the ratio 1 : 2. What is the ratio of their nuclei densities? [*CBSE Delhi 2009*]

Ans. Nuclear density is independent of mass number, so ratio 1 : 1.

Q. 8. What is the nuclear radius of ^{125}Fe, if that of ^{27}Al is 3.6 fermi? [*CBSE (AI) 2008*]

Ans. Nuclear radius, $R = R_0 A^{1/3} \quad \Rightarrow \quad R \propto A^{1/3}$

For Al, $A = 27$, $R_{Al} = 3.6$ fermi, for Fe, $A = 125$

$$\therefore \quad \frac{R_{Fe}}{R_{Al}} = \left(\frac{A_{Fe}}{A_{Al}}\right)^{1/3} = \left(\frac{125}{27}\right)^{1/3}$$

$$\Rightarrow \quad R_{Fe} = \frac{5}{3} R_{Al} = \frac{5}{3} \times 3.6 \text{ fermi} = \mathbf{6.0 \text{ fermi}}$$

Q. 9. Two nuclei have mass numbers in the ratio 1 : 8. What is the ratio of their nuclear radii? [*CBSE (AI) 2009*]

Ans. Nuclear radius, $R = R_0 A^{1/3}$

$$\therefore \qquad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \mathbf{\frac{1}{2}}$$

Q. 10. Which part of electromagnetic spectrum has largest penetrating power? *[CBSE Delhi 2010]*

Ans. γ-rays have largest penetrating power.

Q. 11. Which one of the following cannot emit radiation and why?

Excited nucleus, excited electron *[NCERT Exemplar]*

Ans. Excited electron cannot emit radiation. This is because energy of electronic energy levels is in the range of eV only not in MeV and γ–radiation has energy in MeV.

Q. 12. In pair annihilation, an electron and a positron destroy each other to produce gamma radiation. How is the momentum conserved? *[NCERT Exemplar]*

Ans. 2γ-photons are produced which move in opposite directions to conserve momentum.

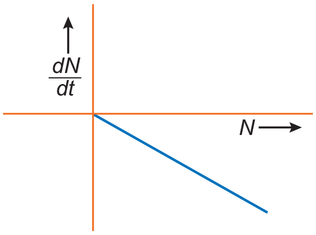
Q. 13. Imagine removing one electron from He^4 and He^3. Their energy levels, as worked out on the basis of Bohr model will be very close. Explain why. *[NCERT Exemplar] [HOTS]*

*This is because both the nuclei are very heavy as compared to electron mass.

Q. 14. $^{3}_{2}He$ and $^{3}_{1}H$ nuclei have the same mass number. Do they have the same binding energy? *[NCERT Exemplar] [HOTS]*

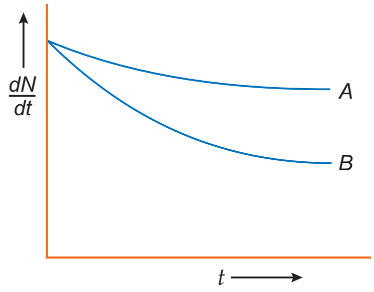
Ans. No, the binding energy of $^{3}_{1}H$ is greater. This is because $^{3}_{2}He$ has 2 proton and 1 neutron, whereas $^{3}_{1}H$ has 1 proton and 2 neutron. Repulsive force between protons in $^{3}_{1}H$ is absent.

Q. 15. Draw a graph showing the variation of decay rate with number of active nuclei. *[NCERT Exemplar] [HOTS]*



Ans. We know that $-\frac{dN}{dt} = \lambda N$, where λ is constant for a given radioactive material. So, the graph between $\frac{dN}{dt}$ and N is a straight line.

Q. 16. Which sample, *A* or *B* as shown in figure has shorter mean-life? *[NCERT Exemplar]*



Ans. *B* has shorter mean life as λ is greater for *B*.

Q. 17. Four nuclei of an element undergo fusion to form a heavier nucleus, with release of energy. Which of the two — the parent or the daughter nucleus — would have higher binding energy per nucleon?

Ans. The daughter nucleus would have a higher binding energy per nucleon.

Short Answer Questions–I [2 marks]

Q. 1. (*i*) **What characteristic property of nuclear force explains the constancy of binding energy per nucleon (*BE/A*) in the range of mass number '*A*' lying 30 < *A* < 170?**

(*ii*) **Show that the density of nucleus over a wide range of nuclei is constant independent of mass number *A*.** *[CBSE Delhi 2012, 2015]*

Ans. (*i*) Saturation or short range nature of nuclear forces.

(*ii*) The radius (size) *R* of nucleus is related to its mass number (*A*) as

$$R = R_0 A^{1/3}, \text{ where } R_0 = 1.1 \times 10^{-15} \text{ m}$$

If m is the average mass of a nucleon, then mass of nucleus = mA, where A is mass number

Volume of nucleus $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$

$\therefore$ Density of nucleus, $\rho_N = \frac{\text{mass}}{\text{volume}} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{m}{\frac{4}{3}\pi R_0^3} = \frac{3m}{4\pi R_0^3}$

Clearly nuclear density ρ_N is independent of mass number A.

Q. 2. ***(i)*** **Write the basic nuclear process involved in the emission of β^+ in a symbolic form, by a radioactive nucleus.**

(ii) **In the reactions given below:**

(a) $^{11}_{6}C \longrightarrow {}^{z}_{y}B + x + \nu$ ***(b)*** $^{12}_{6}C + {}^{12}_{6}C \longrightarrow {}^{20}_{a}Ne + {}^{c}_{b}He$

Find the values of *x, y, z* and *a, b, c*. [*CBSE Central 2016*]

Ans. (*i*) Basic nuclear reaction for β^+ decay is the conversion of proton to neutron.

$p \rightarrow n + e^+ + \nu$

(*ii*) (*a*) $x = \beta^+ / {}^{0}_{1}e, y = 5, z = 11$ (*b*) $a = 10, b = 2, c = 4$

Q. 3. **Calculate the energy in fusion reaction:** [*CBSE Delhi 2016*]

$^{2}_{1}H + {}^{2}_{1}H \longrightarrow {}^{3}_{2}He + n$, **where BE of** $^{2}_{1}H$ **= 2.23 MeV and of** $^{3}_{2}He$ **= 7.73 MeV**

Ans. Initial binding energy

$BE_1 = (2.23 + 2.23) = 4.46$ MeV

Final binding energy

$BE_2 = 7.73$ MeV

$\therefore$ Energy released =(7.73 – 4.46) MeV = **3.27 MeV**

Q. 4. **State three properties of nuclear forces.** [*CBSE Allahabad 2015*]

Ans. Properties of nuclear forces

(1) Nuclear forces are the strongest attractive forces.

(2) Nuclear forces are short ranged upto 10^{-15} m.

(3) Nuclear forces are charge independent.

Q. 5. ***(a)*** **Write the β-decay of tritium in symbolic form.**

(b) **Why is it experimentally found difficult to detect neutrinos in this process?** [*CBSE (F) 2015*]

Ans. (*a*) $^{3}_{1}H \xrightarrow{\beta^-} {}^{3}_{2}He + {}^{0}_{-1}e + \bar{\nu} + Q$

(*b*) It is due to very weak interaction with matter.

Q. 6. **The half-life of $^{238}_{92}U$ against α-decay is 4.5×10^9 years. Calculate the activity of 1g sample of $^{238}_{92}U$. [Given Avogadro's number 6×10^{23} atoms/Kmol]** [*CBSE East 2016*]

Ans. $T_{1/2} = 4.5 \times 10^9$ years $= 4.5 \times 10^9 \times 3.15 \times 10^7$ seconds

Number of atoms in 1 g sample of $^{238}_{92}U$ is $N = 6 \times 10^{23} \times \frac{1}{238}$

Activity of sample $A = \lambda N = \frac{\log_e 2}{T_{1/2}} \times N$

$$= \left(\frac{0.6931}{4.5 \times 10^9 \times 3.15 \times 10^7}\right) \times 6 \times 10^{23} \times \frac{1}{238}$$

$= \mathbf{1.232 \times 10^4}$ **becquerel**

Q. 7. A heavy nucleus X of mass number 240 and binding energy per nucleon 7.6 MeV is split into two fragments Y and Z of mass numbers 110 and 130. The binding energy per nucleon in Y and Z is 8.5 MeV per nucleon. Calculate the energy Q released per fission in MeV.

[*CBSE Delhi 2010*]

Ans. Energy released $Q = (M_Y + M_Z)c^2 - M_X c^2$

$= 8.5\ (110+130)\ \text{MeV} - 7.6 \times 240\ \text{MeV}$

$= (8.5 - 7.6) \times 240\ \text{MeV}$

$= 0.9 \times 240\ \text{MeV} =$ **216 MeV**

Q. 8. When four hydrogen nuclei combine to form a helium nucleus, estimate the amount of energy in MeV released in this process of fusion. (Neglect the masses of electrons and neutrinos) Given:

(*i*) mass of $^1_1H = 1.007825$ u

(*ii*) mass of helium nucleus = 4.002603 u, 1 u = 931 MeV/c^2 [*CBSE (F) 2011*]

Ans. Energy released $= \Delta m \times 931$ MeV

$\Delta m = 4m(^1_1H) - m(^4_2He)$

Energy released $(Q) = [4m(^1_1H) - m(^4_2He)] \times 931$ MeV

$= [4 \times 1.007825 - 4.002603] \times 931$ MeV

= **26.72 MeV**

Q. 9. Prove that the instantaneous rate of change of the activity of a radioactive substance is inversely proportional to the square of its half life. [*HOTS*]

Ans. Activity of a radioactive substance

$$R\left(= -\frac{dN}{dt}\right) = \lambda N$$

Rate of change of activity

$$\frac{dR}{dt} = \lambda\left(\frac{dN}{dt}\right) = \lambda.(-\lambda N) = -\lambda^2 N$$

As $\lambda = \dfrac{\log_e 2}{T_{1/2}}$ $\quad\therefore\quad \dfrac{dR}{dt} = -\left(\dfrac{\log_e 2}{T_{1/2}}\right)^2 N$

$\therefore$ Instantaneous activity, $\dfrac{dR}{dt} \propto \dfrac{1}{T_{1/2}^2}$

Q. 10. Explain how radioactive nuclei can emit β-particles even though atomic nuclei do not contain these particles? Hence explain why the mass number of radioactive nuclide does not change during β-decay? [*HOTS*]

Ans. Radioactive nuclei do not contain electrons (β-particles), but β-particles are formed due to conversion of a neutron into a proton according to equation

$$^1_0n \longrightarrow {}^1_1p + \underset{\beta\text{-particle}}{^{0}_{-1}\beta} + \underset{\text{antineutrino}}{\bar{\nu}}$$

The β-particle so formed is emitted at once. In this process one neutron is converted into one proton; so that the number of nucleons in the nucleus remains unchanged; hence mass number of the nucleus does not change during a β-decay.

Q. 11. Why do stable nuclei never have more protons than neutrons? [*NCERT Exemplar*] [*HOTS*]

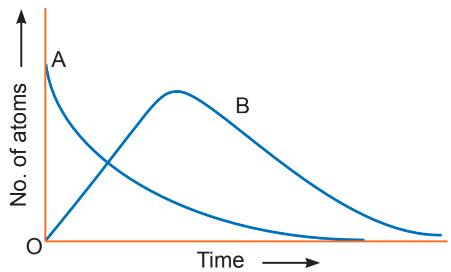
Ans. Protons are positively charged and repel one another electrically. This repulsion becomes so great in nuclei with more than 10 protons or so, that an excess of neutrons which produce only attractive forces, is required for stability.

Q. 12. Consider a radioactive nucleus A which decays to a stable nucleus C through the following sequence:

$$A \rightarrow B \rightarrow C$$

Here B is an intermediate nuclei which is also radioactive. Considering that there are N_0 atoms of A initially, plot the graph showing the variation of number of atoms of A and B versus time. [*NCERT Exemplar*] [*HOTS*]

Ans. At $t = 0$, $N_A = N_0$ while $N_B = 0$. As time increases, N_A falls off exponentially, the number of atoms of B increases, becomes maximum and finally decays to zero at ∞ (following exponential decay law).



Q. 13. A nuclide 1 is said to be the mirror isobar of nuclide 2 if $Z_1 = N_2$ and $Z_2 = N_1$.

(*a*) What nuclide is a mirror isobar of $^{23}_{11}Na$?

(*b*) Which nuclide out of the two mirror isobars has greater binding energy and why?

[*NCERT Exemplar*] [*HOTS*]

Ans. (*a*) $^{23}_{11}Na : Z_1 = 11, N_1 = 12$

$\therefore$ Mirror isobar of $^{23}_{11}Na = ^{23}_{12}Mg$.

(*b*) Since $Z_2 > Z_1$, Mg has greater binding energy than Na.

Q. 14. (*a*) Write two distinguishing features of nuclear forces.

(*b*) Complete the following nuclear reactions for α and β decay:

(*i*) $^{238}_{92}U \longrightarrow ? + ^4_2He + Q$

(*ii*) $^{22}_{11}Na \longrightarrow ^{22}_{10}Ne + ? + \nu$ [*CBSE 2019 (55/3/1)*]

Ans. (*a*) Nuclear force:

(*i*) The nuclear force is much stronger than coulomb's force.

(*ii*) The nuclear force between two nucleons falls rapidly to zero as their distance is more than few femto metres.

(*iii*) Nuclear force does not depend on the electric charge.

(*b*) (*i*) $^{238}_{92}U = ^{234}_{90}Th + ^4_2He + Q$

(*ii*) $^{22}_{11}Na \longrightarrow ^{22}_{10}Ne + e^+ + \nu$

Short Answer Questions–II [3 marks]

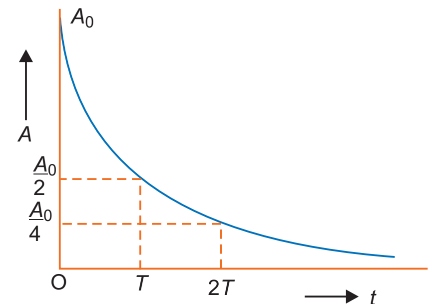
Q. 1. Define the term 'Activity' of a radioactive substance. State its SI unit. Give a plot of activity of a radioactive species versus time. [*CBSE Delhi 2010, (AI) 2009*]

Two different radioactive elements with half lives T_1 and T_2 have N_1 and N_2 (undecayed) atoms respectively present at a given instant. Determine the ratio of their activities at this instant. [*CBSE (F) 2016*]

Ans. The activity of a radioactive element at any instant is equal to its rate of decay at that instant.

SI unit of activity is **becquerel** (= 1 disintegration/second).

The plot is shown in fig.



Activity $\quad R\left(=\frac{dN}{dt}\right)=\lambda N$

Decay constant $\quad \lambda=\frac{\log_e 2}{T}$

$\therefore$ Activity $\quad R=\frac{(\log_e 2)N}{T}$

$$\therefore \quad R_1=\frac{(\log_e 2)N_1}{T_1}, \quad R_2=\frac{(\log_e 2)N_2}{T_2}$$

For two elements $\frac{R_1}{R_2}=\frac{N_1}{T_1}\times\frac{T_2}{N_2}=\left(\frac{N_1}{N_2}\right)\left(\frac{T_2}{T_1}\right)$

Q. 2 (*i*) A radioactive nucleus '*A*' undergoes a series of decays as given below:

$$A\xrightarrow{\alpha}A_1\xrightarrow{\beta}A_2\xrightarrow{\alpha}A_3\xrightarrow{\gamma}A_4$$

The mass number and atomic number of A_2 are 176 and 71 respectively. Determine the mass and atomic numbers of A_4 and A.

(*ii*) Write the basic nuclear processes underlying β^+ and β^- decays. [*CBSE Delhi 2017*]

Ans. (*i*) If we consider β^- decay, the decay scheme may be represented as

$${}^{180}_{72}A\xrightarrow{\alpha}{}^{176}_{70}A_1\xrightarrow{\beta^-}{}^{176}_{71}A_2\xrightarrow{\alpha}{}^{172}_{69}A_3\xrightarrow{\gamma}{}^{172}_{69}A_4$$

A_4 : Mass Number = 172

Atomic Number = 69

A : Mass Number = 180

Atomic Number = 72

If we consider β^+ decay, then

$${}^{180}_{74}A\xrightarrow{\alpha}{}^{176}_{72}A_1\xrightarrow{\beta^+}{}^{176}_{71}A_2\xrightarrow{\alpha}{}^{172}_{69}A_3\xrightarrow{\gamma}{}^{172}_{69}A_4$$

A_4 : Mass Number = 172

Atomic Number = 69

A : Mass Number = 180

Atomic Number = 74

(*ii*) Basic nuclear process for β^+ decay, $p\rightarrow n+{}^{0}_{1}e+\nu$

For β^- decay, $n\rightarrow p+{}^{0}_{-1}e+\bar{\nu}$

Q. 3. (*a*) Write the process of β^--decay. How can radioactive nuclei emit β-particles even though they do not contain them? Why do all electrons emitted during β-decay not have the same energy?

(*b*) A heavy nucleus splits into two lighter nuclei. Which one of the two–parent nucleus or the daughter nuclei has more binding energy per nucleon? [*CBSE (F) 2017*]

Ans. (*a*) In β^- decay, the mass number *A* remains unchanged but the atomic number *Z* of the nucleus goes up by 1. A common example of β^- decay is

$${}^{32}_{15}\text{P} \longrightarrow {}^{32}_{16}\text{S} + e^- + \bar{\nu}$$

A neutron of nucleus decays into a proton, an electron and an antineutrino. It is this electron which is emitted as β^- particle.

$${}^{1}_{0}\text{n} \longrightarrow {}^{1}_{1}\text{p} + {}^{0}_{-1}e + \bar{\nu}$$

In β^-decay, particles like antineutrinos are also emitted along with electrons. The available energy is shared by electrons and antineutrinos in all proportions. That is why all electrons emitted during β^- decay not have the same energy.

(*b*) Parent nucleus has lower binding energy per nucleon compared to that of the daughter nuclei. When a heavy nucleus splits into two lighter nuclei, nucleons get more tightly bound.

Q. 4. In a typical nuclear reaction, *e.g.*,

$${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + \text{n} + 3.27\ \text{MeV},$$

although number of nucleons is conserved, yet energy is released. How? Explain. **[*CBSE Delhi 2013*]**

Ans. In nuclear reaction

$${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + \text{n} + 3.27\ \text{MeV}$$

Cause of the energy released:

(*i*) Binding energy per nucleon of ${}^{3}_{2}\text{He}$ becomes more than the (BE/A) of ${}^{2}_{1}\text{H}$.

(*ii*) Mass defect between the reactant and product nuclei

$$\Delta E = \Delta m\, c^2$$

$$= [2m({}^{2}_{1}\text{H}) - m({}^{3}_{2}\text{He}) + m(\text{n})]c^2$$

Q. 5. (*a*) State the law of radioactive decay. Write the SI unit of 'activity'.

(*b*) There are $4\sqrt{2}\times 10^6$ radioactive nuclei in a given radioactive sample. If the half life of the sample is 20 s, how many nuclei will decay in 10 s? **[*CBSE (F) 2017*]**

Ans. (*a*) The number of nuclei disintegrating per second of a radioactive sample at any instant is directly proportional to the number of undecayed nuclei present in the sample at that instant. The SI unit of 'activity' is becquerel.

(*b*) Given, $t_{1/2} = 20$ s

$$\text{Also, } t_{1/2} = \frac{\ln 2}{\lambda} \quad \Rightarrow \quad \lambda = \frac{\ln 2}{t_{1/2}} \quad \Rightarrow \quad \lambda = \frac{\ln 2}{20}$$

Also, according to equation of radioactivity

$$N = N_0 e^{-\lambda t}$$

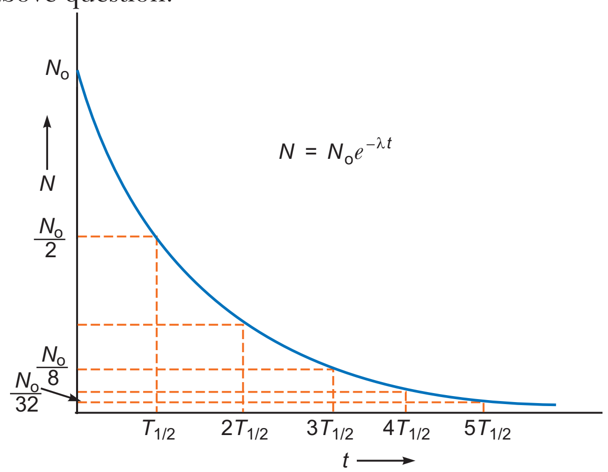
$$N = 4\sqrt{2}\times 10^6 \times e^{-\frac{\ln 2}{20}\times 10}$$

$$= 4\sqrt{2}\times 10^6 \times \frac{1}{\sqrt{2}} = \mathbf{4\times 10^6\ Nuclei}$$

Q. 6. State the law of radioactive decay.

Plot a graph showing the number (*N*) of undebased nuclei as a function of time (*t*) for a given radioactive sample having half life $T_{1/2}$. Depict in the plot the number of undecayed nuclei at (*i*) $t = 3T_{1/2}$ and (*ii*) $t = 5T_{1/2}$. **[*CBSE Delhi 2011*]**

Ans. For the Law refer to above question.



Number of undecayed nuclei at $t = 3T_{1/2}$ is $\frac{N_0}{8}$ and at $t = 5T_{1/2}$, it is $\frac{N_0}{32}$.

Q. 7. (*a*) In the following nuclear reaction

$$n + {}^{235}_{92}U \longrightarrow {}^{144}_{Z}Ba + {}^{A}_{36}X + 3n,$$

assign the values of *Z* and *A*.

(*b*) If both the number of protons and the number of neutrons are conserved in each nuclear reaction, in what way is the mass converted into energy? Explain. [*CBSE Guwahati 2015*]

Ans. (*a*) $n + {}^{235}_{92}U \longrightarrow {}^{144}_{Z}Ba + {}^{A}_{36}X + 3n,$

From law of conservation of atomic number

$$0 + 92 = Z + 36$$

$$\Rightarrow \quad Z = 92 - 36 = \mathbf{56}$$

From law of conservation of mass number,

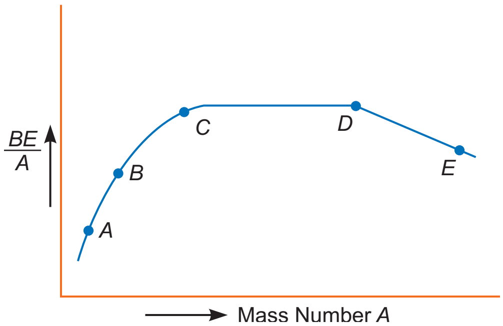
$$1 + 235 = 144 + A + 3 \times 1$$

$$A = 236 - 147 = \mathbf{89}$$

(*b*) (*i*) BE of ${}^{235}_{92}U < BE$ of $({}^{144}_{56}Ba + {}^{89}_{36}X)$ and due to difference in BE of the nuclides. A large amount of the energy will released in the fission of ${}^{235}_{92}U$.

(*ii*) Mass number of the reactant and product nuclides are same but there is an actual mass defect. This difference in the total mass of the nuclei on both sides, gets converted into energy, *i.e.*, $\Delta E = \Delta mc^2$.

Q. 8. (*a*) The figure shows the plot of binding energy (*BE*) per nucleon as a function of mass number *A*. The letters *A, B, C, D* and *E* represent the positions of typical nuclei on the curve. Point out, giving reasons, the two processes (in terms of *A, B, C, D* and *E*), one of which can occur due to nuclear fission and the other due to nuclear fusion.



(*b*) Identify the nature of the radioactive radiations emitted in each step of the decay process given below:

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y \longrightarrow {}^{A-4}_{Z-1}W$$

[*CBSE Ajmer 2015*]

Ans. (*a*) If a heavy nuclei of low $\frac{BE}{A}$ splits up into two fragments, then $\frac{BE}{A}$ of the product nuclei increases and becomes stable. So,

$E \rightarrow C + D$

If two nuclei of low $\frac{BE}{A}$ fuse together, the $\frac{BE}{A}$ of the product nuclei increases and becomes stable. So,

$A + B \rightarrow C$

(*b*) If atomic number decreases by 2 and mass number decreases by 4 an alpha particle is emitted out. So,

$${}^{A}_{Z}X \xrightarrow{\alpha} {}^{A-4}_{Z-2}Y$$

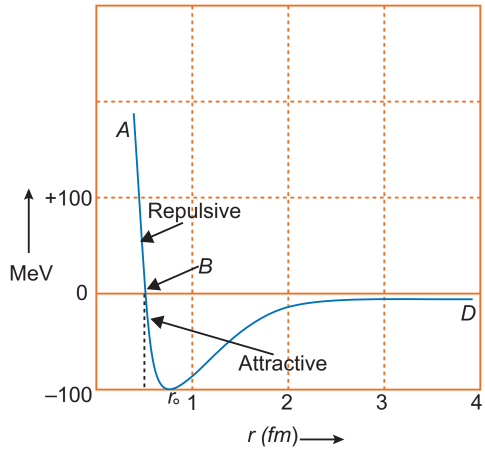
If a β^- is emitted out, the atomic number increases by 1, while mass number remains unchanged. So,

$${}^{A-4}_{Z-2}Y \xrightarrow{\beta^-} {}^{A-4}_{Z-1}W$$

Q. 9. Draw a graph showing the variation of potential energy between a pair of nucleons as a function of their separation. Indicate the regions in which the nuclear force is (*i*) attractive, (*ii*) repulsive.

Write two important conclusions which you can draw regarding the nature of the nuclear forces. **[*CBSE 2019 (55/5/2/1)*]**

Ans.



Conclusions:

(*i*) The potential energy is minimum at a distance r_0 of about 0.8 fm.

(*ii*) Nuclear force is attractive for distance larger than r_0.

(*iii*) Nuclear force is repulsive if two are separated by distance less than r_0.

(*iv*) Nuclear force decreases very rapidly at r_0/equilibrium position.

Q. 10. Define the activity of a radioactive sample. Write its SI unit.

A radioactive sample has activity of 10,000 disintegrations per second (dps) after 20 hours. After next 10 hours its activity reduces to 5,000 dps. Find out its half life and initial activity.

[*CBSE Bhubaneshwar 2015*]

Ans. The activity of a radioactive element at any instant is equal to its rate of decay at that instant. SI unit of activity is becquerel.

Let R_0 be initial activity of the sample, and its activity at any instant '*t*' is

$R = R_0 \, e^{-\lambda t}$

If $t = 20$ h, then $R = 10000$.

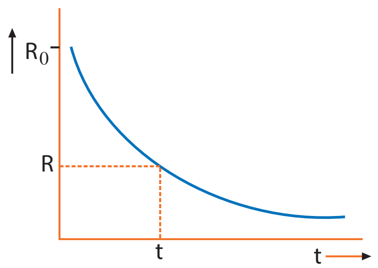
So, $10000 = R_0 \, e^{-\lambda \times 20}$ ...(*i*)

After next 10 h, *i.e.*, at time $t' = 30$ h and $R' = 5000$

$\therefore \quad 5000 = R_0\, e^{-\lambda\times 30} \qquad \ldots(ii)$

Dividing (*i*) by (*ii*), we get

$$\frac{10000}{5000} = \frac{e^{-20\lambda}}{e^{-30\lambda}} = e^{10\lambda}$$



On taking log on both side

$$10\lambda = \log_e 2$$

As we know that

$$\lambda T_{1/2} = \log_e 2$$

$\therefore \quad T_{1/2} = \mathbf{10\ h}$

From initial time $t = 0$ to $t = 20$ h, there are two half lives.

So, $\frac{R}{R_0} = \left(\frac{1}{2}\right)^2$ or $\frac{10,000}{R_0} = \frac{1}{4}$

Initial activity at $t = 0$ is

$$R_0 = 4 \times 10000 = \mathbf{40000\ dps}$$

Q. 11. In a given sample, two radioisotopes, *A* and *B*, are initially present in the ratio of 1:4. The half lives of *A* and *B* are respectively 100 years and 50 years. Find the time after which the amounts of *A* and *B* become equal. **[*CBSE (F) 2012*]**

Ans. We have $N = N_0\, e^{-\lambda t}$

For radio isotopes *A* and *B*, we can write

$$N_A = N_0 e^{-\lambda_A t_A} \quad \ldots(i)$$

$$N_B = 4N_0 e^{-\lambda_B t_B} \quad \ldots(ii)$$

Let t be the time after which $N_A = N_B$

$$t_A = t_B = t (say)$$

$\therefore \quad N_0 e^{-\lambda_A t} = 4N_0 e^{-\lambda_B t} \quad \Rightarrow \quad 4 = e^{\lambda_B t - \lambda_A t}$

$\Rightarrow \log_e 4 = (\lambda_B t - \lambda_A t) \log_e e$

$\Rightarrow \quad 2\log_e 2 = \left[\frac{\log_e 2}{T_{B_{1/2}}} - \frac{\log_e 2}{T_{A_{1/2}}}\right] t \quad [\because \lambda = \frac{\log_e 2}{T}]$

$\Rightarrow \quad 2 = \left(\frac{1}{50} - \frac{1}{100}\right) t \quad \Rightarrow \quad 2 = \left(\frac{2-1}{100}\right) t$

$\Rightarrow \quad t = \mathbf{200\ years}$

Q. 12. (*a*) Distinguish between isotopes and isobars, giving one example for each.

(*b*) Why is the mass of a nucleus always less than the sum of the masses of its constituents? Write one example to justify your answer. **[*CBSE 2019 (55/5/1)*]**

Ans. (*a*) Isotopes have same atomic number but different mass number & isobars have same mass number but different atomic number.

Examples of Isotopes ${}^{12}_{6}C, {}^{14}_{6}C$

Examples of Isobars ${}^{3}_{2}He, {}^{3}_{1}H$

(*b*) Mass of a nucleus is less than its constituents because it is in the bound state.

Some mass is converted into binding energy which is energy equivalent of mass defect *e.g.*, mass of ${}^{16}_{8}O$ nucleus is less than the sum of masses of 8 protons and 8 neutrons.

Q. 13. (*a*) Classify the following six nuclides into (*i*) isotones, (*ii*) isotopes, and (*iii*) isobars:

$${}^{12}_{6}C, {}^{3}_{2}He, {}^{198}_{80}Hg, {}^{3}_{1}H, {}^{197}_{79}Au, {}^{14}_{6}C$$

(*b*) **How does the size of a nucleus depend on its mass number? Hence explain why the density of nuclear matter should be independent of the size of the nucleus.** [*CBSE 2019, 55/5/1*]

Ans. (*a*) (*i*) Isotones: $^{198}_{80}Hg$ and $^{197}_{79}Au$

(*ii*) Isotopes: $^{12}_{6}C$ and $^{14}_{6}C$

(*iii*) For isobars: $^{3}_{2}He$ and $^{3}_{1}H$

(*b*) The radius of a nucleus having mass number A is

$$R = R_0 A^{1/3}$$

R_0 is constant.

Volume of the nucleus $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3$

$$= \frac{4}{3}\pi (R_0)^3 A$$

If 'm' be the average mass of a nucleon then mass of the nucleus= mA

Nuclear density $= \frac{\text{Mass}}{\text{Volume}} = \frac{mA}{\frac{4}{3}\pi (R_0)^3 A} = \frac{3m}{4\pi R_0^3}$

i.e., nuclear density is independent of the size of the nucleus.

Q. 14. The following table shows some measurements of the decay rate of a radionuclide sample. Find the disintegration constant. [*CBSE Sample Paper 2016*]

Time (min)	lnR (Bq)
36	5.08
100	3.29
164	1.52
218	0

Ans. $R = R_0 e^{-\lambda t}$

$\log R = \log R_0 - \lambda t$

$\log R = -\lambda t + \log R_0$

Slope of $\log R$ v/s t is '$-\lambda$'

$$-\lambda = \frac{0 - 1.52}{218 - 164} \quad \Rightarrow \quad \lambda = \mathbf{0.028 \text{ minute}^{-1}}$$

Long Answer Questions [5 marks]

Q. 1. Draw the graph showing the variation of binding energy per nucleon with the mass number for a large number of nuclei 2< A < 240. What are the main inferences from the graph? How do you explain the constancy of binding energy in the range 30 < A < 170 using the property that the nuclear force is short-ranged? Explain with the help of this plot the release of energy in the processes of nuclear fission and fusion.

[*CBSE (AI) 2010, 2011, Chennai 2015, South 2016*]

Ans. The variation of binding energy per nucleon versus mass number is shown in figure.

Inferences from graph

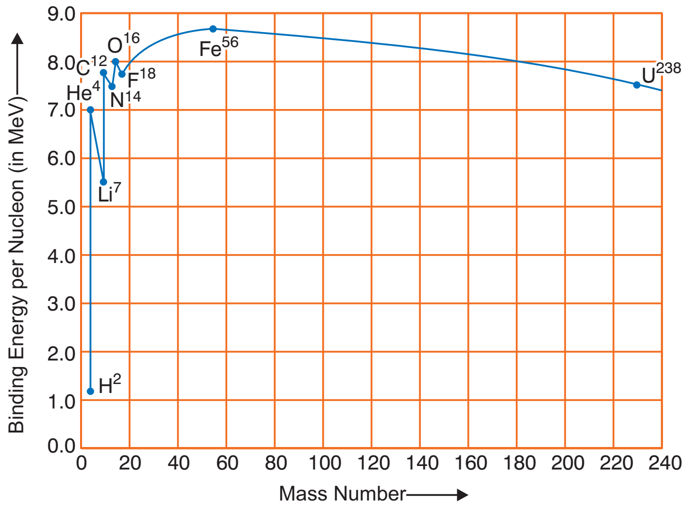
1. The nuclei having mass number below 20 and above 180 have relatively small binding energy and hence they are unstable.
2. The nuclei having mass number 56 and about 56 have maximum binding energy – 8·8 MeV and so they are most stable.

3. Some nuclei have peaks, *e.g.*, ${}_{2}^{4}He, {}_{6}^{12}C, {}_{6}^{12}O$; this indicates that these nuclei are relatively more stable than their neighbours.

(*i*) **Explanation of constancy of binding energy:** Nuclear force is short ranged, so every nucleon interacts with its neighbours only, therefore binding energy per nucleon remains constant.

(*ii*) **Explanation of nuclear fission:** When a heavy nucleus ($A \geq 235$ say) breaks into two lighter nuclei (nuclear fission), the binding energy per nucleon increases *i.e*, nucleons get more tightly bound. This implies that energy would be released in nuclear fission.

(*iii*) **Explanation of nuclear fusion:** When two very light nuclei ($A \leq 10$) join to form a heavy nucleus, the binding is energy per nucleon of fused heavier nucleus more than the binding energy per nucleon of lighter nuclei, so again energy would be released in nuclear fusion.



Q. 2. Derive the expression for the law of radiactive decay of a given sample having initially N_0 nuclei decaying to the number N present at any subsequent time t.

Plot a graph showing the variation of the number of nuclei versus the time t lapsed.

Mark a point on the plot in terms of $T_{1/2}$ value when the number present $N = N_0/16$.

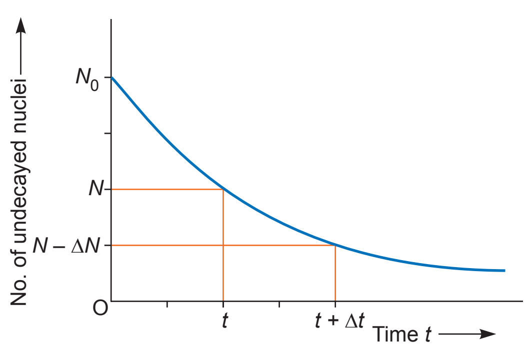
[*CBSE Delhi 2014, (F) 2013*]

Ans. Radioactive decay Law: The rate of decay of radioactive nuclei is directly proportional to the number of undecayed nuclei at that time.

Derivation of formula

Suppose initially the number of atoms in radioactive element is N_0 and N the number of atoms after time t.

After time t, let dN be the number of atoms which disintegrate in a short interval dt then rate of disintegration will be $\frac{dN}{dt}$ this is also called the activity of the substance/element.



According to Rutherford-Soddy law

$$\frac{dN}{dt} \propto N \quad \text{or} \quad \frac{dN}{dt} = -\lambda N \qquad ...(i)$$

where λ is a constant, called decay constant or disintegration constant of the element. Its unit is S^{-1}. Negative sign shows that the rate of disintegration decreases with increase of time. For a given element/substance λ is a constant and is different for different elements. Equation (*i*) may be rewritten as

$$\frac{dN}{N} = -\lambda dt$$

Integrating $\log_e N = -\lambda t + C$ ...(*ii*)

where C is a constant of integration.

At $t = 0, N = N_0$

$\therefore \log_e N_0 = 0 + C \Rightarrow C = \log_e N_0$

$\therefore$ Equation (*ii*) gives $\log_e N = -\lambda t + \log_e N_0$

or $\log_e N - \log_e N_0 = -\lambda t$

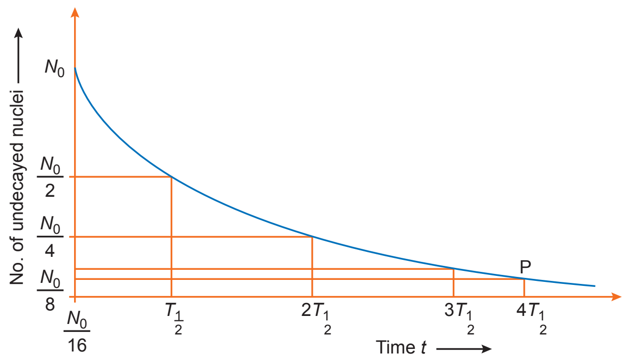
or $\log_e \frac{N}{N_0} = -\lambda t$

or $\frac{N}{N_0} = e^{-\lambda t}$

$\therefore N = N_0 e^{-\lambda t}$ ...(*iii*)

According to this equation, the number of undecayed atoms/nuclei of a given radioactive element decreases exponentially with time (*i.e.*, more rapidly at first and slowly afterwards).

Mark of $N = \frac{N_0}{16}$ in terms of $T_{1/2}$ is shown in figure.



Q. 3. Define the term: Half-life period and decay constant of a radioactive sample. Derive the relation between these terms. **[*CBSE Patna 2015*]**

Ans. Half-life period: The half-life period of an element is defined as the time in which the number of radiactive nuclei decay to half of its initial value.

Decay constant: The decay constant of a radioactive element is defined as the reciprocal of time in which the number of undecayed nuclei of that radioactive element falls to $\frac{1}{e}$ times of its initial value.

Relation between Half-life and Decay constant: The radioactive decay equation is

$$N = N_0 e^{-\lambda t} \quad ...(i)$$

when $t = T, N = \frac{N_0}{2}$

$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T}$

or $e^{-\lambda T} = \frac{1}{2}$ ...(*ii*)

Taking log of both sides

$$-\lambda T \log_e e = \log_e 1 - \log_e 2$$

or $\lambda T = \log_e 2$

$$\therefore \qquad T = \frac{\log_e 2}{\lambda} = \frac{2.3026 \log_{10} 2}{\lambda} = \frac{2.3026 \times 0.3010}{\lambda} \qquad ...(iii)$$

or $$T = \frac{0.6931}{\lambda}$$

Q. 4. Derive expression for average life of a radio nuclei. Give its relationship with half life. **[*CBSE (AI) 2010*]**

Ans. All the nuclei of a radioactive element do not decay simultaneously; but nature of decay process is statistical, *i.e.*, it cannot be stated with certainty which nucleus will decay when. ***The time of decay of a nucleus may be between 0 and infinity. The mean of lifetimes of all nuclei of a radioactive element is called its mean life. It is denoted by τ.***

Expression for mean life

According to Rutherford-Soddy law, rate of decay of a radioactive element

$$R(t) = \left|\frac{dN}{dt}\right| = \lambda N$$

Therefore, the number of nuclei decaying in-between time t and $t + dt$ is

$$dN = \lambda N dt$$

If N_0 is the total number of nuclei at $t = 0$, then mean lifetime

$$\tau = \frac{\text{Total lifetime of all the nuclei}}{\text{Total number of nuclei}} = \frac{\Sigma\, t\,.\,dN}{N_0} = \frac{\Sigma\, t\,\lambda N dt}{N_0}$$

Also we have $N = N_0 e^{-\lambda t}$

$$\therefore \qquad \tau = \frac{\Sigma\, t\lambda(N_0 e^{-\lambda t})dt}{N_0} = \lambda \Sigma\, t\, e^{-\lambda t} dt$$

As nuclei decay indefinitely, we may replace the summation into integration with limits from $t = 0$ to $t = \infty$ *i.e.*,

$$\tau = \lambda \int_0^\infty t\, e^{-\lambda t} dt\,.$$

Integrating by parts, we get

$$\tau = \lambda\left[\left\{\frac{te^{-\lambda t}}{-\lambda}\right\}_0^\infty - \int_0^\infty 1\left(\frac{e^{-\lambda t}}{-\lambda}\right)dt\right] = \lambda\left[0 + \frac{1}{\lambda}\left\{\frac{e^{-\lambda t}}{-\lambda}\right\}_0^\infty\right]$$

$$= -\frac{1}{\lambda}[e^{-\lambda t}]_0^\infty = -\frac{1}{\lambda}[0 - 1] = \frac{1}{\lambda}$$

Thus, $$\tau = \frac{1}{\lambda}$$

i.e., the **mean lifetime of a radioactive element is reciprocal of its decay constant.**

Relation between mean life and half life

Half life $$T = \frac{0.6931}{\lambda} \qquad ...(i)$$

Mean life $$\tau = \frac{1}{\lambda} \qquad ...(ii)$$

Substituting value of λ from (ii) in (i), we get

$$\boldsymbol{T = 0.6931\,\tau} \qquad ...(iii)$$

Q. 5. (*a*) Define the terms (*i*) half-life ($T_{1/2}$) and (*ii*) average life (τ). Find out their relationships with the decay constant (λ).

(*b*) A radioactive nucleus has a decay constant λ=0.3465 (day)$^{-1}$. How long would it take the nucleus to decay to 75% of its initial amount? **[*CBSE (F) 2014*]**

Ans. (*a*) (*i*) Half life ($T_{1/2}$) of a radioactive element is defined as the time taken by a radioactive nuclei to reduce to half of the initial number of radio nuclei.

(*ii*) Average life of a radioactive element is defined as the ratio of total life time of all radioactive nuclei, to the total number of nuclei in the sample.

Relation between half life and decay constant is given by $T_{1/2} = \frac{0.693}{\lambda}$

Relation between average life and decay constant $\tau = \frac{1}{\lambda}$.

(*b*) Let N_0 = the number of radioactive nuclei present initially at time $t = 0$ in a sample of radioactive substance.

N = the number of radioactive nuclei present in the sample at any instant t.

Here, $N = \frac{3}{4}N_0$

From the equation, $N = N_0 e^{-\lambda t}$

$$\frac{3}{4}N_0 = N_0 e^{-0.3465t} \quad \Rightarrow \quad e^{0.3465t} = \frac{4}{3}$$

$$= 2.303\ (0.6020 - 0.4771) = 2.303 \times 0.1249$$

$$\therefore \qquad t = \frac{2.303 \times 0.1249}{0.3465} = \mathbf{0.83\ day}\,.$$

Q. 6. Compare and contrast the nature of α-, β- and γ-radiations.

Ans. **Comparison of properties of α-, β- and γ-rays**

	Property	α-particle	β-particle	γ-rays
1.	Nature	Nucleus of Helium	Very fast-moving electron (e^-)	Electromagnetic wave of wavelength $\approx 10^{-2}$ Å
2.	Charge	$+2e$	$-e$	No charge
3.	Rest mass	6.6×10^{-27} kg	9.1×10^{-31} kg	zero
4.	Velocity	1.4×10^7 m/s to 2.2×10^7 m/s	0.3 c to 0.98 c	$c = 3 \times 10^8$ m/s
5.	Ionising Power	high, 100 times that of β-particle	100 times more than γ-rays	very small
6.	Penetrating Power	very small	high, 100 times more than α-particles	very high, 100 times more than β-particles

Q. 7. State Soddy-Fajan's displacement laws for radioactive transformations.

Ans. The atoms of radioactive element are unstable. When an atom of a radioactive element disintegrates, an entirely new element is formed. This new element possesses entirely new chemical and radioactive properties. The disintegrating element is called the parent element and the resulting product after disintegration is called the daughter element. Soddy and Fajan studied the successive product elements of disintegration of radioactive elements and gave the following conclusions:

1. **Alpha-Emission:** α-particle is nucleus of a helium atom having atomic number 2 and atomic weight 4. It is denoted by $^2He_4$. Therefore when an α-particle is emitted from a radioactive parent atom (X), its atomic number is reduced by 2 and atomic weight is reduced by 4. Thus the daughter element has its place two groups lower in the periodic table. Thus the process of α-emission may be expressed as

$$_ZX^A \longrightarrow {}_{Z-2}Y^{A-4} + \underset{(\alpha\text{-particle})}{{}_2He^4}$$

Examples:

(*i*) $_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4$

(*ii*) $_{80}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_2He^4$

2. **Beta-Emission:** β-particle is an electron (e) and is denoted by $_{-1}\beta^0$. When a β-particle is emitted from a parent atom (X), its atomic number increases by 1, while atomic weight remains unchanged. As a result the daughter element (Y) has a place one group higher in the periodic table. Thus the process of β-emission may be expressed as

$$_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + {}_{-1}\beta^{0} + \bar{\nu}$$

where $\bar{\nu}$ is a fundamental particle called antineutrino which is massless and chargeless.

Example:

$$_{90}\text{Th}^{228} \longrightarrow {}_{89}\text{Ac}^{228} + {}_{-1}\beta^{0} + \bar{\nu}$$

3. **Gamma-Emission:** The emission of γ-ray from a radioactive atom neither changes its atomic number nor its atomic weight. Therefore its place in periodic table remains undisplaced. In natural radioactivity γ-radiation is accompanied with either α or β-emission.

Self-Assessment Test

Time allowed: 1 hour **Max. marks: 30**

1. **Choose and write the correct option in the following questions.** **(3 × 1 = 3)**

   (*i*) How does the binding energy per nucleon vary with the increase in the number of nucleons

   (*a*) decrease continuously with mass number
   (*b*) first decreases and then increases with increase in mass number
   (*c*) first increases and then decreases with increase in mass number
   (*d*) increases continuously with mass number

   (*ii*) α-particles, β-particles and γ-rays are all having same energy. Their penetrating power in a given medium in increasing order will be

   (*a*) γ, α, β (*b*) α, β, γ (*c*) β, α, γ (*d*) β, γ, α

   (*iii*) The half life of a radioactive substance is 30 minutes. The time (in minutes) taken between 40% decay and 85% decay of the same radioactive substance is

   (*a*) 15 (*b*) 30 (*c*) 45 (*d*) 60

2. **Fill in the blanks.** **(2 × 1 = 2)**

   (*i*) Two nuclei have mass number in the ratio 27 : 125. Then the ratio of their radii is ________________.

   (*ii*) Heavy water is a ________________, which slows down fast moving neutrons to thermal velocities so that they can cause fission of $^{235}_{92}\text{U}$ nuclei.

3. A nucleus with mass number A = 240 and BE/A = 7.6 MeV breaks into two fragments each of A = 120 with BE/A = 8.5 MeV. Calculate the released energy. **1**

4. Two nuclei have mass numbers in the ratio 2 : 5. What is the ratio of their nuclear densities? **1**

5. Two nuclei have mass numbers in the ratio 8 : 125. What is the ratio of their nuclear radii? **1**

6. Obtain the relation between the decay constant and half life of a radioactive sample.

   The half life of a certain radioactive material against α– decay is 100 days. After how much time, will the undecayed fraction of the material be 6.25% ? **2**

7. In a given sample, two radioactive nuclei, A and B, are initially present in the ratio of 4:1. The half lives of A and B are respectively 25 years and 50 years. Find the time after which the amounts of A and B become equal. **2**

8. A radioactive nucleus 'A' undergoes a series of decays according to the following scheme :

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of A are 190 and 75 respectively. What are these numbers for A_4? **2**

9. A heavy nucleus X of mass number 240 and binding energy per nucleon 7.6 MeV is split into two fragments Y and Z of mass numbers 110 and 130. The binding energy of nucleons in Y and Z is 8.5 MeV per nucleon. Calculate the energy Q released per fission in MeV. **2**

10. (*a*) Explain the processes of nuclear fission and nuclear fusion by using the plot of binding energy per nucleon (BE/A) versus the mass number A.

(*b*) A radioactive isotope has a half-life of 10 years. How long will it take for the activity to reduce to 3·125%? **3**

11. Distinguish between nuclear fission and fusion. Show how in both these processes energy is released.

Calculate the energy release in MeV in the deuterium-tritium fusion reaction:

$${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + \text{n}$$

Using the data:

$m({}^{2}_{1}\text{H}) = 2.014102$ u $m({}^{3}_{1}\text{H}) = 3.016049$ u $m({}^{3}_{2}\text{He}) = 4.002603$ u

$m_n = 1.008665$ u $1\text{u} = 931.5\ \text{MeV/c}^2$ **3**

12. (*i*) Write symbolically the process expressing the β^+ decay of ${}^{22}_{11}\text{Na}$. Also write the basic nuclear process underlying this decay.

(*ii*) Is the nucleus formed in the decay of the nucleus ${}^{22}_{11}\text{Na}$, an isotope or an isobar? **3**

13. Derive the expression for the law of radiactive decay of a given sample having initially N_0 nuclei decaying to the number N present at any subsequent time t.

Plot a graph showing the variation of the number of nuclei versus the time t lapsed.

Mark a point on the plot in terms of $T_{1/2}$ value when the number present $N = N_0/16$. **5**

Answers

1. (*i*) (*c*) (*ii*) (*b*) (*iii*) (*d*)

2. (*i*) 3: 5 (*ii*) moderator

3. 216 MeV **4.** 1.1 **5.** 2 : 5 **11.** 17.59 MeV