5. Coordinate geometry

Exercise 5.1

1. Question

State whether the following statements are true / false.

i. (5, 7) is a point in the IV quadrant.

ii. (−2, −7) is a point in the III quadrant.

iii. (8, −7) lies below the x–axis.

iv. (5, 2) and (−7, 2) are points on the line parallel to y–axis.

v. (−5, 2) lies to the left of y–axis.

vi. (0, 3) is a point on x–axis.

vii. (−2, 3) lies in the II quadrant.

viii. (−10, 0) is a point on x–axis.

ix. (−2, −4) lies above x–axis.

x. For any point on the x–axis its y–coordinate is zero.

Answer

i. (5,7) is point in the IV quadrant.

False

Reason: X –coordinate(abscissa) and y –coordinate (ordinate) both are positive. When both are positives, then they lie in the I quadrant.

ii. (−2, −7) is point in the III quadrant.

True

Reason: X–coordinate (Abscissa) and y –coordinate (ordinate) both are negative. When both are negatives, then they lie in the III quadrant.

iii. (8, −7) lies below the x–axis.

True

Reason: x – coordinate (Abscissa) is positive and y – coordinate (ordinate) is negative. Hence, this point lies in the IV quadrant. IV quadrant is the area below the x–axis.

iv. (5, 2) and (−7, 2) are points on the line parallel to y–axis.

False

Reason: (5, 2) and (−7, 2) are the line parallel to x–axis. Because, for any points to lie on line parallel to y–axis, the x–coordinates should be same. Hence, these points cannot lie on the line parallel to y–axis.

v. (−5, 2) lies to the left of y–axis.

True

Reason: x – coordinate (Abscissa) is negative and y – coordinate (ordinate) is positive. Hence, this point lies in the II quadrant. II quadrant is the area left of y–axis.

vi. (0, 3) is point on x–axis.

False

Reason: For any point on x–axis, the value of y–coordinate(ordinate) is 0. Hence, this point does not lie on x–axis.

vii. (−2, 3) lies in the II quadrant.

True

Reason: X – coordinate (Abscissa) is negative and y – coordinate (ordinate) is positive. Hence, this point lies in the II quadrant.

viii. (−10, 0) is point on x–axis.

True

Reason: For any point on the x–axis, the value of y–coordinate is zero. Hence, this point lies on the x–axis.

ix. (−2, −4) lies above x–axis

False

Reason: When both coordinates, i.e., x–coordinate and y–coordinate are negative, the point lies in the III quadrant. Therefore (−2, −4) lies in the III quadrant, which is below the axis.

x. For any point on the x–axis its y–coordinate is zero.

True

2. Question

Plot the following points in the coordinate system and specify their quadrant.

i. (5, 2) ii. (−1, −1)

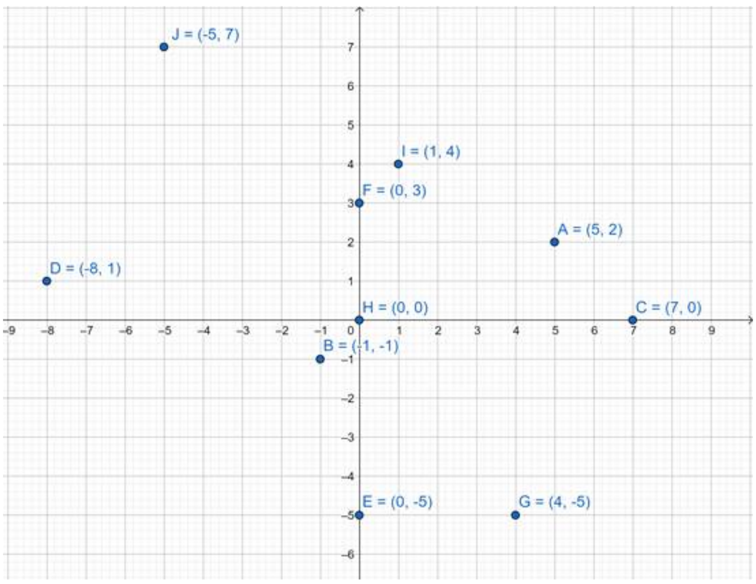
iii. (7, 0) iv. (−8, −1)

v. (0, −5) vi. (0, 3)

vii. (4, −5) viii. (0, 0)

ix. (1, 4) x. (−5, 7)

Answer



i (5, 2) – I quadrant

ii (−1, −1) – III quadrant

iii (7, 0) – on X–axis

iv (−8, 1) – II quadrant

v (0, −5) – on down y–axis

vi (0, 3) – on y – axis

vii (4, −5) IV quadrant

viii (0, 0) – on origin

ix (1, 4) – I quadrant

x (−5, 7) – II quadrant

3. Question

Write down the abscissa for the following points.

i. (−7, 2) ii. (3, 5)

iii. (8, –7) iv. (–5, –3)

Answer

Abscissa is the x–coordinate of any point A (x, y)

i. (–7, 2)

Abscissa of point (–7, 2) is –7

ii. (3, 5)

Abscissa of point (3, 5) is 3

iii. (8, –7)

Abscissa of point (8, –7) is 8

iv. (–5, –3)

Abscissa of point (–5, –3) is –5

4. Question

Write down the ordinate of the following points.

i. (7, 5) ii. (2, 9)

iii. (–5, 8) iv. (–7, –3)

Answer

Ordinate is the y–coordinate of any point A (x, y)

i. (7, 5)

Ordinate of point (7, 5) is 5

ii. (2, 9)

Ordinate of point (2, 9) is 9

iii. (–5, 8)

Ordinate of point (–5, 8) is 8

iv. (–5, –3)

Ordinate of point (–5, –3) is –3

5. Question

Plot the following points in the coordinate plane.

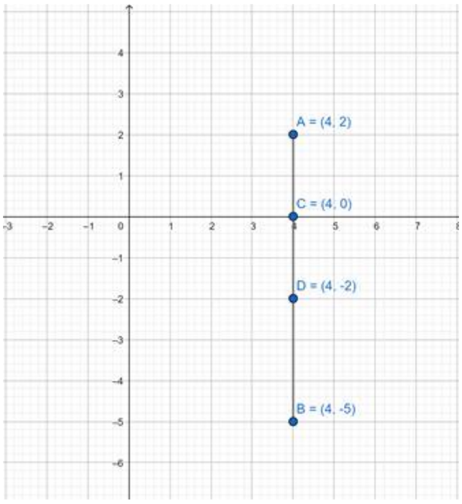
i. (4, 2) ii. (4, –5)

iii. (4, 0) iv. (4, –2)

How is the line joining them situated?

Answer

Let (4, 2) be A, (4, –5) be B, (4,0) be C and (4, –2) be D.



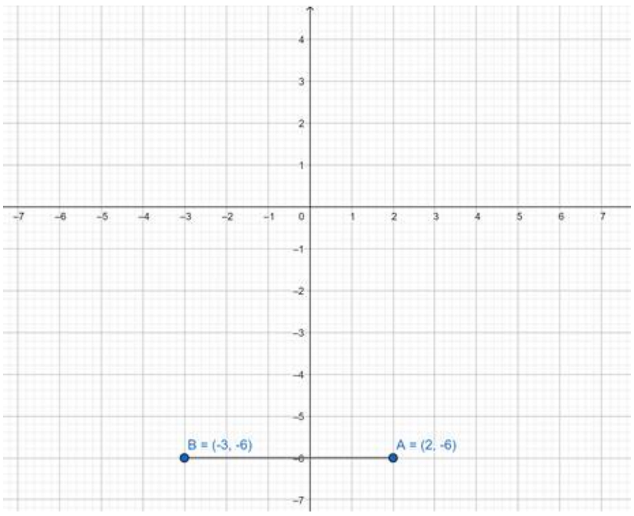
The line joining the coordinates A, B, C and D is parallel to the y–axis.

6. Question

The ordinates of two points are each −6. How is the line joining them related with reference to x–axis?

Answer

Let the coordinates of two points i.e. A and B be (2, −6) and (−3, −6) respectively.



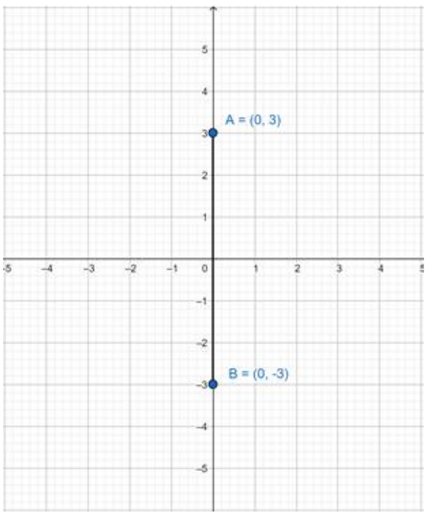
As we can see that, the line joining the point A and B is parallel to x–axis.

7. Question

The abscissa of two points is 0. How is the line joining situated?

Answer

Let the coordinate of two points i.e. A and B are (0, 3) and (0, −3) respectively.



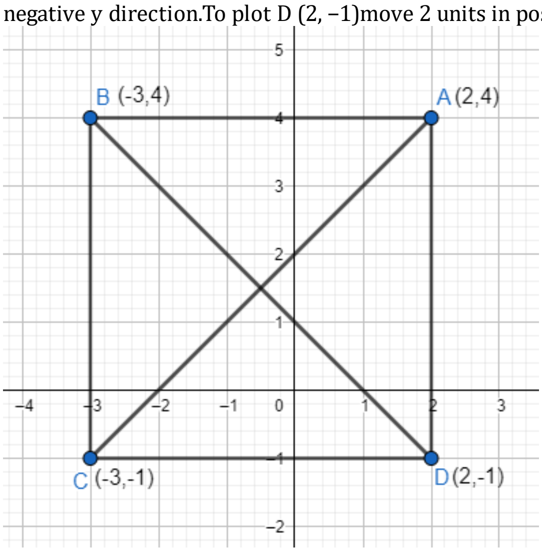
As we can see that, the line joining the point A and B lies on the y–axis.

8. Question

Mark the points A (2, 4), B (−3, 4),C (−3, −1) and D (2, −1) in the cartesian plane. State the figure obtained by joining A and B, B and C, C and D and D and A.

Answer

To plot A (2, 4) move 2 units in positive x direction and 4 units in positive y direction.To plot B (−3, 4) move 3 units in negative x direction and 4 units in positive y direction.To plot C (−3, −1)move 3 units in negative x direction and 1 unit in negative y direction.To plot D (2, −1)move 2 units in positive x direction and 1 unit in negative y direction.



Now use distance formula to find the lengths of each side, $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ For AB,

$AB=\sqrt{(-3-2)^2+(4-4)^2}=\sqrt{(-5)^2+(0)^2}=\sqrt{25}=5\ sq\ units$ For AD,

$AD=\sqrt{(2-2)^2+(-1-4)^2}=\sqrt{(0)^2+(5)^2}=\sqrt{25}=5\ sq\ units$ For CD,

$CD=\sqrt{(2-(-3))^2+(-1-(-1))^2}=\sqrt{(2+3)^2+(1-1)^2}=\sqrt{5^2+0}=\sqrt{25}=5\ sq\ units$

For BC, $BC=\sqrt{(-3-(-3))^2+(-1-4)^2}=\sqrt{(0)^2+(-5)^2}=\sqrt{25}=5\ sq\ units$ Now AC,

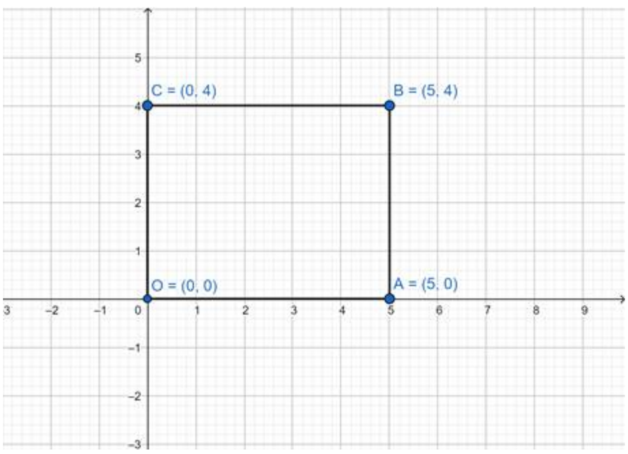
$AC=\sqrt{(-3-2)^2+(-1-4)^2}=\sqrt{(-5)^2+(-5)^2}=\sqrt{25+25}=\sqrt{50}$ For BD,

$BD=\sqrt{(2-(-3))^2+(-1-4)^2}=\sqrt{5^2+(-5)^2}=\sqrt{25+25}=\sqrt{50}$ As AB = AC = BC = CDAlso AC = BDHence the given points make a square.

9. Question

With rectangular axes plot the points O (0, 0), A (5, 0), B (5, 4). Find the coordinate of point C such that OABC forms a rectangle.

Answer



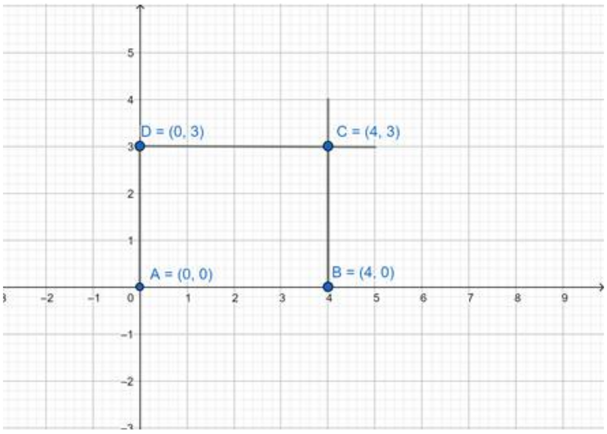
For OABC to be square, the coordinate should be in a line where point B is and where it meets the y–axis. Therefore, the point C should be (0, 4).

10. Question

In a rectangle ABCD, the coordinates of A, B and D are (0, 0) (4, 0) (0, 3). What are the coordinates of C?

Answer

To obtain the coordinate C, extend a line from D towards right and extend a line from the coordinate B. the intersection point is the point C.



Hence, the coordinates of point C is (4, 3).

Exercise 5.2

1 A. Question

Find the distance between the following pairs of points.

(7, 8) and (–2, –3)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(7, 8) and (–2, –3)

$x_1 = 7$ and $x_2 = -2$

$y_1 = 8$ and $y_2 = -3$

⇒ $D = \sqrt{((-2 - 7)^2 + (-3 - 8)^2)}$

⇒ $D = \sqrt{((-9)^2 + (-11)^2)}$

⇒ $D = \sqrt{(81 + 121)}$

⇒ $D = \sqrt{202}$

1 B. Question

Find the distance between the following pairs of points.

(6, 0) and (–2, 4)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(6, 0) and (–2, 4)

$x_1 = 6$ and $x_2 = -2$

$y_1 = 0$ and $y_2 = 4$

⇒ $D = \sqrt{((-2 - 6)^2 + (4 - 0)^2)}$

⇒ $D = \sqrt{((-8)^2 + (4)^2)}$

⇒ $D = \sqrt{(64 + 16)}$

⇒ $D = \sqrt{80}$

⇒ $D = \sqrt{(5 \times 4 \times 4)}$

⇒ $D = 4\sqrt{5}$

1 C. Question

Find the distance between the following pairs of points.

(–3, 2) and (2, 0)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–3, 2) and (2, 0)

$x_1 = -3$ and $x_2 = 2$

$y_1 = 2$ and $y_2 = 0$

⇒ $D = \sqrt{((2 - (-3)^2 + (0 - 2)^2)}$

⇒ $D = \sqrt{((2 + 3)^2 + (0 - 2)^2)}$

⇒ $D = \sqrt{((5)^2 + (-2)^2)}$

⇒ $D = \sqrt{(25 + 4)}$

⇒ $D = \sqrt{29}$

1 D. Question

Find the distance between the following pairs of points.

(–2, –8) and (–4, –6)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–2, –8) and (–4, –6)

$x_1 = -2$ and $x_2 = -4$

$y_1 = -8$ and $y_2 = -6$

⇒ $D = \sqrt{((-4 - (-2))^2 + (-6 - (-8))^2)}$

⇒ $D = \sqrt{((-4 + 2)^2 + (-6 + 8)^2)}$

⇒ $D = \sqrt{((-2)^2 + (2)^2)}$

⇒ $D = \sqrt{(4 + 4)}$

⇒ $D = \sqrt{8}$

⇒ $D = \sqrt{(2 \times 2 \times 2)}$

⇒ $D = 2\sqrt{2}$

1 E. Question

Find the distance between the following pairs of points.

(–2, –3) and (3, 2)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–2, –3) and (3, 2)

$x_1 = -2$ and $x_2 = 3$

$y_1 = -3$ and $y_2 = 2$

⇒ $D = \sqrt{((3 - (-2))^2 + (2 - (-3))^2)}$

⇒ $D = \sqrt{((3 + 2)^2 + (2 + 3)^2)}$

⇒ $D = \sqrt{((5)^2 + (5)^2)}$

⇒ $D = \sqrt{(25 + 25)}$

⇒ D = √ 50

⇒ D = √ (5 × 5 × 2)

⇒ D = 5√ 2

1 F. Question

Find the distance between the following pairs of points.

(2, 2) and (3, 2)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(2, 2) and (3, 2)

$x_1 = 2$ and $x_2 = 3$

$y_1 = 2$ and $y_2 = 2$

⇒ D = √ ((3 – 2)^2 + (2 – 2)^2)

⇒ D = √ ((1)^2 + (0)^2)

⇒ D = √ (1 + 0)

⇒ D = √ 1

⇒ D = 1

1 G. Question

Find the distance between the following pairs of points.

(–2, 2) and (3, 2)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–2, 2) and (3, 2)

$x_1 = -2$ and $x_2 = 3$

$y_1 = 2$ and $y_2 = 2$

⇒ D = √ ((3 – (–2))^2 + (2 – 2)^2)

⇒ D = √ ((5)^2 + (0)^2)

⇒ D = √ (25 + 0)

⇒ D = √ 25

⇒ D = √ (5 × 5)

⇒ D = 5

1 H. Question

Find the distance between the following pairs of points.

(7, 0) and (8, 0)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(7, 0) and (–8, 0)

$x_1 = 7$ and $x_2 = -8$

$y_1 = 0$ and $y_2 = 0$

⇒ D = √ ((–8 – 7)^2 + (0 – 0)^2)

⇒ D = √ ((–15)^2 + (0)^2)

⇒ D = √ (225 + 0)

⇒ D = √ 225

⇒ D = √ (5 × 3 × 5 × 5)

⇒ D = 5 × 3

⇒ D = 15

1 I. Question

Find the distance between the following pairs of points.

(0, 17) and (0, –1)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(0, 17) and (0, –1)

x_1 = 0 and x_2 = 0

y_1 = 17 and y_2 = –1

⇒ D = √ ((0 – 0)^2 + (–1 – 17)^2)

⇒ D = √ ((0)^2 + (–18)^2)

⇒ D = √ (0 + 324)

⇒ D = √ 324

⇒ D = √ (18 × 18)

⇒ D = 18

1 J. Question

Find the distance between the following pairs of points.

(5, 7) and the origin

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(5, 7) and (0, 0)

x_1 = 5 and x_2 = 0

y_1 = 7 and y_2 = 0

⇒ D = √ ((0 – 5)^2 + (0 – 7)^2)

⇒ D = √ ((–5)^2 + (–7)^2)

⇒ D = √ (25 + 49)

⇒ D = √ 74

2 A. Question

Show that the following points are collinear.

(3, 7), (6, 5) and (15, –1)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(3, 7), (6, 5) and (15, –1)

Let the points be A (15, –1), B (6, 5) and C (3, 7)

Distance of AB

⇒ AB = √ (6 – 15)^2 + (5 – (–1))^2

⇒ AB = $\sqrt{(-9)^2 + (6)^2}$

⇒ AB = $\sqrt{(81 + 36)}$

⇒ AB = $\sqrt{117} = \sqrt{3 \times 3 \times 13}$

⇒ AB = $3\sqrt{13}$

Distance of BC

⇒ BC = $\sqrt{(3 - 6)^2 + (7 - 5)^2}$

⇒ BC= $\sqrt{(3)^2 + (2)^2}$

⇒ BC = $\sqrt{(9 + 4)}$

⇒ BC= $\sqrt{13}$

Distance of AC

⇒ AC = $\sqrt{(3 - 15)^2 + (7 - (-1))^2}$

⇒ AC = $\sqrt{(3 - 15)^2 + (7 + 1)^2}$

⇒ AC= $\sqrt{(-12)^2 + (8)^2}$

⇒ AC = $\sqrt{(144 + 64)}$

⇒ AC= $\sqrt{208} = \sqrt{4 \times 4 \times 13}$

⇒ AC = $4\sqrt{13}$

i.e. AB + BC = AC

⇒ $3\sqrt{13} + \sqrt{13} = 4\sqrt{13}$

∴ A, B and C are collinear

2 B. Question

Show that the following points are collinear.

(3, −2), (−2, 8) and (0, 4)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(3, 2), (−2, 8) and (0, 4)

Let A (−2, 8), B (0, 4) and C (3, 2)

Distance of AB

⇒ AB = $\sqrt{((0 - (-2))^2 + (4 - 8)^2)}$

⇒ AB = $\sqrt{(2)^2 + (-4)^2}$

⇒ AB = $\sqrt{(4 + 16)}$

⇒ AB = $\sqrt{20}$

Distance of BC

⇒ BC = $\sqrt{((3 - 0)^2 + (2 - 4)^2)}$

⇒ BC = $\sqrt{(3)^2 + (-2)^2}$

⇒ BC = $\sqrt{(9 + 4)}$

⇒ BC = $\sqrt{13}$

Distance of AC

⇒ AC = $\sqrt{((3 - (-2))^2 + (2 - 8)^2)}$

⇒ AC = $\sqrt{(5)^2 + (-6)^2}$

⇒ AC = $\sqrt{(25 + 36)}$

⇒ AC = $\sqrt{61}$

2 C. Question

Show that the following points are collinear.

(1, 4), (3, –2) and (–1, 10)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(1, 4), (3, –2) and (–1, 10)

Let A (–1, 10), B (1, 4) and C (3, –2)

Distance of AB

⇒ AB =$\sqrt{((1 - (-1))^2 + (4 - 10)^2)}$

⇒ AB = $\sqrt{((1 + 1)^2 + (4 - 10)^2)}$

⇒ AB = $\sqrt{(2)^2 + (-6)^2}$

⇒ AB = $\sqrt{(4 + 36)}$

⇒ AB = $\sqrt{40}$

Distance of BC

⇒ BC =$\sqrt{((3 - 1)^2 + (-2 - 4)^2)}$

⇒ BC = $\sqrt{(2)^2 + (-6)^2}$

⇒ BC = $\sqrt{(4 + 36)}$

⇒ BC = $\sqrt{40}$

Distance of AC

⇒ AC =$\sqrt{((3 - (-1))^2 + (-2 - 10)^2)}$

⇒ AC = $\sqrt{((3 + 1)^2 + (2 - 10)^2)}$

⇒ AC = $\sqrt{(4)^2 + (-8)^2}$

⇒ AC = $\sqrt{(16 + 64)}$

⇒ AC = $\sqrt{80}$

i.e. AB + BC = AC

⇒ $\sqrt{40} + \sqrt{40} = \sqrt{80}$

∴ A, B and C are collinear.

2 D. Question

Show that the following points are collinear.

(6, 2), (2, –3) and (–2, –8)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(6, 2), (2, –3) and (–2, –8)

Let A (6, 2), B (2, –3) and C (–2, –8)

Distance of AB

⇒ AB =$\sqrt{((2 - (6))^2 + (-3 - 2)^2)}$

⇒ AB = $\sqrt{(4)^2 + (-5)^2}$

⇒ AB = $\sqrt{(16 + 25)}$

⇒ AB = $\sqrt{41}$

Distance of BC

⇒ BC =$\sqrt{}$ $((-2 - 2)^2 + (-8 - (-3))^2)$

⇒ BC = $\sqrt{}$ $((-2 - 2)^2 + (-8 + 3)^2)$

⇒ BC = $\sqrt{}$ $(-4)^2 + (-5)^2$

⇒ BC = $\sqrt{}$ (16 + 25)

⇒ BC = $\sqrt{}$ 41

Distance of AC

⇒ AC =$\sqrt{}$ $((-2 - 6)^2 + (-8 - 2)^2)$

⇒ AC = $\sqrt{}$ $(-8)^2 + (-10)^2$

⇒ AC = $\sqrt{}$ (64 + 100)

⇒ AC = $\sqrt{}$ 164 = $\sqrt{}$ 2 × 2 × 41

⇒ AC =2$\sqrt{}$ 41

i.e. AB + BC = AC

⇒ $\sqrt{41} + \sqrt{41} = 2\sqrt{41}$

∴ A, B and C are collinear.

2 E. Question

Show that the following points are collinear.

(4, 1), (5, −2) and (6, −5)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(4, 1), (5, −2) and (6, −5)

Let A (4, 1), B (5, −2) and C (6, −5)

Distance of AB

⇒ AB =$\sqrt{}$ $((5 - 4)^2 + (-2 - 1)^2)$

⇒ AB = $\sqrt{}$ $(1)^2 + (-3)^2$

⇒ AB = $\sqrt{}$ (1 + 9)

⇒ AB = $\sqrt{10}$

Distance of BC

⇒ BC =$\sqrt{}$ $((6 - 5)^2 + (-5 - (-2))^2)$

⇒ BC = $\sqrt{}$ $(6 - 5)^2 + (-5 + 2)^2)$

⇒ BC = $\sqrt{}$ $(1)^2 + (-3)^2$

⇒ BC = $\sqrt{}$ (1 + 9)

⇒ BC = $\sqrt{}$ 10

Distance of AC

⇒ AC =$\sqrt{}$ $((6 - 4)^2 + (-5 - 1)^2)$

⇒ AC = $\sqrt{}$ $(2)^2 + (-6)^2$

⇒ AC = $\sqrt{}$ (4 + 36)

⇒ AC = $\sqrt{20}$ =

i.e. AB + BC = AC

⇒ $\sqrt{10} + \sqrt{10} = \sqrt{20}$

Squaring both sides

⇒ $(\sqrt{10})^2 + (\sqrt{10})^2 = (\sqrt{20})^2$

⇒ 10 + 10 = 20

∴ A, B and C are collinear.

3 A. Question

Show that the following points form an isosceles triangle.

(−2, 0), (4, 0) and (1, 3)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(−2, 0), (4,0) and (1, 3)

Let the point be A (1, 3) B (−2, 0) and C (4, 0)

Distance of AB

⇒ $AB = \sqrt{((-2 - 1)^2 + (0 - 3)^2)}$

⇒ $AB = \sqrt{((-3)^2 + (-3)^2)}$

⇒ $AB = \sqrt{(9 + 9)}$

⇒ $AB = \sqrt{18} = 3\sqrt{2}$

Distance of AC

⇒ $AC = \sqrt{((4 - 1)^2 + (0 - 3)^2)}$

⇒ $AC = \sqrt{((3)^2 + (-3)^2)}$

⇒ $AC = \sqrt{(9 + 9)}$

⇒ $AC = \sqrt{18} = 3\sqrt{2}$

Distance of BC

⇒ $BC = \sqrt{((4 - (-2))^2 + (0 - 0)^2)}$

⇒ $BC = \sqrt{((6)^2 + (0)^2)}$

⇒ $BC = \sqrt{(36 + 0)}$

⇒ $BC = \sqrt{36} = 6$

We notice that $AB = AC = 3\sqrt{2}$

∴ Points A, B and C are coordinates of an isosceles triangle.

3 B. Question

Show that the following points form an isosceles triangle.

(1, −2), (−5, 1) and (1, 4)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(1, −2), (−5, 1) and (1, 4)

Let the point be A (−5, 1) B (1, −2) and C (1, 4)

Distance of AB

⇒ $AB = \sqrt{(1 - (-5))^2 + (-2 - 1)^2)}$

⇒ $AB = \sqrt{(1 + 5)^2 + (-2 - 1)^2}$

⇒ $AB = \sqrt{((6)^2 + (-3)^2)}$

⇒ $AB = \sqrt{(36 + 9)}$

$\Rightarrow AB = \sqrt{45} = 3\sqrt{5}$

Distance of AC

$\Rightarrow AC = \sqrt{((1 - (-5))^2 + (4 - 1)^2)}$

$\Rightarrow AC = \sqrt{((1 + 5)^2 + (4 - 1)^2)}$

$\Rightarrow AC = \sqrt{((6)^2 + (3)^2)}$

$\Rightarrow AC = \sqrt{(36 + 9)}$

$\Rightarrow AC = \sqrt{45} = 3\sqrt{5}$

Distance of BC

$\Rightarrow BC = \sqrt{((1 - 1)^2 + (4 - (-2)^2)}$

$\Rightarrow BC = \sqrt{((1 - 1)^2 + (4 + 2^2)}$

$\Rightarrow BC = \sqrt{((0)^2 + (6)^2)}$

$\Rightarrow BC = \sqrt{(0 + 36)}$

$\Rightarrow BC = \sqrt{36} = 6$

We notice that $AB = AC = 3\sqrt{5}$

∴ Points A, B and C are coordinates of an isosceles triangle.

3 C. Question

Show that the following points form an isosceles triangle.

(−1, −3), (2, −1) and (−1, 1)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(−1, −3), (2, −1) and (−1, 1)

Let the point be A (2, −1) B (−1, −3) and C (−1, 1)

Distance of AB

$\Rightarrow AB = \sqrt{((-1 - 2)^2 + (-3 - (-1))^2)}$

$\Rightarrow AB = \sqrt{((-1 - 2)^2 + (-3 + 1)^2)}$

$\Rightarrow AB = \sqrt{((-3)^2 + (-2)^2)}$

$\Rightarrow AB = \sqrt{(9 + 4)}$

$\Rightarrow AB = \sqrt{13}$

Distance of AC

$\Rightarrow AC = \sqrt{((-1 - 2)^2 + (1 - (-1))^2)}$

$\Rightarrow AC = \sqrt{((-1 - 2)^2 + (1 + 1)^2)}$

$\Rightarrow AC = \sqrt{((-3)^2 + (2)^2)}$

$\Rightarrow AC = \sqrt{(9 + 4)}$

$\Rightarrow AC = \sqrt{13}$

Distance of BC

$\Rightarrow BC = \sqrt{((-1 - (-1))^2 + (1 - (-3))^2)}$

$\Rightarrow BC = \sqrt{((-1 + 1))^2 + (1 + 3)^2)}$

$\Rightarrow BC = \sqrt{((0)^2 + (4)^2)}$

$\Rightarrow BC = \sqrt{(0 + 16)}$

$\Rightarrow BC = \sqrt{16}$

We notice that AB = AC = $\sqrt{13}$

∴ Points A, B and C are coordinates of an isosceles triangle.

3 D. Question

Show that the following points form an isosceles triangle.

(1, 3), (–3, –5) and (–3, 0)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(1, 3), (–3, –5) and (–3, 0)

Let the point be A (–3, 0) B (1, 3) and C (–3, –5)

Distance of AB

$\Rightarrow AB = \sqrt{((1 - (-3))^2 + (3 - 0)^2)}$

$\Rightarrow AB = \sqrt{((1 + 3)^2 + (3 - 0)^2)}$

$\Rightarrow AB = \sqrt{((4)^2 + (3)^2)}$

$\Rightarrow AB = \sqrt{(16 + 9)}$

$\Rightarrow AB = \sqrt{25} = 5$

Distance of AC

$\Rightarrow AC = \sqrt{((-3 - (-3))^2 + (-5 - 0)^2)}$

$\Rightarrow AC = \sqrt{((-3 + 3)^2 + (-5 + 0)^2)}$

$\Rightarrow AC = \sqrt{((0)^2 + (-5)^2)}$

$\Rightarrow AC = \sqrt{(0 + 25)}$

$\Rightarrow AC = \sqrt{25} = 5$

Distance of BC

$\Rightarrow BC = \sqrt{((-3 - 1)^2 + (-5 - 3)^2)}$

$\Rightarrow BC = \sqrt{((-4)^2 + (-8)^2)}$

$\Rightarrow BC = \sqrt{(16 + 64)}$

$\Rightarrow BC = \sqrt{80}$

We notice that AB = AC = 5

∴ Points A, B and C are coordinates of an isosceles triangle.

3 E. Question

Show that the following points form an isosceles triangle.

(2, 3), (5, 7) and (1, 4)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(2, 3), (5, 7) and (1, 4)

Let the point be A (5, 7) B (2, 3) and C (1, 4)

Distance of AB

$\Rightarrow AB = \sqrt{(2 - 5)^2 + (3 - 7)^2)}$

$\Rightarrow AB = \sqrt{((-3)^2 + (-4)^2)}$

$\Rightarrow AB = \sqrt{(9 + 16)}$

$\Rightarrow AB = \sqrt{25} = 5$

Distance of AC

⇒ AC = $\sqrt{((1-5)^2 + (4-7)^2)}$

⇒ AC = $\sqrt{((-4)^2 + (-3)^2)}$

⇒ AC = $\sqrt{(16 + 9)}$

⇒ AC = $\sqrt{25} = 5$

Distance of BC

⇒ BC = $\sqrt{((1-2)^2 + (4-3)^2)}$

⇒ BC = $\sqrt{((-1)^2 + (1)^2)}$

⇒ BC = $\sqrt{(1 + 1)}$

⇒ BC = $\sqrt{2}$

We notice that AB = AC = 5

∴ Points A, B and C are coordinates of an isosceles triangle.

4 A. Question

Show that the following points form a right–angled triangle.

(2, –3), (–6, –7) and (–8, –3)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(2, –3), (–6, –7) and (–8, –3)

Let the points be A (2, –3), B (–6, –7) and C (–8, –3)

Distance of AB

⇒ AB = $\sqrt{((-6-2)^2 + (-7-(-3))^2)}$

⇒ AB = $\sqrt{((-6-2)^2 + (-7+3)^2)}$

⇒ AB = $\sqrt{((-8)^2 + (-4)^2)}$

⇒ AB = $\sqrt{(64 + 16)}$

⇒ AB = $\sqrt{80}$

Distance of BC

⇒ B C= $\sqrt{((-8-(-6))^2 + (-3-(-7))^2)}$

⇒ BC = $\sqrt{((-8+6)^2 + (-3+7)^2)}$

⇒ BC = $\sqrt{((-2)^2 + (4)^2)}$

⇒ BC = $\sqrt{(4 + 16)}$

⇒ BC = $\sqrt{20}$

Distance of AC

⇒ AC = $\sqrt{((-8-2)^2 + (-3-(-3))^2)}$

⇒ AC = $\sqrt{((-8-2)^2 + (-3+3)^2)}$

⇒ AC = $\sqrt{((-10)^2 + (0)^2)}$

⇒ AC = $\sqrt{(100 + 0)}$

⇒ AC = $\sqrt{100}$

i.e. $AB^2 + BC^2$

= $(\sqrt{80})^2 + (\sqrt{20})^2$

= 80 + 20

$= 100 = (AC)^2$

Hence, ABC is a right–angled triangle. Since the square of one side is equal to sum of the squares of the other two sides.

4 B. Question

Show that the following points form a right–angled triangle.

(–11, 13), (–3, –1) and (4, 3)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–11, 13), (–3, –1) and (4, 3)

Let the points be A (–11, 13), B (–3, –1) and C (4, 3)

Distance of AB

$\Rightarrow AB = \sqrt{((-3 - (-11))^2 + (-1 - 13)^2)}$

$\Rightarrow AB = \sqrt{((-3 + 11)^2 + (-1 - 13)^2)}$

$\Rightarrow AB = \sqrt{((8)^2 + (-14)^2)}$

$\Rightarrow AB = \sqrt{(64 + 196)}$

$\Rightarrow AB = \sqrt{260}$

Distance of BC

$\Rightarrow BC = \sqrt{((4 - (-3))^2 + (3 - (-1))^2)}$

$\Rightarrow BC = \sqrt{((4 + 3)^2 + (3 + 1)^2)}$

$\Rightarrow BC = \sqrt{((7)^2 + (4)^2)}$

$\Rightarrow BC = \sqrt{(49 + 16)}$

$\Rightarrow BC = \sqrt{65}$

Distance of AC

$\Rightarrow AC = \sqrt{((4 - (-11))^2 + (3 - 13))^2)}$

$\Rightarrow AC = \sqrt{((4 + 11)^2 + (3 - 13)^2)}$

$\Rightarrow AC = \sqrt{((15)^2 + (-10)^2)}$

$\Rightarrow AC = \sqrt{(225 + 100)}$

$\Rightarrow AC = \sqrt{325}$

i.e. $AB^2 + BC^2$

$= (\sqrt{260})^2 + (\sqrt{65})^2$

$= 260 + 65$

$= 325 = (AC)^2$

Hence, ABC is a right–angled triangle. Since the square of one side is equal to sum of the squares of the other two sides.

4 C. Question

Show that the following points form a right–angled triangle.

(0, 0), (a, 0) and (0, b)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(0, 0), (a, 0) and (0, b)

Let the points be A (0, 0), B (a, 0) and C (0, b)

Distance of AB

⇒ AB = √ ((a – 0)^2 + (0 – 0)^2)

⇒ AB = √ ((a)^2 + (0)^2

⇒ AB = √ a^2

Distance of BC

⇒ BC = √ ((0 – a)^2 + (b – 0)^2)

⇒ BC = √ ((–a)^2 + (b)^2

⇒ BC = √ a^2 + b^2

Distance of AC

⇒ AC = √ ((0 – 0)^2 + (b – 0)^2)

⇒ AC = √ ((0)^2 + (b)^2

⇒ AC = √ b^2

i.e. AB^2 + AC^2

= (√a^2)^2 + (√b^2)^2

= a^2 + b^2 = BC^2

Hence, ABC is a right–angled triangle. Since the square of one side is equal to sum of the squares of the other two sides.

4 D. Question

Show that the following points form a right–angled triangle.

(10, 0), (18, 0) and (10, 15)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(10, 0), (18, 0) and (10, 15)

Let the points be A (10, 15), B (10, 0) and C (18, 0)

Distance of AB

⇒ AB = √ ((10 – 10))^2 + (0 – 15)^2)

⇒ AB = √ ((0)^2 + (–15)^2)

⇒ AB = √ (0 + 225)

⇒ AB = √225

Distance of BC

⇒ B C= √ ((18 – 10)^2 + (0 – 0)^2)

⇒ BC = √ ((8)^2 + (0)^2)

⇒ BC = √ (64 + 0)

⇒ BC = √ 64

Distance of AC

⇒ AC = √ ((18 – 10)^2 + (0 – 15))^2)

⇒ AC = √ ((8)^2 + (–15)^2)

⇒ AC = √ (64 + 225)

⇒ AC = √289

i.e. AB^2 + BC^2

= (√225)^2 + (√64)^2

= 225 + 64

= 289 = $(AC)^2$

Hence, ABC is a right–angled triangle. Since the square of one side is equal to sum of the squares of the other two sides.

4 E. Question

Show that the following points form a right–angled triangle.

(5, 9), (5, 16) and (29, 9)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(5, 9), (5, 16) and (29, 9)

Let the points be A (5, 16), B (5, 9) and C (29, 9)

Distance of AB

⇒ AB = $\sqrt{((5 - 5)^2 + (9 - 16)^2)}$

⇒ AB = $\sqrt{((0)^2 + (-7)^2)}$

⇒ AB = $\sqrt{(0 + 49)}$

⇒ AB = $\sqrt{49}$

Distance of BC

⇒ B C= $\sqrt{((29 - 5)^2 + (9 - 9)^2)}$

⇒ BC = $\sqrt{((24)^2 + (0)^2)}$

⇒ BC = $\sqrt{(576 + 0)}$

⇒ BC = $\sqrt{576}$

Distance of AC

⇒ AC = $\sqrt{((29 - 5)^2 + (9 - 16))^2)}$

⇒ AC = $\sqrt{((24)^2 + (-7)^2)}$

⇒ AC = $\sqrt{(576 + 49)}$

⇒ AC = $\sqrt{625}$

i.e. $AB^2 + BC^2$

= $(\sqrt{49})^2 + (\sqrt{576})^2$

= 49 + 576

= 625 = $(AC)^2$

Hence, ABC is a right–angled triangle. Since the square of one side is equal to sum of the squares of the other two sides.

5 A. Question

Show that the following points form an equilateral triangle.

(0, 0), (10, 0) and (5, 5$\sqrt{3}$)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(0, 0), (10, 0) and (5, 5$\sqrt{3}$)

Let the points be A (0, 0), B (10, 0) and C (5, 5$\sqrt{3}$)

Distance of AB

⇒ AB = $\sqrt{((10 - 0)^2 + (0 - 0)^2)}$

⇒ AB = $\sqrt{((10)^2 + (0)^2)}$

⇒ AB = $\sqrt{(100 + 0)}$

⇒ $AB = \sqrt{100}$

⇒ $AB = 10$

Distance of BC

⇒ $BC = \sqrt{((5 - 10)^2 + (5\sqrt{3} - 0)^2)}$

⇒ $BC = \sqrt{((-5)^2 + (5\sqrt{3})^2)}$

⇒ $BC = \sqrt{(25 + 75)}$

⇒ $BC = \sqrt{100}$

⇒ $BC = 10$

Distance of AC

⇒ $AC = \sqrt{((5 - 0)^2 + (5\sqrt{3} - 0))^2)}$

⇒ $AC = \sqrt{((5)^2 + (5\sqrt{3})^2)}$

⇒ $AC = \sqrt{(25 + 75)}$

⇒ $AC = \sqrt{100}$

⇒ $AC = 10$

∴ $AB = BC = AC = 10$

Since, all the sides are equal the points form an equilateral triangle.

5 B. Question

Show that the following points form an equilateral triangle.

(a, 0), (–a, 0) and (0, $a\sqrt{3}$)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(a, 0), (–a, 0) and (0, $a\sqrt{3}$)

Let the points be A (a, 0), B (–a, 0) and C (0, $a\sqrt{3}$)

Distance of AB

⇒ $AB = \sqrt{((-a - a)^2 + (0 - 0)^2)}$

⇒ $AB = \sqrt{((-2a)^2 + (0)^2)}$

⇒ $AB = \sqrt{(4a^2 + 0)}$

⇒ $AB = \sqrt{4a^2}$

⇒ $AB = 2a$

Distance of BC

⇒ $BC = \sqrt{((0 - a)^2 + (a\sqrt{3} - 0)^2)}$

⇒ $BC = \sqrt{((-a)^2 + (a\sqrt{3})^2)}$

⇒ $BC = \sqrt{(a^2 + 3a^2)}$

⇒ $BC = \sqrt{4a^2}$

⇒ $BC = 2a$

Distance of AC

⇒ $AC = \sqrt{((0 - a)^2 + (a\sqrt{3} - 0))^2)}$

⇒ $AC = \sqrt{((-a)^2 + (a\sqrt{3})^2)}$

⇒ $AC = \sqrt{(a^2 + 3a^2)}$

⇒ $AC = \sqrt{4a^2}$

⇒ AC = 2a

∴ AB = BC = AC = 2a

Since, all the sides are equal the points form an equilateral triangle.

5 C. Question

Show that the following points form an equilateral triangle.

(2, 2), (–2, –2) and (–2√3, 2√3)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(2, 2), (–2, –2) and (–2√3, 2√3)

Let the points be A (2, 2), B (–2, –2) and C (–2√3, 2√3)

Distance of AB

⇒ AB = √ ((–2 – 2)^2 + (–2 – 2)^2)

⇒ AB = √ ((–4)^2 + (–4)^2)

⇒ AB = √ (16 + 16)

⇒ AB = √32

⇒ AB = 4√2

Distance of BC

⇒ B C= √ ((–2√3 – (–2))^2 + (2√3 – (–2))^2)

⇒ B C= √ ((–2√3 + 2))^2 + (2√3 + 2)^2)

⇒ BC = √ (((–2√3)^2 + 2 (–2√3) (2) + (2)^2) + ((2√3)^2 + 2 (2√3) (2) + (2)^2))

⇒ BC = √ (12 – 8√3 + 4 + 12 + 8√3 + 4)

⇒ BC = √ (12 + 4 + 12 + 4

⇒ BC = √ 32

⇒ BC = 4√2

Distance of AC

⇒ AC= √ ((–2√3 – 2))^2 + (2√3 – 2)^2)

⇒ AC = √ (((–2√3)^2 + 2 (–2√3) (–2) + (2)^2) + ((2√3)^2 + 2 (2√3) (–2) + (–2)^2))

⇒ AC = √ (12 + 8√3 + 4 + 12 – 8√3 + 4)

⇒ AC = √ (12 + 4 + 12 + 4)

⇒ AC = √ 32

⇒ AC = 4√2

∴ AB = BC = AC = 4√2

Since, all the sides are equal the points form an equilateral triangle.

5 D. Question

Show that the following points form an equilateral triangle.

(√3, 2), (0,1) and (0, 3)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(√3, 2), (0, 1) and (0, 3)

Let the points be A (√3, 2), B (0, 1) and C (0, 3)

Distance of AB

⇒ AB = $\sqrt{((0 - \sqrt{3})^2 + (1 - 2)^2)}$

⇒ AB = $\sqrt{((\sqrt{3})^2 + (-1)^2)}$

⇒ AB = $\sqrt{(3 + 1)}$

⇒ AB = $\sqrt{4}$

⇒ AB = 2

Distance of BC

⇒ B C= $\sqrt{((0 - 0)^2 + (3 - 1)^2)}$

⇒ BC = $\sqrt{((0)^2 + (2)^2)}$

⇒ BC = $\sqrt{(0 + 4)}$

⇒ BC = $\sqrt{4}$

⇒ BC = 2

Distance of AC

⇒ AC = $\sqrt{((0 - \sqrt{3})^2 + (3 - 2))^2)}$

⇒ AC = $\sqrt{((\sqrt{3})^2 + (1)^2)}$

⇒ AC = $\sqrt{(3 + 1)}$

⇒ AC = $\sqrt{4}$

⇒ AC = 2

∴ AB = BC = AC = 2

Since, all the sides are equal the points form an equilateral triangle.

5 E. Question

Show that the following points form an equilateral triangle.

($-\sqrt{3}$, 1), ($2\sqrt{3}$, –2) and ($2\sqrt{3}$, 4)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

($-\sqrt{3}$, 1), ($2\sqrt{3}$, –2) and ($2\sqrt{3}$, 4)

Let the points be A ($-\sqrt{3}$, 1), B ($2\sqrt{3}$, –2) and C ($2\sqrt{3}$, 4)

Distance of AB

⇒ AB = $\sqrt{((2\sqrt{3} - (-\sqrt{3}))^2 + (-2 - 1)^2)}$

⇒ AB = $\sqrt{((2\sqrt{3} + \sqrt{3}))^2 + (-2 - 1)^2)}$

⇒ AB = $\sqrt{((12 + 12 + 3)^2 + (-3)^2)}$

⇒ AB = $\sqrt{(27 + 9)}$

⇒ AB = $\sqrt{36}$

⇒ AB = 6

Distance of BC

⇒ B C= $\sqrt{((2\sqrt{3} - 2\sqrt{3})^2 + (4 - (-2))^2)}$

⇒ B C= $\sqrt{((2\sqrt{3} - 2\sqrt{3})^2 + (4 + 2)^2)}$

⇒ BC = $\sqrt{((0)^2 + (6)^2)}$

⇒ BC = $\sqrt{(0 + 36)}$

⇒ BC = $\sqrt{36}$

⇒ BC = 6

Distance of AC

⇒ AC = $\sqrt{((2\sqrt{3} - (-\sqrt{3}))^2 + (4 - 1))^2}$

⇒ AC = $\sqrt{((2\sqrt{3} + \sqrt{3}))^2 + (4 - 1))^2}$

⇒ AC = $\sqrt{((3\sqrt{3})^2 + (3)^2)}$

⇒ AC = $\sqrt{(27 + 9)}$

⇒ AC = $\sqrt{36}$

⇒ AC = 6

∴ AB = BC = AC = 6

Since, all the sides are equal the points form an equilateral triangle.

6 A. Question

Show that the following points taken in order form the vertices of a parallelogram.

(–7, –5), (–4, 3), (5, 6) and (2, –2)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–7, –5), (–4, 3), (5, 6) and (2, –2)

Let A, B, C and D represent the points (–7, –5), (–4, 3), (5, 6) and (2, –2)

Distance of AB

⇒ AB = $\sqrt{((-4 - (-7)))^2 + (3 - (-5))^2)}$

⇒ AB = $\sqrt{((-4 + 7))^2 + (3 + 5)^2)}$

⇒ AB = $\sqrt{((3)^2 + (8)^2)}$

⇒ AB = $\sqrt{(9 + 64)}$

⇒ AB = $\sqrt{73}$

Distance of BC

⇒ BC= $\sqrt{((5 - (-4))^2 + (6 - 3)^2)}$

⇒ BC= $\sqrt{((5 + 4))^2 + (6 - 3)^2)}$

⇒ BC = $\sqrt{((9)^2 + (3)^2)}$

⇒ BC = $\sqrt{(81 + 9)}$

⇒ BC = $\sqrt{90}$

Distance of CD

⇒ CD = $\sqrt{((2 - 5)^2 + (-2 - 6)^2)}$

⇒ CD = $\sqrt{((-3)^2 + (-8)^2)}$

⇒ CD = $\sqrt{(9 + 64)}$

⇒ CD = $\sqrt{73}$

Distance of AD

⇒ AD = $\sqrt{((2 - (-7)))^2 + (-2 - (-5))^2)}$

⇒ AD = $\sqrt{((2 + 7))^2 + (-2 + 5)^2)}$

⇒ AD = $\sqrt{((9)^2 + (3)^2)}$

⇒ AD = $\sqrt{(81 + 9)}$

⇒ AD = $\sqrt{90}$

So, AB = CD = $\sqrt{73}$ and BC = AD = $\sqrt{90}$

i.e., The opposite sides are equal. Hence ABCD is a parallelogram.

6 B. Question

Show that the following points taken in order form the vertices of a parallelogram.

(9, 5), (6, 0), (–2, –3) and (1, 2)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(9,5), (6, 0), (–2, –3) and (1, 2)

Let A, B, C and D represent the points (9, 5), (6, 0), (–2, –3) and (1, 2)

Distance of AB

⇒ AB = $\sqrt{((6 - 9))^2 + (0 - 5)^2}$

⇒ AB = $\sqrt{((-3)^2 + (5)^2)}$

⇒ AB = $\sqrt{(9 + 25)}$

⇒ AB = $\sqrt{34}$

Distance of BC

⇒ BC= $\sqrt{((-2 - 6)^2 + (-3 - 0)^2)}$

⇒ BC = $\sqrt{((-8)^2 + (-3)^2)}$

⇒ BC = $\sqrt{(64 + 9)}$

⇒ BC = $\sqrt{73}$

Distance of CD

⇒ CD = $\sqrt{((1 - (-2))^2 + (2 - (-3))^2)}$

⇒ CD = $\sqrt{((1 + 2)^2 + (2 + 3))^2)}$

⇒ CD = $\sqrt{((3)^2 + (5)^2)}$

⇒ CD = $\sqrt{(9 + 25)}$

⇒ CD = $\sqrt{36}$

Distance of AD

⇒ AD = $\sqrt{((1 - 9))^2 + (2 - 5)^2}$

⇒ AD = $\sqrt{((-8)^2 + (-3)^2)}$

⇒ AD = $\sqrt{(64 + 9)}$

⇒ AD = $\sqrt{73}$

So, AB = CD = $\sqrt{36}$ and BC = AD = $\sqrt{73}$

i.e., The opposite sides are equal. Hence ABCD is a parallelogram.

6 C. Question

Show that the following points taken in order form the vertices of a parallelogram.

(0, 0), (7, 3), (10, 6) and (3, 3)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(0,0) (7, 3), (10, 6) and (3, 3)

Let A, B, C and D represent the points (0, 0), (7, 3), (10, 6) and (3, 3)

Distance of AB

⇒ AB = √ ((7 – 0))^2 + (3 – 0)^2)

⇒ AB = √ ((7)^2 + (3)^2)

⇒ AB = √ (49 + 9)

⇒ AB = √ 58

Distance of BC

⇒ BC= √ ((10 – 7)^2 + (6 – 3)^2)

⇒ BC = √ ((3)^2 + (3)^2)

⇒ BC = √ (9 + 9)

⇒ BC = √18

Distance of CD

⇒ CD = √ ((3 – 10)^2 + (3 – 6)^2)

⇒ CD = √ ((–7)^2 + (–3)^2)

⇒ CD = √ (49 + 9)

⇒ CD = √58

Distance of AD

⇒ AD = √ ((3 – 0))^2 + (3 – 0)^2)

⇒ AD = √ ((3)^2 + (3)^2)

⇒ AD = √ (9 + 9)

⇒ AD = √18

So, AB = CD = √58 and BC = AD = √18

i.e., The opposite sides are equal. Hence ABCD is a parallelogram.

6 D. Question

Show that the following points taken in order form the vertices of a parallelogram.

(–2, 5), (7, 1), (–2, –4) and (7, 0)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–2, 5), (7, 1), (–2, –4) and (7, 0)

Let A, B, C and D represent the points (–2, 5), (7, 1), (–2, –4) and (7, 0)

Distance of AB

⇒ AB = √ ((7 – (–2)))^2 + (1 – 5)^2)

⇒ AB = √ ((7 + 2))^2 + (1 – 5)^2)

⇒ AB = √ ((9)^2 + (–4)^2)

⇒ AB = √ (81 + 16)

⇒ AB = √ 97

Distance of BC

⇒ BC= √ ((–2 – 7)^2 + (–4 – 1)^2)

⇒ BC = √ ((–9)^2 + (–5)^2)

⇒ BC = √ (81 + 25)

⇒ BC = √106

Distance of CD

$\Rightarrow$ CD = $\sqrt{((7 - (-2))^2 + (0 - (-4))^2)}$

$\Rightarrow$ CD = $\sqrt{((7 + 2)^2 + (0 + 4))^2)}$

$\Rightarrow$ CD = $\sqrt{((9)^2 + (4)^2)}$

$\Rightarrow$ CD = $\sqrt{(81 + 16)}$

$\Rightarrow$ CD = $\sqrt{97}$

Distance of AD

$\Rightarrow$ AD = $\sqrt{((7 - (-2))^2 + (0 - 5)^2)}$

$\Rightarrow$ AD = $\sqrt{((7 + 2)^2 + (0 - 5)^2)}$

$\Rightarrow$ AD = $\sqrt{((9)^2 + (-5)^2)}$

$\Rightarrow$ AD = $\sqrt{(81 + 25)}$

$\Rightarrow$ AD = $\sqrt{106}$

So, AB = CD = $\sqrt{97}$ and BC = AD = $\sqrt{106}$

i.e., The opposite sides are equal. Hence ABCD is a parallelogram.

6 E. Question

Show that the following points taken in order form the vertices of a parallelogram.

(3, −5), (−5, −4), (7, 10) and (15, 9)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(3, −5), (−5, −4), (7, 10) and (15, 9)

Let A, B, C and D represent the points (3, −5), (−5, −4), (7, 10) and (15, 9)

Distance of AB

$\Rightarrow$ AB = $\sqrt{((-5 - 3)^2 + ((-4 - (-5))^2)}$

$\Rightarrow$ AB = $\sqrt{((-5 - 3))^2 + (-4 + 5)^2)}$

$\Rightarrow$ AB = $\sqrt{((-8)^2 + (1)^2)}$

$\Rightarrow$ AB = $\sqrt{(64 + 1)}$

$\Rightarrow$ AB = $\sqrt{65}$

Distance of BC

$\Rightarrow$ BC= $\sqrt{((7 - (-5))^2 + (10 - (-4))^2)}$

$\Rightarrow$ BC= $\sqrt{((7 + 5)^2 + (10 + 4)^2)}$

$\Rightarrow$ BC = $\sqrt{((12)^2 + (14)^2)}$

$\Rightarrow$ BC = $\sqrt{(144 + 196)}$

$\Rightarrow$ BC = $\sqrt{340}$

Distance of CD

$\Rightarrow$ CD = $\sqrt{((15 - 7)^2 + (9 - 10)^2)}$

$\Rightarrow$ CD = $\sqrt{((8)^2 + (-1)^2)}$

$\Rightarrow$ CD = $\sqrt{(64 + 1)}$

$\Rightarrow$ CD = $\sqrt{65}$

Distance of AD

$\Rightarrow$ AD = $\sqrt{((15 - 3)^2 + (9 - (-5))^2)}$

$\Rightarrow$ AD = $\sqrt{((15 - 3)^2 + (9 + 5)^2)}$

⇒ AD = √ ((12)^2 + (14)^2)

⇒ AD = √ (144 + 196)

⇒ AD = √ 340

So, AB = CD = √65 and BC = AD = √340

i.e., The opposite sides are equal. Hence ABCD is a parallelogram.

7 A. Question

Show that the following points taken in order form the vertices of a rhombus.

(0, 0), (3, 4), (0, 8) and (–3, 4)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(0, 0), (3, 4), (0, 8) and (–3, 4)

Let the vertices be taken as A (0, 0), B (3, 4), C (0, 8) and D (–3, 4).

Distance of AB

⇒ AB = √ ((3 – 0)^2 + ((4 – 0)^2)

⇒ AB = √ ((3)^2 + (4)^2)

⇒ AB = √ (9 + 16)

⇒ AB = √ 25

⇒ AB = 5

Distance of BC

⇒ BC= √ ((0 – 3)^2 + (8 – 4)^2)

⇒ BC = √ ((–3)^2 + (4)^2)

⇒ BC = √ (9 + 16)

⇒ BC = √ 25

⇒ BC = 5

Distance of CD

⇒ CD = √ ((–3 – 0)^2 + (4 – 8)^2)

⇒ CD = √ ((–3)^2 + (–4)^2)

⇒ CD = √ (9 + 16)

⇒ CD = √25

⇒ CD = 5

Distance of AD

⇒ AD = √ ((–3 – 0)^2 + (4 – 0)^2)

⇒ AD = √ ((–3)^2 + (4)^2)

⇒ AD = √ (9 + 16)

⇒ AD = √ 25

⇒ AD = 5

Distance of AC

⇒ AC = √ ((0 – 0)^2 + (8 – 0)^2)

⇒ AC = √ ((0)^2 + (8)^2)

⇒ AC = √ (64)

$\Rightarrow$ AC = 8

Distance of BD

$\Rightarrow$ BD = $\sqrt{((-3-3)^2 + (4-4)^2)}$

$\Rightarrow$ BD = $\sqrt{((-6)^2 + (0)^2)}$

$\Rightarrow$ BD = $\sqrt{(36 + 0)}$

$\Rightarrow$ BD = $\sqrt{36}$

$\Rightarrow$ BD = 6

AB = BC = CD = DA = 5 (That is, all the sides are equal.)

AC ≠ BD (That is, the diagonals are not equal.)

Hence the points A, B, C and D form a rhombus.

7 B. Question

Show that the following points taken in order form the vertices of a rhombus.

(−4, −7), (−1, 2), (8, 5) and (5, −4)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(−4, −7), (−1, 2), (8, 5) and (5, −4)

Let the vertices be taken as A (−4,−7), B (−1, 2), C (8, 5) and D (5, −4).

Distance of AB

$\Rightarrow$ AB = $\sqrt{((-1-(-4))^2 + (2-(-7)^2))}$

$\Rightarrow$ AB = $\sqrt{((-1+4)^2 + (2+7)^2)}$

$\Rightarrow$ AB = $\sqrt{((3)^2 + (9)^2)}$

$\Rightarrow$ AB = $\sqrt{(9 + 81)}$

$\Rightarrow$ AB = $\sqrt{100}$

$\Rightarrow$ AB = 10

Distance of BC

$\Rightarrow$ BC= $\sqrt{((8-(-1))^2 + (5-2)^2)}$

$\Rightarrow$ BC = $\sqrt{((8+1)^2 + (3)^2)}$

$\Rightarrow$ BC = $\sqrt{((9)^2 + 9)}$

$\Rightarrow$ BC = $\sqrt{(81 + 9)}$

$\Rightarrow$ BC = $\sqrt{100}$

$\Rightarrow$ BC = 10

Distance of CD

$\Rightarrow$ CD = $\sqrt{((5-8)^2 + (-4-5)^2)}$

$\Rightarrow$ CD = $\sqrt{((3)^2 + (-9)^2)}$

$\Rightarrow$ CD = $\sqrt{(9 + 81)}$

$\Rightarrow$ CD = $\sqrt{100}$

$\Rightarrow$ CD = 10

Distance of AD

$\Rightarrow$ AD = $\sqrt{((5-(-4))^2 + (-4-(-7))^2)}$

$\Rightarrow$ AD = $\sqrt{((5+4)^2 + (-4+7)^2)}$

⇒ AD = √ $((9)^2 + (3)^2)$

⇒ AD = √ (81+9)

⇒ AD = √ 100

⇒ AD = 10

Distance of AC

⇒ AC = √ $((8 - (-4))^2 + (5 - (-7))^2)$

⇒ AC = √ $((8+4)^2 + (5+7)^2)$

⇒ AC = √ $((12)^2 + (12)^2)$

⇒ AC = √ (144 + 144)

⇒ AC = √ (288)

Distance of BD

⇒ BD = √ $((5 - (-1))^2 + (-4 - 2)^2)$

⇒ BD = √ $((5 + 1))^2 + (-4 - 2)^2)$

⇒ BD = √ $((6)^2 + (-6)^2)$

⇒ BD = √ (36 + 36)

⇒ BD = √ 72

AB = BC = CD = DA = 10 (That is, all the sides are equal.)

AC ≠ BD (That is, the diagonals are not equal.)

Hence the points A, B, C and D form a rhombus.

7 C. Question

Show that the following points taken in order form the vertices of a rhombus.

(1, 0), (5, 3), (2, 7) and (−2, 4)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(1, 0), (5, 3), (2, 7) and (−2, 4)

Let the vertices be taken as A (1, 0), B (5, 3), C (2, 7) and D (−2, 4).

Distance of AB

⇒ AB = √ $((5 - 1)^2 + (3 - 0)^2)$

⇒ AB = √ $((4)^2 + (3)^2)$

⇒ AB = √ (16 + 9)

⇒ AB = √ 25

⇒ AB = 5

Distance of BC

⇒ BC= √ $((2 - 5)^2 + (7 - 3)^2)$

⇒ BC = √ $((3)^2 + (4)^2)$

⇒ BC = √ (9 + 16)

⇒ BC = √ 25

⇒ BC = 5

Distance of CD

⇒ CD = √ $((-2 - 2)^2 + (4 - 7)^2)$

⇒ CD = √ $((-4)^2 + (-3)^2)$

⇒ CD = √ (16 + 9)

⇒ CD = √25

⇒ CD = 5

Distance of AD

⇒ AD = √ $((-2 - 1)^2 + (4 - 0)^2)$

⇒ AD = √ $((-3)^2 + (4)^2)$

⇒ AD = √ (9 + 16)

⇒ AD = √ 25

⇒ AD = 5

Distance of AC

⇒ AC = √ $((2 - 1)^2 + (7 - 0)^2)$

⇒ AC = √ $((1)^2 + (7)^2)$

⇒ AC = √ (1 + 49)

⇒ AC = √ 50

Distance of BD

⇒ BD = √ $((-2 - 5)^2 + (4 - 3)^2)$

⇒ BD = √ $((-7)^2 + (1)^2)$

⇒ BD = √ (49 + 1)

⇒ BD = √ 50

AB = BC = CD = DA = 10 (That is, all the sides are equal.)

AC ≠ BD (That is, the diagonals are not equal.)

Hence the points A, B, C and D form a rhombus.

7 D. Question

Show that the following points taken in order form the vertices of a rhombus.

(2, −3), (6, 5), (−2, 1) and (−6, −7)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(2, −3), (6, 5), (−2, 1) and (−6, −7)

Let the vertices be taken as A (2, −3), B (6, 5), C (−2, 1) and D (−6, −7).

Distance of AB

⇒ AB = √ $((6 - 2)^2 + (5 - (-3)^2))$

⇒ AB = √ $((6 - 2)^2 + (5 + 3)^2)$

⇒ AB = √ $((4)^2 + (8)^2)$

⇒ AB = √ (16 + 64)

⇒ AB = √ 80

Distance of BC

⇒ BC= √ $((-2 - 6)^2 + (1 - 5)^2)$

⇒ BC = √ $((-8)^2 + (-4)^2)$

⇒ BC = √ (64 + 16)

$\Rightarrow BC = \sqrt{80}$

Distance of CD

$\Rightarrow CD = \sqrt{((-6 - (-2))^2 + (-7 - 1)^2)}$

$\Rightarrow CD = \sqrt{((-6 + 2)^2 + (-7 - 1)^2)}$

$\Rightarrow CD = \sqrt{((-4)^2 + (-8)^2)}$

$\Rightarrow CD = \sqrt{(16 + 64)}$

$\Rightarrow CD = \sqrt{80}$

Distance of AD

$\Rightarrow AD = \sqrt{((-6 - (2))^2 + (-7 - (-3))^2)}$

$\Rightarrow AD = \sqrt{((-6 - 2)^2 + (-7 + 3)^2)}$

$\Rightarrow AD = \sqrt{((-8)^2 + (-4)^2)}$

$\Rightarrow AD = \sqrt{(64 + 16)}$

$\Rightarrow AD = \sqrt{80}$

Distance of AC

$\Rightarrow AC = \sqrt{((-2 - 2)^2 + (1 - (-3))^2)}$

$\Rightarrow AC = \sqrt{((-2 - 2)^2 + (1 + 3)^2)}$

$\Rightarrow AC = \sqrt{((-4)^2 + (4)^2)}$

$\Rightarrow AC = \sqrt{(16 + 16)}$

$\Rightarrow AC = \sqrt{32}$

Distance of BD

$\Rightarrow BD = \sqrt{((-6 - 6)^2 + (-7 - 5)^2)}$

$\Rightarrow BD = \sqrt{((-6 - 6))^2 + (-7 - 5)^2)}$

$\Rightarrow BD = \sqrt{((-12)^2 + (-12)^2)}$

$\Rightarrow BD = \sqrt{(144 + 144)}$

$\Rightarrow BD = \sqrt{288}$

$AB = BC = CD = DA = \sqrt{80}$ (That is, all the sides are equal.)

$AC \neq BD$ (That is, the diagonals are not equal.)

Hence the points A, B, C and D form a rhombus.

7 E. Question

Show that the following points taken in order form the vertices of a rhombus.

(15, 20), (–3, 12), (–11, –6) and (7, 2)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(15, 20), (–3, 12), (–11, –6) and (7, 2)

Let the vertices be taken as A (15, 20), B (–3, 12), C (–11, –6) and D (7, 2).

Distance of AB

$\Rightarrow AB = \sqrt{((-3 - 15)^2 + (12 - 20)^2)}$

$\Rightarrow AB = \sqrt{((-18)^2 + (-8)^2)}$

$\Rightarrow AB = \sqrt{(324 + 64)}$

$\Rightarrow AB = \sqrt{388}$

Distance of BC

$\Rightarrow$ BC= $\sqrt{((-11 - (-3))^2 + (-6 - 12)^2)}$

$\Rightarrow$ BC = $\sqrt{(-11 + 3)^2 + (-6 - 12)^2)}$

$\Rightarrow$ BC = $\sqrt{((-8)^2 + (-18)^2)}$

$\Rightarrow$ BC = $\sqrt{(64 + 324)}$

$\Rightarrow$ BC = $\sqrt{388}$

Distance of CD

$\Rightarrow$ CD = $\sqrt{((7 - (-11))^2 + (2 - (-6))^2)}$

$\Rightarrow$ CD = $\sqrt{((7 + 11)^2 + (2 + 6)^2)}$

$\Rightarrow$ CD = $\sqrt{((18)^2 + (8)^2)}$

$\Rightarrow$ CD = $\sqrt{(324 + 64)}$

$\Rightarrow$ CD = $\sqrt{388}$

Distance of AD

$\Rightarrow$ AD = $\sqrt{((7 - 15))^2 + (2 - 20)^2)}$

$\Rightarrow$ AD = $\sqrt{((-8)^2 +(-18)^2)}$

$\Rightarrow$ AD = $\sqrt{(64 + 324)}$

$\Rightarrow$ AD = $\sqrt{388}$

Distance of AC

$\Rightarrow$ AC = $\sqrt{((-11 - 15)^2 + (-6 - 20)^2)}$

$\Rightarrow$ AC = $\sqrt{((-26)^2 +(-26)^2)}$

$\Rightarrow$ AC = $\sqrt{(676 + 676)}$

$\Rightarrow$ AC = $\sqrt{1352}$

Distance of BD

$\Rightarrow$ BD = $\sqrt{((7 - (-3))^2 + (2 - 12)^2)}$

$\Rightarrow$ BD = $\sqrt{((7 + 3))^2 + (2 - 12)^2)}$

$\Rightarrow$ BD = $\sqrt{((10)^2 + (-10)^2)}$

$\Rightarrow$ BD = $\sqrt{(100 + 100)}$

$\Rightarrow$ BD = $\sqrt{200}$

AB = BC = CD = DA = $\sqrt{388}$ (That is, all the sides are equal.)

AC $\neq$ BD (That is, the diagonals are not equal.)

Hence the points A, B, C and D form a rhombus.

8 A. Question

Examine whether the following points taken in order form a square.

(0, −1), (2, 1), (0, 3) and (−2, 1)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(0, −1), (2, 1), (0, 3) and (−2, 1)

Let the vertices be taken as A (0, −1), B (2, 1), C (0, 3) and D (−2, 1).

Distance of AB

$\Rightarrow$ AB = $\sqrt{((2 - 0)^2 + ((1 - (-1))^2)}$

⇒ AB = √ $((2 - 0))^2 + (1 + 1)^2)$

⇒ AB = √ $((2)^2 + (2)^2)$

⇒ AB = √ (4 + 4)

⇒ AB = √ 8

Distance of BC

⇒ BC= √ $((0 - 2)^2 + (3 - 1)^2)$

⇒ BC = √ $((-2)^2 + (2)^2)$

⇒ BC = √ (4 + 4)

⇒ BC = √ 8

Distance of CD

⇒ CD = √ $((-2 - 0)^2 + (1 - 3)^2)$

⇒ CD = √ $((-2)^2 + (-2)^2)$

⇒ CD = √ (4 + 4)

⇒ CD = √8

Distance of AD

⇒ AD = √ $((-2 - 0)^2 + (1 - (-1))^2)$

⇒ AD = √ $((-2 - 0)^2 + (1 + 1)^2)$

⇒ AD = √ $((-2)^2 + (2)^2)$

⇒ AD = √ (4 + 4)

⇒ AD = √ 8

Distance of AC

⇒ AC = √ $((0 - 0)^2 + (3 - (-1))^2)$

⇒ AC = √ $((0 - 0)^2 + (3 + 1)^2)$

⇒ AC = √ $((0)^2 + (4)^2)$

⇒ AC = √ (0 + 16)

⇒ AC = √ 16

⇒ AC = 4

Distance of BD

⇒ AC = √ $((-2 - 2)^2 + (1 - 1)^2)$

⇒ AC = √ $((-4)^2 + (0)^2)$

⇒ AC = √ (16 + 0)

⇒ AC = √ 16

⇒ AC = 4

AB = BC = CD = DA = √8 (That is, all the sides are equal.)

AC = BD = 4. (That is, the diagonals are equal.)

Hence the points A, B, C and D form a square.

8 B. Question

Examine whether the following points taken in order form a square.

(5, 2), (1, 5), (−2, 1) and (2, −2)

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(5, 2), (1, 5), (–2, 1) and (2, –2)

Let the vertices be taken as A (5, 2), B (1, 5), C (–2, 1) and D (2, –2).

Distance of AB

⇒ $AB = \sqrt{((1 - 5)^2 + ((5 - 2)^2)}$

⇒ $AB = \sqrt{((-4)^2 + (3)^2)}$

⇒ $AB = \sqrt{(16 + 9)}$

⇒ $AB = \sqrt{25}$

⇒ AB = 5

Distance of BC

⇒ $BC = \sqrt{((-2 - 1)^2 + (1 - 5)^2)}$

⇒ $BC = \sqrt{((-3)^2 + (-4)^2)}$

⇒ $BC = \sqrt{(9 + 16)}$

⇒ $BC = \sqrt{25}$

⇒ BC = 5

Distance of CD

⇒ $CD = \sqrt{((2 - (-2))^2 + (-2 - 1)^2)}$

⇒ $CD = \sqrt{((2 + 2)^2 + (-2 - 1)^2)}$

⇒ $CD = \sqrt{((4)^2 + (-3)^2)}$

⇒ $CD = \sqrt{(16 + 9)}$

⇒ $CD = \sqrt{25}$

⇒ CD = 5

Distance of AD

⇒ $AD = \sqrt{((2 - 5)^2 + (-2 - 2)^2)}$

⇒ $AD = \sqrt{((-3)^2 + (-4)^2)}$

⇒ $AD = \sqrt{(9 + 16)}$

⇒ $AD = \sqrt{25}$

⇒ AD = 5

Distance of AC

⇒ $AC = \sqrt{((-2 - 5)^2 + (1 - 2)^2)}$

⇒ $AC = \sqrt{((-7)^2 + (-1)^2)}$

⇒ $AC = \sqrt{(49 + 1)}$

⇒ $AC = \sqrt{50}$

⇒ $AC = 5\sqrt{2}$

Distance of BD

⇒ $BD = \sqrt{((2 - 1)^2 + (-2 - 5)^2)}$

⇒ $BD = \sqrt{((1)^2 + (-7)^2)}$

⇒ $BD = \sqrt{(1 + 49)}$

⇒ $BD = \sqrt{50}$

⇒ $BD = 5\sqrt{2}$

AB = BC = CD = DA = 5 (That is, all the sides are equal.)

AC = BD = $5\sqrt{2}$. (That is, the diagonals are equal.)

Hence the points A, B, C and D form a square.

8 C. Question

Examine whether the following points taken in order form a square.

(3, 2), (0, 5), (−3, 2) and (0, −1)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(3, 2), (0, 5), (−3, 2) and (0, −1)

Let the vertices be taken as A (3, 2), B (0, 5), C (−3, 2) and D (0, −1).

Distance of AB

$\Rightarrow AB = \sqrt{((0 - 3)^2 + ((5 - 2)^2)}$

$\Rightarrow AB = \sqrt{((-3)^2 + (3)^2)}$

$\Rightarrow AB = \sqrt{(9 + 9)}$

$\Rightarrow AB = \sqrt{18}$

Distance of BC

$\Rightarrow BC = \sqrt{((-3 - 0)^2 + (2 - 5)^2)}$

$\Rightarrow BC = \sqrt{((-3)^2 + (-3)^2)}$

$\Rightarrow BC = \sqrt{(9 + 9)}$

$\Rightarrow BC = \sqrt{18}$

Distance of CD

$\Rightarrow CD = \sqrt{((0 - (-3))^2 + (-1 - 2)^2)}$

$\Rightarrow CD = \sqrt{((0 + 3)^2 + (-1 - 2)^2)}$

$\Rightarrow CD = \sqrt{((3)^2 + (-3)^2)}$

$\Rightarrow CD = \sqrt{(9 + 9)}$

$\Rightarrow CD = \sqrt{18}$

Distance of AD

$\Rightarrow AD = \sqrt{((0 - 3)^2 + (-1 - 2)^2)}$

$\Rightarrow AD = \sqrt{((-3)^2 + (-3)^2)}$

$\Rightarrow AD = \sqrt{(9 + 9)}$

$\Rightarrow AD = \sqrt{18}$

Distance of AC

$\Rightarrow AC = \sqrt{((-3 - 3)^2 + (2 - 2)^2)}$

$\Rightarrow AC = \sqrt{((-6)^2 + (0)^2)}$

$\Rightarrow AC = \sqrt{(36 + 0)}$

$\Rightarrow AC = \sqrt{36}$

$\Rightarrow AC = 6$

Distance of BD

$\Rightarrow BD = \sqrt{((0 - 0)^2 + (-1 - 5)^2)}$

$\Rightarrow BD = \sqrt{((0)^2 + (-6)^2)}$

⇒ BD = √ (0 + 36)

⇒ BD = √ 36

⇒ BD = 6

AB = BC = CD = DA = √18. (That is, all the sides are equal.)

AC = BD = 6. (That is, the diagonals are equal.)

Hence the points A, B, C and D form a square.

8 D. Question

Examine whether the following points taken in order form a square.

(12, 9), (20, −6), (5, −14) and (−3, 1)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(12, 9), (20, −6), (5, −14) and (−3, 1)

Let the vertices be taken as A (12, 9), B (20, −6), C (5, −14) and D (−3, 1).

Distance of AB

⇒ AB = $\sqrt{((20 - 12)^2 + ((-6 - 9)^2)}$

⇒ AB = $\sqrt{((8)^2 + (-15)^2)}$

⇒ AB = √ (64 + 225)

⇒ AB = √289

Distance of BC

⇒ BC= $\sqrt{((5 - 20)^2 + (-14 - (-6))^2)}$

⇒ BC= $\sqrt{((5 - 20)^2 + (-14 + 6)^2)}$

⇒ BC = $\sqrt{((-15)^2 + (-8)^2)}$

⇒ BC = √ (225 + 64)

⇒ BC = √ 289

Distance of CD

⇒ CD = $\sqrt{((-3 - 5)^2 + (1 - (-14))^2)}$

⇒ CD = $\sqrt{((-3 - 5)^2 + (1 + 14)^2)}$

⇒ CD = $\sqrt{((-8)^2 + (15)^2)}$

⇒ CD = √ (64 + 225)

⇒ CD = √289

Distance of AD

⇒ AD = $\sqrt{((-3 - 12)^2 + (1 - 9)^2)}$

⇒ AD = $\sqrt{((-15)^2 + (-8)^2)}$

⇒ AD = √ (225 + 64)

⇒ AD = √ 289

Distance of AC

⇒ AC = $\sqrt{((5 - 12)^2 + (-14 - 9)^2)}$

⇒ AC = $\sqrt{((-7)^2 + (-23)^2)}$

⇒ AC = √ (49 + 529)

⇒ AC = √578

Distance of BD

$\Rightarrow BD = \sqrt{((-3 - 20)^2 + (1 - (-6))^2)}$

$\Rightarrow BD = \sqrt{((-3 - 20)^2 + (1 + 6)^2)}$

$\Rightarrow BD = \sqrt{((-23)^2 + (7)^2)}$

$\Rightarrow BD = \sqrt{(529 + 49)}$

$\Rightarrow BD = \sqrt{578}$

$AB = BC = CD = DA = \sqrt{289}$ (That is, all the sides are equal.)

$AC = BD = \sqrt{578}$. (That is, the diagonals are equal.)

Hence the points A, B, C and D form a square.

8 E. Question

Examine whether the following points taken in order form a square.

(–1, 2), (1, 0), (3, 2) and (1, 4)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–1, 2), (1, 0), (3, 2) and (1, 4)

Let the vertices be taken as A (–1, 2), B (1, 0), C (3, 2) and D (1, 4).

Distance of AB

$\Rightarrow AB = \sqrt{((1 - (-1))^2 + ((0 - 2)^2)}$

$\Rightarrow AB = \sqrt{((1 + 1)^2 + (0 - 2)^2)}$

$\Rightarrow AB = \sqrt{((2)^2 + (-2)^2)}$

$\Rightarrow AB = \sqrt{(4 + 4)}$

$\Rightarrow AB = \sqrt{8}$

Distance of BC

$\Rightarrow BC= \sqrt{((3 - 1)^2 + (2 - 0)^2)}$

$\Rightarrow BC = \sqrt{((2)^2 + (2)^2)}$

$\Rightarrow BC = \sqrt{(4 + 4)}$

$\Rightarrow BC = \sqrt{8}$

Distance of CD

$\Rightarrow CD = \sqrt{((1 - 3)^2 + (4 - 2))^2)}$

$\Rightarrow CD = \sqrt{((-2)^2 + (2)^2)}$

$\Rightarrow CD = \sqrt{(4 + 4)}$

$\Rightarrow CD = \sqrt{8}$

Distance of AD

$\Rightarrow AD = \sqrt{((1 - (-1))^2 + (4 - 2)^2)}$

$\Rightarrow AD = \sqrt{((1 + 1)^2 + (4 - 2)^2)}$

$\Rightarrow AD = \sqrt{((2)^2 + (2)^2)}$

$\Rightarrow AD = \sqrt{(4 + 4)}$

$\Rightarrow AD = \sqrt{8}$

Distance of AC

$\Rightarrow AC = \sqrt{((3 - (-1))^2 + (2 - 2)^2)}$

⇒ AC = √ ((3 + 1) + (2 – 2)^2)

⇒ AC = √ ((4)^2 + (0)^2)

⇒ AC = √ (16 + 0)

⇒ AC = √16

⇒ AC = 4

Distance of BD

⇒ BD = √ ((1 – 1)^2 + (4 – 0)^2)

⇒ BD = √ ((0)^2 + (4)^2)

⇒ BD = √ (0 + 16)

⇒ BD = √16

⇒ BD = 4

AB = BC = CD = DA = √8 (That is, all the sides are equal.)

AC = BD = 4. (That is, the diagonals are equal.)

Hence the points A, B, C and D form a square.

9 A. Question

Examine whether the following points taken in order form a rectangle.

(8, 3), (0, −1), (−2, 3) and (6, 7)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(8, 3), (0, −1), (−2, 3) and (6, 7)

Let the vertices be taken as A (8, 3), B (0, −1), C (−2, 3) and D (6, 7).

Distance of AB

⇒ AB = √ ((0 – 8)^2 + ((−1 – 3)^2)

⇒ AB = √ ((−8)^2 + (−4)^2)

⇒ AB = √ (64 + 16)

⇒ AB = √ 80

Distance of BC

⇒ BC= √ ((−2 – 0)^2 + (3 – (−1))^2)

⇒ BC = √ ((−2 – 0)^2 + (3 + 1)^2)

⇒ BC = √ ((−2)^2 + (4)^2)

⇒ BC = √ (4 + 16)

⇒ BC = √ 20

Distance of CD

⇒ CD = √ ((6 – (−2))^2 + (7 – 3)^2)

⇒ CD = √ ((6 + 2)^2 + (7 – 3)^2)

⇒ CD = √ ((8)^2 + (4)^2)

⇒ CD = √ (64 + 16)

⇒ CD = √80

Distance of AD

⇒ AD = √ ((6 – 8)^2 + (7 – 3)^2)

⇒ AD = √ $((-2)^2 + (4)^2)$

⇒ AD = √ (4 + 16)

⇒ AD = √ 20

Distance of AC

⇒ AC = √ $((-2 - 8)^2 + (3 - 3)^2)$

⇒ AC = √ $((-10)^2 + (0)^2)$

⇒ AC = √ (100 + 0)

⇒ AC = √ 100

⇒ AC = 10

Distance of BD

⇒ BD = √ $((6 - 0)^2 + (7 - (-1))^2)$

⇒ BD = √ $((6 - 0)^2 + (7 + 1)^2)$

⇒ BD = √ $((6)^2 + (8)^2)$

⇒ BD = √ (36 + 64)

⇒ BD = √ 100

⇒ BD = 10

AB = CD = √80 and BC = AD = √ 20 (opposite sides of rectangle are equal).

AC = BD = 10 (Diagonals of rectangle are equal)

Hence the points A, B, C and D form a square.

9 B. Question

Examine whether the following points taken in order form a rectangle.

(–1, 1), (0, 0), (3, 3) and (2, 4)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–1, 1), (0, 0), (3, 3) and (2, 4)

Let the vertices be taken as A (–1, 1), B (0, 0), C (3, 3) and D (2, 4).

Distance of AB

⇒ AB = √ $((0 - (-1))^2 + (0 - 1)^2)$

⇒ AB = √ $((0 + 1)^2 + (0 - 1)^2)$

⇒ AB = √ $((1)^2 + (-1)^2)$

⇒ AB = √ (1 + 1)

⇒ AB = √ 2

Distance of BC

⇒ BC= √ $((3 - 0)^2 + (3 - 0)^2)$

⇒ BC = √ $((3)^2 + (3)^2)$

⇒ BC = √ (9 + 9)

⇒ BC = √ 18

Distance of CD

⇒ CD = √ $((2 - 3)^2 + (4 - 3)^2)$

⇒ CD = √ $((1)^2 + (1)^2)$

⇒ CD = √ (1 + 1)

⇒ CD = √2

Distance of AD

⇒ AD = √ $((2 – (–1))^2 + (4 – 1)^2)$

⇒ AD = √ $((2 + 1)^2 + (4 – 1)^2)$

⇒ AD = √ $((3)^2 + (3)^2)$

⇒ AD = √ (9 + 9)

⇒ AD = √ 18

Distance of AC

⇒ AC = √ $((3 – (–1))^2 + (3 – 1)^2)$

⇒ AC = √ $((3 + 1)^2 + (3 – 1)^2)$

⇒ AC = √ $((4)^2 + (2)^2)$

⇒ AC = √ (16 + 4)

⇒ AC = √ 20

Distance of BD

⇒ BD = √ $((2 – 0)^2 + (4 – 0)^2)$

⇒ BD = √ $((2)^2 + (4)^2)$

⇒ BD = √ (4 + 16)

⇒ BD = √ 20

AB = CD = √2 and BC = AD = √ 18 (opposite sides of rectangle are equal).

AC = BD = √ 20 (Diagonals of rectangle are equal)

Hence the points A, B, C and D form a square.

9 C. Question

Examine whether the following points taken in order form a rectangle.

(–3, 0), (1, –2), (5, 6) and (1, 8)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(–3, 0), (1, –2), (5, 6) and (1, 8)

Let the vertices be taken as A (–3, 0), B (1, –2), C (5, 6) and D (1, 8).

Distance of AB

⇒ AB = √ $((1 – (–3))^2 + ((–2 – 0)^2)$

⇒ AB = √ $((1 + 3)^2 + (–2 – 0)^2)$

⇒ AB = √ $((4)^2 + (–2)^2)$

⇒ AB = √ (16 + 4)

⇒ AB = √ 20

Distance of BC

⇒ BC= √ $((5 – 1)^2 + (6 – (–2))^2)$

⇒ BC = √ $((5 – 1)^2 + (6 + 2)^2)$

⇒ BC = √ $((4)^2 + (8)^2)$

⇒ BC = √ (16 + 64)

⇒ BC = √ 80

Distance of CD

⇒ CD = √ $((1 – 5)^2 + (8 – 6)^2)$

⇒ CD = √ $((–4)^2 + (2)^2)$

⇒ CD = √ (16 + 4)

⇒ CD = √ 20

Distance of AD

⇒ AD = √ $((1 – (–3))^2 + (8 – 0)^2)$

⇒ AD = √ $((1 + 3)^2 + (8 – 0)^2)$

⇒ AD = √ $((4)^2 + (8)^2)$

⇒ AD = √ (16 + 64)

⇒ AD = √ 80

Distance of AC

⇒ AC = √ $((5 – (–3))^2 + (6 – 0)^2)$

⇒ AC = √ $((5 + 3)^2 + (6 – 0)^2)$

⇒ AC = √ $((8)^2 + (6)^2)$

⇒ AC = √ (64 + 36)

⇒ AC = √ 100

⇒ AC = 10

Distance of BD

⇒ BD = √ $((1 – 1)^2 + (8 – (–2))^2)$

⇒ BD = √ $((1 – 1)^2 + (8 + 2)^2)$

⇒ BD = √ $((0)^2 + (10)^2)$

⇒ BD = √ (0 + 100)

⇒ BD = √ 100

⇒ BD = 10

AB = CD = √20 and BC = AD = √ 80 (opposite sides of rectangle are equal).

AC = BD = 10 (Diagonals of rectangle are equal)

Hence the points A, B, C and D form a square.

10. Question

If the distance between two points (x,7) and (1, 15) is 10, find x.

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Given: Distance = 10 and coordinates of two points is A (x, 7) and B (1, 15)

AB = √ $(x_2 – x_1)^2 + (y_2 – y_1)^2$

⇒ 10 = √ $(1 – x)^2 + (15 – 7)^2$

⇒ 10 = √ $(1 – x)^2 + 8^2$

Squaring both sides

⇒ $10^2 = (1 – x)^2 + 8^2$

⇒ $100 = 1 – 2x + x^2 + 64$

$\Rightarrow 100 = x^2 - 2x + 65$

$\Rightarrow x^2 - 2x + 65 - 100 = 0$

$\Rightarrow x^2 - 2x - 35 = 0$

$\Rightarrow x^2 - 7x + 5x - 35 = 0$

$\Rightarrow x (x - 7) + 5(x - 7) = 0$

$\Rightarrow (x - 7) (x + 5) = 0$

$x - 7 = 0$ or $x + 5 = 0$

$x = 7$ or $x = -5$

11. Question

Show that (4, 1) is equidistant from the points (−10, 6) and (9, −13).

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the points be A (4, 1), B (−10, 6) and C (9, −13)

Distance of AB

$\Rightarrow AB = \sqrt{((-10 - 4)^2 + (6 - 1)^2)}$

$\Rightarrow AB = \sqrt{((-14)^2 + (5)^2)}$

$\Rightarrow AB = \sqrt{(196 + 25}$

$\Rightarrow AB = \sqrt{221}$

Distance of BC

$\Rightarrow BC = \sqrt{((9 - 4)^2 + (-13 - 1)^2)}$

$\Rightarrow BC = \sqrt{((5)^2 + (-14)^2)}$

$\Rightarrow BC = \sqrt{(25 + 196}$

$\Rightarrow BC = \sqrt{221}$

$\therefore AB = BC = \sqrt{221}$

12. Question

If two points (2, 3) and (−6, −5) are equidistant from the point (x, y), show that $x + y + 3 = 0$.

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the points be A (x, y), B (2, 3) and C (−6, −5)

Distance of AB

$\Rightarrow AB = \sqrt{((2 - x)^2 + (3 - y)^2)}$

$\Rightarrow AB = \sqrt{((4 - 4x + x^2) + (9 - 6y + y^2))}$

$\Rightarrow AB = \sqrt{(4 - 4x + x^2 + 9 - 6y + y^2)}$

$\Rightarrow AB = \sqrt{x^2 + y^2 - 4x - 6y + 13}$

Distance of BC

$\Rightarrow BC = \sqrt{((-6 - x)^2 + (-5 - y)^2)}$

$\Rightarrow BC = \sqrt{((36 + x^2 + 12x) + (25 + y^2 + 10y))}$

$\Rightarrow BC = \sqrt{(36 + x^2 + 12x + 25 + y^2 + 10y)}$

$\Rightarrow BC = \sqrt{(x^2 + y^2 + 12x + 10y + 61)}$

i.e. AB = BC ($\because$ Given)

$\Rightarrow \sqrt{x^2 + y^2 - 4x - 6y + 13} = \sqrt{x^2 + y^2 + 12x + 10y + 61}$

Squaring both sides

$\Rightarrow x^2 + y^2 - 4x - 6y + 13 = x^2 + y^2 + 12x + 10y + 61$

$\Rightarrow x^2 + y^2 - 4x - 6y + 13 - x^2 - y^2 - 12x - 10y - 61 = 0$

$\Rightarrow -16x - 16y - 48 = 0$

$\Rightarrow -4(x + y + 3) = 0$

$\Rightarrow x + y + 3 = 0$

Hence proved.

13. Question

If the length of the line segment with end points (2, −6) and (2, y) is 4, find y.

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Given: Distance = 4 and coordinates of two points is A (2, −6) and B (2, y)

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow 4 = \sqrt{(2 - 2)^2 + (y - (-6))^2}$

$\Rightarrow 4 = \sqrt{(0) + (y + 6)^2}$

Squaring both sides

$\Rightarrow 4^2 = (y + 6)^2$

$\Rightarrow 16 = y^2 + 12y + 36$

$\Rightarrow y^2 + 12y + 36 - 16 = 0$

$\Rightarrow y^2 + 12y + 20 = 0$

$\Rightarrow y^2 + 10y + 2y + 20 = 0$

$\Rightarrow y(y + 10) + 2(y + 10) = 0$

$\Rightarrow (y + 2)(y + 10) = 0$

$y + 2 = 0$ or $y + 10 = 0$

$y = -2$ or $y = -10$

$\therefore y = -2$ or -10

14. Question

Find the perimeter of the triangle with vertices (i) (0, 8), (6, 0) and origin; (ii) (9, 3), (1, −3) and origin.

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

i). (0, 8), (6, 0) and (0, 0)

Let the points be A (0, 8), B (6, 0) and C (0, 0)

Distance of AB

$\Rightarrow AB = \sqrt{(6 - 0)^2 + (0 - 8)^2}$

$\Rightarrow AB = \sqrt{(6)^2 + (-8)^2}$

$\Rightarrow AB = \sqrt{36 + 64}$

$\Rightarrow AB = \sqrt{100}$

$\Rightarrow AB = 10$

Distance of BC

$\Rightarrow BC = \sqrt{((0 - 6)^2 + (0 - 0)^2)}$

$\Rightarrow BC = \sqrt{((-6)^2 + (0)^2)}$

$\Rightarrow BC = \sqrt{(36 + 0)}$

$\Rightarrow BC = \sqrt{36}$

$\Rightarrow BC = 6$

Distance of AC

$\Rightarrow AC = \sqrt{((0 - 0)^2 + (0 - 8)^2)}$

$\Rightarrow AC = \sqrt{((0)^2 + (-8)^2)}$

$\Rightarrow AC = \sqrt{(0 + 64)}$

$\Rightarrow AC = \sqrt{64}$

$\Rightarrow AC = 8$

Perimeter of $\Delta ABC = AB + BC + AC$

$= 10 + 6 + 8$

$= 24$

ii). (9, 3), (1, –3) and (0, 0)

Let the points be A (9, 3), B (1, –3) and C (0, 0)

Distance of AB

$\Rightarrow AB = \sqrt{((1 - 9)^2 + (-3 - 3)^2)}$

$\Rightarrow AB = \sqrt{((-8)^2 + (-6)^2)}$

$\Rightarrow AB = \sqrt{(64 + 36)}$

$\Rightarrow AB = \sqrt{100}$

$\Rightarrow AB = 10$

Distance of BC

$\Rightarrow BC = \sqrt{((0 - 1)^2 + (0 - (-3))^2)}$

$\Rightarrow BC = \sqrt{((0 - 1)^2 + (0 + 3)^2)}$

$\Rightarrow BC = \sqrt{((-1)^2 + (3)^2)}$

$\Rightarrow BC = \sqrt{(1 + 9)}$

$\Rightarrow BC = \sqrt{10}$

Distance of AC

$\Rightarrow AC = \sqrt{((0 - 9)^2 + (0 - 3)^2)}$

$\Rightarrow AC = \sqrt{((-9)^2 + (-3)^2)}$

$\Rightarrow AC = \sqrt{(81 + 8)}$

$\Rightarrow AC = \sqrt{90}$

$\Rightarrow AC = 3\sqrt{10}$

Perimeter of $\Delta ABC = AB + BC + AC$

$= 10 + \sqrt{10} + 3\sqrt{10}$

$= 10 + 4\sqrt{10}$

15. Question

Find the point on the y–axis equidistant from (–5, 2) and (9, –2) (Hint: A point on the y–axis will have its x–coordinate as zero).

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point A (–5, 2), B (9, –2) and C be the point on y–axis i.e. (0, y)

Distance of AC

⇒ AC = $\sqrt{((0 - (-5))^2 + (y - 2)^2)}$

⇒ AC = $\sqrt{((0 + 5)^2 + (y - 2)^2)}$

⇒ AC = $\sqrt{((5)^2 + (y - 2)^2)}$

⇒ AC = $\sqrt{(25 + y^2 - 4y + 4)}$

⇒ AC = $\sqrt{}\ y^2 - 4y + 29$

Distance of BC

⇒ BC = $\sqrt{((0 - 9)^2 + (y - (-2)^2)}$

⇒ BC = $\sqrt{((0 - 9)^2 + (y + 2)^2)}$

⇒ BC = $\sqrt{((9)^2 + (y + 2)^2)}$

⇒ BC = $\sqrt{(81 + y^2 + 4y + 4)}$

⇒ BC = $\sqrt{}\ y^2 + 4y + 85$

i.e. AC = BC (∵ Given)

⇒ $\sqrt{}\ y^2 - 4y + 29 = \sqrt{}\ y^2 + 4y + 85$

Squaring both sides

⇒ $y^2 - 4y + 29 = y^2 + 4y + 8$

⇒ $y^2 - 4y + 29 - y^2 - 4y - 85 = 0$

⇒ $-8y - 56 = 0$

⇒ $-8 (y + 7) = 0$

⇒ $y + 7 = 0$

$y = -7$

∴ the point on y–axis is (0, –7).

16. Question

Find the radius of the circle whose center is (3, 2) and passes through (–5, 6).

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point be A (–5, 6) and O (3, 2)

Distance of OA

⇒ OA = $\sqrt{((-5 - 3)^2 + (6 - 2)^2)}$

⇒ OA = $\sqrt{((-8)^2 + (4)^2)}$

⇒ OA = $\sqrt{(64 + 16)}$

⇒ OA = $\sqrt{80}$

⇒ OA = $4\sqrt{5}$

17. Question

Prove that the points (0, –5), (4, 3) and (–4, –3) lie on the circle centered at the origin with radius 5.

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point A (0, –5), B (4, 3) and C (–4, –3) lie on the circle with center O (0, 0)

Distance of AO

⇒ AO = √ ((0 – 0)^2 + (0 – (–5))^2)

⇒ AO = √ ((0 – 0)^2 + (0 + 5)^2)

⇒ AO = √ ((0)^2 + (5)^2)

⇒ AO = √ (0 + 25)

⇒ AO = √ 25

⇒ AO = 5

Distance of BO

⇒ BO = √ ((0 – 4)^2 + (0 – 3)^2)

⇒ BO = √ ((–4)^2 + (–3)^2)

⇒ BO = √ (16 + 9)

⇒ BO = √ 25

⇒ BO = 5

Distance of CO

⇒ CO = √ ((0 – (–4))^2 + (0 – (–3))^2)

⇒ CO = √ ((0 + 4)^2 + (0 + 3)^2)

⇒ CO = √ ((4)^2 + (3)^2)

⇒ CO = √ (16 + 9)

⇒ CO = √ 25

⇒ CO = 5

∴ AO = BO = CO = 5 = Radius

Hence, point A, B and C lie on the circle.

18. Question

In the Fig. 5.20, PB is perpendicular segment from the point A (4, 3). If PA = PB then find the coordinates of B.

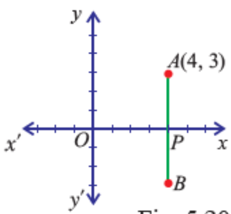


Fig. 5.20

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point P (4, 0)

PB is perpendicular segment from point A to B

∴ let B be (4, –y)

Distance of PA

⇒ PA = √ ((4 – 4)^2 + (3 – 0)^2)

⇒ PA = √ ((0)^2 + (3)^2)

⇒ PA = √ (0 + 9)

⇒ PA= √ 9

⇒ PA = 3

Distance of PB

⇒ PB = $\sqrt{}$ ((4 – 4)^2 + (–y – 0)^2)

⇒ PB = $\sqrt{}$ ((4 – 4)^2 + (–y)^2)

⇒ PB = $\sqrt{}$ ((0)^2 + (–y)^2)

⇒ PB = $\sqrt{}$ 0 + y^2

⇒ PB = y^2

i.e. AP = BP

⇒ 3 = $\sqrt{}$ y^2

Squaring both sides

⇒ 9 = y^2

⇒ y = $\sqrt{9}$

⇒ y = 3

∴ Point B is (4, –3)

19. Question

Find the area of the rhombus ABCD with vertices A (2, 0), B (5, –5), C (8, 0) and D (5, 5). [Hint: Area of the rhombus ABCD = 1/2d_1 d_2]

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Coordinates of rhombus are A (2, 0), B (5, –5), C (8, 0) and D (5, 5)

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

Distance of AC(d_1)

⇒ AC = $\sqrt{}$ ((8 – 2)^2 + (0 – 0)^2)

⇒ AC = $\sqrt{}$ ((6)^2 + (0)^2)

⇒ AC = $\sqrt{}$ (36 + 0)

⇒ AC = $\sqrt{}$ 36

⇒ AC = 6

Distance of BD(d_2)

⇒ BD = $\sqrt{}$ ((5 – 5)^2 + (5 – (–5))^2)

⇒ BD = $\sqrt{}$ ((5 – 5)^2 + (5 + 5)^2)

⇒ BD = $\sqrt{}$ ((0)^2 + (10)^2)

⇒ BD = $\sqrt{}$ (0 + 100)

⇒ BD = $\sqrt{}$ 100

⇒ BD = 10

∴ Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

⇒ Area = $\frac{1}{2} \times 6 \times 10$

⇒ Area = 3 × 10

⇒ Area = 30 units sq.

20. Question

Can you draw a triangle with vertices (1, 5), (5, 8) and (13, 14)? Give reason.

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the points A (1, 5) B (5, 8) and C (13, 14)

Distance of AB

⇒ AB = $\sqrt{((5 - 1)^2 + (8 - 5)^2)}$

⇒ AB = $\sqrt{((4)^2 + (3)^2)}$

⇒ AB = $\sqrt{(16 + 9)}$

⇒ AB = $\sqrt{25}$

⇒ AB = 5

Distance of BC

⇒ BC = $\sqrt{((13 - 5)^2 + (14 - 8)^2)}$

⇒ BC = $\sqrt{((8)^2 + (6)^2)}$

⇒ BC = $\sqrt{(64 + 36)}$

⇒ BC = $\sqrt{100}$

⇒ BC = 10

Distance of AC

⇒ AC = $\sqrt{((13 - 1)^2 + (14 - 5)^2)}$

⇒ AC = $\sqrt{((12)^2 + (9)^2)}$

⇒ AC = $\sqrt{(144 + 81)}$

⇒ AC = $\sqrt{225}$

⇒ AC = 15

Now, we can see that AB + BC = AC.

∴ A, B and C are collinear. Hence, we cannot draw triangle using these coordinates.

21. Question

If origin is the center of a circle with radius 17 units, find the coordinates of any four points on the circle which are not on the axes. (Use the Pythagorean triplets)

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point be A (x, y)

Center is at origin (0, 0)

Distance of OA

⇒ OA = $\sqrt{((x - 0)^2 + (y - 0)^2)}$

⇒ OA = $\sqrt{((x)^2 + (y)^2)}$

⇒ OA = $\sqrt{x^2 + y^2}$

Squaring both sides

⇒ $(0A)^2 = x^2 + y^2$

⇒ $(17)^2 = x^2 + y^2$

Using Pythagorean triplet

x and y can 8 and 5 or vice–a–versa.

∴ x = ± 8 or ±15

y = ± 8 or ±15

Hence, coordinate on circle other than coordinates on axis are

(8, 15), (−8, −15), (−8, 15) and (8, −15)

22. Question

Show that (2, 1) is the circum−center of the triangle formed by the vertices (3, 1), (2, 2) and (1, 1).

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the points be A (3, 1), B (2, 2), C (1, 1) and S(2, 1)

Distance of SA

⇒ $SA = \sqrt{((3 - 2)^2 + (1 - 1)^2)}$

⇒ $SA = \sqrt{((1)^2 + (0)^2}$

⇒ $SA = \sqrt{(1 + 0)}$

⇒ $SA = \sqrt{1} = 1$

Distance of SB

⇒ $SB = \sqrt{((2 - 2)^2 + (2 - 1)^2)}$

⇒ $SB = \sqrt{((0)^2 + (1)^2}$

⇒ $SB = \sqrt{(0 + 1)}$

⇒ $SB = \sqrt{1} = 1$

Distance of SC

⇒ $SC = \sqrt{((1 - 2)^2 + (1 - 1)^2)}$

⇒ $SC = \sqrt{((-1)^2 + (0)^2}$

⇒ $SC = \sqrt{(1 + 0)}$

⇒ $SC = \sqrt{1} = 1$

It is known that the circum−centre is equidistant from all the vertices of a triangle.

Since S is equidistant from all the three vertices, it is the circum−centre of the triangle ABC.

23. Question

Show that the origin is the circum−center of the triangle formed by the vertices (1, 0), (0, −1) and $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$.

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the points be A (1, 0), B (0, −1), C $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and S (0, 0)

Distance of SA

⇒ $SA = \sqrt{((1 - 0)^2 + (0 - 0)^2)}$

⇒ $SA = \sqrt{((1)^2 + (0)^2}$

⇒ $SA = \sqrt{(1 + 0)}$

⇒ $SA = \sqrt{1} = 1$

Distance of SB

⇒ $SB = \sqrt{((0 - 0)^2 + (-1 - 0)^2)}$

⇒ $SB = \sqrt{((0)^2 + (-1)^2}$

⇒ $SB = \sqrt{(0 + 1)}$

$\Rightarrow SB = \sqrt{1} = 1$

Distance of SC

$$\Rightarrow SC = \sqrt{\left(\left(-\frac{1}{2}-0\right)^2 + \left(\frac{\sqrt{3}}{2}-0\right)^2\right)}$$

$$\Rightarrow SC = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow SC = \sqrt{\left(\frac{1}{4}+\frac{3}{4}\right)}$$

$$\Rightarrow SC = \sqrt{\frac{4}{4}}$$

$\Rightarrow SC = \sqrt{1} = 1$

It is known that the circum–centre is equidistant from all the vertices of a triangle.

Since S is equidistant from all the three vertices, it is the circum–centre of the triangle ABC.

24. Question

If the points A (6, 1), B (8, 2), C (9, 4) and D (p, 3) taken in order are the vertices of a parallelogram, find the value of p using distance formula.

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let A, B, C and D represent the points (6, 1), (8, 2), (9, 4) and (p, 3)

Distance of AB

$\Rightarrow AB = \sqrt{((8 - 6))^2 + (2 - 1)^2)}$

$\Rightarrow AB = \sqrt{((2)^2 + (1)^2)}$

$\Rightarrow AB = \sqrt{(4 + 1)}$

$\Rightarrow AB = \sqrt{5}$

Distance of CD

$\Rightarrow CD = \sqrt{((p - 9)^2 + (3 - 4)^2)}$

$\Rightarrow CD = \sqrt{((p - 9)^2 + (1)^2)}$

$\Rightarrow CD = \sqrt{(p^2 + 81 - 18p + 1)}$

$\Rightarrow CD = \sqrt{p^2 - 18p + 82}$

i.e., The opposite sides are equal.

$\therefore AB = CD$

$\Rightarrow \sqrt{5} = \sqrt{p^2 - 18p + 82}$

Squaring both sides

$\Rightarrow 5 = p^2 - 18p + 82$

$\Rightarrow p^2 - 18p + 82 - 5 = 0$

$\Rightarrow p^2 - 18p + 77 = 0$

$\Rightarrow p^2 - 11p - 7p + 77 = 0$

$\Rightarrow p(p - 11) - 7(p - 11) = 0$

$\Rightarrow (p - 11)(p - 7) = 0$

$p - 11 = 0$ or $p - 7 = 0$

$p = 11$ or $p = 7$

25. Question

The radius of the circle with center at the origin is 10 units. Write the coordinates of the point where the circle intersects the axes. Find the distance between any two of such points.

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point be A (x, 0) and B (0, y)

Given center O (0, 0) and radius = 10

Distance of OA

⇒ 5 = √ ((x – 0)^2 + (0 – 0)^2)

⇒ 5 = √ ((x)^2 + (0)^2)

⇒ 5= √ (x^2 + 0)

⇒ 5 = √ x^2

⇒ 5 = x

∴ point A is (5, 0)

Distance of OB

⇒ 5 = √ ((0 – 0)^2 + (y – 0)^2)

⇒ 5 = √ ((0)^2 + (y)^2)

⇒ 5 = √ (0 + y^2)

⇒ 5 = √ y^2

⇒ 5 = y

∴ point B is (0, 5)

Now,

Distance AB = √ ((0 – 5)^2 + (5 – 0)^2)

= √ ((–5)^2 + (5)^2)

= √ (25 + 25)

= √ (50)

= 5√2

Exercise 5.3

1. Question

The point (–2, 7) lies is the quadrant

A. I

B. II

C. III

D. IV

Answer

Option A: value to lie in I quadrant, both should be positive. Hence, this is not correct.

Option B: value to lie in II quadrant, x–coordinate should be negative and y–coordinate should be positive. Hence, this is correct.

Option C: value to lie in III quadrant, both should be negative. Hence, this is not correct.

Option D: value to lie in I quadrant, x–coordinate should be positive and y– coordinate should be negative. Hence, this is not correct.

2. Question

The point (x, 0) where x < 0 lies on

A. OX

B. OY

C. OX'

D. OY'

Answer

Option A: point on OX, x–coordinate will be greater than 0 i.e. x > 0.

Option B: point on OY, y–coordinate will be greater than 0 i.e. y > 0.

Option C: point on OX', x–coordinate will be lesser than 0 i.e. x < 0.

Option D: point on OY', y–coordinate will be lesser than 0 i.e. y < 0.

3. Question

For a point A (a, b) lying in quadrant III

A. a > 0, b < 0

B. a < 0, b < 0

C. a > 0, b > 0

D. a < 0, b > 0

Answer

Option A: point with a > 0, b < 0, lies in the IV quadrant.

Option B: point with a < 0, b < 0, lies in the III quadrant.

Option C: point with a > 0, b > 0, lies in the I quadrant.

Option D: point with a < 0, b > 0, lies in the II quadrant.

4. Question

The diagonal of a square formed by the points (1, 0) (0, 1) (–1, 0) and (0, –1) is

A. 2

B. 4

C. √2

D. 8

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the points be A (1, 0), B (0, 1), C (–1, 0) and D (0, –1)

Distance of diagonal AC

⇒ $AC = \sqrt{((-1 - 1)^2 + (0 - 0)^2)}$

⇒ $AC = \sqrt{((-2)^2 + (0)^2)}$

⇒ $AC = \sqrt{(4 + 0)}$

⇒ $AC = \sqrt{4}$

⇒ AC = 2

∴ Option A is correct.

5. Question

The triangle obtained by joining the points A (–5, 0) B (5, 0) and C (0, 6) is

A. an isosceles triangle

B. right triangle

C. scalene triangle

D. an equilateral triangle

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point be A (–5, 0) B (5, 0) and C (0, 6)

Distance of AB

⇒ AB = $\sqrt{(5 - (-5))^2 + (0 - 0)^2}$

⇒ AB = $\sqrt{(5 + 5)^2 + (0 - 0)^2}$

⇒ AB = $\sqrt{(10)^2 + (0)^2}$

⇒ AB = $\sqrt{100 + 0}$

⇒ AB = $\sqrt{100}$ = 10

Distance of AC

⇒ AC = $\sqrt{(0 - (-5))^2 + (6 - 0)^2}$

⇒ AC = $\sqrt{(5)^2 + (6)^2}$

⇒ AC = $\sqrt{25 + 36}$

⇒ AC = $\sqrt{61}$

Distance of BC

⇒ BC = $\sqrt{(0 - 5)^2 + (6 - 0)^2}$

⇒ BC = $\sqrt{(-5)^2 + (6)^2}$

⇒ BC = $\sqrt{25 + 36}$

⇒ BC = $\sqrt{61}$

We notice that BC = AC = $\sqrt{61}$

∴ Points A, B and C are coordinates of an isosceles triangle.

6. Question

The distance between the points (0, 8) and (0, –2) is

A. 6

B. 100

C. 36

D. 10

Answer

Formula used: Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point be A (0, 8) and B (0, –2)

Distance of AB

⇒ AB = $\sqrt{(0 - 0)^2 + (-2 - 8)^2}$

⇒ AB = $\sqrt{(0)^2 + (-10)^2}$

⇒ AB = $\sqrt{0 + 100}$

⇒ AB = $\sqrt{100}$

⇒ AB = 10

∴ , Option B is correct.

7. Question

(4, 1), (–2, 1), (7, 1) and (10, 1) are points

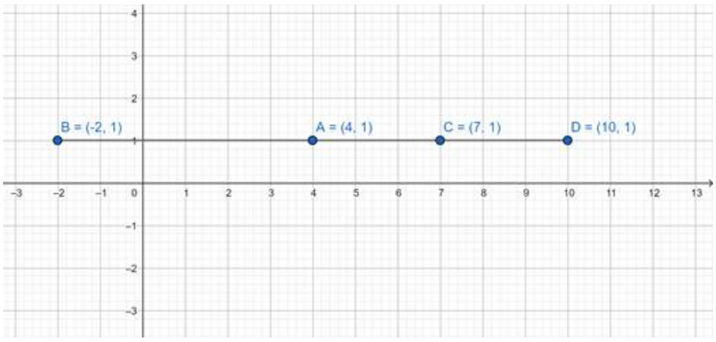
A. on x–axis

B. on a line parallel to x–axis

C. on a line parallel to y–axis

D. on y–axis

Answer



8. Question

The distance between the points (a, b) and (–a, –b) is

A. 2a

B. 2b

C. 2a + 2b

D. $2\sqrt{a^2 + b^2}$

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point be A (a, b) and B (–a, –b)

Distance of AB

⇒ AB = $\sqrt{}$ (–a – a)^2 + (–b – b)^2)

⇒ AB = $\sqrt{}$ ((–2a)^2 + (–2b)^2)

⇒ AB = $\sqrt{}$ (4a^2 + 4b^2)

⇒ AB = $\sqrt{}$ 4(a^2 + b^2)

⇒ AB = 2$\sqrt{}$ (a^2 + b^2)

∴ Option D is correct.

9. Question

The relation between p and q such that the point (p, q) is equidistant from (–4, 0) and (4, 0) is

A. p = 0

B. q = 0

C. p + q = 0

D. p + q = 8

Answer

Formula used: $\text{Distance Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let the point be A (p, q), B (–4, 0) and C (4, 0)

Distance of AB

⇒ AB = $\sqrt{}$ (–4 – p)^2 + (0 – q)^2)

$\Rightarrow AB = \sqrt{((-4 - p)^2 + (-q)^2)}$

$\Rightarrow AB = \sqrt{(16 + p^2 + 8p + q^2)}$

Distance of AC

$\Rightarrow AC = \sqrt{(4 - p)^2 + (0 - q)^2)}$

$\Rightarrow AC = \sqrt{((4 - p)^2 + (-q)^2)}$

$\Rightarrow AC = \sqrt{(16 + p^2 - 8p + q^2)}$

i.e. AB = AC (Given)

$\Rightarrow 16 + p^2 + 8p + q^2 = 16 + p^2 - 8p + q^2$

Squaring both sides

$\Rightarrow 16 + p^2 + 8p + q^2 = 16 + p^2 - 8p + q^2$

$\Rightarrow 16 + p^2 + 8p + q^2 - 16 - p^2 + 8p - q^2 = 0$ …

$\Rightarrow 16\ p = 0$

$\Rightarrow p = 0$

∴ Option A is correct.

8. Question

The point which is on y–axis with ordinate –5 is

A. (0, –5)

B. (–5, 0)

C. (5, 0)

D. (0, 5)

Answer

For any point on y–axis, x–coordinate is 0.

∴ the point is (0, –5).