Line and Angles

Introduction

In our daily life we observe different geometrical shapes. These geometrical shapes are not only the matter of study of mathematics but are directly related with our daily, life basic geometrical figures which make these geometrical shapes are lines and angles.

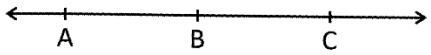
Some Terms Related to Lines

Point

It is a dimensionless figure which represents exact position. It is represented by a fine dot. We denote point by capital letters like A, B, C.....

Line Segment

It is the straight path between two points. In other words we can say that it has two end points and is of finite length.

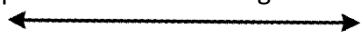


Ray

When line segment extends infinitely in one direction is a ray. Simply we can say that a ray has one end point and definite length.

Line

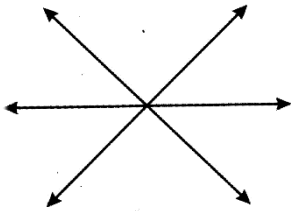
When both end of line segment extended infinitely is known as a line. Simply we can say that line has no end point and definite length



- A line contains infinitely many points.
- Through a given point infinitely many lines can be drawn.
- One and only line can be drawn between two fixed points.

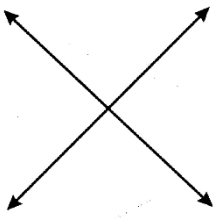
Concurrent Lines

If three or more than three lines intersect at a point then the lines are known a concurrent lines,



Intersecting Lines

When two lines having one common point is called intersecting lines.



Angle

If two rays have common end point then the inclination between two rays is called angle. In the figure 0 is the vertex, $\overline{OP}$ and $\overline{OQ}$ are called arm of the angle. It is represented by notation $\angle$.

Types of angle

Acute Angle

Tangle whose measure is more than $0°$ and less than $90°$.

Right Angle

The angle of measure $90°$

Obtuse Angle

The angle whose measure is more than 90° and less than $180°$.

Straight Angle

The angle whose measure is $180°$

Reflex Angle

The angle whose measure is more than $180°$ and less than $360°$.

Complete Angle

The angle whose measure is 360°.

Equal Angles

Two angles are said to be equal if they are of same measure.

Complementary Angles

If the sum of measure of two angles is 90^{o} then they are said to be complementary angles .e.g $75°$ and $15°$ are complementary angles and they are said to be complement of each other.

Supplementary Angles

If the sum of measure of two angles is $180°$ then they are said to be supplementary angles, e.g $107°$ and $73°$ are said to be supplement of each other.

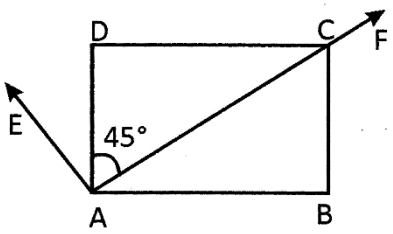
Commonly Asked QUESTIONS

Which one of the following statements is not true?
(i) A line segment has finite length
(ii) A line has only one dimension
(iii) A line $\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ represents the same
(iv) A ray $\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ represents the same

(a) i, ii
(b) ii and iii
(c) Only iv
(d) iii and iv
(e) None of these

Answer: (c)
Explanation
$\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays. They are started from different end points A and B respectively.
Therefore, option (c) is correct and rest of the options is incorrect.

In the following AD is the bisector of $\angle EAF$



Which one of the following statements is incorrect?
(a) $\angle EAD$ is an acute angle
(b) $\angle BAE$ is an obtuse angle
(c) $\angle FAD$ and $\angle DAE$ are complement to each other
(d) $\angle CAD$ and $\angle DAE$ are not complement to each other
(e) None of these

Answer: (d)
Explanation
Since $\angle EAD = \angle CAD = 45$ degree hence, option (a) is correct.
$\angle BAE$ is more than 90° therefore, it is obtuse hence, option (b) is also correct.
The sum of $\angle FAD$ and $\angle DAE$ is $90°$ hence, option (c) is also correct
$(\angle CAD and\ \angle DAE)$ and $(\angle FAD\ and\ \angle DAE)$ are the same. Therefore, they are also complement.

If the difference of two supplementary angles is $50°$ then find the measurement of the smaller angle.
(a) 67° (b) $75°$
(c) $65°$ (d) $90°$
(e) None of these

Answer: (c)

Explanation
Let the one angle be x then other angle will be $180° - x$.
By given condition $x-(180°-x)=50°$
$2x=180°+50° \Rightarrow 2x=230° \Rightarrow x=115°$
Hence, the measurement of smaller angle $= 180°-115°=65°$

Find the supplement of an angle which is 8 times of its complement.
(a) 90° (b) 100°
(c) 80° (d) 70°
(e) None of these

Answer: (b)

If the angle and its complement are x and ^/x respectively then find the angle.

(a) $11°,\ 12°$ (b) $13°,\ 14°$

(c) $-15°, 1°$ (d) $81°$

(e) None of these

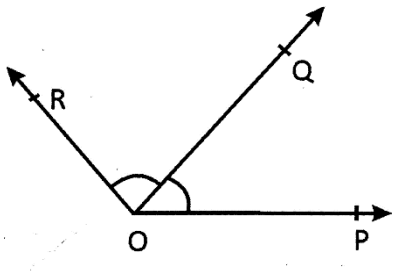
Answer: (d)

Adjacent Angles

Two angles are said to be adjacent angles, if

- They have a common vertex
- They have common arm and
- Non-common arms are opposite to the common arm.

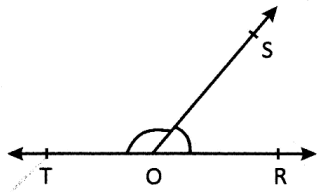
In the given figure $\angle POQ,\ \angle QOR$ are adjacent angles



Linear Pair of Angles

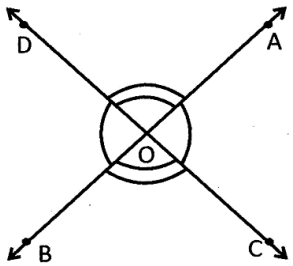
If the sum of measure of two adjacent angles is $180°$ then they are said to be linear pair of angles. In the linear pair non-common arms are opposite to each other.

In the figure $\angle ROS$ and $\angle TOS$ are linear pairs.



Vertically Opposite Angles

It is the pair of angle which is formed by two intersecting lines having no common arms. In the given figure $\angle AOC$ and $\angle BOD$ are vertically opposite angles. Similarly $\angle AOD$ and $\angle BOC$ are also vertically opposite angles.

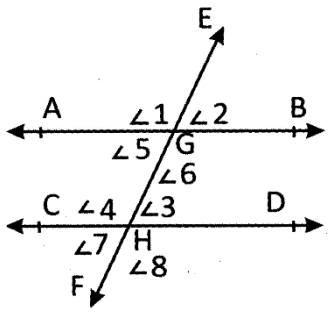


Parallel Lines

Two lines are said to be parallel if the distance among them always remains same at each and every point. The parallel lines never intersect each other.
In other words we can say that if two lines do not have any common point than they are said to be parallel.
In the figure l and m are parallel lines.

Concept of Transversal

Transversal is a line which intersects two or more parallel lines. In the figure, n is a transversal line.



Alternate Interior Angles

In the above figure
$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$ are alternate interior angles.

Alternate Exterior Angles

In the above figure $\angle 7$ and $\angle 2$ are alternate exterior angles.

Corresponding Angles

$\angle 2$ and $\angle 3$ in the above figure are corresponding angles.
They are also the angles on the same side of transversal.

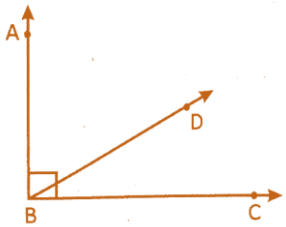
Properties of Angles

When the parallel lines are intersected by a transversal:

- Corresponding angles are equal.
- Alternate interior angles are equal.
- The sum of interior angles on the same side of transversal is $180°$.

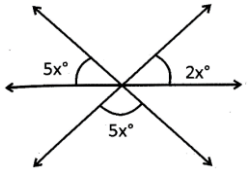
Commonly Asked QUESTIONS

Using the figure below which one of the following statements is true?



(a) Z ABD and $\angle ABC$ are adjacent angles
(b) $\angle ABD$ and $\angle DBC$ are complementary
(c) $\angle DBC$ is half of the measure of $\angle ABC$
(d) $\angle ABC$ and Z DBC are congruent
(e) None of these
Answer: (b)
Explanation
From the figure only option (b) is correct because $\angle ABD + \angle DBC = 90^{O}$

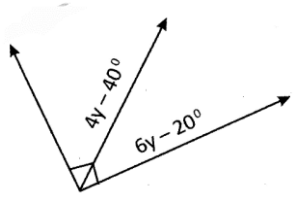
Find the value of x in the figure given below.



(a) $15°$ (b) $20°$
(c) $9°$ (d) $15°$
(e) None of these

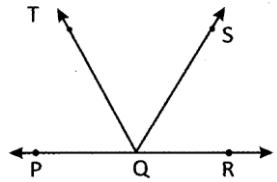
Answer: (d)
Explanation
From the figure $5x° + 5x° + 2x° = 180°$
$\Rightarrow 12x° = 180° \Rightarrow X° = 15°$

Find the difference between two angles in the figure given below.



(a) $15°$ (b) $50°$
(c) $70°$ (d) $20°$
(e) None of these
Answer: (b)

Read the following statements.



(i) $\angle PQT$ and $\angle TQS$ are adjacent angles
(ii) $\angle TQR$ and $\angle RQS$ are adjacent angles
(iii) $\angle TQP$ and $\angle TQR$ are linear pairs
(iv) $\angle SQR$ and $\angle SQT$ are linear pairs

Which one of the following options represents the incorrect statement?

(a) (i), (ii) (b) (ii), (iii)
(c) (ii), (iv) (d) (i), (iv)
(e) None of these

Answer: (c)

The angle formed between the bisectors of linear pair is always:

(a) An acute angle
(b) An obtuse angle
(c) An angle which is half of the double of the right angle
(d) An acute angle greater than the half of the right angle
(e) None of these

Answer: (c)

SUMMARY

- One and only line can be drawn between two fixed points.
- If the sum of measure of two angles is 90° then they are said to be complementary angles.
- If the sum of measure of two angles is 180° then they are said to be supplementary angles.
- Two angles are said to be adjacent angles, if
  (i) They have a common vertex.
  (ii) They have common arm.
  (iii) Non common arms is opposite to the common arm.
- If the sum of measure of two adjacent angles is 180° then they are said to be linear pair of angles. In the linear pair non-common arms are opposite to each other.

You Must KNOW

- In the third century BC Euclid was the mathematician who put geometry into an axiomatic form
- Archimedes developed ingenious techniques for calculating areas and volumes, in many ways anticipating modern integral calculus.
- The field of astronomy, especially mapping the positions of the stars and planets on the celestial sphere and describing the relationship between movements of celestial bodies served as an important source of geometric problems during the next one and a half millennia.
- A mathematician who works in the field of geometry is called a geometer.

Self Evaluation TEST

Duration 10 Minutes

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1. **The complement of 80° is:**

(a) 20° (b) 90°
(c) 10° (d) 30°
(e) None of these

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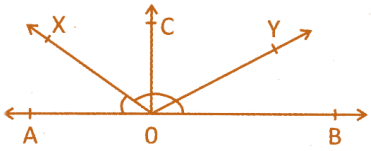
2. **In the following figure, AOB is a straight line. If OX and OY are bisector of $\angle AOC$ and $\angle BOC$ then find $\angle XOY$.**



(a) $95°$ (b) $90°$
(c) $98°$ (d) $92°$
(e) None of these

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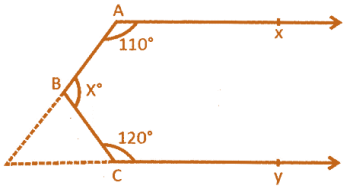
3. **In the following figure then find X, if AX is parallel to CY.**



(a) $130°$ (b) $170°$
(c) $190°$ (d) $180°$
(e) None of these

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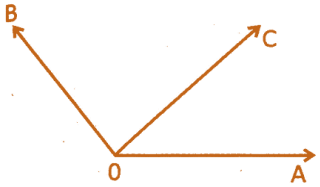
4. **In the following figure, if $\angle BOC = 7x° + 20°$ and $\angle COA = 3x°$ then the value of x for which AOB becomes a straight line is:**



(a) $16°$ (b) $14°$
(c) $20°$ (d) $21°$
(e) None of these

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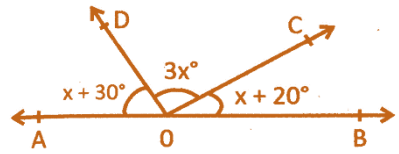
5. **Find x, from the figure given below.**



(a) 24° (b) 26°
(c) 28° (d) 30°
(e) None of these

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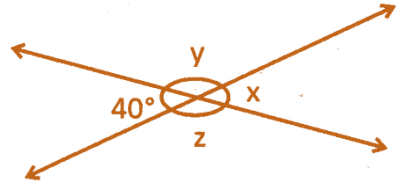
6. **The value of x, y and z respectively in the following figure is:**



(a) 40°, 140°, 140° (b) 80°,180°,180°
(c) 40°,120°,120° (d) 80°,100°,140°
(e) None of these

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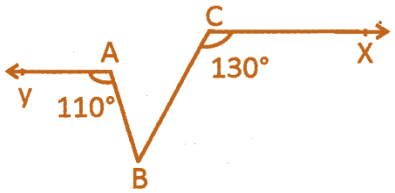
7. **If AY parallel to CX, then find $\angle ABC$.**



(a) 45° (b) 60°
(c) 75° (d) 90°
(e) None of these

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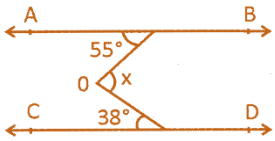
8. **Determine x, if AB is parallel to CD in the following figure.**



(a) 63° (b) 78°
(c) 93° (d) 112°
(e) None of these

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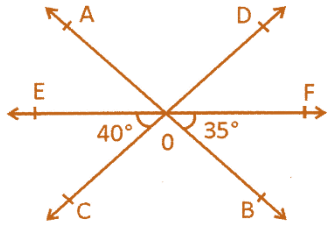
9. **In the following figure, lines AB, CD and EF intersect at 0. The measures of $\angle AOE$ and $\angle AOD$ respectively are:**



(a) $40°,120°$ (b) $140°,110°$
(c) $35°,120°$ (d) $35°,105°$
(e) None of these

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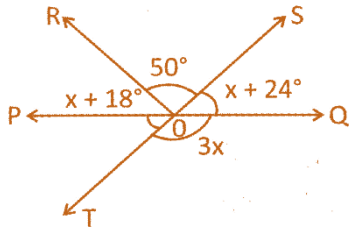
10. **In the given figure, if POQ is a straight line then x is equal to:**



(a) $144°$ (b) $145°$
(c) $44°$ (d) $50°$
(e) None of these

Answers – Self Evaluation Test																			
1.	C	**2.**	B	**3.**	A	**4.**	A	**5.**	B	**6.**	A	**7.**	B	**8.**	C	**9.**	D	**10.**	C

Self Evaluation Test

SOLUTIONS

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2. AOB is a straight line

$\therefore \angle AOC + \angle BOC = 180°$

$\Rightarrow \frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = \frac{1}{2} \times 180^O$

$\Rightarrow \angle XOC + \angle YOC = 90^O \Rightarrow \angle XOY = 90HO$

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3. Extend AB& YC as shown so that they meet in P. in ΔBCP

$\angle BCP = 180^O - 120 = 60^0$ ($\therefore$ CYP is a straight line

$\angle BPC = 180^o - 110^0 = 70^0$

($\therefore \angle BAX \ \& \angle BPY = 60^o 70^o = 130^o$

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4. The condition when AOB becomes a straight line.

$\Rightarrow \angle BOC + \angle COA = 180^o$

$\Rightarrow 7x^o + 20^o + 3x^o = 180^o$

$\Rightarrow 10x = 16^o \Rightarrow x^1 = 16^o$

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5. $x + 30^o + 3x + x + 20^o = 180^o$

$\Rightarrow 5x + 50^o = 180^0 \Rightarrow 5x = 130^o \Rightarrow x = 26^o$

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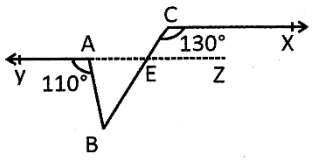
6. $x = 40^o$ (because it is vertically opposite angle)

$40 + z = 180^o \Rightarrow z = 140^o$ (Linear Pair)

7.



$\angle CEZ = 50^o$

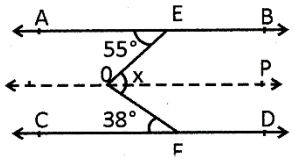
$\angle CEZ = \angle AEB = 50^o$ (Vertically opposite angles)

and $\angle CEZ = \angle AEB = 50^o$ (Linear pair)

$\angle BAE = 70^o \Rightarrow \angle ABC = 180^o - (70^o + 50) = 180^o - 120^o = 60^o \ \angle BAE = 70^o \Rightarrow \angle ABC$

$= 180^o - (70^o + 50) = 180^o - 120^o = 60^o$

8.



$\angle AEO = \angle EOP = 55^o$ (Alternate angles)

Similarly, $\angle CFO = \angle FOP = 38^o$ (Alternate angle)

$\Rightarrow x = \angle EOP + \angle FOP = 55^o + 38^o = 93^o$

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9. $\angle AOE = \angle BOF = 35^o$

$\angle COB = 180^o - 75^o$

$= \angle COB = 105^o = \angle AOB = \angle AOD = 105$

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10. $x + 18^o + 50^o + x + 24^o = 180^o$

$\Rightarrow 2x + 92^o = 180 \Rightarrow 2x = 180^o - 92^o \Rightarrow 2x = 88^o$

$\Rightarrow x = 44^o$
