Class XI Session 2024-25
Subject - Mathematics
Sample Question Paper -9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If cos A = m cos B, then cot $\frac{A+B}{2}$ cot $\frac{B-A}{2}$ = **[1]**

   a) $\frac{m-1}{m+1}$ b) $\frac{m-2}{m+2}$

   c) $\frac{m+2}{m-2}$ d) $\frac{m+1}{m-1}$

2. If R is a relation on the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9} given by x R y ⇔ y = 3x, then R = **[1]**

   a) {(3, 1), (2, 6), (3, 9)} b) {(3, 1), (6, 2), (9, 3)}

   c) {(3, 1), (6, 2), (8, 2), (9, 3)} d) none of these

3. If P(E) denotes the probability of an event E, then E is called certain event, if **[1]**

   a) P(E) is either 0 or 1 b) P(E) = 0

   c) P(E) = 1 d) P(E) = $\frac{1}{2}$

4. $\lim\limits_{x \to 0} \frac{|\sin x|}{x}$ is equal to **[1]**

   a) 0 b) Does not exist

   c) 1 d) -1

5. The acute angle between the lines y = 2x and y = - 2x is **[1]**

   a) greater than 60^o b) 90^o

   c) less than 60^o d) 60^o

6. If A = {(x, y) : $x^2 + y^2 = 25$} and B = {(x, y) : $x^2 + 9y^2 = 144$} then A ∩ B contains **[1]**

   a) three points b) two points

c) one point

d) four points

7. If $\left(\frac{1+i}{1-i}\right)^{x}$ = 1, then **[1]**

a) x = 4n

b) x = 2n

c) x = 2n+1

d) x = 4n + 1, where n $\in$ N

8. If R = {(x, y): x, y $\in$ Z, $x^2 + y^2 \leq 4$} is a relation on Z, then domain of R is **[1]**

a) {-2, -1, 0, 1, 2}

b) {0, -1, -2}

c) {-1, 0, 1}

d) {0, 1, 2}

9. The solution set of the inequation 3x < 5, when x is a natural number is **[1]**

a) {1, 2}

b) {1}

c) {4}

d) {0, 1}

10. cos 405° = ? **[1]**

a) $\frac{-1}{\sqrt{2}}$

b) $-\sqrt{2}$

c) $\sqrt{2}$

d) $\frac{1}{\sqrt{2}}$

11. Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are. **[1]**

a) 5, 1

b) 7, 6

c) 8, 7

d) 6, 3

12. The sum of first 10 terms of a G.P. is equal to 244 times the sum of its first five terms. Then the common ratio is **[1]**

a) 7

b) 5

c) 4

d) 3

13. If C_r denotes ${}^nC_r$ in the expansion of $(1 + x)^n$, then $C_0 + C_1 + C_2 + ... + C_n$ = ? **[1]**

a) 2n

b) 2^n

c) $\frac{1}{3}$ n(2n + 1)

d) 2^n

14. The solution set of 6x - 1 > 5 is : **[1]**

a) {x : x > 1, x $\in$ N}

b) {x : x > 1, x $\in$ R}

c) {x : x < 1, x $\in$ N}

d) {x : x < 1, x $\in$ W}

15. If A and B are two given sets , then $A \cap (A \cap B)^c$ is equal to **[1]**

a) B

b) A

c) $A \cap B^c$

d) ϕ

16. Mark the Correct alternative in the following: 8 $\sin\frac{x}{8}\cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}$ is equal to **[1]**

a) sin x

b) 8 cos x

c) cos x

d) 8 sin x

17. For any positive integer n, $(-\sqrt{-1})^{4n+3}$ = ? **[1]**

a) 1

b) i

c) -i

d) -1

18. In how many ways can a committee of 5 members be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies? **[1]**

a) 50

b) 200

c) 25

d) 100

19. **Assertion (A):** The expansion of $(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$. **[1]**

Reason (R): If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If each of the observations $x_1, x_2, ..., x_n$ is increased by a, where a is a negative or positive number, then the variance remains unchanged. **[1]**

Reason (R): Adding or subtracting a positive or negative number to (or from) each observation of a group does not affect the variance.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. If A = {a, b}, B = {c, d} and C = {d, c}, then find A $\times$ (B $\cup$ C). **[2]**

OR

Draw the graph of the signum function, f :R $\rightarrow$ R, defined by f(x) = $\begin{cases} \frac{x}{|x|}, \text{ when } x \neq 0 \\ 0, \text{ when } x = 0 \end{cases}$

or f(x) = $\begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{ if } x < 0 \end{cases}$

22. Evaluate $\lim\limits_{x \rightarrow 0} \frac{e^{bx} - 1}{x}$. **[2]**

23. Find equation of circle whose end points of its diameter are (-2, 3) and (0, -1). **[2]**

OR

Find the equations to the circles which pass through the origin and cut off equal chords of length 'a' from the straight lines y = x and y = -x.

24. Write down all possible subsets of A = (1, {2, 3}). **[2]**

25. If O is the origin and Q is a variable point on $y^2 = x$. Find the locus of the mid-point of OQ. **[2]**

Section C

26. Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R into R. Determine the range of f. **[3]**

27. Solve system of linear inequation: 1 < lx - 2l < 3 **[3]**

28. Find the equation of the curve formed by the set of all points which are equidistant from the points A (-1, 2, 3) and B(3, 2, 1). **[3]**

OR

What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and three edges passing through the origin coincides with the positive direction of the axes through the origin?

29. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$. **[3]**

OR

Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem.

30. Express $(1 - 2i)^{-3}$ in the form of (a + ib). **[3]**

OR

Evaluate $\left[\frac{1}{1-4i}-\frac{2}{1+i}\right]\left[\frac{3-4i}{5+i}\right]$ to the standard form.

31. If u = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 24} **[3]**

A = {x : x is prime and $x \leq 10$}

B = {x : x is a factor of 24}

Verify the following result

i. A - B = A $\cap B'$
ii. $(A \cup B)' = A' \cap B'$
iii. $(A \cap B)' = A' \cup B'$

Section D

32. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that: **[5]**

i. one is red and two are white
ii. two are blue and one is red
iii. one is red.

33. Differentiate If y = $\sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}$ show that $\frac{dy}{dx}=$ sec x (tan x + sec x) **[5]**

OR

Differentiate $\frac{\cos x}{x}$ from first principle.

34. Find the sum of the following series up to n terms: **[5]**

i. 5 + 55 + 555 + ……
ii. 6 + .66 + .666 + …..

35. Prove that: tan 20° tan 30° tan 40° tan 80° = 1 **[5]**

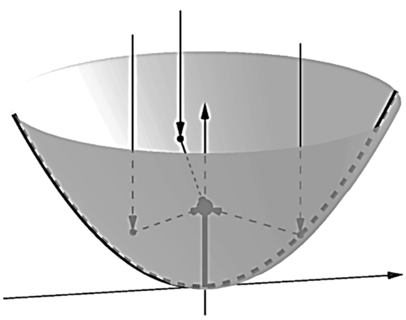
OR

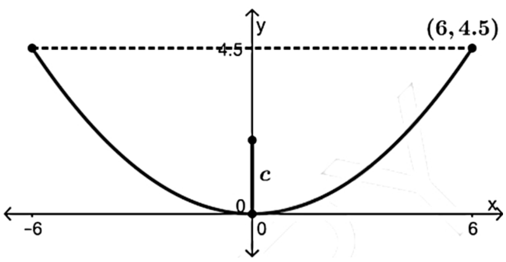
Prove the following identity: $\cos^3 2x + 3\cos 2x = 4(\cos^6 x - \sin^6 x)$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A satellite dish has a shape called a paraboloid, where each cross section is parabola. Since radio signals (parallel to axis) will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. The dish is 12 ft across, and 4.5 ft deep at the vertex.





i. Name the type of curve given in the above paragraph and find the equation of curve? (1)

ii. Find the equation of parabola whose vertex is (3, 4) and focus is (5, 4). (1)

iii. Find the equation of parabola Vertex (0, 0) passing through (2, 3) and axis is along x-axis. and also find the length of latus rectum. (2)

OR

iv. Find focus, length of latus rectum and equation of directrix of the parabola $x^2 = 8y$. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Consider the data

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

i. Find the standard deviation. (1)

ii. Find the variance. (1)

iii. Find the mean. (2)

OR

Write the formula of variance? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

A state cricket authority has to choose a team of 11 members, to do it so the authority asks 2 coaches of a government academy to select the team members that have experience as well as the best performers in last 15 matches. They can make up a team of 11 cricketers amongst 15 possible candidates. In how many ways can the final eleven be selected from 15 cricket players if:

i. Two of them being leg spinners, in how many ways can be the final eleven be selected from 15 cricket players if one and only one leg spinner must be included? (1)

ii. If there are 6 bowlers, 3 wicketkeepers, and 6 batsmen in all. In how many ways can be the final eleven be selected from 15 cricket players if 4 bowlers, 2 wicketkeepers and 5 batsmen are included. (1)

iii. In how many ways can be the final eleven be selected from 15 cricket players if there is no restriction? (2)

OR

In how many ways can be the final eleven be selected from 15 cricket players if one particular player must be included. (2)

Solution

Section A

1.

(d) $\frac{m+1}{m-1}$

Explanation: Given:

cos A = m cos B

$\Rightarrow \frac{\cos A}{\cos B} = \frac{m}{1}$

$\Rightarrow \frac{\cos A+\cos B}{\cos A-\cos B} = \frac{m+1}{m-1}$

$\Rightarrow \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)} = \frac{m+1}{m-1}$ [$\because$ cos A + cos B = 2 cos $\left(\frac{A-B}{2}\right)$ cos $\left(\frac{A+B}{2}\right)$ and cos A - cos B = -2 sin $\left(\frac{A+B}{2}\right)$ cos $\left(\frac{A-B}{2}\right)$]

$\Rightarrow \frac{\cos\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{-\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)} = \frac{m+1}{m-1}$

$\Rightarrow$ - cot $\left(\frac{A+B}{2}\right)$ cot $\left(\frac{A-B}{2}\right)$ = $\frac{m+1}{m-1}$

$\Rightarrow$ cot $\left(\frac{A+B}{2}\right)$ cot $\left(\frac{A-B}{2}\right)$ = $\frac{1+m}{1-m}$

2.

(d) none of these

Explanation: $\therefore$ For A = {1, 2, 3, 4, 5, 6, 7, 8, 9} the satisfying complete relation is:

R = {(1, 3), (2, 6), (3, 9)}

3.

(c) P(E) = 1

Explanation: P(E) = 1

4.

(b) Does not exist

Explanation: Given, $\lim\limits_{x\to 0}\frac{|\sin x|}{x}$

LHL = $\lim\limits_{x\to 0^-}\frac{-\sin x}{x} = -1 \quad \left[\because \lim\limits_{x\to 0}\frac{\sin x}{x} = 1\right]$

RHL = $\lim\limits_{x\to 0^+}\frac{\sin x}{x} = 1$

LHL $\neq$ RHL, So the limit does not exist.

5.

(c) less than 60^o

Explanation: The angle between two straight lines is given by $\frac{|m_1-m_2|}{1+m_1m_2}$

Here $m_1 = 2$ and $m_2 = -2$

Sustituting the values we get,

tan θ = $\frac{2-(-2)}{1+2.(-2)}$

= $\frac{4}{5}$ < 60^0

6.

(d) four points

Explanation: From A, $x^2 + y^2 = 25$ and from B, $x^2 + 9y^2 = 144$

$\therefore$ From B, $(x^2 + y^2) + 8y^2 = 144$

$\Rightarrow 25 + 8y^2 = 144$

$\Rightarrow 8y^2 = 119$

$\Rightarrow y = \pm\sqrt{\frac{119}{8}}$

$\therefore x^2 + y^2 = 25 \Rightarrow x^2 = 25 - y^2 = 25 - \frac{119}{8} = \frac{81}{8}$

$\Rightarrow x = \pm\sqrt{\frac{81}{8}}$

Since we solved equations simultaneously, therefore $A \cap B$ has four points A has 2 elements & B has 2 elements.

7. **(a)** x = 4n

Explanation: $\Rightarrow \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^x = 1 \Rightarrow \left[\frac{1+2i+i^2}{1-i^2}\right]^x = 1 \Rightarrow \left[\frac{2i}{1+1}\right]^x = 1$

$\Rightarrow I^x = 1$

$\Rightarrow x = 4n, n \in N$

8. **(a)** {-2, -1, 0, 1, 2}

Explanation: Domain of R is a set constituting all values of x.

Here, possible values for x by equation $x^2 + y^2 \leq 4$ will be 0, 1, -1, 2, -2.

So, Domain of R is : {-2, -1, 0, 1, 2}.

9.

(b) {1}

Explanation: 3x < 5

$\Rightarrow x < \frac{5}{3}$

$\Rightarrow x < 1\frac{2}{3}$

Hence solution set = $\left\{x : x < 1\frac{2}{3}, x \in N\right\}$ = {1}

10.

(d) $\frac{1}{\sqrt{2}}$

Explanation: cos 405° = cos(360° + 45°) = cos 45° $= \frac{1}{\sqrt{2}}$

11.

(d) 6, 3

Explanation: Since, let A and B be such sets, i.e., n (A) = m, and n(B) = n

Thus, n (P(A)) = 2^m , n (P(B)) = 2^n

Therefore, n (P(A)) – n (P(B)) = 56, i.e., $2^m - 2^n = 56$

$\Rightarrow 2^n (2^{m-n} - 1) = 2^3\ 7$

$\Rightarrow n = 3, 2^{m-n} - 1 = 7$

$\Rightarrow m = 6$

12.

(d) 3

Explanation: Let r be the common ratio of the G.P.

Given $S_{10} = 244\ S_5$

$\Rightarrow \frac{S_{10}}{S_5} = 244$

We have $S_n = \frac{a(r^n-1)}{r-1}$

$\Rightarrow \frac{\frac{a\left(r^{10}-1\right)}{r-1}}{\frac{a\left(r^5-1\right)}{r-1}} = 244, r - 1 \neq 0$

$\Rightarrow \frac{r^{10}-1}{r^5-1} = 244$

$\Rightarrow r^{10} - 1 - 244r^5 + 244 = 0$

$\Rightarrow (r^5)^2 - 244r^5 + 243 = 0$

$\Rightarrow (r5)^2 - 243r^5 - 1r^5 + 243 = 0$

$\Rightarrow r^5(r^5 - 1) - 243(r^5 - 1) = 0$

$\Rightarrow r^5 = 243$ or $r^5 = 1$

Since $r - 1 \neq 0$, r cannot be 1

$\Rightarrow r = \sqrt[5]{243} = 3$

13.

(d) 2^n

Explanation: Here,we know that $C_0 + C_1 + C_2 + ... + C_n = 2^n$

14.

(b) {x : x > 1, x ∈ R}

Explanation: 6x - 1 > 5

⇒ 6x - 1 + 1 > 5 + 1

⇒ 6x > 6

⇒ x > 1

Hence the solution set is {x : x > 1, x ∈ R}

15.

(c) $A \cap B^c$

Explanation: $A \cap B^c$

A and B are two sets.

$A \cap B$ is the common region in both the sets.

$(A \cap B^c)$ is all the region in the universal set except $A \cap B$

Now, $A \cap (A \cap B)^c = A \cap B^c$

16. **(a)** sin x

Explanation: $8\sin\frac{x}{8}\cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}$

$4(2\sin\frac{x}{8}\cos\frac{x}{8})\cos\frac{x}{2}\cos\frac{x}{4}$ [by rearranging terms]

$4(2\sin\frac{x}{8}\cos\frac{x}{8})\cos\frac{x}{2}\cos\frac{x}{4}$ [using the formula $\sin 2\theta = 2\sin\theta\cos\theta$]

$= 4(\sin\frac{x}{4})\cos\frac{x}{2}\cos\frac{x}{4}$

$= 2(2\sin\frac{x}{4}\cos\frac{x}{4})\cos\frac{x}{2}$

$= 2(\sin\frac{2x}{4})\cos\frac{x}{2}$

$= (2\sin\frac{x}{2}\cos\frac{x}{2})$

= sin x

Hence $\sin\frac{x}{8}\cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8} = \sin x$

17.

(b) i

Explanation: $(-\sqrt{-1})^{4n+3} = (-i)^{4n+3} = \{(-i)^4\}^n (-1)^3 = 1\times(-i)\times(-i)\times(-i) = i^2\times(-i) = -1\times(-i) = i$

18.

(b) 200

Explanation: Number of ways of selecting 3 men out of 6 and 2 ladies out of 5 $= \left({}^6C_3 \times {}^5C_2\right)$

$= \left(\frac{6\times5\times4}{3\times2\times1} \times \frac{5\times4}{2\times1}\right) = 200.$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$

Reason:

$(1 + (-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2 + ... + {}^nc_n(1)^{n-n}(-1)^n$

$= n_{c_8} - n_{c_1} + n_{c_2} - n_{c_3} + ... (-1)^n n_{c_n}$

Each term will cancel each other

$\therefore (1 + (-1))^n = 0$

Reason is also the but not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: Let $\bar{x}$ be the mean of $x_1, x_2 ..., x_n$. Then, variance is given by

$\sigma_1^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

If a is added to each observation, the new observations will be

$y_i = x_i + a$

Let the mean of the new observations be $\bar{y}$.

Then,

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n}(x_i + a)$

$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} a\right]$

$= \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{na}{n} = \bar{x} + a$

i.e. $\bar{y} = \bar{x} + a$ ...(ii)

Thus, the variance of the new observations is $\sigma_2^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + a - \bar{x} - a)^2$ [using Eqs. (i) and (ii)]

$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sigma_1^2$

Thus, the variance of the new observations is same as that of the original observations.

Reason: We may note that adding (or subtracting) a positive number to (or from) each observation of a group does not affect the variance.

Section B

21. Given, $A = \{a, b\}$, $B = \{c, d\}$

and $C = \{d, c\}$.

Now, B∪C = {c,d}

∴ A × (B ∪ C) = $\{a, b\} \times \{c, d\}$

= {(a, c), (a, d), (b, c) (b, d)}

OR

Here we have, f :R → R, defined by f(x) = $\begin{cases} \frac{x}{|x|}, \text{ when } x \neq 0 \\ 0, \text{ when } x = 0 \end{cases}$

or f(x) = $\begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{ if } x < 0 \end{cases}$

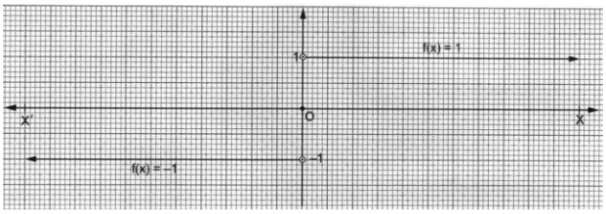
Clearly, we have

x < 0 ⇒ f(x) = - l

x = 0 ⇒ f(x) = 0

x > 0 ⇒ f(x) = 1

We may now draw the graph as shown below.



Graph of signum function

22. We have, $\lim_{x \to 0} \frac{e^{bx}-1}{x} = \lim_{x \to 0} \frac{e^{bx}-1}{x} \times \frac{b}{b}$

[multiplying numerator and denominator by b]

$= \lim_{x \to 0} \frac{b\left(e^{bx}-1\right)}{bx}$

On putting h = bx and as x → 0, then h → 0, we get

$\lim_{x \to 0} \frac{e^{bx}-1}{x} = b \lim_{h \to 0} \frac{\left(e^{h}-1\right)}{h}$

$= b \times 1 \left[\because \lim_{x \to 0} \frac{e^{x}-1}{x} = 1\right]$

= b

23. The equation of a circle with (x_1, y_1) and (x_2, y_2) as end points of one of its diameter is

$(x - x_1)(x - x_2) + (y - y_1)\cdot(y - y_2) = 0$ ...(i)

Given, $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (0, -1)$

(x - 2)(x - 0) + (y - 3)(y + 1) = 0

$\Rightarrow x^2 + 2x + y^2 - 2y - 3 = 0$

$\Rightarrow x^2 + y^2 + 2x - 2y - 3 = 0$, is the required equation of the circle.

OR

We see that there will be four such circles which pass through the origin and cut off equal chords of length a from the straight lines y = $\pm$ x.

Now, $\angle \mathrm{XOA} = \pi/4$ and, $\mathrm{OA} = a$

$AC_1 = asin\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$ and $OC_1 = a\cos\frac{\pi}{4} = \frac{a}{\sqrt{2}}$

So, the coordinates of A $(a/\sqrt{2}, a/\sqrt{2})$

Similarly, the coordinates B.C D are $(-a/\sqrt{2}, a/\sqrt{2}), (-a/\sqrt{2}, -a/\sqrt{2})$ and $(a/\sqrt{2}, -a/\sqrt{2})$ respectively.

The equation of the circle with AD as diameter is

$\left(x - \frac{a}{\sqrt{2}}\right)\left(x - \frac{a}{\sqrt{2}}\right) + \left(y - \frac{a}{\sqrt{2}}\right)\left(y + \frac{a}{\sqrt{2}}\right) =$ 0 or $x^2 + y^2 - \sqrt{2}ax = 0$

Similarly, the equation of the required circle with BC, CD and AB as diameter are

$\left(x + \frac{a}{\sqrt{2}}\right)\left(x + \frac{a}{\sqrt{2}}\right) + \left(y - \frac{a}{\sqrt{2}}\right)\left(y + \frac{a}{\sqrt{2}}\right)$ = 0 or $x^2 + y^2 + \sqrt{2}ax = 0$

$\left(x + \frac{a}{\sqrt{2}}\right)\left(x - \frac{a}{\sqrt{2}}\right) + \left(y + \frac{a}{\sqrt{2}}\right)\left(y + \frac{a}{\sqrt{2}}\right)$ = 0 or $x^2 + y^2 + \sqrt{2}ay = 0$

$\left(x - \frac{a}{\sqrt{2}}\right)\left(x + \frac{a}{\sqrt{2}}\right) + \left(y - \frac{a}{\sqrt{2}}\right)\left(y - \frac{a}{\sqrt{2}}\right) =$ 0 of $x^2 + y^2 - \sqrt{2}ay = 0$

24. Here, we have,

A contains two elements, namely 1 and {2, 3}

{2, 3} = B, then A = {1, B}

$\therefore$ P(A) = {ϕ, {1}, {B}, {1, B}}

$\Rightarrow$ P(A) = {ϕ , ,{1}, {{2, 3}}, {1, {2, 3}}}.

25. Let the coordinates of Q be (a, b), which lies on the parabola.

$y^2 = x$

$\Rightarrow b^2 = a$ .....(i)

Let P(h, k) be the mid-point of OQ.

Now,we have

h = $\frac{0+a}{2}$ and k = $\frac{0+b}{2}$

$\Rightarrow$ a = 2h and b = 2k

Substituting a = 2h and b = 2k in equation (i), we obtain

$(2k)^2 = 2h$

$\Rightarrow 2k^2 = h$

Therefore, the required locus of the mid-point of OQ is $2y^2 = x$.

Section C

26. Here $f(x) = \frac{x^2}{1+x^2}$

Put $y = \frac{x^2}{1+x^2} \Rightarrow y + yx^2 = x^2 \Rightarrow x^2(1 - y) = y$

$\Rightarrow x^2 = \frac{y}{1-y} \Rightarrow x = \pm\sqrt{\frac{y}{1-y}}$

$\frac{y}{1-y} \geq 0$

$\Rightarrow \frac{y}{y-1} \leq 0$

$\Rightarrow 0 \leq y < 1$

$\Rightarrow y \in [0, 1)$

$\therefore$Range of f(x) = [0, 1)

27. When,

$|x - 2| \leq 1$

Then,

$x - 2 \leq -1$ and $x - 2 \geq 1$

Now when,

$x - 2 \leq -1$

Adding 2 to both the sides in above equation

==> $x - 2 + 2 \leq -1 + 2$

==> $x \leq 1$

Now when,

$x - 2 \geq 1$

Adding 2 to both the sides in above equation

==> $x - 2 + 2 \geq 1 + 2$

==> $x \geq 3$

For $|x - 2| \geq 1$ <==> $x \leq 1$ or $x \geq 3$

When,

$|x - 2| \leq 3$

Then,

$x - 2 \geq -3$ and $x - 2 \leq 3$

Now when,

$x - 2 \geq -3$

Adding 2 to both the sides in above equation

==> $x - 2 + 2 \geq -3 + 2$

==> $x \geq -1$

Now when,

$x - 2 \leq 3$

Adding 2 to both the sides in above equation

==> $x - 2 + 2 \leq 3 + 2$

==> $x \leq 5$

For $|x - 2| \leq 3$: $x \geq -1$ or $x \leq 5$

Combining the intervals:

$x \leq 1$ or $x \geq 3$ and $x \geq -1$ or $x \leq 5$

Merging the overlapping intervals:

$-1 \leq x \leq 1$ and $3 \leq x \leq 5$

Therefore,

$x \in [-1, 1] \cup [3, 5]$

28. Consider, C(x, y, z) point equidistant from points A(-1, 2, 3) and B(3, 2, 1).

$\therefore$ AC = BC

$$\sqrt{(x+1)^2 + (y-2)^2 + (z-3)^2} = \sqrt{(x-3)^2 + (y-2)^2 + (z-1)^2}$$

Squaring both sides,

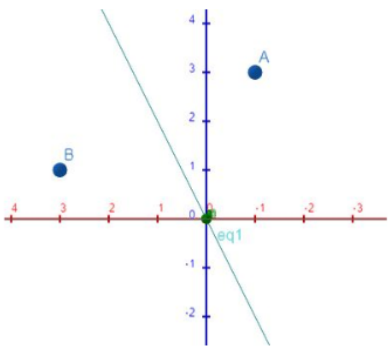
$\Rightarrow (x + 1)^2 + (y - 2)^2 + (z - 3)^2 = (x - 3)^2 + (y - 2)^2 + (z - 1)^2$

$\Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 - 2z + 1$

$\Rightarrow 8x - 4z = 0$

$\Rightarrow 2x - z = 0$

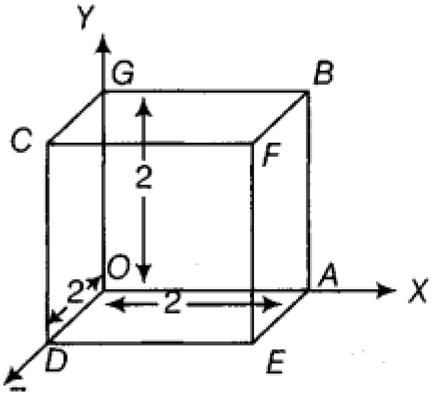
$\Rightarrow z = 2x$



Equation of curve is z = 2x

OR

Given, edge of a cube is 2 unit. It is clear that



coordinate of O = (0, 0, 0)
coordinate of A = (2, 0, 0)
coordinate of G = (0, 2, 0)
coordinate of D = (0, 0, 2)
coordinate of B = (2, 2, 0)
coordinate of F = (2, 2, 2)
coordinate of P = (2, 0, 2)
coordinate of C = (0, 2, 2)

29. We have $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^{n}$

5 th term from the beginning $={}^{n}C_{4}(\sqrt[4]{2})^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^{4}$

5 th term from the end = (n + 1 - 5 + 1)th term from beginning

= (n - 3)th term from beginning

$={}^{n}C_{n-4}(\sqrt[4]{2})^{4}\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$

Now $\frac{{}^{n}C_{4}(\sqrt[4]{2})^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^{4}}{{}^{n}C_{n-4}(\sqrt[4]{2})^{4}\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}}=\frac{\sqrt{6}}{1}$

$\Rightarrow(2)^{\frac{n-8}{4}}\cdot(3)^{\frac{n-8}{4}}=2^{\frac{1}{2}}\times 3^{\frac{1}{2}}$

$\frac{n-8}{4}=\frac{1}{2}\Rightarrow n-8=2$

$\Rightarrow n=10$

OR

We have

$(3x^2 - 2ax + 3a^2)^3 = [(3x^2 - 2ax) + 3a^2)]^3$

$={}^{3}C_{0}(3x^{2}-2ax)^{3}+{}^{3}C_{1}(3x^{2}-2ax)^{2}(3a^{2})+{}^{3}C_{2}(3x^{2}-2ax)(3a^{2})^{2}+{}^{3}C_{3}(3a^{2})^{3}$

$=(3x^{2}-2ax)^{3}+3\times 3a^{2}(3x^{2}-2ax)^{2}+3\times 9a^{4}(3x^{2}-2ax)+27a^{6}$

$=(27x^{6}-8a^{3}x^{3}-54ax^{5}+36a^{2}x^{4})+9a^{2}(9x^{4}+4a^{2}x^{2}-12ax^{3})+27a^{4}(3x^{2}-2ax)+27a^{6}$

$=27x^{6}-8a^{3}x^{3}-54ax^{5}+36a^{2}x^{4}+81a^{2}x^{4}+36a^{4}x^{2}-108a^{3}x^{3}+81a^{4}x^{2}-54a^{5}x+27a^{6}$

$=27x^{6}-54ax^{5}+117a^{2}x^{4}-116a^{3}x^{3}+117a^{4}x^{2}-54a^{5}x27a^{6}$

30. Let z = $(1 - 2i)^{-3}$

= $\frac{1}{(1-2i)^{3}}=\frac{1}{1-8i^{3}-6i+12i^{2}}$ [$\because$ $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$]

= $\frac{1}{1-8i^{2}\cdot i-6i+12(-1)}$

= $\frac{1}{1+8i-6i-12}$ [$\because$ $i^2 = - 1$]

= $\frac{1}{-11+2i}=\frac{1}{-11+2i}\times\frac{-11-2i}{-11-2i}$ [multiplying numerator and denominator by - 11 - 2i]

= $\frac{-11-2i}{(-11)^{2}-(2i)^{2}}=\frac{-11-2i}{121+4}$ [$\because$ $(a - b)(a + b) = a^2 - b^2$]

= $\frac{-11-2i}{125}=\frac{-11}{125}-\frac{2i}{125}$ = a + ib [say]

where, a = $\frac{-11}{125}$ and b = $\frac{-2}{125}$

OR

$\left[\frac{1}{1-4i}-\frac{2}{1+i}\right]\left[\frac{3-4i}{5+i}\right]=\left[\frac{1+i-2+8i}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$
$=\left[\frac{-1+9i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right]=\left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$
$=\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}=\frac{33+31i}{28-10i}\times\frac{28+10i}{28+10i}$
$=\frac{924+330i+868i+310i^2}{(28)^2-(10i)^2}=\frac{614+1198i}{784+100}(\because i^2=-1)$
$=\frac{2(307+599i)}{884}=\frac{307+599i}{442}$

31. Given, U = {1,2,3,4,5,6,7,8,9,10,12,24}

A = {2,3,5,7} B = {1,2,3,4,5,6,8,12,24}

Now, A' = {1,4,6,8,9,10,12,24} B' = {5,7,9,10}

A ∪ B = {1,2,3,4,5,6,7,8,12,24}

(A ∪ B)' = {9,10}

A ∩ B = {2,3} (A ∪ B)' = {1,4,5,6,7,8,9,10,12,24}

(i) A – B = A ∩ B'

L.H.S = A – B = {2,3,5,7} – {1,2,3,4,6,8,12,24} = {5,7} R.H.S = A ∩ B' = {2,3,5,7} ∩ {5,7,9,10} = {5,7}

∴ L.H.S = R.H.S,

(ii) (A ∪ B)' = A ∩ B'

L.H.S = (A ∪ B)' = {9,10}

R.H.S = A'∩ B' = {1,4,6,8,9,10,12,24} ∩ {5,7,9,10}

= {9,10}

∴ L.H.S = R.H.S,

(iii) (A ∩ B)' = A' ∩ B'

L.H.S = (A ∩ B)' = {1,4,5,6,7,8,9,10,12,24}

R.H.S = A' ∩ B' = {1,4,6,8,9,10,12,24} ∩ {5,7,9,10}

= {1,4,5,6,7,8,9,10,12,34}

∴ L.H.S = R.H.S

Section D

32. Bag contains:

6 -Red balls

4 -White balls

8 -Blue balls

Since three ball are drawn,

$\therefore n(S)={}^{18}C_3$

i. Let E be the event that one red and two white balls are drawn.

$\therefore n(E)={}^{6}C_1\times{}^{4}C_2$

$\therefore P(E)=\frac{{}^{6}C_1\times{}^{4}C_2}{{}^{18}C_3}=\frac{6\times4\times3}{2}\times\frac{3\times2}{18\times17\times16}$

$P(E)=\frac{3}{68}$

ii. Let E be the event that two blue balls and one red ball was drawn.

$\therefore n(E)={}^{8}C_2\times{}^{6}C_1$

$\therefore P(E)=\frac{{}^{8}C_2\times{}^{6}C_1}{{}^{18}C_3}=\frac{8\times7}{2}\times6\times\frac{3\times2\times1}{18\times17\times16}=\frac{7}{34}$

$P(E)=\frac{7}{34}$

iii. Let E be the event that one of the ball must be red.

∴ E = {(R,W,B) or (R,W,W) or (R,B,B)}

$\therefore n(E)={}^{6}C_1\times{}^{4}C_1\times{}^{8}C_1+{}^{6}C_1\times{}^{4}C_2+{}^{6}C_1\times{}^{8}C_2$

$\therefore P(E)=\frac{{}^{6}C_1\times{}^{4}C_1\times{}^{8}C_1+{}^{6}C_1\times{}^{4}C_2+{}^{6}C_1\times{}^{8}C_2}{{}^{18}C_3}=\frac{6\times4\times8+\frac{6\times4\times3}{2\times1}+\frac{6\times8\times7}{2\times1}}{\frac{18\times17\times16}{3\times2\times1}}$

$=\frac{396}{816}=\frac{33}{68}$

33. We have to show that $\frac{dy}{dx}=$ (sec x tan x + sec x)

where, it is given that

y = $\sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}$

y = $\sqrt{\frac{\frac{1}{\cos x}\frac{\sin x}{1}}{\cos x}+\frac{\sin x}{\cos x}}=\sqrt{\frac{1-\sin x}{1+\sin x}}$

u = 1 - sin x, v = 1 + sin x, $x=\frac{1-\sin x}{1+\sin x}$

if z = $\frac{u}{v}$

$\frac{dz}{dx}=\frac{\mathrm{v}\times\frac{\mathrm{du}}{\mathrm{dx}}-\mathrm{u}\times\frac{\mathrm{dv}}{\mathrm{dx}}}{\mathrm{v}^2}$

$=\frac{(1+\sin x)\times(-\cos x)-(1-\sin x)\times(\cos x)}{(1+\sin x)^2}$

$=\frac{-\cos x-\sin x\cos x-\cos x+\sin x\cos x}{(1+\sin x)^2}$

$=\frac{-2\cos x}{(1+\sin x)^2}$

According to the chain rule of differentiation

$\frac{dy}{dx}=\frac{\mathrm{dy}}{\mathrm{dz}}\times\frac{\mathrm{dz}}{\mathrm{dx}}$

$=\left[-\frac{\cos x}{1}\times\left(\frac{1-\sin x}{1}\right)^{-\frac{1}{2}}\right]\times\left[\frac{1}{(1+\sin x)^{2-\frac{1}{2}}}\right]$

$=\left[\cos x\times(1+\sin x)^{-\frac{1}{2}}\right]\times(1-\sin x)^{-\frac{3}{2}}\times\left(\frac{1+\sin x}{1+\sin x}\right)^{\frac{3}{2}}$

Multiplying and dividing by $(1+\sin x)^{\frac{3}{2}}$

$=\left[\cos x\times(1+\sin x)^{\frac{2}{2}-\frac{1}{2}}\right]\times(1-\sin x)^{-\frac{2}{2}}\times\left(\frac{1}{1+\sin x}\right)^{\frac{3}{2}}$

$=\left[\cos x\times(1+\sin x)^{\frac{2}{2}-\frac{1}{2}}\right]\times(1-\sin x)^{-\frac{2}{2}}\times(1+\sin x)^{-\frac{2}{2}}$

$=\left[\cos x\times(1+\sin x)^{1}\right]\times\left(1-\sin^2 x\right)^{-\frac{3}{2}}$

$=\left[\cos x\times(1+\sin x)^{1}\right]\times\left(\cos^2 x\right)^{-\frac{3}{2}}$

$=\left[\cos x\times(1+\sin x)^{1}\right]\times(\cos x)^{-3}$

$=\left[(1+\sin x)^{1}\right]\times(\cos x)^{-3+1}$

$=\frac{1+\sin x}{\cos^2 x}$

$=\frac{1}{\cos^2 x}\times\frac{1+\sin x}{\cos^2 x}$

$=\sec x\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)$

= sec x (sec x + tan x)

Hence proved

OR

We have to find the derivative of f(x) = $\frac{\cos x}{x}$

Derivative of a function f(x) is given by f’(x) = $\lim\limits_{h\to 0}=\frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ Derivative of f(x) = $\frac{\cos x}{x}$ is given as f’(x) = $\lim\limits_{h\to 0}=\frac{f(x+h)-f(x)}{h}$

$\Rightarrow \mathrm{f(x)}=\lim\limits_{\mathrm{h}\to 0}\frac{\frac{\cos(x+h)}{x+h}-\frac{\cos x}{x}}{h}$

$\Rightarrow \mathrm{f(x)}=\lim\limits_{\mathrm{h}\to 0}\frac{\frac{x\cos(x+h)-(x+h)\cos x}{x(x+h)}}{h}$=$\lim\limits_{h\to 0}\frac{x\cos(x+h)-(x+h)\cos x}{h(x)(x+h)}$

Using the algebra of limits we have:

$\Rightarrow \mathrm{f(x)}=\lim\limits_{\mathrm{h}\to 0}\frac{x\cos(x+h)-(x+h)\cos x}{h}\times\lim\limits_{h\to 0}\frac{1}{x(x+h)}$

$\Rightarrow \mathrm{f(x)}=\lim\limits_{\mathrm{h}\to 0}\frac{x\cos(x+h)-(x+h)\cos x}{h}\times\frac{1}{x(x+o)}$

$\Rightarrow \mathrm{f(x)}=\frac{1}{\mathrm{x}^2}\lim\limits_{\mathrm{h}\to 0}\frac{x\cos(x+h)-(x+h)\cos x}{h}$

$\Rightarrow \mathrm{f(x)}=\frac{1}{\mathrm{x}^2}\lim\limits_{\mathrm{h}\to 0}\frac{x\cos(x+h)-x\cos x-h\cos x}{h}$

Using the algebra of limits, we have:

$\Rightarrow \mathrm{f(x)}=\frac{1}{\mathrm{x}^2}\left\{\lim\limits_{h\to 0}\frac{-h\cos x}{h}+\lim\limits_{h\to 0}\frac{x\cos(x+h)-x\cos x}{h}\right\}$

$\Rightarrow$ f(x) $= \frac{1}{x^2}\left\{-\lim_{h\to 0}\cos x + \lim_{h\to 0}\frac{x(\cos(x+h)-\cos x)}{h}\right\}$

Using the algebra of limits we have:

$\therefore$ f'(x) = $-\frac{\cos x}{x^2} + \frac{1}{x}\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$

We can't evaluate the limits at this stage only as on putting value it will take $\frac{0}{0}$ form. So, we need to do little modifications.

Use: cos A – cos B = – 2 sin $(\frac{(A+B)}{2})$ sin $(\frac{(A-B)}{2})$

$\therefore$ f'(x) = $-\frac{\cos x}{x^2} + \frac{1}{x}\lim_{h\to 0}\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

$\Rightarrow$ f'(x) = $-\frac{\cos x}{x^2} + \frac{1}{x}\lim_{h\to 0}\frac{\sin\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Using algebra of limits:

$\Rightarrow$ f'(x) = $-\frac{\cos x}{x^2} + \frac{1}{x}\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h\to 0}\sin\left(x+\frac{h}{2}\right)$

By using the formula we get : $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$\Rightarrow$f'(x) =$-\frac{\cos x}{x^2} + \frac{1}{x}\lim_{h\to 0}\sin\left(x+\frac{h}{2}\right)$

Put the value of h to evaluate the limit:

$\therefore$ f'(x) =$-\frac{\cos x}{x^2} + \frac{1}{x}$ $\times$sin (x + 0) = $-\frac{\cos x}{x^2} - \frac{\sin x}{x}$

Hence,

Derivative of f(x) = (cos x)/x is $-\frac{\cos x}{x^2} - \frac{\sin x}{x}$

34. i. S_n = 5 + 55 + 555 + ………. up to n terms

= 5 [1 + 11 + 111 + ………. up to n terms]

$= \frac{5}{9}$ [9 + 99 + 999 + ... up ton terms]

$= \frac{5}{9}$ [(10 - 1) + (10^2 - 1) + (10^3 - 1) + ... up to n terms]

$= \frac{5}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$

$= \frac{5}{9}\left[\frac{10}{9}(10^n-1) - n\right]$

$= \frac{50}{81}(10^n-1) - \frac{5}{9}n$

ii. S_n = .6 + .66 + .666 + …………. up to n terms

= 6 [.1 + .11 + .111 + ………. up to n terms]

$= \frac{6}{9}$ [.9 + .99 + .999 + ... up to n terms]

$= \frac{6}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ldots\ldots \text{ up to } n \text{ terms}\right]$

$= \frac{6}{9}\left[\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{10^2}\right) + \left(1-\frac{1}{10^3}\right)\ldots\ldots \text{ up to } n \text{ terms}\right]$

$= \frac{6}{9}\left[n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \ldots\ldots\ldots \text{ up to } n \text{ terms}\right)\right]$

$= \frac{2}{3}\left[n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$

$= \frac{2}{3}\left[n - \frac{1}{9}\left(1-\frac{1}{10^2}\right)\right]$

$= \frac{2n}{3} - \frac{2}{27}\left(1-\frac{1}{10^n}\right)$

35. LHS = tan 20° tan 30° tan 40° tan 80°

= $\frac{1}{\sqrt{3}}(\tan 20^\circ \tan 40^\circ \tan 80^\circ)$ $\left[\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$

$= \frac{(\sin 20^\circ \sin 40^\circ \sin 80^\circ)}{(\cos 20^\circ \cos 40^\circ \cos 80^\circ)\sqrt{3}}$

$= \frac{(2\sin 20^\circ \sin 40^\circ)\sin 80^\circ}{\sqrt{3}(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ}$

Applying

$\Rightarrow$ 2 sin A sin B = cos (A - B) - cos (A + B) and 2 cos A cos B = cos(A + B) + cos (A - B), we get

$= \frac{[\cos(40^\circ-20^\circ)-\cos(20^\circ+40^\circ)]\sin 80^\circ}{[\cos(20^\circ+40^\circ)+\cos(40^\circ-20^\circ)]\cos 80^\circ\sqrt{3}}$

$= \frac{(\cos 20^\circ - \cos 60^\circ)\sin 80^\circ}{\sqrt{3}(\cos 60^\circ+\cos 20^\circ)\cos 80^\circ}$

$= \frac{\left(\cos 20^\circ - \frac{1}{2}\right)\sin 80^\circ}{\sqrt{3}\left(\frac{1}{2} + \cos 20^\circ\right)\cos 80^\circ}$

$= \frac{2\cos 20^\circ \sin 80^\circ - \sin 80^\circ}{\sqrt{3}(\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ)}$

Now,

$\Rightarrow$ 2 sin A cos B = sin (A + B) + sin (A - B)

$= \frac{\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - \sin 80^\circ}{\sqrt{3}[\cos 80^\circ + \cos(20^\circ + 80^\circ) + \cos(80^\circ - 20^\circ)]}$

$= \frac{\sin 100^\circ + \sin 60^\circ - \sin 80^\circ}{\sqrt{3}(\cos 80^\circ + \cos 100^\circ + \cos 60^\circ)}$

$= \frac{\sin 100^\circ + \sin 60^\circ - \sin(180^\circ - 100^\circ)}{\sqrt{3}(\cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos 60^\circ)}$

$= \frac{\sin 100^\circ + \frac{\sqrt{3}}{2} - \sin 100^\circ}{\sqrt{3}(\cos 80^\circ - \cos 80^\circ + \cos 60^\circ)}$

$= \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}\left(\frac{1}{2}\right)}$ = 1 = RHS

OR

We have to prove that $\cos^3 2x + 3\cos 2x = 4(\cos^6 x - \sin^6 x)$

Let us consider RHS = $4(\cos^6 x - \sin^6 x)$

$= 4((\cos^2 x)^3 - (\sin^2 x)^3)$

$= 4(\cos^2 x - \sin^2 x)(\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x)$ ... $\{\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)\}$

$= 4\cos 2x(\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x + \cos^2 x \sin^2 x - \cos^2 x \sin^2 x)$ ... $\{\because \cos 2x = \cos^2 x - \sin^2 x\}$

$= 4\cos 2x(\cos^4 x + \sin^4 x + 2\cos^2 x \sin^2 x - \cos^2 x \sin^2 x)$

$= 4\cos 2x\{(\cos^2 x)^2 + (\sin^2 x)^2 + 2\cos^2 x \sin^2 x - \cos^2 x \sin^2 x)\}$ ... $\{\because a^2 + b^2 + 2ab = (a + b)^2\}$

$= 4\cos 2x\{(\cos^2 x + \sin^2 x)^2 - \cos^2 x \sin^2 x)\}$ ... $\{\because \cos^2 x + \sin^2 x = 1\}$

$= 4\cos 2x\{(1)^2 - \frac{1}{4}(4\cos^2 x \sin^2 x)\}$

$= 4\cos 2x\{(1)^2 - \frac{1}{4}(2\cos x \sin x)^2\}$ ... $\{\because \sin 2x = 2\sin x\cos x\}$

$= 4\cos 2x\{(1)^2 - \frac{1}{4}(2\sin 2x)^2\}$

$= 4\cos 2x(1 - \frac{1}{4}\sin^2 2x)$ ... $\{\because \sin^2 x = 1 - \cos^2 x\}$

$= 4\cos 2x(1 - \frac{1}{4}(1 - \cos^2 2x))$

$= 4\cos 2x(1 - \frac{1}{4} + \frac{1}{4}\cos^2 2x))$

$= 4\cos 2x(\frac{3}{4} + \frac{1}{4}\cos^2 2x))$

$= 4(\frac{3}{4}\cos 2x + \frac{1}{4}\cos^3 2x))$

$= 3\cos 2x + \cos^3 2x$

RHS = LHS

Hence Proved.

Section E

36. i. Given curve is a parabola

Equation of parabola is $x^2 = 4ay$

It passes through the point (6, 4.5)

$\Rightarrow 36 = 4 \times a \times 4.5$

$\Rightarrow 36 = 18a$

$\Rightarrow a = 2$

Equation of parabola is $x^2 = 8y$

ii. Distance between focus and vertex is = a = $\sqrt{(4-4)^2 + (5-3)^2}$ = 2

Equation of parabola is $(y - k)^2 = 4a(x - h)$

where (h, k) is vertex

$\Rightarrow$ Equation of parabola with vertex (3, 4) & a = 2

$\Rightarrow (y - 4)^2 = 8(x - 3)$

iii. Equation of parabola with axis along x - axis

$y^2 = 4ax$

which passes through (2, 3)

$\Rightarrow 9 = 4a \times 2$

$\Rightarrow 4a = \frac{9}{2}$

hence required equation of parabola is

$y^2 = \frac{9}{2}x$

$\Rightarrow 2y^2 = 9x$

Hence length of latus rectum = 4a = 4.5

OR

$x^2 = 8y$

a = 2

Focus of parabola is (0, 2)

length of latus rectum is $4a = 4 \times 2 = 8$

Equation of directrix y + 2 = 0

37. i. By using formula,

$\sigma^2 = \frac{1}{N}\left[\sum_{i=1}^{n} f_i(x_i - \bar{x})^2\right]$

x_i	f_i	f_ix_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
Total	30	420			1374

Given, N = $\sum f_i$ = 30, $\sum f_ix_i$ = 420 and $\sum f_i(x_i - \bar{x})^2$ = 1374

$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_ix_i}{N}$ = $\frac{420}{30}$ = 14

Variance (σ^2) = $\frac{1}{N}\sum_{i=1}^{7} f_i(x_i - \bar{x})^2$ = $\frac{1}{30} \times 1374 = 45.8$

Standard deviation, σ = $\sqrt{\sigma^2}$ = $\sqrt{45.8}$ = 6.77

ii. Variance (σ^2) = $\frac{1}{N}\sum_{i=1}^{7} f_i(x_i - \bar{x})^2$ = $\frac{1}{30} \times 1374 = 45.8$

iii. Given, N = $\sum f_i$ = 30, $\sum f_ix_i$ = 420 and $\sum f_i(x_i - \bar{x})^2$ = 1374

$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_ix_i}{N}$ = $\frac{420}{30}$ = 14

OR

$\sigma^2 = \frac{1}{N}\Sigma(x_i - \bar{x})$

38. i. Two of them being leg spinners, one and only one leg spinner must be included

Let's first find out possible ways to select players which are not leg spinner

There are two leg spinners out of 15 and one players must be leg spinner.

So, we have to select 10 players out of 13

Total possible ways to select 11 players out of 15 out of which one must be leg spinner out of 2 are ${}^{13}C_{10} \times {}^{2}C_1$

${}^nC_r = \frac{n!}{(n-r)!r!}$

$\Rightarrow {}^{13}C_{10} = \frac{13!}{(13-10)!10!}$

$\Rightarrow {}^{13}C_{10} = \frac{13!}{3!10!} = \frac{13\times12\times11\times10!}{3\times2\times1\times10!}$

$\Rightarrow {}^{13}C_{10} = \frac{13\times12\times11}{3\times2\times1} = 13 \times 6 \times 11$

$\Rightarrow {}^{13}C_{10}$ = 858

${}^{2}C_{1} \times {}^{13}C_{10}$

$\Rightarrow$ 2 $\times$ 858 = 1716

Total possible ways to select 11 players out of 15 out of which one must be leg spinner out of 2 = 1716

ii. number of ways of selecting 4 bowlers out of 6 = ${}^{6}C_{4}$

$\Rightarrow {}^{6}C_{4} = \frac{6!}{(6-4)!4!} = \frac{6!}{2!4!} = \frac{6\times5\times4!}{2\times1\times4!}$ = 15

number of ways of selecting 5 batsmen out of 6 = ${}^{6}C_{5}$ = 6

number of ways of selecting 2 wicket keepers out of 3 = ${}^{3}C_{2}$ = ${}^{3}C_{1}$ = 3

$\Rightarrow {}^{6}C_{4} \times {}^{6}C_{5} \times {}^{3}C_{2}$

$\Rightarrow$ 15 $\times$ 6 $\times$ 3 = 270

Total ways to select 4 bowlers, 2 wicketkeepers and 5 batsmen out of 6 bowlers, 3 wicketkeepers, and 6 batsmen in all are 270.

iii. Here, we have to select 11 players out of 15 and there are no restrictions and here the order of the players doesn't matter. So, we will here apply combination

${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$

$\Rightarrow {}^{15}C_{11} = \frac{15!}{(15-11)!11!}$

$\Rightarrow {}^{15}C_{11} = \frac{15!}{4!11!} = \frac{15\times14\times13\times12\times11!}{4\times3\times2\times1\times11!}$

$\Rightarrow {}^{15}C_{11} = \frac{15\times14\times13\times12}{4\times3\times2\times1} = 15 \times 13 \times 7$

$\Rightarrow {}^{15}C^{11}$ = 1365

In 1365 many ways can be the final eleven be selected from 15 cricket players if there is no restriction

OR

If one player must always be included, then we have to select 10 players from 14

${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$

$\Rightarrow {}^{14}C_{10} = \frac{14!}{(14-10)!10!}$

$\Rightarrow {}^{14}C_{10} = \frac{14!}{4!10!} = \frac{14\times13\times12\times11\times10!}{4\times3\times2\times1\times10!}$

$\Rightarrow {}^{14}C_{10} = \frac{14\times13\times12\times11}{4\times3\times2\times1} = 13 \times 11 \times 7$

$\Rightarrow {}^{14}C_{10}$ = 1001

In 1001 ways can be the final eleven be selected from 15 cricket players if one particular player must be included.