

Topics : Solution of Triangle, Application of Derivatives, Straight Line

Type of Questions

M.M., Min.

Single choice Objective (no negative marking) Q. 1,2 (3 marks, 3 min.)

[6, 6]

Subjective Questions (no negative marking) Q.3,4,5,6,7,8 (4 marks, 5 min.)

[26, 30]

1. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A **false** statement among the following is

(A) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$. (B) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$.

(C) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$. (D) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$.

2. If in triangle ABC, $r_1 = 2r_2 = 3r_3$, D is the middle point of BC. Then $\cos \angle ADC$ is equal to

(A) $\frac{7}{25}$ (B) $-\frac{7}{25}$ (C) $\frac{24}{25}$ (D) $-\frac{24}{25}$

3. Two men P and Q start with velocities v at the same time from the junction of two roads inclined at 45° to each other. If they travel by different roads, find the rate at which they are being separated.

4. ABC is a triangle and D is the middle point of BC. If AD is perpendicular to AC, prove that

$$\cos A \cdot \cos C = \frac{2(c^2 - a^2)}{3ac}$$

5. With usual notation In a $\triangle ABC$, a, c, A are given and $b_2 = 2b_1$, where b_1, b_2 are two values of the third side, then prove that $3a = c\sqrt{1+8\sin^2 A}$

6. If $2f(x) = f(xy) + f\left(\frac{x}{y}\right)$ for all $x, y, \in \mathbb{R}^+$, $f(1) = 0$ and $f'(1) = 1$, then find $f(e)$ and $f'(2)$.

7. Through the origin O, a straight line is drawn to cut the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ at Q and R respectively. Find the locus of the point P on this variable line, such that OP is the geometric mean between OQ and OR.

8. The circle $x^2 + y^2 = 1$ cuts the x-axis at P & Q. Another circle with centre at Q and variable radius intersects the first circle at R above x-axis and the line segment PQ at S. Find the maximum area of the $\triangle QSR$.

Answers Key

1. (B) 2. (B) 3. $v\sqrt{(2-\sqrt{2})}$

6. $f(e) = 1, f'(2) = \frac{1}{2}$ 7. $(y - m_1x)(y - m_2x) = c_1c_2$

8. $\frac{4\sqrt{3}}{9}$