Exponents and Powers

MATHEMATICAL REASONING

1. The value of $\frac{2^{2001}+2^{1999}}{2^{2000}-2^{1998}}$, is _____.

(a) 2 (b) 10 / 3

(c) $2^{1000}+1$ (d) 10

2. When simplified, $(x^{-1}+y^{-1})^{-1}$ is equal to_____.

(a) $x+y$

(b) $\frac{xy}{x+y}$

(c) xy

(d) $\frac{1}{xy}$

3. The value of $x+x(x^x)$ at $x=2$ is __.

(a) 10 (b) 16

(c) 18 (d) 36

4. Simplify;

$$\left(\frac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+c^2+cb)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+a^2+ca)}$$

(a) 1 (b) $(a+b+c)^3$

(c) $a^2+b^2+c^2$ (d) 0

5. If $((6x)^6 = 6^{2^3}$, then find the value of x.

(a) 1 (b) $\sqrt{6}$

(c) $\sqrt[3]{6}$ (d) $\sqrt[6]{6}$

6. Find the value of $(0.000064)^{\frac{5}{6}} \div (0.00032)^{\frac{6}{5}}$.

(a) 0.2 (b) 0.4

(c) 5 (d) 2.5

7. If $x=\left(\frac{3}{2}\right)^2 \times \left(\frac{2}{3}\right)^{-4}$, find the value of x^2.

(a) $\left(\frac{4}{3}\right)^{12}$ (b) $\left(\frac{2}{3}\right)^{12}$

(c) $\left(\frac{3}{2}\right)^{12}$ (d) $\left(\frac{3}{4}\right)^{12}$

8. If $a^x=b^y=c^z$ and $b^2=ac$, then y equals

(a) $\frac{2xz}{x+z}$ (b) $\frac{xz}{2(x-z)}$

(c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(x-z)}$

9. If $\frac{10}{3}\times 3^x - 3^{x-1} = 81$, then the value of x is____.

(a) 2 (b) 1

(c) 3 (d) 0

10. Which of the following values are equal?

(P) 1^4 (Q) 4^0

(R) 0^4 (S) 4^1

(a) P and Q (b) Q and R

(c) P and R (d) P and S

11. The value of $(7^{-1}-8^{-1})^{-1}-(3^{-1}-4^{-1})^{-1}$ is

(a) 56 (b) 12

(c) 68 (d) 44

12. When $5\times10^{-3}-2000\times10^{-6}$ is expressed in standard form, we will get

(a) 3×10^{-2} (b) 3×10^{-3}

(c) 3×10^{3} (d) 3×10^{2}

13. If $3^{(x-y)}=27$ and $3^{(x+y)}=243$, then x is equal to ____.

(a) 0 (b) 2

(c) 4 (d) 6

14. Number of prime factors in $(216)^{\frac{3}{5}} \times (2500)^{\frac{2}{5}} \times (300)^{\frac{1}{5}}$ is _____.

(a) 6 (b) 9

(c) 8 (d) None of these

15. Find x, if $\left(\frac{5}{11}\right)^{-3} \times \left(\frac{5}{11}\right)^{5} = \left(\frac{5}{11}\right)^{x}$

(a) 3 (b) 4

(c) 8 (d) 2

EVERYDAY MATHEMATICS

16. The size of a red blood cell is 0.000007 m and the size of a plant cell is 0.00001275 m. Find the ratio of the size of red blood cell to that of plant cell.

(a) 13 : 56 (b) 28 : 51

(c) 31 : 39 (d) 22 : 31

17. Pluto is 5913000000 km away from the Sun. Express this distance in standard form.

(a) 5.913×10^{-11} (b) 5.913×10^{-9}

(c) 5.913×10^{6} (d) 5.913×10^{9}

18. The cell of bacteria doubles itself after every 1 hour. How many cells will there be after 8 hours?

(a) 200 times of the original

(b) 2^{10} times of the original

(c) 2^{8} times of the original

(d) 2^{6} times of the original

19. At the end of the 20th century, the world's population was approximately 6.125×10^{9}. Express this population in usual form.

(a) 6.125×10^{10} (b) 6125000000

(c) 6125×10^{5} (d) 612500000

20. Weight of moon is $(7.346 \times 10^{22})kg$ and weight of Earth is $5,9724 \times 10^{24}$ 7 kg. What is the total weight of both in standard form?

(a) 6.04×10^{24}kg (b) 7.08×10^{22}kg

(c) 5.98×10^{24}kg (d) 6.44×10^{24} kg

ACHIEVERS SECTION (HOTS)

21. Simplify: $\dfrac{\left[\frac{2}{3}\right]^{3} \times \left[\frac{2}{3}\right]^{-2} \times \left[\left(\frac{1}{2}\right)^{2}\right]^{-2} \times \frac{1}{24}}{\left(\frac{2}{3}\right)^{-5} \times \left(\frac{3}{2}\right)^{-12}}$

(a) $\left(\frac{2}{3}\right)^{-4}$

(b) $\frac{32}{3}$

(c) $\frac{243}{16}$

(d) $\frac{243}{32}$

22. Solve for y, if

$$\frac{\left(\frac{1}{9}\right)^{2y-1} (.0081)^{1/3}}{\sqrt{243}} = \left(\frac{1}{3}\right)^{2y-5} \sqrt[3]{\frac{27^{y-1}}{10000}}$$

(a) $\frac{1}{2}$

(b) $-\frac{19}{18}$

(c) $\frac{3}{10}$

(d) $\frac{12}{17}$

23. Fill in the blanks.

(i) If $m^2 = 27^{2/3} \times 16^{-3/2}$, then m = __P__.

(ii) If $ab = 1$, then $\frac{1}{1+a^{-1}} + \frac{1}{1+b^{-1}} =$ __Q__.

(iii) If x = $(8^{2/3} \cdot 32^{-2/5})$, then $x^{-5} =$ __R__.

	P	Q	R
(a)	5/4	0	10/7
(b)	1	1	5/16
(c)	3/8	1	1
(d)	7/8	0	7/8

24. Match the following.

Column – I	Column – II
(P) $\left(6^{-1}+\left(\frac{3}{2}\right)^{-1}\right)^{-1}$	(i) $-\frac{4}{13}$
(Q) $\left\{\left(\frac{4}{3}\right)^{-1}\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$	(ii) $\frac{9}{32}$
(R) $\left[\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right]\div\left(\frac{1}{4}\right)^{-3}$	(iii) $\frac{6}{5}$
(S) $(3^{-1}\times 4^{-1})\times\left(\frac{2}{3}\right)^{-3}$	(iv) $\frac{19}{64}$

(a) (P) → (iii); (Q) → (i); (R) → (iv); (S) → (ii)
(b) (P) → (iv): (Q) → (i); (R) → (ii); (S) → (iii)
(c) (P) → (ii); (Q) → (iii); (R) → (iv); (S) → (i)
(d) (P) → (iii); (Q) → (i); (R) → (ii); (S) → (iv)

25. If $a=(2^{-2}-2^{-3})$, $b=(2^{-3}-2^{-4})$ and $c=(2^{-4}-2^{-2})$ then find :

(i) $a^3+b^3+c^3$ (ii) 10 abc

	(i)	(ii)
(a)	9/2048	7/2048
(b)	3/1024	5/2048
(c)	– 3/1024	– 10/2048
(d)	– 9/2048	– 15/1024

ANSWER KEY									
1.	B	**2.**	B	**3.**	A	**4.**	A	**5.**	C
6.	C	**7.**	B	**8.**	A	**9.**	C	**10.**	A
11.	D	**12.**	B	**13.**	C	**14.**	D	**15.**	D
16.	B	**17.**	D	**18.**	C	**19.**	B	**20.**	A
21.	D	**22.**	B	**23.**	C	**24.**	A	**25.**	D

HINTS & EXPLANATIONS

1. (b) : $\frac{2^{2001}+2^{1999}}{2^{2000}-2^{1998}}=\frac{2^{1999}(2^2+1)}{2^{1998}(2^2-1)}$

$=2^{1999-1998}\left(\frac{4+1}{4-1}\right)$ $\left[\because \frac{a^m}{a^n}=a^{m-n}\right]$

$=2\left(\frac{5}{3}\right)=\frac{10}{3}$

2. (b) : $(x^{-1}+y^{-1})^{-1}=\left(\frac{1}{x}+\frac{1}{y}\right)^{-1}=\left(\frac{x+y}{xy}\right)^{-1}$

$=\frac{xy}{x+y}\left[\because a^{-n}=\frac{1}{a^n}\right]$

3. (a) : At $x=2$,

$x+x(x^x)=2+2(2^2)=2+2(4)=2+8=10$

4. (a) :

$\left(\frac{x^a}{x^b}\right)^{a^2+b^2+ab}\times\left(\frac{x^b}{x^c}\right)^{b^2+c^2+cb}\times\left(\frac{x^c}{x^a}\right)^{c^2+a^2+ca}$

$=(x^{a-b})^{a^2+b^2+ab}\times(x^{b-c})^{b^2+c^2+cb}$

$\times(x^{c-a})^{c^2+a^2+ca}$

$=x^{(a-b)(a^2+b^2+ab)+(b-c)(b^2+c^2+cb)+(c-a)(c^2+a^2+ca)}$

$=x^{(a^3-b^3+b^3-c^3+c^3-a^3)=x^0=1}$.

5. (c) : We have, $(6x)^6=6^{2^3}$

$\Rightarrow x^6=\frac{6^8}{6^6}\Rightarrow x^6=6^{8-6}=6^2$

$\Rightarrow (x^6)^{1/6}=(6^2)^{1/6}\Rightarrow x=(6)^{1/3}$

6. (c) : We have, $(0.000064)^{5/6}\div(0.00032)^{6/5}$

$=\left(\frac{64}{1000000}\right)^{5/6}\div\left(\frac{32}{100000}\right)^{6/5}$

$=\frac{2^5}{10^5}\div\frac{2^6}{10^6}=\frac{2^5}{10^5}\times\frac{10^6}{2^6}=\frac{10}{2}=5$

7. (b) : We have, $x=\left(\frac{3}{2}\right)^2\times\left(\frac{2}{3}\right)^{-4}$

$=\left(\frac{3}{2}\right)^2\times\left(\frac{3}{2}\right)^4=\left(\frac{3}{2}\right)^6$

So, $x^{-2}=\left(\left(\frac{3}{2}\right)^6\right)^{-2}=\left(\frac{3}{2}\right)^{-12}=\left(\frac{2}{3}\right)^{12}$

8. (a) : We have, $a^x=b^y=c^z$ and $b^2=ac$

For $a^x=b^y\Rightarrow a^{x/y}=b^{y/y}$

$\Rightarrow b=a^{x/y}$ …(i)

Also, $c^z = a^x \Rightarrow c^{z/z} = a^{x/z}$

$\Rightarrow c = a^{x/z}$ ...(ii)

Now, $b^2 = ac$

$\Rightarrow (a^{x/y})^2 = a \times a^{x/z}$ [From (i) and (ii)]

$\Rightarrow a^{2x/y} = a^{((x/z)+1)}$

On comparing, we get $\frac{2x}{y} = \frac{x}{z} + 1$

$\Rightarrow \frac{2x}{y} = \frac{x+z}{z} \Rightarrow y = \frac{2xz}{x+z}$

9. (c) : We have, $\frac{10}{3} \times 3^x - 3^{x-1} = 81$

$\Rightarrow 10 \times 3^{x-1} - 3^{x-1} = 81$

$\Rightarrow 3^{x-1}(10-1) = 81 \Rightarrow 3^{x-1} = 81$

$\Rightarrow 3^{x-1} = \frac{81}{9} = 9 \Rightarrow 3^{x-1} = 3^2$

On comparing, we get

$x - 1 = 2 \Rightarrow x = 2 + 1 = 3$

10. (a) :

11. (d) : We have, $(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1}$

$= \left(\frac{1}{7} - \frac{1}{8}\right)^{-1} - \left(\frac{1}{3} - \frac{1}{4}\right)^{-1} = \left(\frac{8-7}{56}\right)^{-1} - \left(\frac{4-3}{12}\right)^{-1}$

$= \left(\frac{1}{56}\right)^{-1} - \left(\frac{1}{12}\right)^{-1} = 56 - 12 = 44$

12. (b) : We have, $5 \times 10^{-3} - 2000 \times 10^{-6}$

$= 5 \times 10^{-3} - 2 \times 10^3 \times 10^{-6}$

$= 5 \times 10^{-3} - 2 \times 10^{3-6} = 5 \times 10^{-3} - 2 \times 10^{-3}$

$= (5-2) \times 10^{-3} = 3 \times 10^{-3}$

13. (c) : We have, $3^{(x-y)} = 27$ or $3^{(x-y)} = (3)^3$

$\therefore\ x - y = 3$ ...(i)

Also, $3^{(x+y)} = 243$ or $3^{(x+y)} = (3)^5$

$\therefore x + y = 5$ ...(ii)

On adding (i) and (ii), we get

$2x = 8 \Rightarrow x = 4$

14. (d) : $(216)^{3/5} = (6^3)^{3/5} = 6^{9/5}$

$= (2 \times 3)^{9/5} = 2^{9/5} \times 3^{9/5}$

$(2500)^{\frac{2}{5}} = (50)^{2 \times \frac{2}{5}} = (50)^{\frac{4}{5}}$

$= (5^2 \times 2)^{\frac{4}{5}} = 5^{8/5} \times 2^{4/5}$

$(300)^{\frac{1}{5}} = (3 \times 10^2)^{\frac{1}{5}} = 3^{\frac{1}{5}} \times 2^{\frac{2}{5}} \times 5^{\frac{2}{5}}$

$\therefore (216)^{\frac{3}{5}} \times (2500)^{\frac{2}{5}} \times (300)^{\frac{1}{5}}$

$= 2^{\frac{9}{5}} \times 3^{\frac{9}{5}} \times 5^{\frac{8}{5}} \times 2^{\frac{4}{5}} \times 3^{\frac{1}{5}} \times 2^{\frac{2}{5}} \times 5^{\frac{2}{5}}$

$= 2^{15/5} \times 3^{10/5} \times 5^{10/5} = 2^3 \times 3^2 \times 5^2$

15. (d) : We have, $\left(\frac{5}{11}\right)^{-3} \times \left(\frac{5}{11}\right)^5 = \left(\frac{5}{11}\right)^x$

$\Rightarrow \left(\frac{5}{11}\right)^{-3+5} = \left(\frac{5}{11}\right)^x \Rightarrow \left(\frac{5}{11}\right)^2 = \left(\frac{5}{11}\right)^x$

$\therefore x = 2$

16. (b) : Size of red blood cell = 0.000007 m

Size of plant cell = 0.00001275 m

$\text{Required ratio} = \frac{\text{Size of Red blood cell}}{\text{Size of Plant cell}}$

$= \frac{0.000007}{0.00001275} = \frac{7 \times 10^{-6}}{1.275 \times 10^{-5}}$

$= \frac{7}{1.275} \times 10^{-6+5} = \frac{0.7}{1.275} = \frac{28}{51}$

17. (d) :

18. (c) : Let no. of cells al present be x

No. of cells after 1 hour = $2x$

No. of cells after 2 hours = $2(2x) = 4x = 2^2 x$

No. of cells after 3 hours = $2(4x) = 8x = 2^3 x$

$\therefore$ No. of cells after 8 hours $= 2^8 x$

19. (b) :

20. (a) : Weight of moon $= (7.346 \times 10^{22})$ kg

Weight of Earth $= (5.9724 \times 10^{24})kg$

Total weight $= 7.346 \times 10^{22} + 5.9724 \times 10^{24}$

$= 0.07346 \times 10^{24} + 5.9724 \times 10^{24}$

$= (0.07346 + 5.9724) \times 10^{24}$

$= 6.04586 \times 10^{24} kg$

21. (d): We have,

$$\frac{\left[\frac{2}{3}\right]^3 \times \left[\frac{2}{3}\right]^{-2} \times \left[\left(\frac{1}{2}\right)^2\right]^{-2} \times \frac{1}{24}}{\left(\frac{2}{3}\right)^{-5} \times \left(\frac{3}{2}\right)^{-12}}$$

$$= \frac{\left(\frac{2}{3}\right)^{3-2} \times \left(\frac{1}{2}\right)^{-4} \times \frac{1}{24}}{\left(\frac{2}{3}\right)^{12-5}} = \frac{\frac{2}{3} \times 2^4 \times \frac{1}{24}}{\left(\frac{2}{3}\right)^7}$$

$$= \frac{2^5}{3} \times \frac{1}{2^3 \times 3} \times \frac{3^7}{2^7} = \frac{3^{7-2}}{2^{7+3-5}} = \frac{3^5}{2^5} = \frac{243}{32}$$

22. (b) : We have,

$$\frac{(3^{-2})^{2y-1}(3^4 \cdot 10^{-4})^{1/3}}{3^{5/2}} = \frac{3^{-(2y-5)} \cdot 3^{3\left(\frac{y-1}{3}\right)}}{10^{4/3}}$$

$$\Rightarrow \frac{3^{-4y+2+\frac{4}{3}-\frac{5}{2}}}{10^{4/3}} = \frac{3^{-2y+5+y-1}}{10^{4/3}}$$

$$\Rightarrow 3^{-4y+\frac{5}{6}} = 3^{-y+4}$$

On comparing, we get $-4y + \frac{5}{6} = -y + 4$

$$\Rightarrow -3y = 4 - \frac{5}{6} \Rightarrow -3y = \frac{24-5}{6}$$

$$\Rightarrow y = \frac{-19}{18}$$

23. (c) : (i) We have, $m^2 = 27^{2/3} \times 16^{-3/2}$

$$\Rightarrow m^2 = (3^3)^{2/3} \times (4^2)^{-3/2} \Rightarrow m^2 = 3^2 \times 4^{-3}$$

$$\Rightarrow m^2 = \frac{3^2}{4^3} = \frac{9}{64} \Rightarrow m = \frac{3}{8}$$

(ii) We have, $\frac{1}{1+a^{-1}} + \frac{1}{1+b^{-1}}$

$$= \frac{1}{1+\frac{1}{a}} + \frac{1}{1+\frac{1}{b}} = \frac{1}{\frac{a+1}{a}} + \frac{1}{\frac{b+1}{b}}$$

$$= \frac{a}{a+1} + \frac{b}{b+1} = \frac{a(b+1)+b(a+1)}{(a+1)(b+1)}$$

$$= \frac{ab+a+ab+b}{(a+1)(b+1)} = \frac{1+a+1+b}{ab+a+b+1}$$

$(\because ab = 1)$

$$= \frac{1+a+1+b}{1+a+b+1} = 1$$

(iii) We have, $x = 8^{2/3} \cdot 32^{-2/5}$

$$= (2^3)^{2/3} \cdot (2^5)^{-2/5} = 2^2 \cdot 2^{-2} = 1$$

$\therefore\ x^{-5} = (1)^{-5} = 1$

24. (a) : (P) We have,

$$\left(6^{-1} + \left(\frac{3}{2}\right)^{-1}\right)^{-1} = \left(\frac{1}{6} + \frac{2}{3}\right)^{-1}$$

$$= \left(\frac{1+4}{6}\right)^{-1} = \left(\frac{5}{6}\right)^{-1} = \frac{6}{5}$$

(Q) We have,

$$\left\{\left(\frac{4}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \left(\frac{3}{4} - \frac{4}{1}\right)^{-1}$$

$$= \left(\frac{3-16}{4}\right)^{-1} = \left(\frac{-13}{4}\right)^{-1} = \frac{-4}{13}$$

(R) We have, $\left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-3}$

$$= (3^3 - 2^3) \div 4^3 = (27-8) \div 64 = \frac{19}{64}$$

(S) We have, $(3^{-1} \times 4^{-1}) \times \left(\frac{2}{3}\right)^{-3}$

$$= \left(\frac{1}{3} \times \frac{1}{4}\right) \times \left(\frac{3}{2}\right)^3 = \frac{1}{12} \times \frac{27}{8} = \frac{9}{32}$$

25. (d) : We have, $a = 2^{-2} - 2^{-3}$

$$= \frac{1}{2^2} - \frac{1}{2^3} = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$$

Also, $b = 2^{-3} - 2^{-4}$

$$= \frac{1}{2^3} - \frac{1}{2^4} = \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$$

And $c = 2^{-4} - 2^{-2}$

$$= \frac{1}{2^4} - \frac{1}{2^2} = \frac{1}{16} - \frac{1}{4} = \frac{-3}{16}$$

(i) $a^3 + b^3 + c^3 = \left(\frac{1}{8}\right)^3 + \left(\frac{1}{16}\right)^3 + \left(\frac{-3}{16}\right)^3$

$$= \frac{1}{512} + \frac{1}{4096} - \frac{27}{4096} = \frac{8+1-27}{4096}$$

$$= \frac{-18}{4096} = \frac{-9}{2048}$$

(ii) $10abc = 10\left(\frac{1}{8}\right)\left(\frac{1}{16}\right)\left(\frac{-3}{16}\right) = -\frac{15}{1024}$