

Integers

Location of Integers on Number Line

A line which is used to represent numbers graphically is called a **number line**. This line can be of any length and it has both positive and negative numbers along with zero. The numbers on number line are marked off at equal distances from each other.

However, we do not know where to mark negative and positive numbers on this number line.

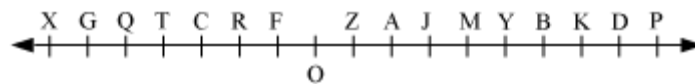
We can also find the predecessor and successor of a number using a number line. Let us see how.

1. To find the predecessor of a number using a number line, we have to move 1 unit to the left of the given number. The result of this activity gives us the predecessor of the number.
2. To find the successor of a number using a number line, we have to move 1 unit to the right of the given number. The result of this activity gives us the successor of the number.

Let us discuss some examples based on the location of integers on the number line.

Example 1:

The following figure is a horizontal number line representing integers.



Observe the number line and give answers to the following questions.

- (a) If M is 4, then which points represent the integers -4 , -6 , and 7?
- (b) Which point on the number line represents neither a negative number nor a positive number?
- (c) Write the integers for the points J, D, R, and Q.
- (d) Is X a positive or a negative integer?

Solution:

(a) It is given that point M represents the integer 4 i.e., M represents +4. By moving 1 unit to the left of M, we will reach at point J. This point J represents the location of the integer 3. When we keep on moving 3 units to the left, we will be at point O. This point O represents the location of the integer 0.

To locate the integer -4 on the number line, we will move 4 units to the left of O. On doing so, we will reach at point T. Thus, point T represents the location of the integer -4 on the number line. In this way, we will locate the integers -6 and 7 on the given number line. After doing so, we will obtain point G as -6 and K as 7 .

Thus, the points T, G, and K represent the location of the integers -4 , -5 , and 7 respectively on the number line.

(b) We know that 0 can be written as $+0$ or -0 . Therefore, 0 is such a number that is neither negative nor positive. On the given number line, the position of 0 is represented by point O.

Therefore, O is the only point on the number line that represents neither a negative number nor a positive number.

(c) For the given number line, point O represents the integer 0. In order to reach J, we will move 3 steps to the right of the point O i.e., 0. Thus, point J represents the integer 3 on the number line.

In this way, we can find the integers represented by the points D, R, and Q with reference to point O. They are 8, -2 , and -5 respectively.

Therefore, the integers for the points J, D, R, and Q are 3, 8, -2 , and -5 respectively.

(d) If we move 7 points to the left of O, then we will reach at point X. Thus, point X represents the integer -7 . However, -7 is a negative integer. Hence, X is a negative integer.

Example 2:

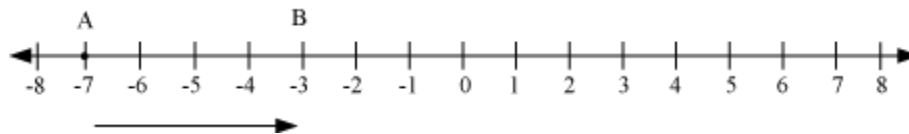
Answer the following questions by drawing a number line.

(a) If we are at -7 on the number line, then in which direction should we move to reach -1 ?

(b) If we are at -2 on the number line, then in which direction should we move to reach -8 ?

Solution:

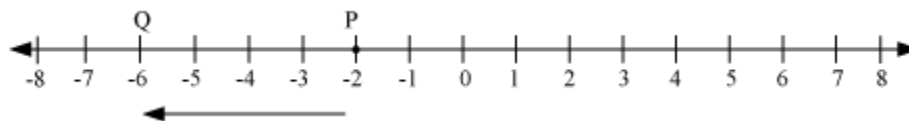
(a) Consider the following number line.



On the number line, the integer -1 is to the right of -7 .

Therefore, to reach -1 , we should move towards the right of -7 .

(b) Consider the following number line.



On the number line, the integer -8 is to the left of -2 .

Therefore, to reach -8 , we should move towards the left of -2 .

Example 3:

Draw a number line and answer the following questions.

(a) Which number will we reach, if we move 3 units to the right of -5 ?

(b) Which number will we reach, if we move 4 units to the left of 3?

Solution:

(a) Let us consider the following number line.

Thus, the predecessor and successor of 5 are 4 and 6 respectively.

(b) If we observe the number line carefully, then we will find that we will be

(i) at -1 , if we move 1 unit towards the left of 0 and

(ii) at 1, if we move 1 unit towards the right of 0

Thus, the predecessor and successor of 0 are -1 and 1 respectively.

(c) If we observe the number line carefully, then we will find that we will be

(i) at -8 , if we move 1 unit towards the left of -7 and

(ii) at -6 , if we move 1 unit towards the right of -7

Thus, the predecessor and successor of -7 are -8 and -6 respectively.

Comparing and Ordering Integers

We use mathematical symbols such as $>$ (Greater than), $<$ (Less than), and $=$ (Equal to) to make a comparison between two numbers.

How will we compare two numbers with the help of a number line?

We use this concept to compare between integers and put the symbols ($>$, $<$, and $=$) accordingly.

If we compare between -8 and -2 with the help of number line, then we will find that -8 is to the left of -2 . Hence, we can write $-8 < -2$.

Sometimes it is difficult to locate large integers such as 5093, 805, etc. and small integers such as -596 , -8053 , etc. on a number line. In such cases, we cannot compare the integers using number line. We will solve the problem by keeping some facts in our mind. They are as follows.

1. All positive integers are greater than 0 and the negative numbers. For example, $20 > 0$ and $30 > -3$.
2. 0 is always greater than negative integers. For example, $0 > -5$.

3. To compare two negative integers, we first neglect the negative sign and then if the integer without negative sign is greater than the other integer, then that negative integer is smaller than the other negative integer.

For example, if we compare between -112 and -535 , then first of all we have to neglect the negative sign of both the numbers. After neglecting the negative sign, the numbers are 112 and 535 . Here, $535 > 112$. Hence, $-535 < -112$

So far we have learnt the comparison of two integers. Now, let us discuss ordering of more than two integers. For ordering of integers, we use the concept of comparison of integers.

Let us discuss some examples based on comparison and ordering of integers.

Example 1:

Fill in the boxes with appropriate signs ($>$ or $<$ or $=$) using a number line.

(a) -5 8

(b) -3 -7

(c) -8 -4

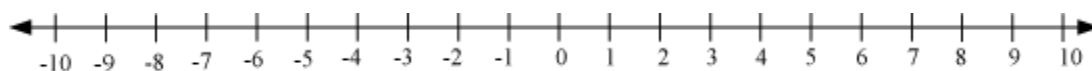
(d) 2 -9

(e) -3 0

(f) -6 -6

Solution:

Let us draw a number line as follows.



- (a) On comparing -5 and 8 on the number line, we observe that -5 is to the left of 8 .

Therefore, -5 8

- (b) On comparing -3 and -7 on the number line, we observe that -3 is to the right of -7 .

Therefore, $-3 \boxed{>} -7$

(c) On comparing -8 and -4 on the number line, we observe that -8 is to the left of -4 .

Therefore, $-8 \boxed{<} -4$

(d) Oncomparing 2 and -9 on the number line, we observe that 2 is to the right of -9 .

Therefore, $2 \boxed{>} -9$

(e) Oncomparing -3 and 0 on the number line, we observe that -3 is to the left of 0 .

Therefore, $-3 \boxed{<} 0$

(f) -6 is equal to -6 itself. It is neither greater nor smaller than itself.

Therefore, $-6 \boxed{=} -6$

Example 2:

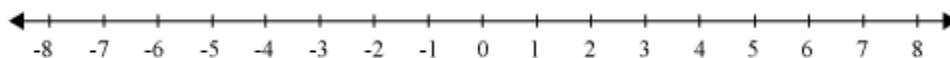
The temperature of Shimla is recorded to be maximum of 5°C and minimum of -4°C . Also, the temperature of Dehradun is recorded to be maximum of 8°C and minimum of -2°C . Which of the two places has

(i) lower maximum temperature

(ii) higher minimum temperature

Solution:

We can easily compare the lower maximum temperatures and the higher minimum temperatures of the two cities with the help of a number line. Let us draw a number line, which contains the maximum as well as the minimum temperature of the cities as shown below.



(i) Maximum temperature of Shimla = 5°C

Maximum temperature of Dehradun = 8°C

From the number line, we can observe that $5^{\circ}\text{C} < 8^{\circ}\text{C}$.

Therefore, the maximum temperature of Shimla is lower than that of Dehradun.

(ii) Minimum temperature of Shimla = -4°C

Minimum temperature of Dehradun = -2°C

From the number line, we can observe that $-2^{\circ}\text{C} > -4^{\circ}\text{C}$.

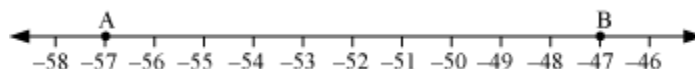
Therefore, the minimum temperature of Dehradun is higher than that of Shimla.

Example 3:

Calculate all integers between -57 and -47 by drawing a number line. Write them in increasing order as well as in decreasing order. Also, find the smallest and the greatest numbers between them.

Solution:

Let us consider a number line which contains all the integers between -57 and -47 as shown below.



On the above number line, the integer -57 is represented by point A and -47 is represented by point B. From the above number line, we can clearly observe that the integers lying between points A and B are $-56, -55, -54, -53, -52, -51, -50, -49$, and -48 .

If we arrange the above integers in increasing order, then we will obtain

$$-56 < -55 < -54 < -53 < -52 < -51 < -50 < -49 < -48$$

If we arrange the integers in decreasing order, then we will obtain

$$-48 > -49 > -50 > -51 > -52 > -53 > -54 > -55 > -56$$

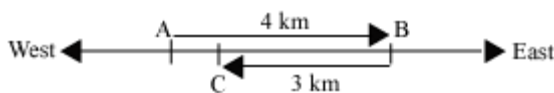
From the number line, we can observe that the highest integer is -48 and the lowest integer is -56 between the given integers.

Addition of Integers

Rahul walked 4 km towards East. Then, he walked back 3 km towards West. Now, he is wondering about the distance between his initial and final positions.

Can we help him?

To find the distance between the initial and final positions of Rahul, let us consider that initially Rahul was at the position A. He walked 4 km towards East to reach the position B. From B, he moved 3 km towards West to reach the final position C. It can be shown by a figure as:



We will use ‘+’ sign when Rahul moves towards East. When he moves towards West, i.e., opposite to East, we will use “-” sign. Thus, we will take the distance AB as +4 km and the distance BC as -3 km.

Now, we are required to calculate the distance between the initial position and the final position. We can calculate this value if we can calculate the distance AC.

But from the above figure, it is clear that the distance AC is equal to the addition of +4 km and -3 km i.e. $[(+4) + (-3)]$ km. We can find this value if we can find the value of $(+4) + (-3)$.

We know how to add counting numbers. But the expression $[(+4) + (-3)]$ km is the addition of integers. We can calculate the value of this expression if we know the method of addition of integers. First of all, let us know this method. Then we will find the value of the expression $[(+4) + (-3)]$.

We can use three different methods for addition of integers. They are as follows:

1. Using number line

2. Using concrete material and
3. Using Standard algorithm

Let us start with *addition of integers using number line*.

In this method, we add integers by drawing a number line. We perform addition by following these steps:

Step 1: We always start addition from 0. If the first integer is a positive integer, then we move towards the right of 0 on the number line with number of units equivalent to the given number. Similarly, if the first integer is a negative integer, then we move towards the left of 0 on the number line with number of units equivalent to the given number. Now, we mark this point.

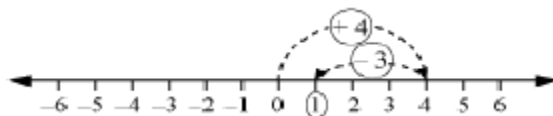
Step 2: When we add a positive integer to the previous integer, we move towards the right of the above marked point on the number line with number of units equivalent to this integer. Similarly, when we add a negative integer to the previous integer, we move towards the left of the above marked point on the number line with number of units equivalent to this integer. Now we mark this point.

Step 3: We continue this process. The final marked point on the number line will represent the result of addition of integers.

Suppose we have to carry out the addition of integers -2 and -7 i.e., we have to find the value of the expression $[(-2) + (-7)]$.

Now let us find the distance between the initial and final positions of Rahul using this method.

We have to find the value of the expression $(+4) + (-3)$. Here, $(+4)$ is a positive integer and (-3) is a negative integer. Therefore, we move 4 steps to the right of 0 on the number line. By doing so, we will reach at the point 4. The second integer is (-3) . Now, we will move 3 steps to the left of 4 on the number line. By doing so, we will reach at the point 1. This activity can be represented on a number line as follows:



Here, the final point is 1. This is the result of expression $4 + (-3)$.

Now, $4 + (-3) = 1$

Therefore, $[4 + (-3)] \text{ km} = 1 \text{ km}$

Thus, the distance between the initial and final positions of Rahul is 1 km.

Now, let us discuss the second method of addition of integer i.e., ***addition of integers using concrete material.***

In this method, we add integers by using some materials such as different colours of sticks, buttons etc. We can take a red stick for positive integer +1 and a green stick for negative integer -1 or vice-versa. We can also take 1 black coloured button for positive integer +1 and one white coloured button for negative integer -1 or vice-versa. After doing so, we will add the integers easily. It can easily be understood with the help of an example.

Let us now look how do we add these integers $\{(-5) \text{ and } 3\}$ using concrete material.

Let us find the distance between the initial and final positions of Rahul using this method.

We have to find the value of the expression $(+4) + (-3)$. Here, $(+4)$ is a positive integer and (-3) is a negative integer. Therefore, we take 4 black coloured buttons and three white coloured buttons.

Now, $(+4) + (-3)$



=



$(+1) + (+3) + (-3)$



$= (+1) + 0 = (+1)$

Thus, the distance between the initial and final positions of Rahul is 1 km.

Let us now discuss how integers can be added by using **Standard Algorithm Method** and in what cases we should use this method.

Let us find the distance between the initial and final positions of Rahul using this method.

Now, we have to find the value of the expression $(+4) + (-3)$.

First, we have to ignore the signs and subtract 3 from 4

i.e., $4 - 3 = 1$

Between the two numbers, 4 is bigger. But the sign of 4 is positive.

Thus, putting positive sign to the previous result i.e., 1, we obtain

$$(+4) + (-3) = (+1)$$

Therefore, the distance between the initial and final positions of Rahul is 1 km.

What happen if we add 9 and (-9) ?

If we add 9 and -9 , we have, $9 + (-9) = 0$

The sum of 9 and -9 is zero. These integers i.e., 9 and -9 are known as **additive inverse** of each other.

Thus, additive inverse can be defined as follows:

“Two integers are said to be additive inverse of each other if their sum is equal to zero”.

Writing Integers:

We have studied about positive and negative integers and learned to write the same with their respective signs. For example, $+4$, -9 etc. Conventionally, positive integers are written without using sign. So, if a number is written without any sign, it is

considered to be positive. Thus, we write 4 instead of +4 and read it as "four" rather than "positive four".

Similarly, we can avoid + sign of positive integers while writing and reading the expressions involving addition of integers.

For example,

$(+4) + (-3)$ can be written as $4 + (-3)$ and read as "four plus negative three".

$(-98) + (+45)$ can be written as $(-98) + 45$ and read as "negative ninety-eight plus forty-five".

$(-65) + (-9)$ can be written as $(-65) + (-9)$ and read as "negative sixty-five plus negative nine".

$(+115) + (+13)$ can be written as $115 + 13$ and read as "hundred and fifteen plus thirteen".

From these examples, it is clear that + sign is used to represent positive integers as well as to add integers. It can be easily identified from the expression that what + sign means at a particular place.

Basically, + sign is written with positive integers at learning level, but when we become familiar with the concept, we write positive integers without using sign. Henceforth, + sign is used only for addition.

Let us discuss some examples based on addition of integers.

Example 1:

Find the value of the following expressions using number line.

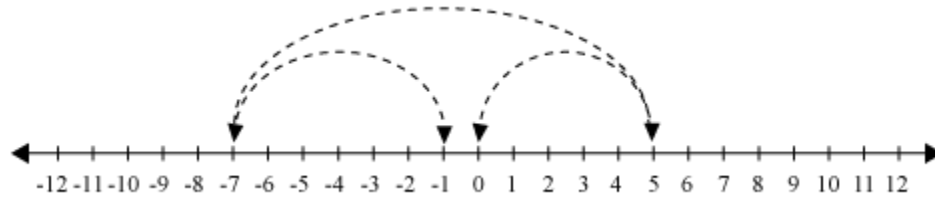
(i) $(+5) + (-12) + (+6)$

(ii) $(-4) + (+9) + (-5)$

Solution:

(i) To find the value of $(+5) + (-12) + (+6)$, we have to add the integers (+5), (-12) and (+6).

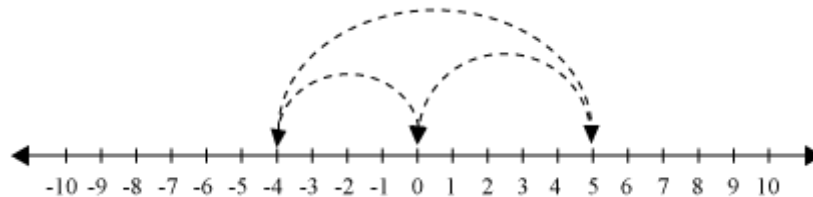
First, we move 5 steps towards right from 0 to reach 5. Then, we move 12 steps towards left to reach -7 . At last, we move 6 steps towards right to reach the final position -1 , which can be shown on a number line as follows.



Thus, $(+5) + (-12) + (+6) = -1$

(ii) To find the value of $(-4) + (+9) + (-5)$, we have to add the integers (-4) , $(+9)$ and (-5) .

First we move 4 steps towards left starting from 0 to reach -4 . Then, we move 9 steps towards right to reach $+5$. At last, we move 5 steps towards left to reach the final position 0, which can be shown on a number line as follows.



Thus, $(-4) + (+9) + (-5) = 0$

Example 2:

Find the values of the following expressions.

(i) $176 + (-312)$

(ii) $(-12) + (-9) + 27$

(iii) $(-32) + (-19) + 46 + (-52)$

Solution:

(i) $176 + (-312)$

Here, one integer is positive and the other is negative. Therefore, by ignoring the sign and by subtracting the smaller number from the bigger number, we obtain

$$312 - 176 = 136$$

But the bigger number has “−” sign,

$$\text{Thus, } 176 + (-312) = -136$$

$$\text{(ii) } (-12) + (-9) + 27$$

$$= \{(-12) + (-9)\} + 27$$

(Arranging all the negative integers and positive integers in groups)

$$= (-21) + 27$$

$$= 6$$

$$\text{(iii) } (-32) + (-19) + 46 + (-52)$$

$$= \{(-32) + (-19) + (-52)\} + 46$$

(Arranging all the negative integers and positive integers in groups)

$$= (-103) + 46$$

$$= -57$$

Example 3:

Find the missing number.

$$\text{(i) } \underline{\hspace{1cm}} + (-35) = 0$$

$$\text{(ii) } (+26) + \underline{\hspace{1cm}} + (-19) = -19$$

Solution:

(i) The missing number is +35. (Additive inverse of −35)

(ii) The missing number is −26. (Additive inverse of 26)

Example 4:

Write the following expressions without using + sign of positive integers. Also, write how to read the new expressions.

(i) $(+2) + (-19)$

(ii) $(-28) + (+19)$

(iii) $(-71) + (-34)$

(iv) $(+156) + (+64)$

Solution:

The given expression can be written without + sign of positive integers as follows:

(i) $(+2) + (-19)$ can be written as $2 + (-19)$ and read as "two plus negative nineteen".

(ii) $(-28) + (+19)$ can be written as $(-28) + 19$ and read as "negative twenty-eight plus nineteen".

(iii) $(-71) + (-34)$ can be written as $(-71) + (-34)$ and read as "negative seventy-one plus negative thirty-four".

(iv) $(+156) + (+64)$ can be written as $156 + 64$ and read as "hundred and fifty-six plus sixty-four".

Subtraction of Integers

On a particular day, the temperature of a place was recorded as -2°C . On the next day, the temperature of the place was recorded as -5°C . By how many degrees did the temperature decrease?

It is given that the temperature of the place was -2°C . On the next day, the temperature was -5°C . The decrease in degree of temperature is the difference of the temperatures -2°C and -5°C .

Thus, decrease in temperature = $(-2)^{\circ}\text{C} - (-5)^{\circ}\text{C} = [(-2) - (-5)]^{\circ}\text{C}$

We can find the result, if we can find the value of $[(-2) - (-5)]$.

We know how to carry out the subtraction of counting numbers. However, the expression $[(-2) - (-5)]$ is the expression of subtraction of integers. We can calculate the value of this expression if we know the method of subtraction of integers. First of all, let us know this method. Then, we will find the value of the expression $[(-2) - (-5)]$.

Two integers can be subtracted by two methods.

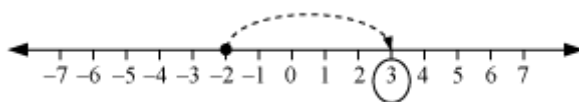
1. Using number line
2. Using standard algorithm

Let us start with the method of *subtraction of two integers using number line*.

In this method, we subtract integers by drawing a number line. Let us simplify $(-2) - 4$ on the number line.

Now, we can find the decrease in temperature between the two days, which was mentioned in the beginning.

The decrease in temperature was $[(-2) - (-5)]^{\circ}\text{C}$. Here, we have to subtract (-5) from (-2) . For this, first of all, we will locate (-2) on the number line. The integer (-5) is a negative integer. Therefore, we will move 5 units to the right of (-2) . Now, we will reach 3. This can be shown on a number line as follows.



We have, $[(-2) - (-5)] = 3$

Therefore, $[(-2) - (-5)]^{\circ}\text{C} = 3^{\circ}\text{C}$

Thus, the decrease in temperature between the two days is 3°C .

Can we find the value of $(-502) - (-705)$ using number line?

Since these are large numbers and it is quite difficult to perform their subtraction on number line, we use standard algorithm method in such cases.

Now, we can find the difference in temperatures of the two days, which was mentioned in the beginning of this learning section, using standard algorithm method.

$$\begin{aligned} & [(-2) - (-5)] ^\circ\text{C} \\ & = [(-2) + (+5)] ^\circ\text{C} \text{ \{Additive inverse of } (-5) \text{ is } (+5)\}} \\ & = 3^\circ\text{C} \end{aligned}$$

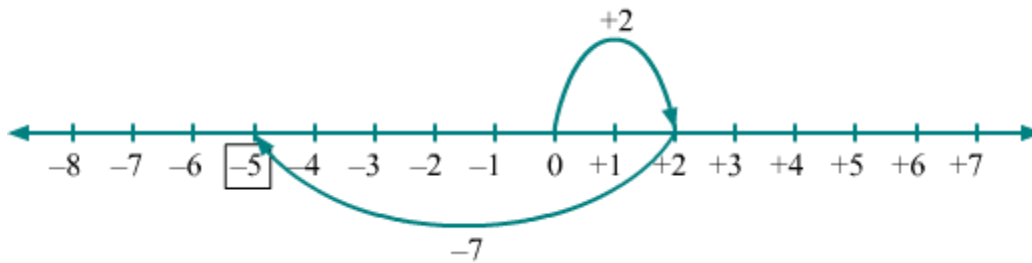
Let us discuss some more examples based on subtraction of integers.

Example 1:

Subtract 7 from 2 by using number line.

Solution:

In order to subtract 7 from 2, firstly locate the position of +2 on number line. Then, from +2; move 7 units to the left. This can be done as,



Here, the tip of the finale arrow is at -5 .

$$\text{So, } 2 - 7 = -5$$

Example 2:

Find the value of the following expressions.

(a) $48 - (-52) - (-25)$

(b) $(-74) + 32 - 19 - 13$

(c) $(-63) - 37 - (-100)$

Solution:

(a) $48 - (-52) - (-25)$

$= 48 + (+52) + (+25)$ [Additive inverse of (-52) is $(+52)$ and additive inverse of (-25) is $(+25)$]

$$= 125$$

(b) $(-74) + 32 - 19 - 13$

$= (-74) + (32) + (-19) + (-13)$ {Additive inverse of 19 is (-19) and 13 is (-13) }

$= \{(-74) + (-19) + (-13)\} + (32)$ (Arranging all the negative integers and positive integers in groups)

$$= (-106) + (32)$$

$$= -74$$

(c) $(-63) - 37 - (-100)$

$= (-63) + (-37) + (+100)$ {Additive inverse of 37 is (-37) and (-100) is 100>}

$= \{(-63) + (-37)\} + (+100)$ (Arranging all the negative integers and positive integers in groups)

$$= (-100) + (+100)$$

$$= 0$$

Example 3:

Find the missing numbers.

(i) $(+14) - \underline{\hspace{1cm}} + (-10) = (+11)$

(ii) $(-22) - (+15) - \underline{\hspace{1cm}} = (-6)$

Solution:

- (i) Let the missing number be x .

Thus, the given question becomes

$$(+14) - x + (-10) = (+11)$$

Now, on adding x to both sides, we obtain

$$(+14) - x + (-10) + x = (+11) + x$$

$$\Rightarrow (+14) + (-10) = (+11) + x$$

$$\Rightarrow (+4) = (+11) + x$$

Now, on subtracting $+11$ from both sides, we obtain

$$(+4) - (+11) = (+11) + x - (+11)$$

On subtracting $+11$ from both sides, we obtain

$$(+4) - (+11) = (+11) + x - (+11)$$

$$\Rightarrow (+4) + (-11) = 11 + x + (-11) \quad [\text{Additive inverse of 11 is } -11]$$

$$\Rightarrow (-7) = 11 + (-11) + x$$

$$\Rightarrow (-7) = x$$

$$\Rightarrow x = -7$$

$$\text{Hence, } (+14) - \underline{(-7)} + (-10) = (+11)$$

- (ii) Let the missing number be x .

Thus, the given question becomes

$$(-22) - (+15) - x = (-6)$$

On adding x to both sides, we obtain

$$(-22) - (+15) - x + x = (-6) + x$$

$$\Rightarrow (-22) + (-15) = x + (-6) \quad [\text{Additive inverse of } +15 \text{ is } -15]$$

$$\Rightarrow (-37) = x + (-6)$$

On adding 6 to both sides, we obtain

$$(-37) + 6 = x + (-6) + 6$$

$$\Rightarrow (-31) = x \quad [\text{Additive inverse of } -6 \text{ is } +6]$$

$$\Rightarrow x = -31$$

$$\text{Hence, } (-22) - (+15) - \underline{(-31)} = (-6)$$

Example 4:

Fill in the blanks with $>$, $<$ or $=$ sign.

(a) $(-13) + (-16)$ _____ $(-13) - (-16)$

(b) $(-18) - (-20)$ _____ $(-17) - (-11)$

(c) $92 - (-19)$ _____ $117 + (-5)$

Solution:

To put $>$, $<$ or $=$ sign, we have to simplify both the sides.

(a) LHS = $(-13) + (-16) = -29$

RHS = $(-13) - (-16) = (-13) + (16) = 3$

However, $-29 < 3$

Hence, $(-13) + (-16) < (-13) - (-16)$

(b) LHS = $(-18) - (-20) = (-18) + 20 = 2$

RHS = $(-17) - (-11) = (-17) + 11 = -6$

However, $2 > -6$

Hence, $(-18) - (-20) > (-17) - (-11)$

(c) LHS = $92 - (-19) = 92 + 19 = 111$

RHS = $117 + (-5) = 112$

However, $111 < 112$

Hence, $92 - (-19) < 117 + (-5)$