Circular Motion

Exercise Solutions

Solution 1:

Distance between the earth and the moon = Radius of Revolution = R = 385×10^5 m

Angular frequency, $\omega = 2\pi f = 2\pi/T$

Time taken by moon to complete one revolution = T = 273 days = $273 \times 24 \times 60 \times 60$ secs = 23587200 secs

Acceleration of the moon = $\omega^2 R = (2\pi/T)^2 R = 4\pi^2 R/T^2$

Substituting the values, we have

Acceleration of the moon = 2.73×10^{-7} m/s^2

Solution 2:

Radius of earth = R = half of diameter = Diameter of earth/2 = 12800/2 km = 6400 km = 6.4×10^6 m

Time taken for one rotation = T = 24 hours = $24 \times 60 \times 60$ sec = 86400 secs

Angular velocity of the particle, $\omega = 2\pi/T$

Therefore, acceleration of the particle = $R\omega^2$

Substituting the values, we have

acceleration of the particle = 0.0338 m/s^2

Solution 3:
V = 2t and Radius = r = 1.0 cm

(a) At t = 1s

Velocity of the particle, u = 2.0 t = 2.0×1 = 2.0 cm/s

Radial acceleration of the particle at t = 1 is $v^2/r = (2.0)^2/1 = 4\ cm/s^2$

(b)
We know, a = dv/dt = d/dt(2t) = 2 cm/s^2

Tangential acceleration at t=1 is 2 cm/s^2

(c) Magnitude of acceleration at t = 1sec

$$\text{Magnitude of acceleration} = \sqrt{(\text{Radial acceleration})^2 + (\text{Tangential acceleration})^2}$$

= $\sqrt{20}\ cm/s^2$

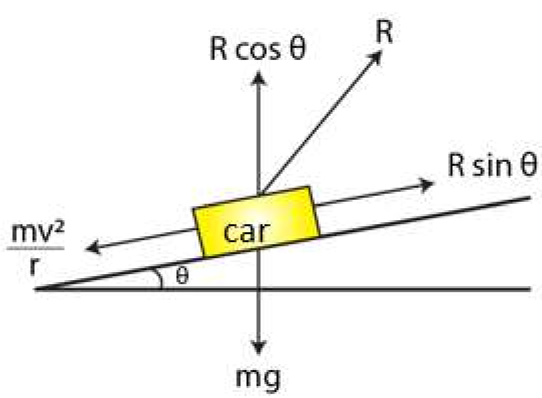
Solution 4:
Mass of the scooter = M = 150 kg
Velocity = v = 36 km/hr = 36 × 5/18 m/s = 10 m/s
and Radius = r = 30 m

Horizontal force required = centripetal force = Mv^2/r

=150×10×10/30

= 500 N

Solution 5:



Velocity of the car = v = 10m/s
Radius = r = 30m
Consider,
mass of the car=m, acceleration due to gravity = g, Angle of banking = θ and Normal reaction = R

From the diagram,
$R\cos\theta = mg$

$R\sin\theta = mv^2/r$
Now,
$\tan\theta = v^2/rg$

or $\theta = \tan^{-1}(v^2/rg)$

or $\theta = \tan^{-1}(10\times10/30)$

or $\theta = \tan^{-1}(1/3)$: Angle of banking

Solution 6:
Given:
Average speed = v = 18km/hr or 5 m/s

Radius of park = r = 10 m

Acceleration due to gravity = g = $10m/s^2$

We know, angle of banking, $\theta = \tan^{-1}(v^2/rg)$

$= \tan^{-1}(1/4)$

Solution 7:
The road is horizontal (no banking)

$mv^2/R = \mu N$ and $N = mg$

So, $mv^2/R = \mu mg$

=> $25/10 = \mu g$

=> $\mu = 25/100 = 0.25$

Solution 8:
We know that, $\tan\theta = v^2/rg$

Radius of the circular path = r = 50 m
Acceleration due to gravity = g = $10 m/s^2$
Angle of banking = θ = 30°

=> $\tan 30° = v^2/(50x10)$

=> v = 17 m/s

Vehicle should go with a speed of 17 m/s.

Solution 9:
Centripetal force is provided by coulomb attraction.

Mass of electron, $m = 9.1 \times 10^{-31}$ kg

Radius of the circular path, $r = 5.3 \times 10^{-11}$ m

$k = 1/(4\pi\varepsilon 0) = 9\times10^9$

q_1 = charge on an electron= q_2 = charge on the proton = $q = 1.6 \times 10^{-19}$ C

We know, $v^2 = kq^2/mr$

=> $v^2 = [9\times10^9\times1.6\times10^{-19}\times1.6\times10^{-19}] / [9.31\times10^{-31}\times5.3\times10^{-11}]$

or $v = 2.2 \times 10^6$ m/s (approx)

Which is the speed of the electron in ground state.

Solution 10:

At the highest point of a vertical circle.

$mv^2/R = mg$

=> $v = \sqrt{Rg}$

Solution 11:
Diameter = d = 120 cm = 1.2 m

Radius = r = d/2 = 0.6 m

Frequency = f = 1500 rpm = 1500/60 rps = 25 s^{-1}

Now,
Angular frequency, $\omega = 2\pi f$ = 157.14 rad s^{-1}
[putting value of f]

Again,
Mass of the particle, m = 1g =0.001 kg

Force on the particle on the blade = $mr\omega^2$

= 0.001 × 0.6 × 157.14 ×157.14
= 14.81 N
The fan runs at the full speed in circular path. This exerts the force on the particle.
Force exerted by the particle on the blade = Force exerted on the particle = 14.81 N

Solution 12:

A mosquito is sitting on an L.P. record disc rotating on a turn table at 33_1/3 revolutions per minute. (given)

say, n = 33_1/3 rpm = 100/(3x60) rps

we know, $\omega = 2\pi n = 10\pi/9$ rad/sec

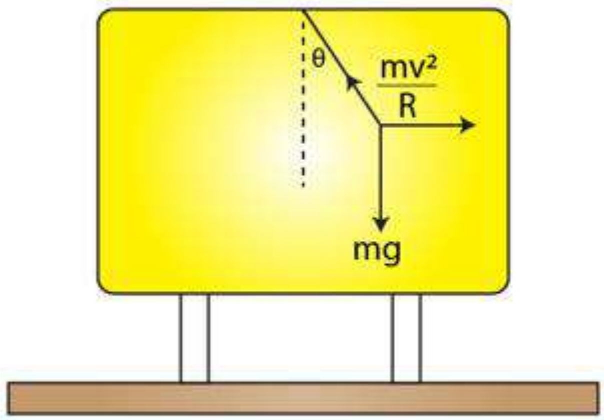
Also, we are given with
radius = r = 10 cm or 0.1 m
and g = 10 m/sec^2

$$\mu mg \geq mr\omega^2 \Rightarrow \mu = \frac{r\omega^2}{g} \geq \frac{0.1\times\left(\frac{10\pi}{9}\right)^2}{10}$$

$$\Rightarrow \mu \geq \frac{\pi^2}{81}$$

Solution 13:

Radius = r = 10 m, speed = v = 36 km/h and g = 10 m/s^2



The speed of the car is not changing. So, there is no force acting in the tangential direction of motion.

From the figure:

$T \sin \theta = mv^2/r$ ....(1)
and $T \cos \theta = mg$ ....(2)

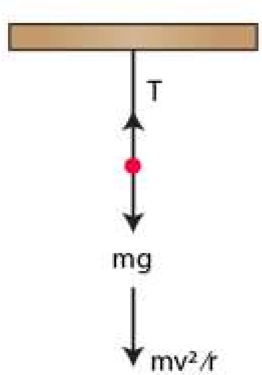
=> $\tan \theta = v^2/rg$

or $\theta = \tan^{-1}(v^2/rg)$

After substituting and solving the above equation, we have

$\theta = 45^o$

Solution 14:
A simple pendulum always constitutes some velocity at the lowest point. This proves that there exists a non-zero central force in the system. This central force will contribute to increase tension.



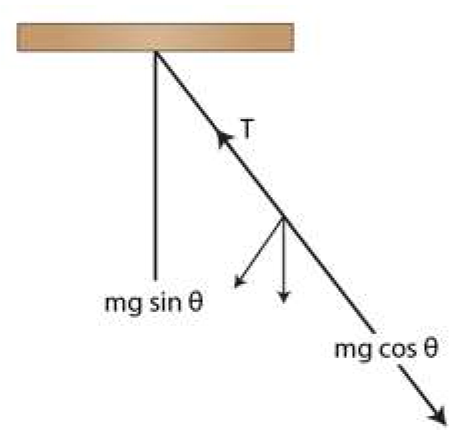
At the lowest point:

$T = mg + v^2/r$

Here m = 100g or 1/10 kg, r = 1m and v = 1.4 m/s

$T = 1/10 \times 9.8 \times (1.4)^2/10 = 1.2$ N

Solution 15:



The bob has a velocity 1.4 m/s, when the string makes an angle of 0.2 radian.

Here m = 100g or 0.1 kg and r = 1m and v = 1.4 m/s

From figure,

$T - mg\cos\theta = mv^2/r$

$\Rightarrow T = 0.196 + 9.8(1-0.04/2)$
$[\cos\theta = 1 - \theta^2/2$ for small $\theta]$

$\Rightarrow T = 0.196 + (0.98)(0.98)$

= 1.16N (approx)

Solution 16:
When a simple pendulum reaches the extreme position, velocity of the pendulum is zero.
=> no centrifugal force.

$\Rightarrow T = mg\cos\theta_o$

Solution 17:

(a) As person is standing at one position, he is in an equilibrium with the reaction force.

Net force on the spring balance:

$R = mg - m\omega^2 r$

The actual weight of the body,

$W = mg$

So, the decrement will be,

$$\frac{R - W}{W} = \frac{mg - mg + m\omega^2 R}{mg}$$

$$= \frac{\omega^2 R}{g}$$

and, Angular speed of earth, $\omega = 2\pi/(24x60^2)$ Hz

So, it will decrease at a rate,

$$\frac{\left\{\left(\frac{2\pi}{24 \times 60 \times 60}\right)^2 \times 6400 \times 10^3\right\}}{10}$$

$= 3.5 \times 10^{-3}$

(b) When the balance reading is half the true weight

$mg - m\omega^2 R = 1/2\ mg$

$(1/2)\ g = \omega^2 R$

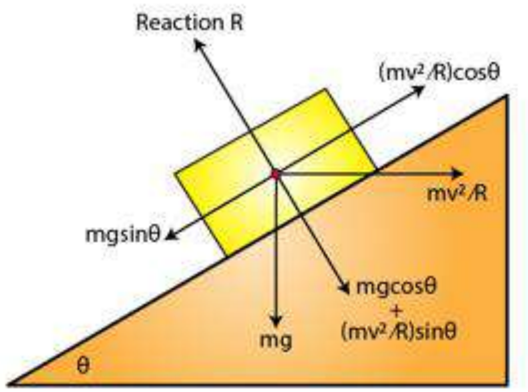
$$\omega = \sqrt{\left(\frac{g}{2R}\right)}$$

$$\frac{2\pi}{T} = \sqrt{\left(\frac{g}{2R}\right)}$$

$$T = 2\pi\sqrt{\left(\frac{2R}{g}\right)}$$

= 7105 sec or 2 hrs(approx)

Solution 18:



Let θ be the path has an inclination. There is no movement in vertical direction. So,

$R = mg \cos\theta + mv^2/R \sin\theta$

If the vehicle is not to be slide up then the friction must reinforce the overall force. Condition must be

$\mu R = mg \sin\theta - mv^2/R \cos\theta$

From above equations, we have

$$\mu\left(mg\cos\theta + \frac{mv^2}{R}\sin\theta\right) = mg\sin\theta - \frac{mv^2}{R}\cos\theta$$

$$\mu\left(g\cos\theta + \frac{v^2}{R}\sin\theta\right) = g\sin\theta - \frac{v^2}{R}\cos\theta$$

$$v = \sqrt{\frac{(-\mu\cos\theta + \sin\theta)gR}{\cos\theta + \mu\sin\theta}}$$

$$v = \sqrt{\frac{(-\mu + \tan\theta)gR}{1 + \mu\tan\theta}}$$

Find tanθ:

$\tan\theta = v^2/Rg$

Putting, $v = (36x1000)/(60x60)$, $R = 20$ and $g = 10$, we have

$\tan\theta = 0.5$

=> $v = 14.7$ km/h

The condition for not to skid down will be

$\mu R = mv^2/R \cos\theta - mg \sin\theta$

Solving above equation, we get

$v = 54$ km/h

The possible speeds are between 14.7 km/h and 54 km/h.

Solution 19:

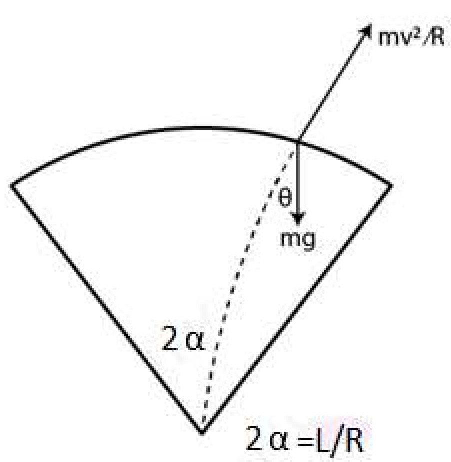
Let us consider, L be the total length of the over bridge and R be the radius of the bridge.

(a) At the highest point

$mg = mv^2/R$

or $v = \sqrt{(Rg)}$

(b) Distance $\pi R/3$ along the bridge from the highest point,



The condition for just flying off will be,

$$\frac{\left(\frac{1}{\sqrt{2}} mv\right)^2}{R} = mg\cos\theta$$

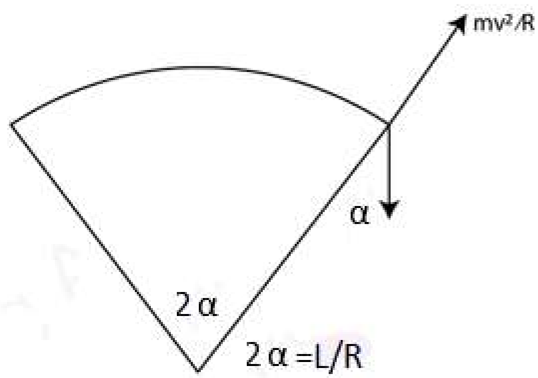
or cos θ = 1/2

or θ = π/3

we can also write, θ = x/R

or x = πR/3, is the distance where it lose the contact when it moves with 1/√2 times the maximum.

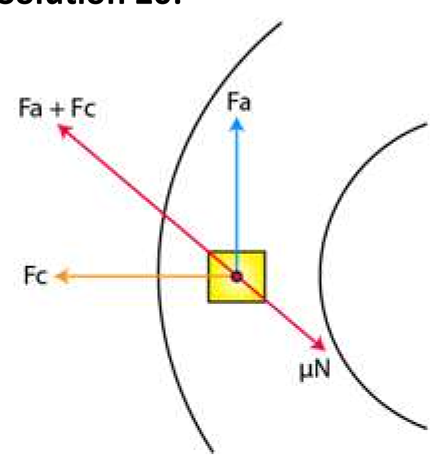
(c) The chances of losing contact is maximum at the end of the bridge for which α = L/2R



=> mg cos α = mv²/R

$$v = \sqrt{Rg\cos\left(\frac{L}{2R}\right)}$$

Solution 20:



The body is experiencing a force in the tangential direction as there exists some an acceleration along this direction.

friction force = F = μN

As the path is circular the body will also experience centrifugal force in the radial direction.

$F_c = mv^2/R$

As there is no motion in vertical direction, normal reaction force will be mg

N = mg

Magnitude of the tangential force:

$F_a = m\,dv/dt$

=> $F_a = ma$

The car will skid when the friction will not be enough to reinforce $F_a + F_c$.

Condition for just skidding:

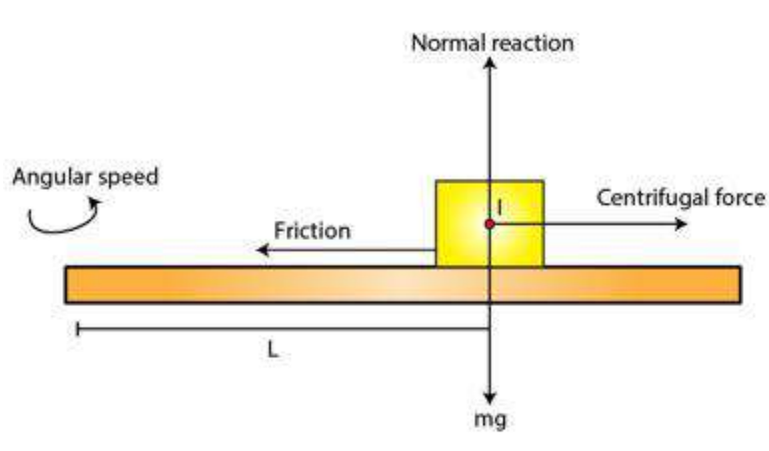
$$\mu N = \sqrt{F_a^{\,2} + F_c^{\,2}}$$

$$\mu mg = m\sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$$

$$\mu^2 g^2 = a^2 + \left(\frac{v^2}{R}\right)^2$$

$$v = \sqrt[4]{\{R^2(\mu^2 g^2 - a^2)\}}$$

Solution 21:



(a)
When the system rotates the block will experience centrifugal force which will be balanced by friction.
Normal reaction will be balance the force of gravitation.

$N = mg$

$F_C = f$

$\Rightarrow m\omega^2 L = \mu mg$

or $\omega = \sqrt{(\mu g)/L}$

(b)
When there comes some amount of acceleration, friction starts working in the opposite direction of motion.

Radial acceleration will be due to centrifugal force.

$a_R = \omega^2 L$

and tangential acceleration, $a_T = dv/dt$

$= d/dt\,(L\omega)$

$= L\,d/dt\,(\omega)$

$\Rightarrow a_T = L\alpha$

When friction reaches its maximum value, it will start slipping.

$$\mu g = \sqrt{(\omega^2 L)^2 + (L\alpha)^2}$$

$$(\mu g)^2 = (\omega^2 L)^2 + (L\alpha)^2$$

$$\omega = \left\{\frac{\mu^2 g^2}{L^2} - \alpha^2\right\}^{\frac{1}{4}}$$

Solution 22:
Radius of curve = R = 100 m
Weight = m = 100 kg

Velocity = v = 18km/h or 5m/sec
and $g = 10\ m/s^2$.

(a) At point B

$mg - mv^2/R = N$

on substituting the values.

=> N = 975N

(b) At points B and D, the cycle has no tendency to slide.
So at these points friction of force is zero.

At Point C,
$mg \sin \theta = F$

=> $F = 1000 \times 1/\sqrt{2} = 707$ N

(c)
Before point C: $mg \cos\theta - N = mv^2/R$

=> $N = mg \cos\theta - mv^2/R = 707 - 25 = 683N$
and

$N - mg \cos \theta = mv^2/R$

=> $N = mv^2/R + mg \cos \theta = 25 + 707 = 732N$

(d) Consider a point just before C, where N is minimum.

$\mu N = mg \sin\theta$

=> $\mu \times 682 = 707$

=> $\mu = 1.037$

Solution 23:

Radius= R = diameter/2 = 3/2 = 1.5 m
where R is the distance from the centre to one of the kids

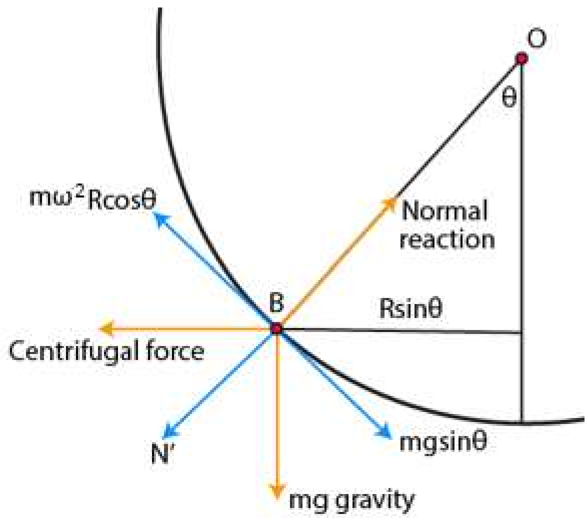
Here, $\omega = 2\pi R = 2\pi/3$
m = 15 kg (given)

Now, the frictional force on one of the kids = $F = mR\omega^2$

$= 15 \times 1.5 \times (2\pi/3)^2$

$= 10\pi^2$

Solution 24:



Here gravity force and the centrifugal force are used.

Normal reaction force is balanced as,

$N = mg\cos\theta + m\omega^2 R\sin^2\theta$

Here two cases of friction aries, one is along mgsinθ and the another in opposite direction as friction is a variable.

When friction is along mg sinθ

$f = m\omega^2 R\cos\theta\sin\theta - mg\sin\theta$

when friction is at its maximum limit, we have

$f = \mu N$

$$\mu(mg\cos\theta + m\omega^2 R\sin^2\theta) = -mg\sin\theta + m\omega^2 R\cos\theta\sin\theta$$

or

$$\omega = \sqrt{\frac{g(\sin\theta + \mu\cos\theta)}{R\sin\theta(\cos\theta - \mu\sin\theta)}}$$

…..(1)

When Friction is in the opposite direction of mgsinθ

f = mg sinθ - mω² R cosθ sinθ

when friction is at its maximum limit, we have

f = μN

$$\mu(mg\cos\theta + m\omega^2 R\sin^2\theta) = mg\sin\theta - m\omega^2 R\cos\theta\sin\theta$$

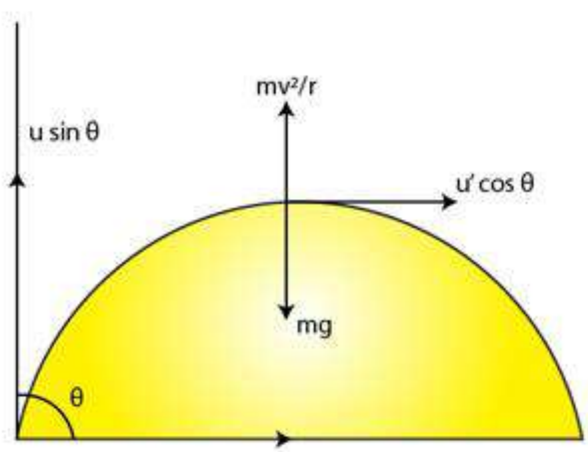
or

$$\omega = \sqrt{\frac{g(\sin\theta - \mu\cos\theta)}{R\sin\theta(\cos\theta + \mu\sin\theta)}}$$

…..(2)

The angular speed is lies in the range from (2) to (1) then the block will not slip.

Solution 25:



A particle is projected with a speed u at an angle θ with the horizontal.
Vertical component of velocity is zero at the highest point.

consider at any point velocity is u cosθ

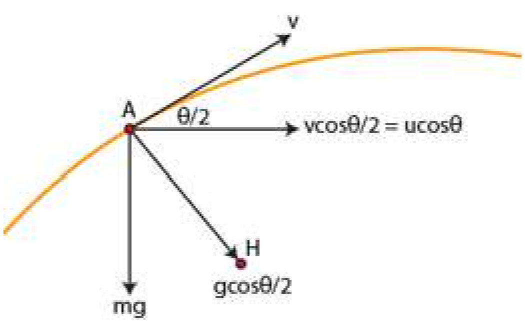
Centripetal force = $mu^2 \cos^2 (\theta/2)$

So, the radius of the circle at highest point is

$mg = mv^2/r$

=> $r = (u^2 \cos^2\theta)/g$

Solution 26:



Here g is in downwards direction. Hence, some component to g will act as central force which will be changing at each point depending upon θ.

Horizontal component of velocity will not change due to absence of force.

$v_H = u \cos\theta$

or

$v_H = v \cos(\theta/2)$

or $v = u\cos\theta/\cos(\theta/2)$

We know, central force = $F_c = mv^2/R$

=> $m\, g \cos(\theta/2) = mv^2/R$

=> $R = (mu^2 \cos^2\theta)/(g\cos^3(\theta/2))$

Solution 27:

(a) When normal reaction will be balancing the centrifugal force.

$F_c = mv^2/R$

and normal reaction, $N = mv^2/R$

(b) Friction force is at maximum as the body is moving,

Force of friction = $f = \mu N = \mu mv^2/R$

(c)

The force changing the velocity will be friction.

$f = \mu mv^2/R$

mass x acceleration = $\mu mv^2/R$

or acceleration = $-\mu v^2/R$

(d) dv/dt = v dv/dv

=> $v(dv/ds) = -\mu v^2/R$

=> $dv/v = -\mu/R\, ds$

for one complete revolution, $s = 2\pi R$

$$(\ln v)\binom{v}{v_0} = -\frac{\mu}{R}s$$

$$v = v_0 e^{-\frac{\mu}{R}s}$$

$$v = v_0 e^{-\frac{\mu}{R}2\pi R}$$

$$v = v_0 e^{-2\pi\mu}$$

Solution 28:

The cabin rotates with angular velocity ω and radius R.
The particle experience a force $mR\omega^2$.
The component of $mR\omega^2$ along groove provides the required force to the particle to move along AB.

$mR\omega^2 \cos\theta = ma$

$\Rightarrow a = R\omega^2 \cos\theta$

If length of groove be L then

$L = ut + 1/2\ at^2$

$L = 1/2\ R\ \omega^2 \cos\theta\ t^2$

$$t^2 = \frac{2L}{\omega^2 R\cos\theta}$$

$$t = \sqrt{\frac{2L}{\omega^2 R\cos\theta}}$$

Solution 29:

(a) The speed in MKS = v = 10m/s
The normal force will be balancing the centrifugal force.

$F_c = N$

$N = mv^2/R$

N = 0.2 N

(b) The plate is turned so the angle between normal t the plate and the radius of the road slowly increases.

$N = mv^2/R \cos\theta$ ...(1)

$\mu N = mv^2/R \sin\theta$ ...2)

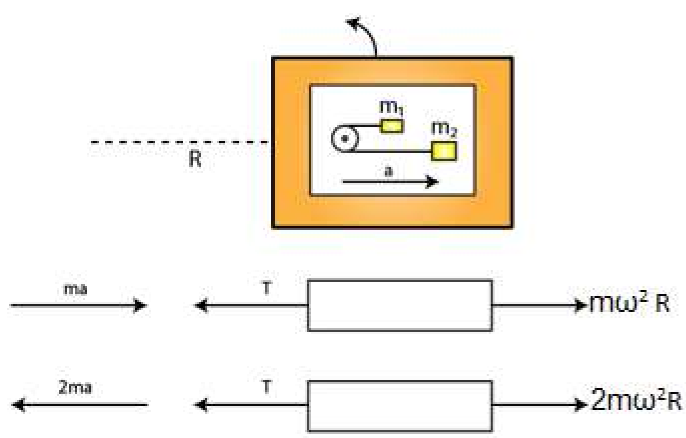
Solving (1) and (2), we have

$\mu = \tan\theta$

μ = friction coefficient between plate and body = 0.58 (given)

or $\theta = \tan^{-1} (0.58) = 30^o$

Solution 30:



From the free body diagrams,

$T + 2ma - 2m\omega^2R = 0$ ....(2)

on subtracting (2) from (1)

$\Rightarrow 3\ ma = m\omega^2\ R$

$\Rightarrow a = m\omega^2R/3$

(1)$\Rightarrow T = 4/3\ m\omega^2\ R$