Chapter 1

Network Elements and Basic Laws

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Basic concepts
- Basic quantities
- Classification of network elements
- Independent sources
- Network terminology
- DC network
- AC network
- Kirchhoff's laws
- Circuit elements are connected in parallel
- Nodal analysis
- Mesh analysis

Basic Concepts

The most basic quantity used in the analysis of electrical circuits is the electric charge.

Basic Quantities

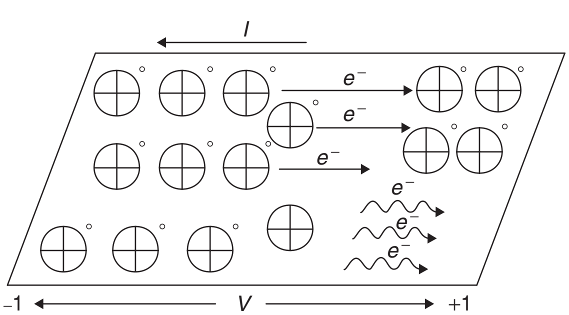
Electron

Electron is a mobile charge carrier. The electron (e^-) is measured in coulombs (C). 1 $e^- = 1.6 \times 10^{-19}$ C

- Multiples of electrons constitute charge (q)
- The movement of charge (q) over time causes current.

Current

There are free electrons available in all semi-conductive and conductive materials. These free e^-'s move at random in all directions with in the structure in the absence of external pressure or voltage. If a certain amount of voltage is applied across the material, all the free e^-'s move in one direction depending on the polarity of the applied voltage.



The voltage

According to the structure of an atom, there are two types of charges; positive and negative charge. A force of attraction exists between these charges. A certain amount of energy is required to overcome the force and move the charges through a specific distance. All opposite charges possess a certain amount of potential energy because of the separation between them. The difference in potential energy of the charges is called the potential difference.

The potential difference in electrical terminology is known as voltage and is denoted by V. It is expressed in terms of energy (W) per unit charges (Q)

$$\therefore \quad V = \frac{W}{Q} \text{ or } \upsilon = \frac{dW}{dQ}$$

The voltage is defined as the work (or) energy required to move a unit charge through an element.

The time rate of change of charge produces an electrical current

$$i(t) = \frac{dq(t)}{dt}$$

The electric current is measured in Ampere (A).

$$1\text{ A} = \frac{1\text{ C}}{1\text{ sec}}$$

Power and energy

Energy is the capacity for doing work, i.e., energy is nothing but stored work.

Energy can be expressed as,

$$W(t) = \int_{t_1}^{t_2} p(t) \cdot dt = \int_{at_1}^{t_2} v(t) \cdot i(t) \cdot dt$$

Power is the rate of change of energy and is denoted by P. If W amount of energy is used over a t amount of time, then

$$\text{Power } (P) = \frac{\text{Energy}}{\text{Time}} = \frac{W}{t} = \frac{dW}{dt'}$$

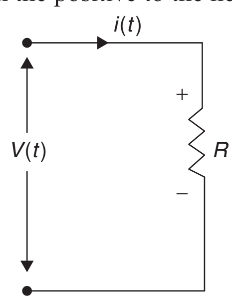
$$P = \frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt}$$

$$\therefore P = V \cdot I \text{ W.}$$

⇒ One watt is the amount of power generated when one Joule of energy is consumed in one second.

Positive sign convention

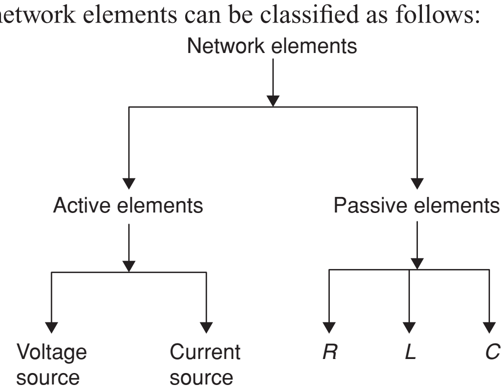
Current flows from the positive to the negative terminals.



⇒ Power is absorbed by elements if the sign of power is positive. That is, current enters from the +ve terminal and leaving from –ve terminal of the element. Power is supplied or delivered by element or source if the sign of power is negative i.e., current enters from the –ve terminal of the element.

Classification of Network Elements

The network elements can be classified as follows:



Where, $R \rightarrow$ resistance

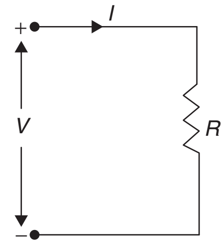
$L \rightarrow$ inductance and

$C \rightarrow$ capacitance

Circuit elements: The basic elements of circuits are resistance, inductance, and capacitance.

Resistance (R)

Electrical resistance is the property of material. It opposes the flow of electrons through the material. Thus, resistance restricts the flow of current through the material.



The unit of resistance (R) is 'ohm' (Ω).

According to Ohm's law,

$$J = \sigma E$$

$$J = \frac{I}{A} \text{ and electric field } E = \frac{V}{\ell}$$

$$\frac{I}{A} = \sigma \times \frac{V}{\ell} \Rightarrow V = \frac{\ell}{\sigma} * \frac{I}{A}$$

$$V = \frac{\rho \ell}{A} \cdot I$$

$$\therefore V = R \cdot I \Rightarrow$$

Ohm's law in circuit theory

$$\therefore \quad R = \frac{\rho \ell}{A} \,\Omega$$

Where,

$\rho \rightarrow$ Resistivity (Ω–m)

$\ell \rightarrow$ Length of conductor (m)

A→ cross sectional area (m^2)

When current flows through any conductor, heat is generated due to collision of free electrons with atoms.

The power absorbed by the resistor is given by:

$$P = V \cdot I = I^2 R = \frac{V^2}{R} \text{ W}$$

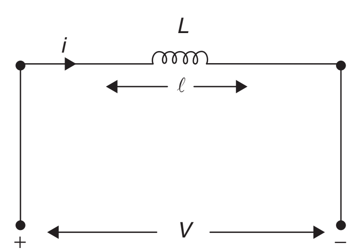
$$P = \frac{dW}{dt}$$

$$dW = P \cdot dt$$

$$W = \int_0^t P \cdot dt = \int_0^t V \cdot I dt = \frac{V^2 t}{R} \text{ J.}$$

Inductance (L)

A wire of finite length, when twisted into a coil, it becomes an inductor.



When a time varying current is flowing through the coil, magnetic flux will be produced.

The total flux $\psi(t) = N\phi$

$\therefore \quad \psi(t) = N \cdot \phi(t) wb$

$$V = \frac{d\psi(t)}{dt}$$

$$\psi(t) = L \cdot i$$

$$V = \frac{d}{dt}\{L \cdot i\}$$

$$V_L = L \cdot \frac{di_L(t)}{dt} \text{ V}$$

$$\Rightarrow \quad i_L = \frac{1}{L}\int_{-\infty}^{t} V \cdot dt \text{ A}$$

$\Rightarrow$ power $P = V \cdot i.$

$$W = \int P \cdot dt$$

$$W = \int L \cdot i \cdot di$$

$$\Rightarrow \quad \boxed{W = \frac{1}{2}Li^2 \text{ J.}}$$

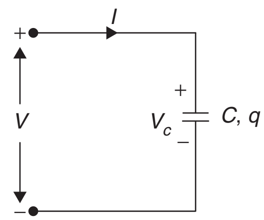
$\therefore$ The energy stored in the inductor at any instant will depend on the current flow through the inductor at that instant.

$$L = \frac{\mu_0 N^2 A}{\ell}$$

Where,
$\ell \rightarrow$ Length of the inductor
$N \rightarrow$ Number of turns
$A \rightarrow$ Cross-sectional area of coil

Capacitance (C)

It is the capability of an element to store electric charge within it



$$i = \frac{dq}{dt} \text{ A.}$$

$$q \;\alpha\; V \quad \Rightarrow \quad q = CV$$

$$C = \frac{q}{V} \text{ F}$$

$$\Rightarrow \quad i_c = \frac{d}{dt}CV$$

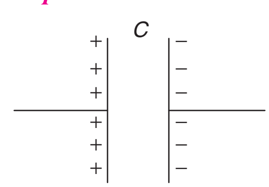
$$\boxed{\Rightarrow \quad i_c = C\frac{dV}{dt} \text{ A}}$$

$$V_c = \frac{1}{C}\int i_c dt \text{ V}$$

$$\Rightarrow P = V i_c$$

$$\boxed{W_c = \int Pdt = \frac{1}{2}CV^2 \text{ J}}$$

For parallel plate capacitance



$$\boxed{C = \frac{\varepsilon A}{d} \text{ F}}$$

Where,
$\varepsilon \rightarrow$ permittivity of material
$A \rightarrow$ cross-sectional area of plates
$d \rightarrow$ distance between plates.

Table 1 *Summary of relationships for the parameters*

Parameter	Basic Relationship	Voltage-current Relationships	Energy
R $G = \frac{1}{R}$	$V = iR$	$V_R = i_R \cdot R, i_R = G \cdot V_R$	$W_R = \int_{-\infty}^{t} p \cdot dt; W_R = \int_{-\infty}^{T} V_R \cdot i_R dt$
L	$\Psi = L \cdot i$	$V_L = L \cdot \frac{di}{dt}; i_L = \frac{1}{L}\int_{-\infty}^{t} V_L dt$	$W_L = \frac{1}{2}Li^2 = \frac{1}{2}\psi \cdot i$
C	$q = CV$	$V_C = \frac{1}{C}\int_{-\infty}^{t} i_c \cdot dt; i_C = C \cdot \frac{dV_C}{dt}$	$W_C = \frac{1}{2}CV^2 = \frac{1}{2}q \cdot V$

Note: R, L, and C elements are a linear, passive, bilateral and time-invariant, at a constant temperature.

Solved Example

Example 1: In the interval $0 > t > 4\pi$ ms, a 10 μF capacitance has a voltage $V = 25 \sin 200t$ V. The charge, power, and energy are:

Solution:
Charge $q = C \cdot V$

$= 10 \times 25 \sin 200t$ (μC)

$= 250 \text{ Sin } 200t$ (μC)

Power, $P = V \cdot i$

But, $i_c = C \cdot \dfrac{dV_c}{dt}$

$= 10 \times 25 \times 200 \cos 200t$ μA

$i_c = 50 \cos 200t$ mA

$\therefore \quad P = V \cdot i = 25 \sin 200t \cdot 50 \cos 200t$ mW

$= 25 \times 25 \sin 400t$ mW

$= 0.625 \sin 400t$ W

Energy, $W_c = \int_{t1}^{t2} P \cdot dt$

$$W_c = \int_0^{4\pi\times10^{-3}} 0.625 \sin 400t \, dt$$

$$= -\frac{0.625}{400}[\cos 400t]_0^{4\pi\times10^{-3}}$$

$= 1.5$ nJ.

Example 2: A capacitor of 100 μF stores 10 mJ of energy. Obtain the amount of charge stored in it. How much time does it take to build up this charge if the charging current is 0.2A?

(A) $Q = 1.414$ mC, $t = 1.4$ ms
(B) $Q = 1.414$ mC, $t = 7.07$ ms
(C) $Q = 2$ μC, $t = 0.1$ ms
(D) $Q = 2$ mC, $t = 10$ ms

Solution: (B)
The energy stored in a capacitor is given by, $W_c = \dfrac{CV^2}{2} = \dfrac{1}{2}QV = \dfrac{Q^2}{2C}$

$$\therefore \quad Q = \sqrt{2CW} = \sqrt{2\times10\times10^{-3}\times100\times10^{-6}}$$

$= 1.414\ mC$. We know, $i = \dfrac{dq}{dt}$

$\therefore Q = I \cdot t$

$$t = \frac{Q}{I} = \frac{1.414\times10^{-3}}{0.2} = 7.07 \text{ ms.}$$

Example 3: The strength of current in 2 H inductor changes at a rate of 3 A/s. The voltage across it and the magnitude of energy stored in an inductor after 4 seconds are

(A) $V_L = 6$ V, $W_L = 144$ J
(B) $V_L = 1.5$ V, $W_L = 12$ J
(C) $V_L = 6$ V, $W_L = 77$ J
(D) $V_L = 1.5$ V, $W_L = 144$ J

Solution: (A)
From the given data

$$L = 2H, \frac{di_L}{dt} = 3 \text{ A/sec}$$

$$V_L = L \cdot \frac{di_L}{dt} = 2 \times 3 = 6 \text{ V.}$$

$$W = \frac{1}{2} L \cdot i^2$$

$$\frac{di}{dt} = 3 \text{ A/sec}$$

$di = 3\ dt$

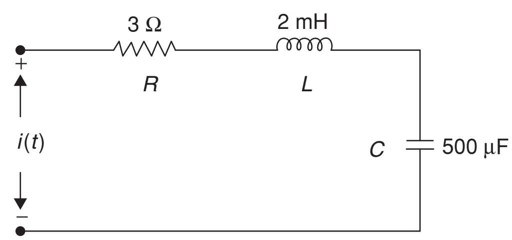
$I = 3t$ A/sec But $t = 4$ sec

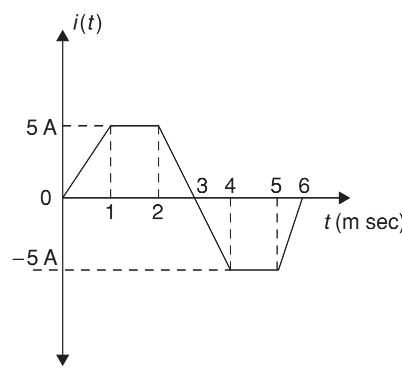
$I = 3 \times 4 = 12$ A

$$W = \frac{1}{2} \times 2 \times (12)^2$$

$= 144$ J.

Example 4: A current source $i(t)$ is applied to a series RLC circuit shown in below figures





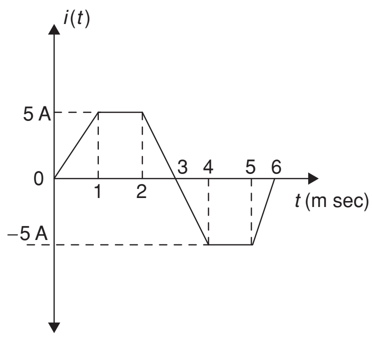
The maximum voltage across the resistor is

(A) 10 V
(B) 15 V
(C) 5 V
(D) 20 V

Solution: (B)
The drop across resistance $V_R(t) = i(t)\ R$

$V_{R(\max)} = i_{\max} \cdot R$ V

$V(t) = 5 \times 3 = 15$ V



Example 5: The total voltage across indicator is

(A) 0 V (B) –10 V
(C) 10 V (D) None of these

Solution: (C)

$$V_L = L \cdot \frac{di(t)}{dt}$$

(i) For $0 \le t \le 1$ ms

$$i(t) = \frac{5}{1\times10^{-3}} t \text{ A} = 5\times10^{3} t \text{ A}$$

$$V_L = 2 \times 10^{-3} \times 5 \times 10^3 = 10 \text{ V}$$

(ii) For 1 ms $\le t \le$ 2 ms:

$i(t) = 5$ A constant

$\therefore V_L = 0$ V

(iii) For $2 \le t \le 4$ ms:

$A(2 \times 10^{-3}, 5)$ $B(4 \times 10^{-3}, -5)$

$$I(t) = \frac{-5-5}{2\times10^{-3}}(t - 2\times10^{-3}) + 5$$

$i(t) = 5 - 5 \times 10^3 (t - 2 \times 10^{-3})$ A

$= 15 - 5 \times 10^3\, t$ A

$V_L = L$

$$\frac{di(t)}{dt} = 2\times10^{-3}[0 - 5\times10^{3}] = -10 \text{ V}$$

(iv) For 4 ms $\le t \le$ 5 ms:

$i(t) = 5$ and $\Rightarrow$ constants

so $V_L = 0$

(v) For 5 ms $\le t \le$ 6 ms:

$A(5 \times 10^{-3}, -5)$ $B(6 \times 10^{-3}, 0)$

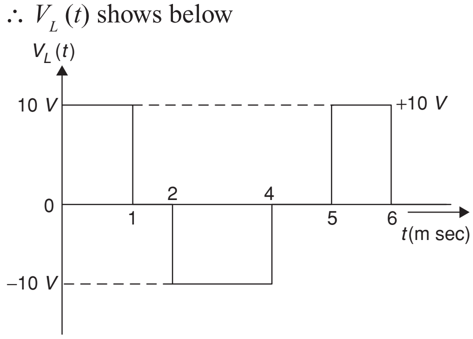
$i_L(t) = 5 \times 10^3\, t$ A

$$V_L(t) = L\frac{di_{L(t)}}{dt}$$

$= 2 \times 10^{-3} \times 5 \times 10^3$

$= 10$ V

$\therefore V_L(t)$ shows below



$\Rightarrow \quad \sum V_L = 10 + 0 + (-10) + 0 + 10$

$V_L = 10$ V

Example 6: A voltage $V(t) = 2 \sin \omega t$ V is applied across a capacitor having time varying capacitance given by $C(t) = (2 + 0.5 \sin t)$ F. Find $i(t) = ?$

Solution: $Q = C \cdot V \quad \Rightarrow \quad q(t) = C(t) \times V(t)$

$$i(t) = \frac{dq(t)}{dt}$$

$$i(t) = \frac{d}{dt}\{(2 + 0.5\sin t)\times 2\sin\omega t\}$$

$$= \frac{d}{dt}\{4\sin\omega t + \sin t \times \sin\omega t\}$$

$= 4 \cos \omega t \times \omega + \sin t \times \cos \omega t \times \omega + \sin \omega t \times \cos t$

$I(t) = \sin \omega t \times \cos t + \omega\, [4 + \sin t] \cos \omega t$

Example 7: If $\omega = 2$ rad/sec, the value of $i(t)$ at $i(t)$ at $t = \frac{1}{2}$ sec in the above problem is,

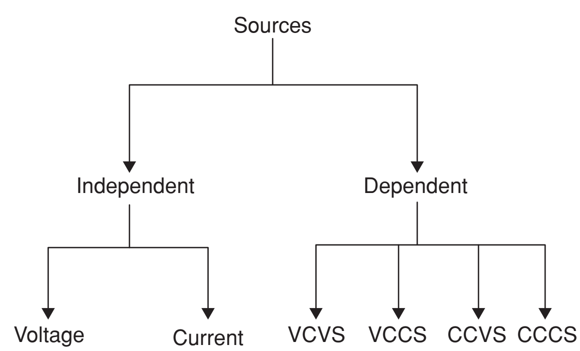
(A) 2 A (B) 4 A
(C) 6 A (D) 8 A

Solution: $i(t) = [4 + \sin t]\ \omega \cos \omega t + \sin \omega t \cdot \cos t$

$= [4 + \sin 0.5\}\ 2 \times \cos 1 + \sin 1 \times \cos 0.5$

$= 8.016 + 0.174 = 8.033 \approx 8$ A.

Energy Sources



Independent Sources

Ideal voltage source

Terminal voltage of an ideal voltage source is independent of the current supplied by it. Internal resistance of an ideal voltage source is zero.

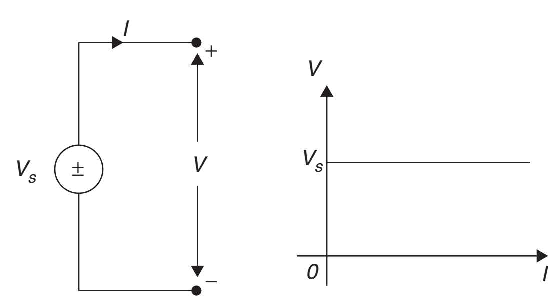


Figure 1 Ideal voltage source and *V–I* characteristics

Practical voltage source

Practical voltage source has some finite internal resistance. Due to the presence of an internal resistance, the terminal voltage of a practical voltage source reduces with the increase in current supplied.

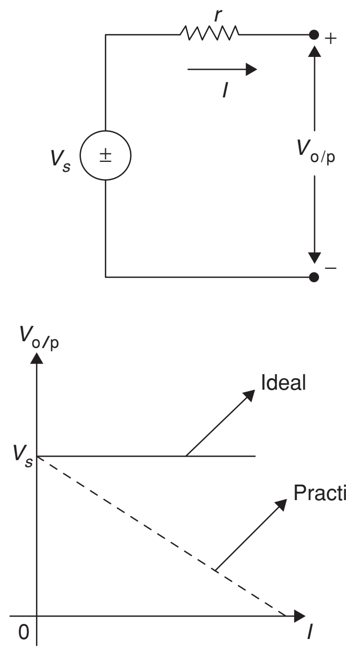


Figure 2 Practical voltage Source and *V*–*I* characteristics

By KVL

$$V_s - I \cdot r = V_{o/p}$$
$$V_{o/p} = V_s - I \cdot r$$

When current through any element is zero, then the potential difference is also zero.

Ideal current source

Current delivered by any ideal current source is independent of voltage across its terminals. Internal resistance of an ideal current source is infinite.

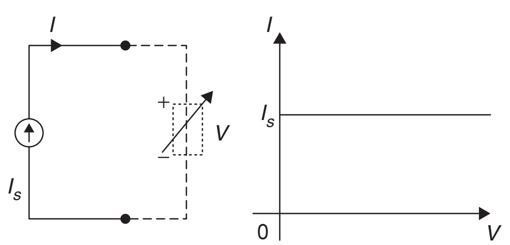
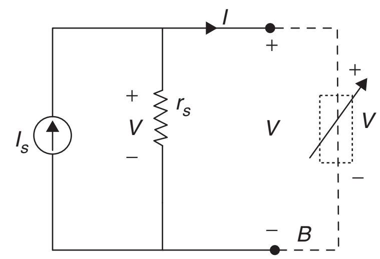


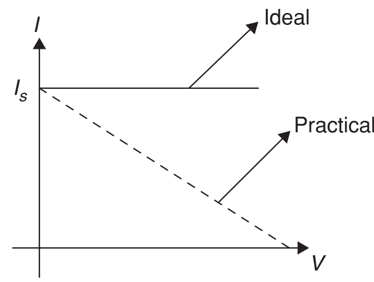
Figure 3 Ideal current source and *I* – *V* characteristics

That is, $I = I_s$ for all V.

Practical current source

Practical current source has some finite internal resistance. Due to the presence of an internal resistance, current delivered by the practical current source reduces with increase in its terminal voltage.





By KCL

$$I_s = \frac{V}{r_s} + I$$

$$\boxed{I = I_s - \frac{V}{r_s}}$$

→ current always choose minimum resistance path.

Dependent Sources

Voltage controlled voltage source (VCVS)

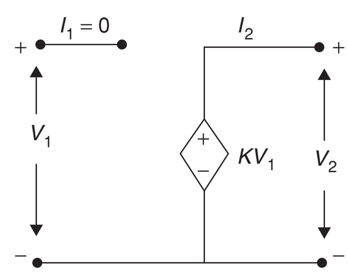
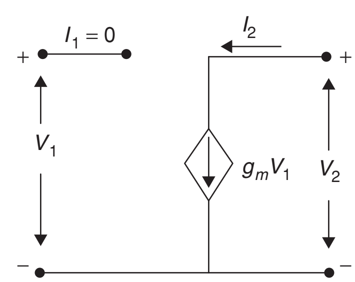


Figure 4 VCVS

Voltage controlled current source (VCCS)

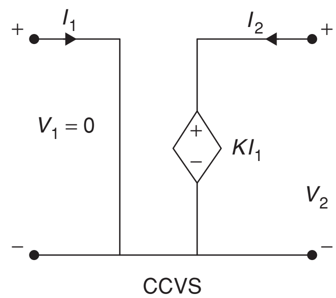


VCCS

That is, $I_2 = g_m V_1$

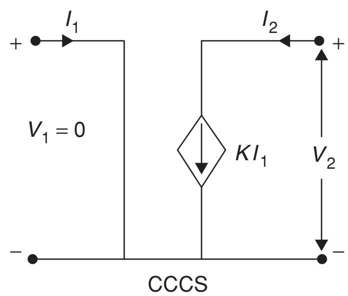
Because, o/p current depends on the i/p voltage

Current controlled voltage source



i.e., $V_2 = KI_1$

Current controlled current source



$$\therefore\ I_2 = KI_1$$

Because o/p current depends on the i/p current so it is called current controlled current source

Where, $K \Rightarrow$ constant.

Basic Definitions

Network Terminology

In this section some of the basic terms which are commonly associated with a network are defined.

Network element

Network elements can be either active elements (or) passive elements.

Active elements ⇒ which supply power or energy to the network (outside world).

Example: Voltage source and current source.

Passive elements ⇒ which either store the energy or dissipate energy in the form of heat

Example: R, L and C

Branch

A part of the network which connects the various points of the network with one another is called a branch

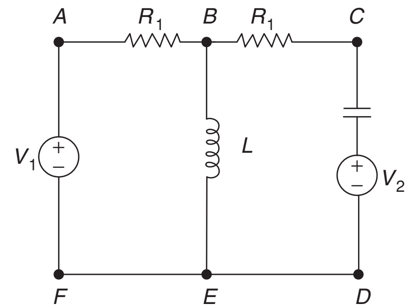
Node

A point at which two or more elements are connected together is called node. The junction points are also the nodes of network.

Mesh or Loop

Mesh is a set of branches forming a closed path in a network

Example:



Branches ⇒ $A - B$, $B - E$, … etc.
Nodes ⇒ A, B, C, … etc.
Mesh ⇒ $ABEFA$, $ABCDEFA$, $BCDEB$

Types of Elements

1. Linear and Non-linear
2. Active and passive
3. Bilateral and unilateral
4. Distributed and lumped
5. Time invariant and time variant

Linear and non-linear

A two terminal element is said to be linear for all time 't', if its characteristics is a straight line through the origin, otherwise it is non-linear.

Example: A linear element must satisfy superposition and homogeneity principles.

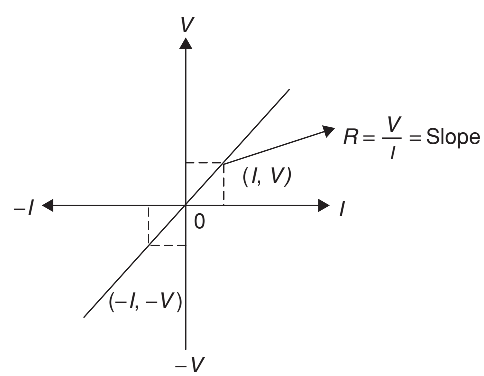


Figure 5 A bilateral, linear characteristics

Active network

A circuit which contains at least one source of energy is called Active. An energy source may be a voltage or current.

An element is said to be active if it deliveres a net amount of energy to the outside world is non-zero.

Example: Transistors, op-amps, batteries, … etc.

Passive network

A circuit which contains no energy source is called passive circuit. These networks consists of passive elements only.

Example: *R*, *L*, *C* and thermistors ... etc.

Bilateral and unilateral networks

A circuit whose characteristics, behaviors is same irrespective of the direction of current through various elements it is called bilateral network. Otherwise it is said to be unilateral.

Example: Diode (Unilateral)
Resistors (Bilateral)

Lumped and distributed networks

A network in which all the network elements are physically separable is known as lumped network.

Example: simple *RLC* circuits.
Otherwise it is called distributed network

Example: Transmission lines.

DC network A network consist of DC sources which are fixed polarity sources with time invariant is called a DC network.

AC network A network consist of AC sources which are alternating sources and periodically varying with time, is called an AC network.

Kirchhoff's Laws

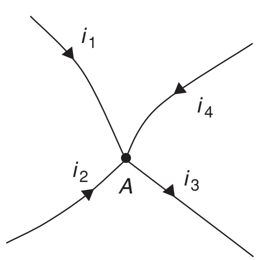
In 1847, a German physicist, Kirchhoff, formulated two fundamental Laws as given below:

1. Kirchhoff's Current law (KCL)
2. Kirchhoff's Voltage law (KVL)

Kirchhoff's current law (KCL)

In any network, the algebraic sum of currents meeting at a point or node is always zero i.e., the total current leaving a junction is equal to the total current entering that junction.

Example:



Assume currents entering a junction is positive and currents leaving away from the junction is negative. (vice-versa)

Apply KCL at node A

By KCL ⇒ Σ Leaving currents = 0

$i_1 + i_2 + i_4 - i_3 = 0 \Rightarrow i_3 = i_1 + i_2 + i_4.$

we know, $i = \dfrac{dq}{dt}$

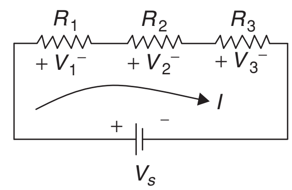
$$\frac{dq_3}{dt} = \frac{dq_1}{dt} + \frac{dq_2}{dt} + \frac{dq_4}{dt}$$

$$\boxed{q_3 = q_1 + q_2 + q_4}$$

∴ sum of entering charges is equal to sum of leaving charges, KCL is based on the principle of law of conservation of charge.

Kirchoff's voltage law (KVL)

The algebraic sum of all branch voltages around any closed path is always zero at all instants of time.



Apply KVL to the above circuit

$$-V_1 - V_2 - V_3 + V_S = 0$$

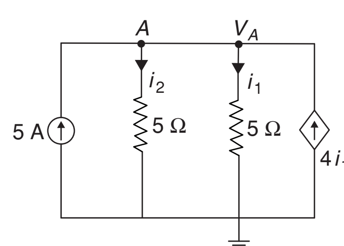
or
$$V_S = V_1 + V_2 + V_3$$
$$V_S = IR_1 + IR_2 + IR_3$$
$$V_S = I(R_1 + R_2 + R_3)$$

Properties

1. KCL and KVL can be applied to any lumped electric circuits.
2. KCL expresses conservation of charge at each and every node. And KVL expresses conservation of flux or energy in every loop of electric circuit.

Note: Ohm's law is not applicable for active elements. It is applicable only for linear, passive elements.

Example 8:



The values of i_1 and i_2 are respectively,

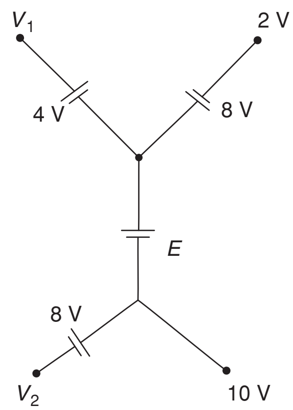
(A) $i_1 = -10$ A, $i_2 = 5$ A (B) $i_1 = i_2 = -2.5$ A
(C) $i_1 = 2.5$ A and $i_2 = -2.5$ A (D) None of the above

Solution: (B)

Applying KCL at node 'A'

$$5 + 4i_1 = i_1 + i_2$$
$$3i_1 - i_2 + 5 = 0$$
$$3\left[\frac{V_A}{5}\right] - \frac{V_A}{5} + 5 = 0$$
$$\frac{2V_A}{5} + 5 = 0$$
$$V_A = -12.5 \text{ V}$$
$$i_1 = -\frac{12.5}{5} = -2.5 \text{ A}$$
$$i_2 = i_1 = -2.5 \text{ A}$$

Example 9: Consider the following circuit



The node voltages V_1, V_2 and E are respectively,

(A) $V_1 = -14$ V, $V_2 = 18$ V, $E = -2$ V
(B) $V_1 = +14$ V, $V_2 = -2$ V, $E = -2$ V
(C) $V_1 = +14$ V, $V_2 = 2$ V, $E = 0$ V
(D) $V_1 = -14$ V, $V_2 = -2$ V, $E = 0$ V

Solution: (C)

Apply KVL in loop1

$$V_2 + 8 - 10 = 0$$
$$V_2 = 2 \text{ V}$$

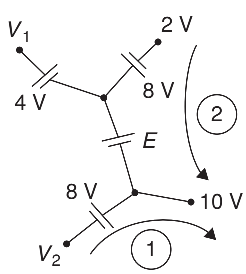
Apply KVL in loop 2

$$2 + 8 - E - 10 = 0$$
$$E = 0 \text{ V}$$

Apply KVL in Loop 3

$$V_1 - 4 - 0 - 10 = 0$$
$$V_1 = 14 \text{ V}$$

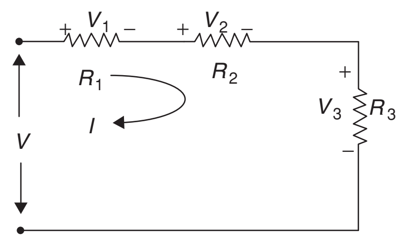
$\therefore$ $V_1 = 14$ V, $V_2 = 2$ V and $E = 0$ V



Circuit Elements in Series

When sending end of an element is connected to receiving end of another element and no other element is connected at that node then those two elements are said to be connected in series. Current flowing through series connected elements is equal.

Series Connected Resistors



In the above network R_1, R_2, R_3 are connected in series. By apply KVL for the loop,

$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$= [R_1 + R_2 + R_3] \cdot I$$

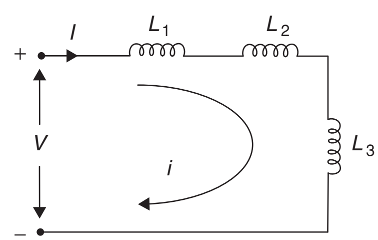
But $V = I.\ R_{eq}$.

$$\therefore\ I \cdot R_{eq} = I\,[R_1 + R_2 + R_3]$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \cdots R_n}$$

Series Connected Inductors

L_1, L_2, L_3 are the three inductances connected in series as shown in the fiure.



Apply KVL to the circuit,

$$V = L_1\frac{di}{dt} + L_2 \cdot \frac{di}{dt} + L_3\frac{di}{dt}$$
$$= (L_1 + L_2 + L_3)\frac{di}{dt}$$

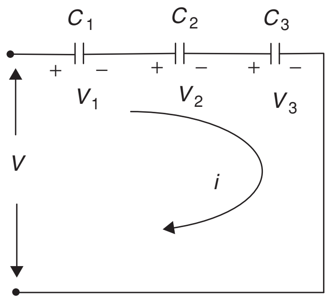
But we know,

$$V_L = L_{eq} \cdot \frac{di}{dt}$$

$$\boxed{\therefore\ \ L_{eq} = L_1 + L_2 + L_3}$$

Series Connected Capacitors

If three circuit elements are capacitances connected in series, assuming zero initial charges. Apply KVL for the circuit



$$V = V_1 + V_2 + V_3$$

$$= \frac{1}{C_1}\int idt + \frac{1}{C_2}\cdot\int i\cdot dt + \frac{1}{C_3}\int idt$$

$$= \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]\int idt$$

$$V = \frac{1}{C_{eq}}\cdot\int idt$$

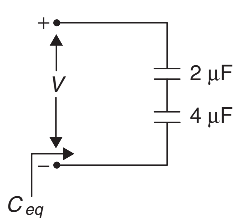
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}}$$

Example 10: Two capacitors C_1 = 2 μF, C_2 = 4 μF are connected in series. The equivalent capacitance is

(A) 6 μF (B) 8 μF

(C) 2 μF (D) $\frac{4}{3}$ μF

Solution: (D)



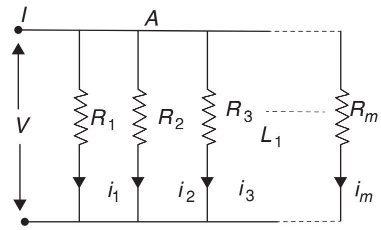
$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{4} \Rightarrow C_{eq} = \frac{2\times 4}{2+4} = \frac{8}{6}\,\mu\text{F}$$

C_{eq} = 4/3 μF.

Circuit Elements are Connected in Parallel

Resistors in Parallel

Resistors R_1, R_2, R_3…R_n are connected in parallel as shown in the figure update.



Apply KCL at node A,

$$I = i_1 + i_2 + i_3 + \cdots + i_m$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_m}$$

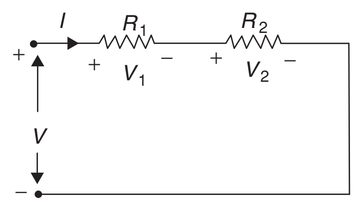
$$\boxed{\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_m}}$$

Let $m = 2$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

Voltage division

A set of series connected resistors as shown in figure is referred as a voltage divider.



Applied voltage V is divided into V_1 and V_2 across R_1 and R_2 respectively.

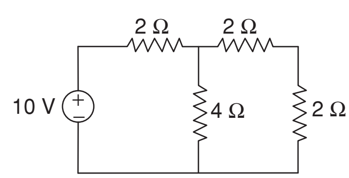
$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = I\cdot R_1\,;\; V_2 = I\cdot R_2$$

$$\boxed{V_1 = \frac{V}{R_1 + R_2}\cdot R_1\text{ V}}$$

$$V_2 = \frac{V}{R_1 + R_2} R_2\text{V}$$

Example 11:

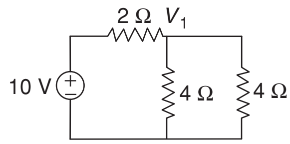


The voltage across 4Ω resistance is

(A) 5 V (B) 2.5 V

(C) 7.5 V (D) None of the above

Solution: (A)

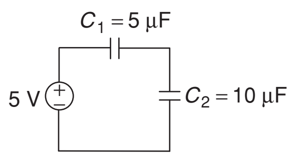


$$\frac{V_1 - 10}{2} + \frac{V_1}{4} + \frac{V_1}{4} = 0$$

$$2\,(V_1 - 10) + 2V_1 = 4V_1 = 20$$

$$V_1 = 5\text{ V.}$$

Example 12:



The value of V_{C2} and V_{C1} are

(A) $V_1 = \frac{5}{3}\text{ V}, V_2 = \frac{10}{3}\text{ V.}$

(B) $V_1 = \frac{10}{3}\text{ V}, V_2 = \frac{5}{3}\text{ V.}$

(C) $V_1 = 2.5\text{ V}, V_2 = 2.5\text{ V}$

(D) $V_1 = \frac{3}{5}\text{ V}, V_2 = \frac{10}{3}\text{ V.}$

Solution: (B)

$$V_{c2} = \frac{V}{C_1 + C_2} \times C_1$$

$$= \frac{5}{15} \times 5$$

$$= \frac{5}{3}\text{ V}$$

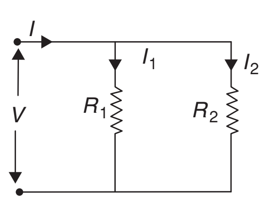
$$V_{c1} = \frac{V}{C_1 + C_2} \times C_2$$

$$= \frac{5}{15} \times 10$$

$$= \frac{10}{3}\text{ V}$$

Current division

A parallel arrangement of resistors shown in figure results in a current divider.

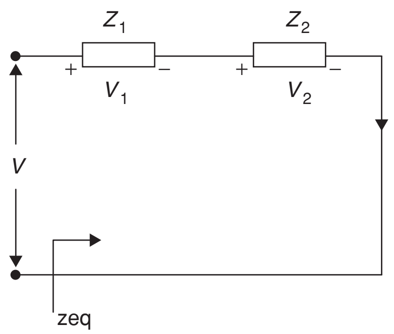


Total current I is divided into I_1 and I_2 through R_1 and R_2 respectively. I_1 and I_2 are expressed as shown below.

$$\boxed{I_1 = \frac{R_2}{R_1 + R_2} \cdot I}$$

$$I_2 = \frac{R_1}{R_1 + R_2} \cdot I$$

Note 1: When two impedances are connected in series, voltage division across those impedances depends on elements in it.



(i) If $Z = R$ or $Z = L$

$$Z_{eq} = Z_1 + Z_2$$

$$V_1 = \frac{V}{Z_{eq}} \times Z_1\text{ V}$$

$$V_2 = \frac{V}{Z_{eq}} \times Z_2\text{ V}$$

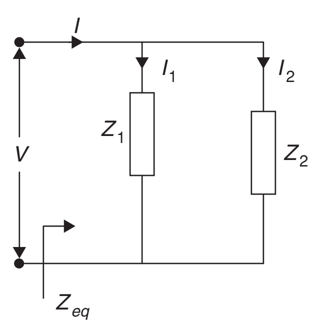
(ii) If $Z_1 = C_1$ and $Z_2 = C_2$

then

$$Z_{eq} = \frac{C_1 \times C_2}{C_1 + C_2}$$

$$\boxed{\begin{aligned} V_2 &= \frac{V}{C_1 + C_2} \times C_1 \\ V_1 &= \frac{V}{C_1 + C_2} \times C_2 \end{aligned}}$$

Note 2: When two impedances are connected in parallel, current division through those impedances depends on the elements present in it.



$I = I_1 + I_2$.

(i) If $Z_1 = R_1\ \Omega$ and $Z_2 = R_2\ \Omega$ or

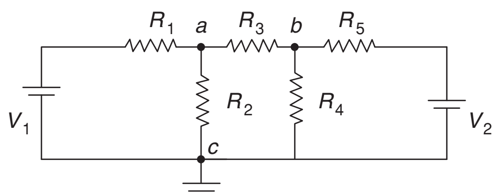
$$Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} Z_1 = L_1 \text{ and } Z_2 = L_2$$

(ii) If $Z_1 = C_1$ and $Z_2 = C_2$

Then,

$Z_{eq} = C_1 + C_2$

Nodal Analysis



KCL at node a,

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{V_b}{R_3} = \frac{V_1}{R_1} \quad (1)$$

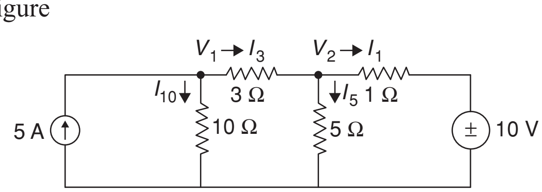
KCL at node b,

$$\frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} + \frac{V_b - V_2}{R_5} = 0$$

$$\left(\frac{-1}{R_3}\right)V_a + \left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right]V_b = \frac{V_2}{R_5} \quad (2)$$

Solving (1) and (2), the currents can be estimated.

Example 13: Write the node voltage equations and determine the currents in each branch of the network shown in figure



Solution:

KCL at node 1

$$5 = \frac{V_1}{10} + \frac{V_1 - V_2}{3}$$

$$5 = V_1\left[\frac{1}{10} + \frac{1}{3}\right] - V_2\left[\frac{1}{3}\right] \quad (3)$$

KCL at node 2

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

$$-V_1\left[\frac{1}{3}\right] + V_2\left[\frac{1}{3} + \frac{1}{5} + 1\right] = 10 \quad (4)$$

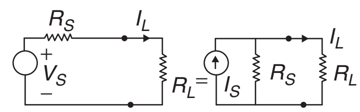
Solving (3) and (4)

$V_1 = 19.85$ V, $V_2 = 10.9$ V

$$I_{10} = \frac{V_1}{10} = 1.985 \text{ A}, \ I_3 = \frac{V_1 - V_2}{3} = 2.98 \text{ A}$$

$$I_5 = \frac{V_2}{5} = \frac{10.9}{5} = 2.18 \text{ A}, \ I_1 = \frac{V_2 - 10}{1} = 0.9 \text{ A}$$

Source Transformation



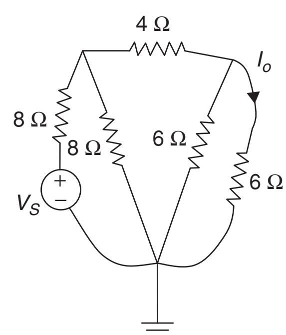
V_S is the voltage and R_S is the series resistance.

$$I_L = \frac{V_S}{R_S + R_L}$$

When transformed to a current source

$$I_S = \frac{V_S}{R_S} \text{ and } I_L = \frac{V_S}{R_S + R_L}$$

Example 14: For the network shown in fig. find V_s when $I_o = 0.5$ A

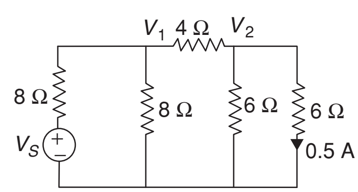


(A) $V_s = 22$ V (B) $V_s = 20$ V

(C) $V_s = -22$ V (D) $V_s = +70$ V

Solution: (A)

Redraw the above network



Given $I_o = 0.5$ A

So $$I_o = \frac{V_2}{6} \Rightarrow V_2 = 6 \times 0.5 = 3 \text{ V}$$

Apply KCL at node V_1

$$\frac{V_1 - V_S}{8} + \frac{V_1}{8} + \frac{V_1 - 3}{4} = 0$$

$$V_1 - V_S + V_1 + 2(V_1 - 3) = 0$$

$$4V_1 - V_S = 6 \quad (5)$$

Apply KCL at node (6)

$$\frac{V_2 - V_1}{4} + \frac{V_2}{3} = 0$$

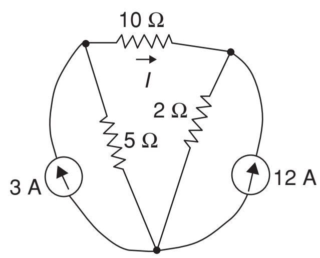
$$3(V_2 - V_1) + 4V_2 = 0$$

$$7V_2 = 3V_1$$

$$V_1 = \frac{7 \times 3}{3} = 7 \text{ V}$$

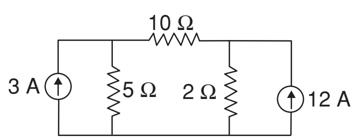
$$V_S = 4\ V_1 - 6$$
$$= 4 \times 7 - 6 = 22 \text{ V}$$

Examples 15: In the network shown in figure find the current in the 10 Ω resistor

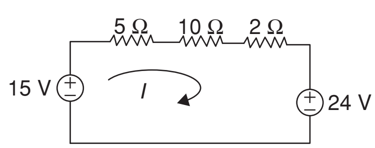


(A) $I = 0.53$ A (B) $I = 9$ A
(C) $I = -1$ A (D) $I = -0.53$ A

Solution: (D)
Redraw the above circuit



Apply source transformation

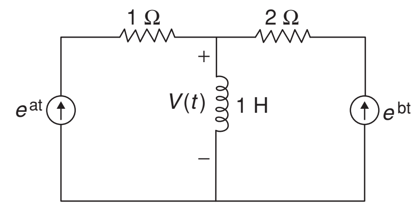


$$I = \frac{15 - 24}{17} = -\frac{9}{17}$$
$$= -0.53 \text{ A}$$

Notes:

1. **Simplenode:** It is an Inter connection of only two branches
2. **Principle node:** It is an inter connection of at least three branches.
3. If a branch between two essential non-reference node contain a voltage source this is called 'super-node'

Example 16: In the circuit given below, the voltage $V(t)$ is,



(A) $e^{at} - e^{bt}$ (B) $e^{at} + e^{bt}$
(C) $a.e^{at} - b.e^{bt}$ (D) $a.e^{at} + b \cdot e^{bt}$

Solution: (D)

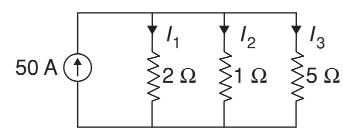
$$V_L = L \cdot \frac{di_L}{dt}$$

$$I_L = e^{at} + e^{bt}$$

$$V_L(t) = 1 \times \frac{d}{dt}[e^{at} + e^{bt}]$$

$$= a.e^{at} + b.e^{bt}.$$

Example 17: Determine the current in all resistors in the circuit shown in figure



Solution:
By KCL

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{2} + \frac{V}{1} + \frac{V}{5}$$

$$50 = V\left[\frac{1}{2} + \frac{1}{1} + \frac{1}{5}\right]$$

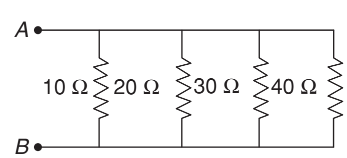
$$V = \frac{50}{1.7} = 29.41 \text{ V}$$

$$I_1 = \frac{29.41}{2} = 14.705 \text{ A}$$

$$I_2 = \frac{29.41}{1} = 29.41 \text{ A}$$

$$I_3 = \frac{29.41}{5} = 5.882 \text{ A}$$

Example 18:



Determine the parallel resistance between points A and B of the circuit shown in figure.

(A) 8 Ω (B) 6 Ω
(C) 4.8 Ω (D) 3 Ω

Solution: (C)

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

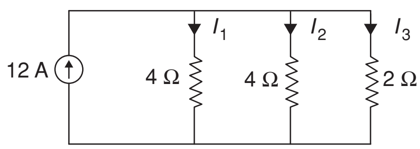
$$= \frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40}$$

$$= 0.1 + 0.05 + 0.033 + 0.025$$

$$\frac{1}{R_T} = 0.208$$

$$R_T = 4.8\ \Omega$$

Example 19: Determine the current through each resistor in the circuit shown in figure



Solution:

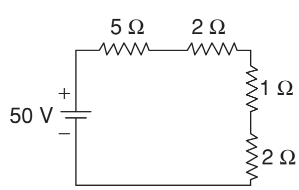
$$I_1 = 12 \times \frac{4 \parallel 2}{4 \parallel 2 + 4}$$

$$= 12 \times \frac{\frac{4 \times 2}{4+2}}{\frac{4 \times 2}{4+2} + 4}$$

$$= 12 \times \frac{1.3333}{1.3333 + 4} = 3\text{ A}$$

$$I_2 = 3\text{ A}$$
$$I_3 = 12 - 3 - 3$$
$$I_3 = 6\text{ A}$$

Example 20: Determine the total amount of power dissipated in the circuit shown in figure



(A) 100 W (B) 250 W (C) 150 W (D) 200 W

Solution: (B)

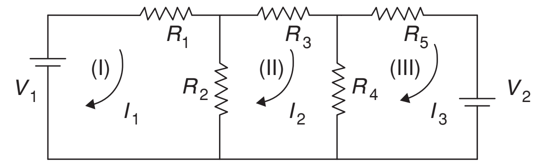
Total resistance, $R = 5 + 2 + 1 + 2 = 10\ \Omega$

$$\text{Power} = \frac{V^2}{R}$$

$$= \frac{(50)^2}{10}$$

$$= 250\text{ W}$$

Mesh Analysis



Consider the network shown in figure. V_1 and V_2 are the voltage sources. The loop currents are I_1, I_2 and I_3 in their direction as shown in figure

KVL in loop I

$$-I_1R_1 - I_2R_2 + V_1 = 0$$

$$V_1 = I_1(R_1 + R_2) - I_2R_2 \quad (7)$$

KVL in loop II ,

$$-I_2R_3 - I_2R_4 + I_3R_4 - I_2R_2 + I_1R_2 = 0$$

$$-I_1R_2 + I_2(R_2 + R_3 + R_4) - I_3R_4 = 0 \quad (8)$$

KVL in loop III,

$$-I_3R_5 - V_2 - I_3R_4 + I_2R_4 = 0$$

$$-I_2R_4 + I_3(R_4 + R_5) = -V_2 \quad (9)$$

from (7), (8) and (9)

$$\begin{bmatrix} R_1 + R_2 & -R_2 & 0 \\ -R_2 & R_2 + R_3 + R_4 & -R_4 \\ 0 & -R_4 & R_4 + R_5 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ V_2 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} V_1 & -R_2 & 0 \\ 0 & R_2 + R_3 + R_4 & -R_4 \\ -V_2 & -R_4 & R_4 + R_5 \end{vmatrix}}{|R|}$$

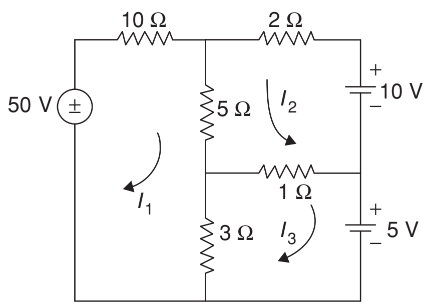
Where,

$$[R] = \begin{bmatrix} R_1 + R_2 & -R_2 & 0 \\ -R_2 & R_2 + R_3 + R_4 & -R_4 \\ 0 & -R_4 & R_4 + R_5 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} R_1 + R_2 & V_1 & 0 \\ -R_2 & 0 & -R_4 \\ 0 & -V_2 & R_4 + R_5 \end{vmatrix}}{|R|}$$

$$I_3 = \frac{\begin{vmatrix} R_1 + R_2 & -R_2 & V_1 \\ -R_2 & R_2 + R_3 + R_4 & 0 \\ 0 & -R_4 & -V_2 \end{vmatrix}}{|R|}$$

Example 21: Determine the mesh currents I_1, I_2 and I_3 is the circuit shown in figure



Solution:

KVL in loop I

$$50 = (10 + 5 + 3)\, I_1 + 5I_2 - 3I_3 \qquad (10)$$

KVL in loop II

$$10 = 2I_2 + 5(I_1 + I_2) + 1(I_2 + I_3)$$

$$10 = 5\, I_1 + 8\, I_2 + I_3 \qquad (11)$$

KVL in loop III

$$1(i_2 + i_3) + 5 + 3\,(i_3 - i_1) = 0$$

$$-3i_1 + i_2 + 4i_3 = -5 \qquad (12)$$

By Cramer's rule

$$I_1 = \frac{\begin{vmatrix} 50 & 5 & -3 \\ 10 & 8 & 1 \\ -5 & 1 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}}$$

$I_1 = 3.3$ A

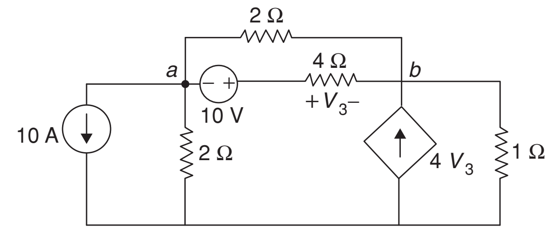
$$I_2 = \frac{\begin{vmatrix} 18 & 50 & -3 \\ 5 & 10 & 1 \\ -3 & -5 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}}$$

$I_2 = -0.997$ A

$$I_3 = \frac{\begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}}$$

$I_3 = 1.47$ A

Example 22:



Write nodal equations for the circuit shown in figure and find the power supplied by the 10 V source.

Solution:

KCL at node 'a':

$$10 + \frac{V_a}{2} + \frac{V_a - V_b}{2} + \frac{V_a + 10 - V_b}{4} = 0$$

$$1.25\ V_a - 0.75\ V_b = -12.5 \qquad (13)$$

KCL at node 'b':

$$\frac{V_b - 10 - V_a}{4} + \frac{V_b - V_a}{2} - 4V_3 + \frac{V_b}{1} = 0$$

$$-4.75\ V_a + 5.75\ V_b = 42.b \qquad (14)$$

$$V_3 = V_a + 10 - V_b \qquad (15)$$

Solving (13) and (14) and (15)

$V_a = -11.03$ V, $V_b = -1.724$ V

The current deliverred by 10 V source is

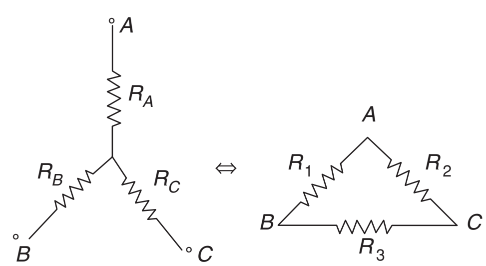
$$I_{10} = \frac{V_a - V_b + 10}{4}$$

The power supplied by the 10 V source is

$$P_{10} = (10)I_{10} = 10\left(\frac{V_a - V_b + 10}{4}\right)$$

$$= 1.735 \text{ W}$$

Star–Delta transformation



If delta is given, corresponding star elements are:

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

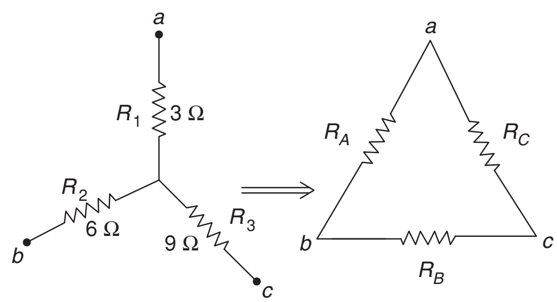
If star elements are given, corresponding delta network elements are:

$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

Example 23: A star connected network which is equivalent to the delta network is shown in the below figure. The R_A, R_B and R_C (in ohms) are respectively,



(A) 99 Ω, 33 Ω, 16.5 Ω
(B) 11 Ω, 16.5 Ω, 33 Ω
(C) 11 Ω, 33 Ω, 16.5 Ω
(D) 1 Ω, 3 Ω, 1.5 Ω

Solution: (C)

$$\Sigma R_A = R_1R_2 + R_2R_3 + R_3R_1$$
$$= 6 \times 3 + 3 \times 9 + 9 \times 6$$
$$= 99$$

$$R_A = \frac{\Sigma R}{R_3} = \frac{99}{9} = 11\,\Omega$$

$$R_B = \frac{\Sigma R}{R_1} = \frac{99}{3} = 33\,\Omega$$

$$R_C = \frac{\Sigma R}{R_2} = \frac{99}{6} = 16.5\,\Omega.$$

Note: Twelve Z Ω impedances are used as edges to form a cube. The equivalent impedance seen between the two diagonally opposite corners of the cube is

(i) If $Z = R$ Ω, then $z_{eq} = \frac{5}{6} R$ Ω

(ii) If $Z = L$, then $z_{eq} = \frac{5}{6} L$ H

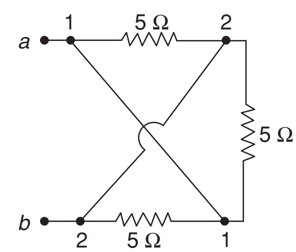
(iii) If $Z = C$ then $z_{eq} = \frac{6}{5} C$ F

Example 24: Twelve 3H inductors are used as edges to form a cube, determine the equivalent inductance seen b/w the two diagonally opposite corners of the cube.

Solution:
We know that,

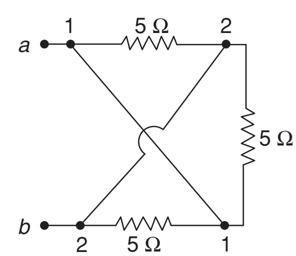
$$Z_{eq} = \frac{5}{6}\text{L} = \frac{5}{6} \times 3 = 2.5\text{ H}$$

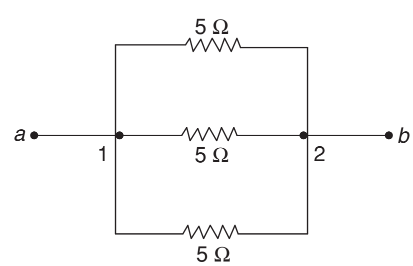
Example 25: Consider the circuit shown in figure below and determine R_{ab},



(A) 2.5 Ω (B) $\frac{5}{3}$ Ω
(C) 7.5 Ω (D) None of the above

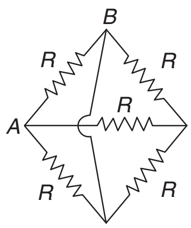
Solution: (B)
Redraw the above circuit





$$R_{ab} = (5 \,\|\, 5 \,\|\, 5)\ \Omega$$
$$= \frac{2.5 \times 5}{7.5} = \frac{5}{3}\ \Omega.$$

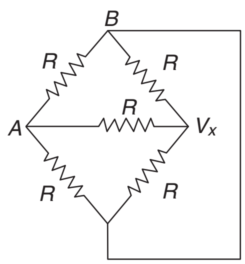
Example 26: Consider the circuit shown in figure determine R_{AB}



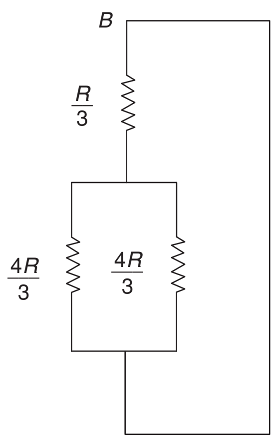
(A) $R_{AB} = 0\ \Omega$ (B) $R_{AB} = R\ \Omega$

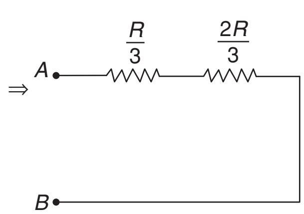
(C) $R_{AB} = 2\,R\ \Omega$ (D) $R_{AB} = \dfrac{R}{2}\ \Omega$

Solution: (B)
Redraw the given circuit



Convert $\Delta - y$

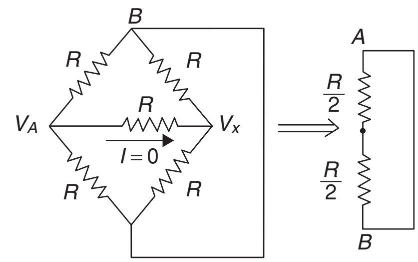




$$R_{AB} = R\Omega$$

for all values of 'R'

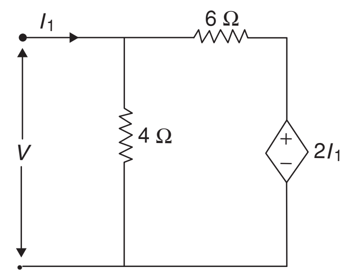
2nd Method:
From the given circuit
Since $R_1 \times R_4 = R_2 R_3$
Bridge is in Balanced condition
$R \times R = R \times R$



$$\Rightarrow \quad R_{AB} = \frac{R}{2} + \frac{R}{2} = R\ \Omega$$

Here $V_x = V_A$
So $I = 0$.

Example 27: The circuit shown in figure will act as a load resistor of



(A) $\dfrac{1}{5}\ \Omega$ (B) $\dfrac{16}{5}\ \Omega$

(C) $\dfrac{5}{16}\ \Omega$ (D) None of the above

Solution: (B)
From given circuit

$$I_1 = \frac{V}{4} + \frac{V - 2I_1}{6}$$

$$12I_1 = 3V + 2V - 2V - 4I_1$$

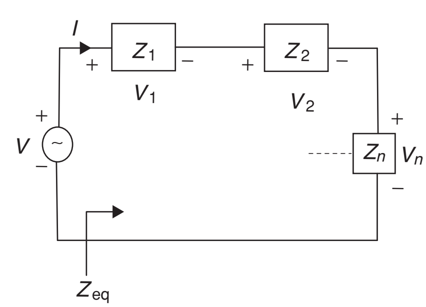
$$16_1 = 5V$$

$$\frac{V}{I_1} = R = \frac{16}{5}\ \Omega.$$

Equivalent Circuits wrt Passive R, L, Cs

Two elements are said to be in series only when currents through the elements are same and two elements are said to be in parallel only when voltages across the elements are same

Series circuits



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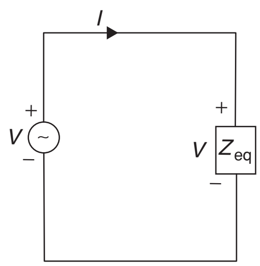


Let $n = 2$

$\Rightarrow Z = Z_R = R\ \Omega$

$\Rightarrow Z = Z_L = j\omega L\ \Omega$

$$\Rightarrow \quad Z = Z_C = \frac{1}{j\omega c}\ \Omega$$

$$z_{eq} = Z_1 + Z_2$$

If Z equal to

1. R: $R_{eq} = R_1 + R_2$
1. L: $L_{eq} = L_1 + L_2$
3. C: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Voltage division

$$V = Z_{eq} I$$

$$I = \frac{V}{Z_{eq}}$$

$\Rightarrow V_1 = I Z_1$ (by ohm's law)

$$\Rightarrow \quad \boxed{V_1 = \frac{V}{Z_1 + Z_2} Z_1}$$

$$\Rightarrow V_2 = Z_2 I$$

$$\Rightarrow \quad \boxed{V_2 = \frac{V}{Z_1 + Z_2} Z_2}$$

1. $Z = R$:

$$V_1 = \frac{V \cdot R_1}{R_1 + R_2}:$$

$$V_2 = \frac{V \cdot R_2}{R_1 + R_2}$$

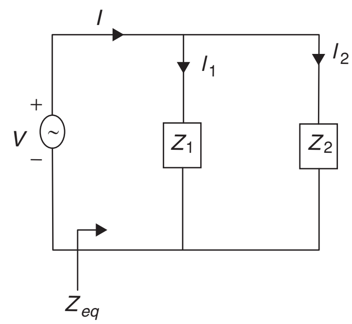
2. $Z = j\omega L$: $V_1 = \frac{V \cdot L_1}{L_1 + L_2}; V_2 = \frac{V \cdot L_2}{L_1 + L_2}$

3. $Z = \frac{1}{j\omega c}$

$$V_1 = \frac{V \cdot C_2}{C_1 + C_2}:$$

$$V_2 = \frac{V \cdot C_1}{C_1 + C_2}$$

Parallel circuits



≡

$$\frac{1}{Z_{eq}} = \frac{1}{z_1} + \frac{1}{z_2}$$

$$Z_{eq} = \frac{z_1 z_2}{z_1 + z_2}$$

If $Z = R$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

If $R_1 = R_2 = R\ \Omega$

$$R_{eq} = \frac{R}{2}\ \Omega$$

If $Z = j\omega L : \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$

$$L_{eq} = \frac{L_1 \cdot L_2}{L_1 + L_2}$$

If $Z = Z_c$: $C_{eq} = C_1 + C_2$

Current division

$$V = z_{eq} \cdot I$$

$$\Rightarrow \quad V = \frac{Z_1 Z_2}{Z_1 + Z_2} \times I$$

$$\boxed{I_1 = \frac{V}{Z_1}}$$

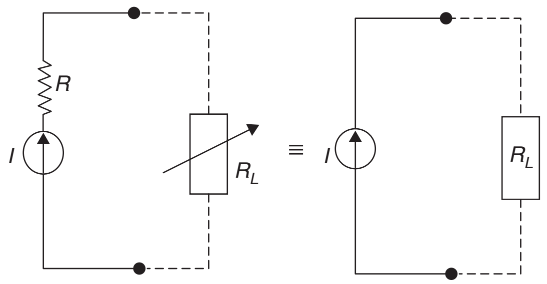
Examples:

$$Z_1 = \frac{1}{j\omega c1}; Z_2 = \frac{1}{j\omega c2}$$

$$I_1 = \frac{I \cdot C_1}{C_1 + C_2}; I_2 = \frac{I \cdot C_2}{C_1 + C_2}$$

Notes:

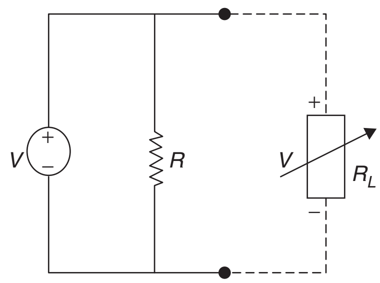
1. Resistance connected in series with an ideal current source (internal resistance of an ideal current source) does not have any effect on current supplied by it. So it can be neglected.



$R \neq \infty$

That is, the load current is independent of R value $i^2R \neq 0$

2. Resistance connected in parallel with an ideal voltage source does not have any effect on voltage offered by it. So it can be neglected.

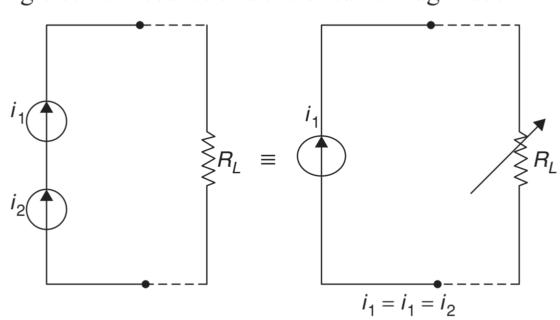


Here $R \neq 0$

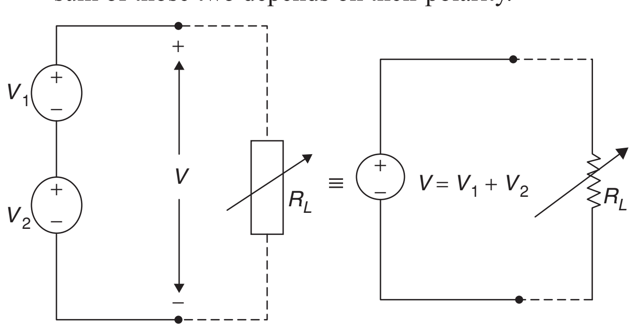
So, a resistor in parallel with an ideal voltage source can be neglected in the analysis.

That is, the load voltage is independent of R value $\frac{V^2}{R} \neq 0$

3. Equivalent of series connected current sources is a single current source and are of same magnitude.



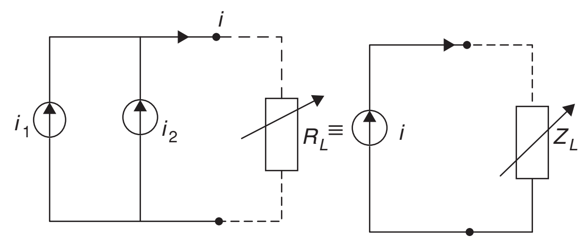
4. Equivalent of two series connected voltage sources is sum of those two depends on their polarity.



By KVL

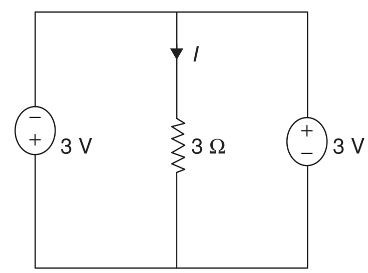
$$\Rightarrow V_2 + V_1 - V = 0$$
$$V = V_1 + V_2$$

5. Equivalent of two parallel connected current sources is equal to sum of those two with their relevant polarity.



$\therefore i = i_1 + i_2$

Example 28:



The current I is

(A) 1 A

(B) −1 A

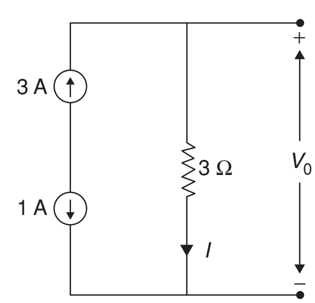
(C) 2 A

(D) Indeterminate

Solution: (D)

Two voltage sources are in parallel and different this is violates KVL and KCL, so indeterminate.

Examples 29:



Determine voltage V_o

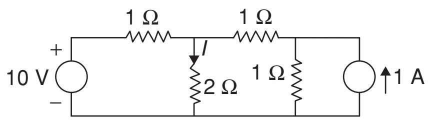
$$V_o = I \times 3$$
$$= (3 - 1)\ 3$$
$$= 6\text{ V}$$

Exercises

Practice Problems I

Directions for questions 1 to 21: Select the correct alternative from the given choices.

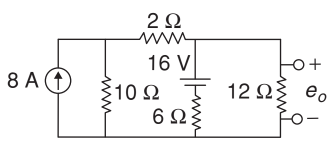
1. The current in the 2 Ω resistor '*I*' is



(A) $\frac{1}{8}$ A (B) $\frac{3}{8}$ A

(C) $\frac{21}{8}$ A (D) $\frac{23}{8}$ A

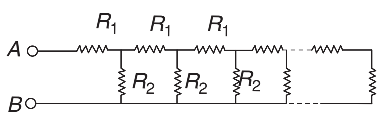
2. The voltage e_o in the figure is



(A) 48 V (B) 24 V
(C) 36 V (D) 28 V

3. The driving point impedance of the infinite ladder network shown in the figure is _________.

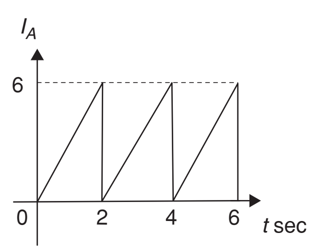
Given $R_1 = 3$ Ω, $R_2 = 2$ Ω



(A) $\frac{3}{2}$ Ω (B) $\frac{2}{3}$ Ω

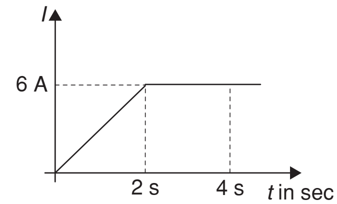
(C) $\left[3+\frac{2}{3}\right]$ Ω (D) $\sqrt{21}$

4. The current wave formed in a pure resistor of 5 Ω is shown in the fig. The power dissipated in the resistor is



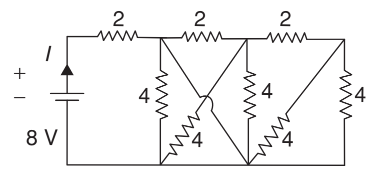
(A) 20 W (B) 45 W
(C) 60 W (D) 90 W

5. Figure shows the waveform of the current passing through an inductor of resistance 1 Ω and inductance 2H. The energy absorbed by the inductor in the first four seconds is _________.



(A) 132 J (B) 98 J
(C) 144 J (D) 168 J

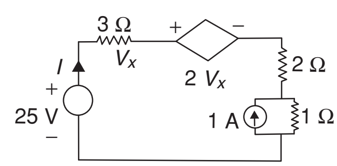
6. In the circuit of the given figure the source current '*I*' is



(A) 2 A (B) 4 A

(C) 1 A (D) $\frac{8}{3}$ A

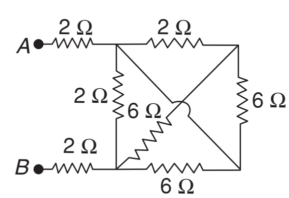
7. In the circuit shown in The figure the current *I* is



(A) $\frac{25}{3}$ A (B) $\frac{25}{6}$ A

(C) 2 A (D) 3 A

8. In the circuit shown in figure below

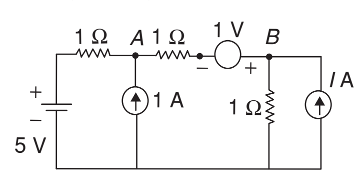


R_{AB} =?

(A) $\frac{21}{4}$ Ω (B) $\frac{5}{6}$ Ω

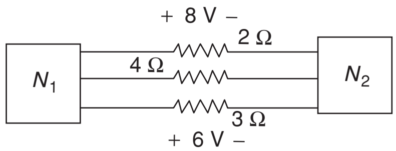
(C) 10 Ω (D) 8 Ω

9. What should be the value of current '*I*' to have zero current flowing through *AB*.



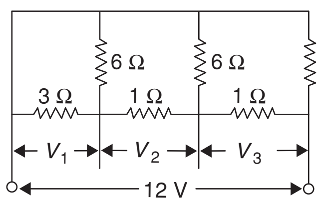
(A) 2 A (B) 4 A
(C) –4 A (D) –2 A

10. The two electrical sub networks N_1 and N_2 are connected through three resistors as shown in the figure. The voltage across the 2 Ω resistor is 8 V and the 3 Ω resistor is 6 V. The voltage across the 4 Ω resistor is



(A) 24 V (B) –24 V
(C) 8 V (D) –8 V

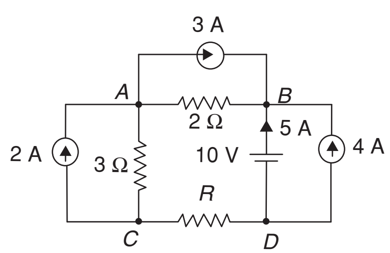
11. In the circuit shown in figure the voltages V_1, V_2 and V_3 are



(A) $\frac{8}{3}, \frac{16}{3}, 4$ (B) $\frac{16}{3}, \frac{8}{3}, \frac{8}{3}$

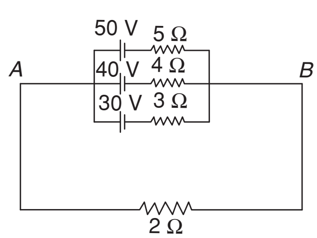
(C) $\frac{16}{3}, \frac{8}{3}, 4$ (D) $4, \frac{16}{3}, \frac{8}{3}$

12. In the network shown in the figure the current in resistor R is



(A) 2 A (B) 3 A
(C) 4 A (D) 9 A

13. In the circuit shown the voltage AB is

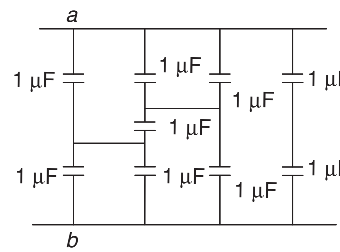


(A) 35 V (B) 28.2 V
(C) 38.3 V (D) 42.6 V

14. A network contains linear resistors which are connected in series across an ideal voltage source. If all the resistances are halved and the voltage is doubled then the voltage across each resistor becomes

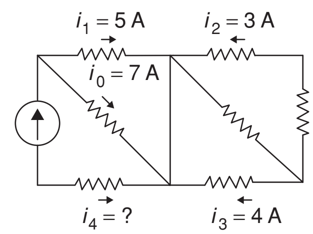
(A) Doubled (B) halved
(C) not changed (D) none

15. Obtain the equivalent capacitance of the network given



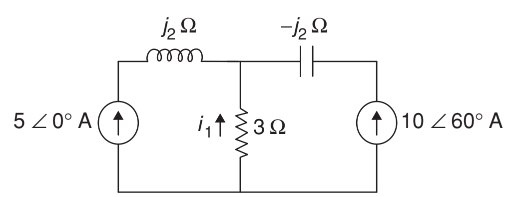
(A) 1 μF (B) 0.8 μF
(C) 1.9 μF (D) 2.6 μF

16. The current i_4 in the circuit of the figure equal to



(A) 12 A (B) −12 A
(C) 4 A (D) None of these

17.

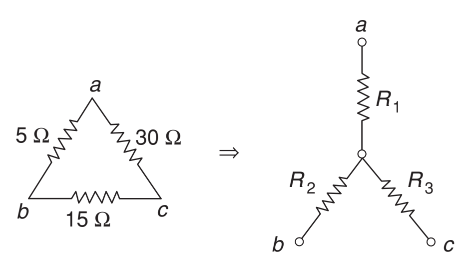


For the circuit shown in the figure, the instantaneous current $i_1(t)$ is

(A) $\frac{10\sqrt{3}}{2}\angle 90°\text{A}$ (B) $\frac{10\sqrt{3}}{2}\angle -90°\text{A}$

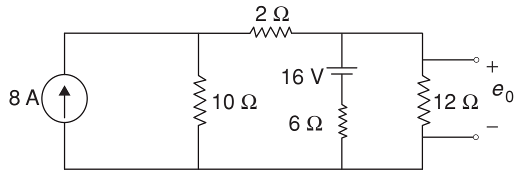
(C) $5\angle 60°$ A (D) $5\angle -60°$ A

18. A Delta - connected network with its Wye - equivalent is shown in the given figure. The resistance R_1, R_2 and R_3 (in Ohm) are respectively



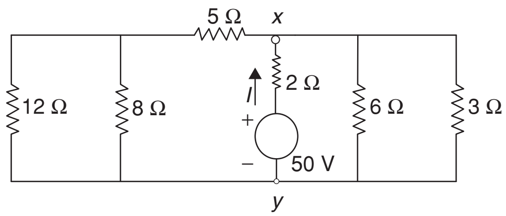
(A) 1.5, 3 and 9 (B) 3, 9 and 1.5
(C) 9, 3 and 1.5 (D) 3, 1.5 and 9

19. The voltage e_0 in the figure



(A) 48 V (B) 24 V (C) 36 V (D) 28 V

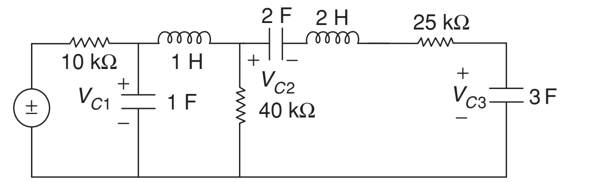
20.



The current *I* supplied by the source 50 V is

(A) 25 A (B) 13.7 A
(C) 9.8 A (D) 3.66 A

21. The voltages V_{C1}, V_{C2}, and V_{C3} across the capacitors in the circuit in the given figure, understeady state are respectively



(A) 80 V, 32 V, 48 V (B) 80 V, 48 V, 32 V
(C) 20 V, 8 V, 12 V (D) 20 V, 12 V, 8 V

Practice Problems 2

Directions for questions 1 to 19: Select the correct alternative from the given choices.

1. A network contains linear resistors which are connected in series across an ideal voltage source. If all the resistances are halved and the voltage is doubled then the voltage across each resistor becomes

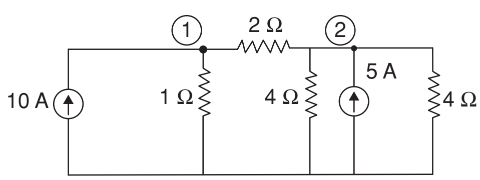
(A) doubled (B) halved
(C) not changed (D) none

2. Twelve similar conductors of 1 Ω resistance form a cubical frame work. Then the resistance between two adjacent corners, two opposite corners of one face and two opposite corners of the cube are

(A) $\frac{3}{4}, \frac{5}{6}, \frac{7}{12}$ (B) $\frac{7}{4}, \frac{5}{6}, \frac{3}{4}$

(C) $\frac{7}{12}, \frac{3}{4}, \frac{5}{6}$ (D) $\frac{12}{7}, \frac{4}{3}, \frac{6}{5}$

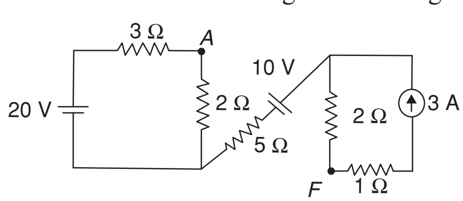
3. In the network shown in the figure



The voltage at node 2 is

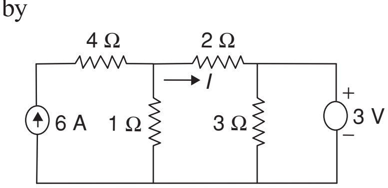
(A) 2 V (B) 10 V (C) 6 V (D) 4 V

4. In the network shown in the figure the voltage AF is



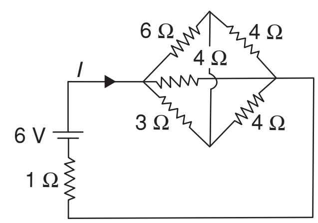
(A) 4 V (B) –4 V
(C) 6 V (D) 2 V

5. For the circuit shown in the figure the current '*I*' is given by



(A) 2 A (B) 3 A
(C) 1 A (D) zero

6. The current '*I*' supplied by the source in the figure is



(A) 2 A (B) 3 A

(C) $\frac{3}{2}$ A (D) $\frac{2}{3}$ A

7. A resistance of 10 Ω is connected in series with two resistances of 20 Ω arranged in parallel what resistance should be shunted across this parallel combination so that the total current taken shall be 2 A with 30 V applied

(A) 5 Ω (B) 10 Ω
(C) 20 Ω (D) 25 Ω

8. The resistance of a strip of conductor is *R* Ω. If the strip is elongated such that its length is doubled the resistance of the strip is given by

(A) 4*R* (B) 2*R*

(C) $\frac{R}{2}$ (D) *R*

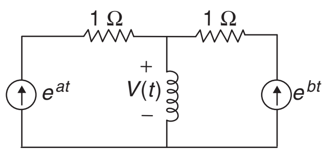
9. The current $i(t)$ through a 10 Ω resistor in series with an inductance, is given by $i(t) = 3 + 4\sin(100t + 45°) + 4\sin(300t + 60°)$ Amperes

The RMS (root mean square) value of the current and the power dissipated in the circuit are

(A) $\sqrt{41}$ A, 410 W, respectively
(B) $\sqrt{35}$ A, 350 W, respectively
(C) 5 A, 250 W, respectively
(D) 11 A, 1210 W, respectively

10. The nodal method of circuit analysis is based on
(A) KVL and Ohm's law
(B) KCL and Ohm's law
(C) KCL and KVL
(D) KCL, KVL and Ohm's law

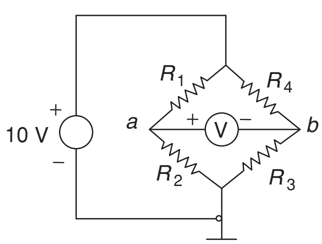
11. In the given circuit, the voltage $v(t)$ is



(A) $e^{at} - e^{bt}$
(B) $e^{at} + e^{bt}$
(C) $ae^{at} - be^{bt}$
(D) $ae^{at} + be^{bt}$

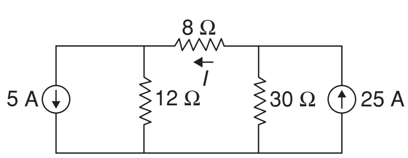
12. The rms value of the voltage defined by $v(t) = 5 + 5\sin\left(314t + \frac{\pi}{6}\right)$ is
(A) 5 V
(B) 2.5 V
(C) 6.12 V
(D) 10 V

13. If $R_1 = R_2 = R_4 = R$ and $R_3 = 1.1\,R$ in the bridge circuit shown in the figure, then the reading in the ideal voltmeter connected between a and b is



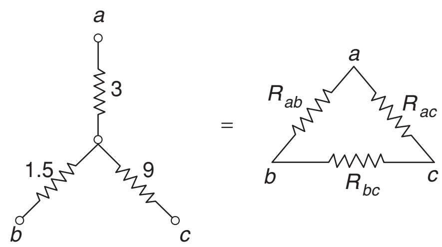
(A) 0.238 V
(B) 0.138 V
(C) –0.238 V
(D) 1 V

14.



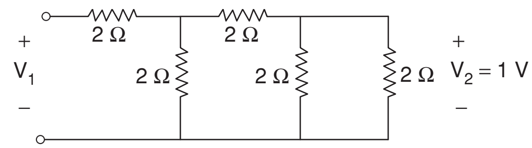
The current I in the above circuit is
(A) 20 A
(B) –20 A
(C) 16.2 A
(D) –16.12 A

15.



The value of R_{ab}, R_{ac} and R_{bc} are
(A) 30 Ω, 15 Ω, 5 Ω
(B) 5 Ω, 30 Ω, 15 Ω
(C) 5 Ω, 30 Ω, 15 Ω
(D) 15 Ω, 5 Ω, 30 Ω

16.

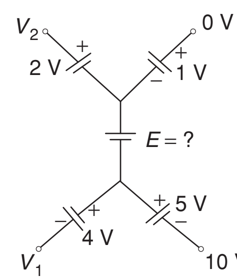


If $V_2 = 1$ V in the network shown, the value of V_1 will be
(A) 2.5 V
(B) 4 V
(C) 5 V
(D) 8 V

17. If each branch of a Delta circuit has impedance $\sqrt{3}Z$, then each branch of the equivalent Wye circuit has impedance

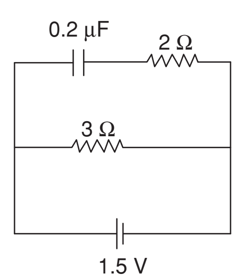
(A) $\frac{Z}{\sqrt{3}}$
(B) $3Z$
(C) $3\sqrt{3}Z$
(D) $\frac{Z}{3}$

18. In the given circuit, the value of the voltage source E is



(A) –16 V
(B) 4 V
(C) –6 V
(D) 16 V

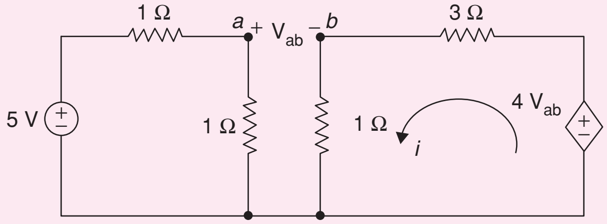
19. What is the current through resistor 2 Ω in the circuit given?



(A) 0.6 V
(B) 1.8
(C) 0.9 V
(D) 0 V

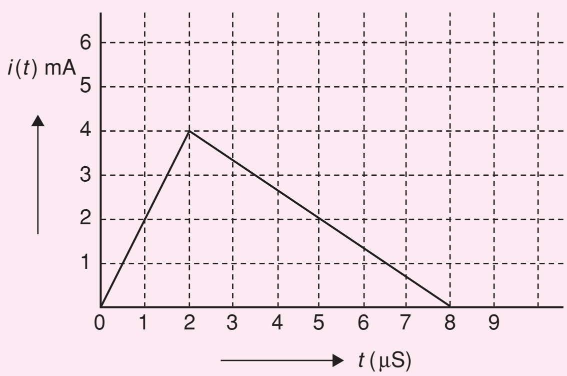
Previous Years' Questions

1. In the circuit shown in the figure, the value of the current i will be given by **[2008]**



(A) 0.31 A (B) 1.25 A
(C) 1.75 A (D) 2.5 A

Common Data for Questions 2 and 3: The current $i(t)$ sketched in the figure flows through an initially uncharged 0.3 nF capacitor.



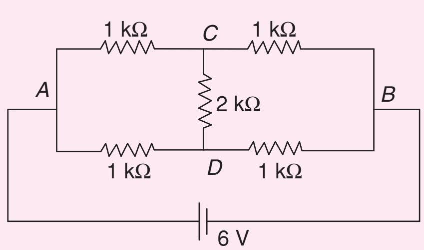
2. The charge stored in the capacitor at $t = 5$ μs, will be **[2008]**

(A) 8 nC (B) 10 nC
(C) 13 nC (D) 16 nC

3. The capacitor charged upto 5 μs, as per the current profile given in the figure, is connected across an inductor of 0.6 mH. Then the value of voltage across the capacitor after 1μs will approximately be **[2008]**

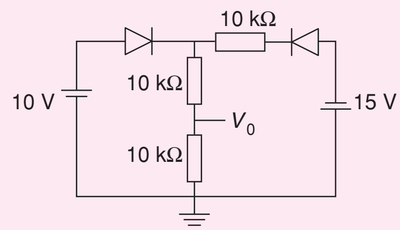
(A) 18.8 V (B) 23.5 V
(C) −23.5 V (D) −30.6 V

4. The current through the 2 kΩ resistance in the circuit shown is **[2009]**



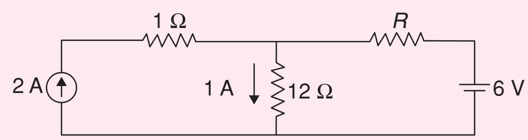
(A) 0 mA (B) 1 mA
(C) 2 mA (D) 6 mA

5. Assuming that the diodes in the given circuit are ideal, the voltage V_0 is **[2010]**



(A) 4 V (B) 5 V
(C) 7.5 V (D) 12.12 V

6. If the 12 Ω resistor draws a current of 1 A as shown in the figure, the value of resistance R is **[2010]**

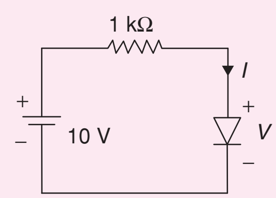


(A) 4 Ω (B) 6 Ω
(C) 8 Ω (D) 18 Ω

7. The $I-V$ characteristics of the diode in the circuit given below are **[2012]**

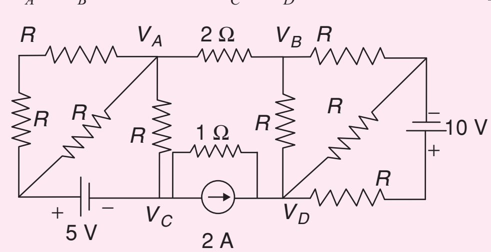
$$I = \begin{cases} \dfrac{V-0.7}{500} \text{ A}, & V \geq 0.7 \text{ V} \\ 0 \text{ A}, & V < 0.7 \text{ V} \end{cases}$$

The current in the circuit is



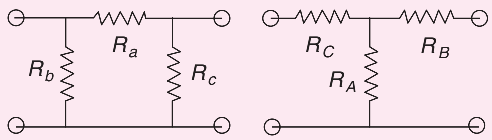
(A) 10 mA (B) 9.3 mA
(C) 6.67 mA (D) 6.2 mA

8. If $V_A - V_B = 6$ V, then $V_C - V_D$ is **[2012]**



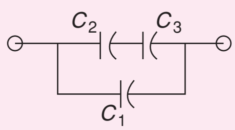
(A) −5 V (B) 2 V
(C) 3 V (D) 6 V

9. Consider a delta connection of resistors and its equivalent star connection as shown below. If all elements of the delta connection are scaled by a factor k, $k > 0$, the elements of the corresponding star equivalent will be scaled by a factor of **[2013]**



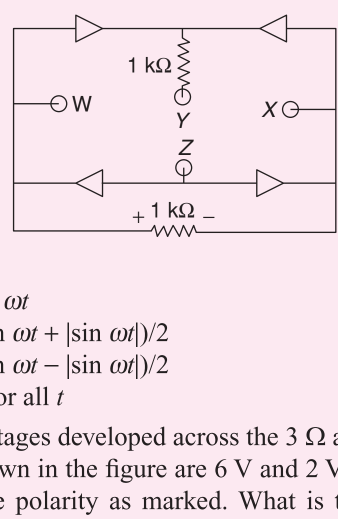
(A) k^2
(B) k
(C) $1/k$
(D) $\sqrt{k}$

10. Three capacitors C_1, C_2 and C_3, whose values are 10 μF, 5 μF, and 2 μF respectively, have breakdown voltages of 10 V, 5 V, and 2 V respectively. For the interconnection shown, the maximum safe voltage in Volts that can be applied across the combination and the corresponding total charge in μC stored in the effective capacitance across the terminals are respectively, **[2013]**



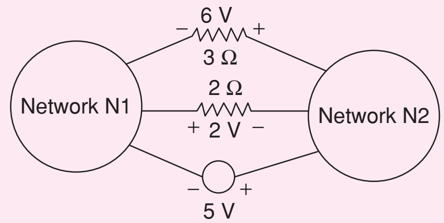
(A) 2.8 and 36
(B) 7 and 119
(C) 2.8 and 32
(D) 7 and 80

11. A voltage 1000 sin ωt V is applied across YZ. Assuming ideal diodes, the voltage measured across WX in volts is **[2013]**



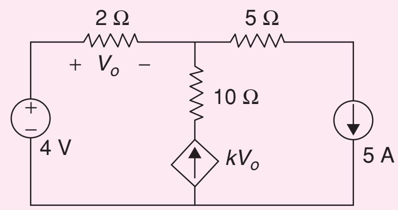
(A) sin ωt
(B) $(\sin \omega t + |\sin \omega t|)/2$
(C) $(\sin \omega t - |\sin \omega t|)/2$
(D) 0 for all t

12. The voltages developed across the 3 Ω and 2 Ω resistors shown in the figure are 6 V and 2 V respectively, with the polarity as marked. What is the power (in Watt) delivered by the 5 V voltage source? **[2015]**

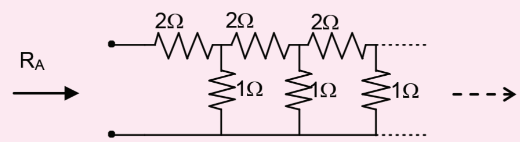


(A) 5
(B) 7
(C) 10
(D) 14

13. In the given circuit, the parameter k is positive, and the power dissipated in the 2 Ω resistor is 12.5 W. The value of k is ______ **[2015]**

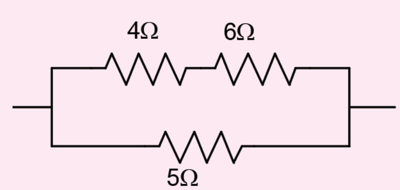


14. R_A and R_B are the input resistances of circuits as shown below. The circuits extend infinitely in the direction shown. Which one of the following statements is TRUE? **[2016]**



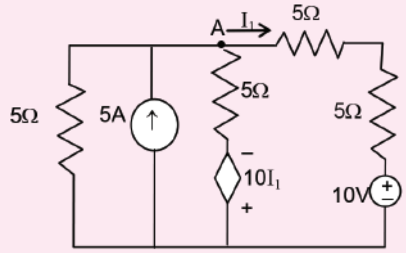
(A) $R_A = R_B$
(B) $R_A = R_B = 0$
(C) $R_A < R_B$
(D) $R_B = R_A/(1+R_A)$

15. In the portion of a circuit shown, if the heat generated in 5 Ω resistance is 10 calories per second, then heat generated by the 4 Ω resistance, in calories per second, is _____. **[2016]**



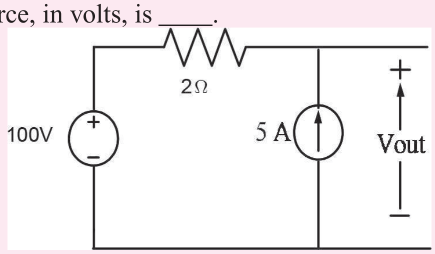
16. In the given circuit, the current supplied by the battery, in ampere, is ___________. **[2016]**

17. In the circuit shown below, the node voltage VA is_______V. **[2016]**



(A) 0 (B) 5
(C) 10 (D) 20

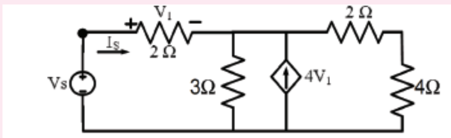
18. In the circuit shown below, the voltage and current sources are ideal. The voltage (V_{out}) across the current source, in volts, is _____. **[2016]**



19. The graph associated with an electrical network has 7 branches and 5 nodes. The number of independent KCL equations and the number of independent KVL equations, respectively, are **[2016]**

(A) 2 and 5 (B) 5 and 2
(C) 3 and 4 (D) 4 and 3

20. The driving point input impedance seen from the source V_s of the circuit shown below, in Ω is__________. **[2016]**



Answer Keys

Exercises

Practice Problems 1

1. C	**2.** D	**3.** D	**4.** C	**5.** C	**6.** B	**7.** C	**8.** A	**9.** B	**10.** B
11. C	**12.** D	**13.** C	**14.** A	**15.** C	**16.** B	**17.** A	**18.** D	**19.** D	**20.** B
21. B									

Practice Problems 2

1. A	**2.** C	**3.** B	**4.** A	**5.** B	**6.** A	**7.** B	**8.** A	**9.** C	**10.** B
11. D	**12.** C	**13.** C	**14.** C	**15.** C	**16.** D	**17.** A	**18.** A	**19.** D	

Previous Years' Questions

1. B	**2.** C	**3.** D	**4.** A	**5.** B	**6.** B	**7.** D	**8.** A	**9.** B	**10.** C
11. D	**12.** A	**13.** 0.5	**14.** D	**15.** 2	**16.** 0.5	**17.** 11.428	**18.** D	**19.** D	**20.** 20