Chapter 16

probability

Exercise 16.1

Q. 1 In each of the following, describe the sample space for the indicated experiment.

A coin is tossed three times.

Answer:

A coin can either give a head or a tail

So, when 1 coin is tossed once the sample space = 2

when the coin is tossed 3 times sample space = 23 = 8

The sample space can be found by relative combination of the events

Let H denote the event of a head and T denote the event of a tail

We define the possible outcomes by an ordered set (x, y, z) where

x denotes the outcomes when the coin is tossed for the first time

y denotes the outcomes when the coin is tossed for the second time

z denotes the outcomes when the coin is tossed for the third time

The sample spaces are:

S= {(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)}

Q. 2 In each of the following, describe the sample space for the indicated experiment.

A die is thrown two times.

Answer:

Let 1,2,3,4,5,6 denote the event the respective numbers come when the die is thrown

The total number of sample space = (6×6) = 36

We define the possible outcomes by an ordered set (x, y) where

x denotes the outcome of the first time the die is thrown

y denotes the outcome of the second time the die is thrown

The sample spaces

S= {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3) (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

Q. 3 In each of the following, describe the sample space for the indicated experiment.

A coin is tossed four times.

Answer:

Let H denote the event of a head and T denote the event of a tail

Total number of sample space= (24) =16

We define the possible outcomes by an ordered set (w, x, y, z)

w denotes the event the coin is tossed for the first time

x denotes the event the coin is tossed for the second time

y denotes the event the coin is tossed for the third time

z denotes the event the coin is tossed for the fourth time Sample space

S= {(H, H, H, H), (H, H, H, T), (H, H, T, H), (H, T, H, H), (T, H, H, H), (H, H, T, T), (H, T, H, T), (T, H, H, T), (T, H, T, H), (T, T, H, H), (H, T, T, H), (H, T, T, T), (T, H, T, T), (T, T, H, T), (T, T, T, H), (T, T, T, T)}

Q. 4 In each of the following, describe the sample space for the indicated experiment.

A coin is tossed and a die is thrown.

Answer:

Let H denote the event of a head and T denote the event of a tail,

1,2,3,4,5,6 denote the event the respective numbers come when the die is thrown

Total Number of space = $(2 \times 6) = 12$

We define the possible outcomes by an ordered set (x, y)

x denotes the first event when the coin is tossed

y denotes the second event when the die is thrown

Sample Space S={(H,1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

Q. 5 In each of the following, describe the sample space for the indicated experiment.

A coin is tossed and then a die is rolled only in case a head is shown on the coin.

Answer:

Let H denote the event of a head and T denote the event of a tail,

1,2,3,4,5,6 denote the event the respective numbers come when the die is thrown

The problem can be solved by breaking it into two cases

Case 1: Head is encountered

We define the possible outcomes by an ordered set (x, y)

x denotes the first event when the head is encountered y denotes the second event when the die is thrown

Sample Space S_1= {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)}

Case 2: Tail is encountered

Sample Space S_2={(T)}

The overall sample space S= {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T)}

Q. 6 In each of the following, describe the sample space for the indicated experiment.

2 boys and 2 girls are in Room X, and 1 boy and 3 girls in Room Y. Specify the sample space for the experiment in which a room is selected and then a person.

Answer:

Let X denote the event Room X is selected, Y denote the event Room Y is selected

B_1, B_2 denote the event a boy is selected from room X, B_3 denote the event a boy is selected from Room Y and G_1, G_2 denote the event a girl is selected from Room X and G_3, G_4, G_5 denote the event a girl is selected among the three girls from Room Y

The problem is solved by dividing into two cases

Case 1: Room X is selected

We define the possible outcomes by an ordered set (x, y)

x denotes the event Room X is chosen

y denotes the second event a student is selected

Sample Space S_1= {(X, B_1), (X, B_2), (X, G_1), (X, G_2)}

Case 2: Room Y is selected

We define the possible outcomes by an ordered set (x, y)

x denotes the event Room Y is chosen

y denotes the second event a student is selected

Sample Space S_2= {(Y, B_3), (Y, G_3), (Y, G_4), (Y, G_5)}

The overall sample space

S= {(X, B_1), (X, B_2), (X, G_1), (X, G_2), (Y, B_3), (Y, G_3), (Y, G_4), (Y, G_5)}

Q. 7 In each of the following, describe the sample space for the indicated experiment.

One die of red color one of white color and one of blue color are placed in a bag. One die is selected at random and rolled, its color and the number on its uppermost face is noted. Describe the sample space.

Answer:

Let R denote the event the red die is chosen W denote the event the white die is chosen B denote the event the Blue die is chosen

1,2,3,4,5,6 denote the event the respective numbers come when the die is thrown

We define the possible outcomes by an ordered set (x, y)

x denotes the first event a die is chosen

y denotes the second event the die is thrown and a number comes up

Total number of possible outcomes= $(6 \times 3) = 18$

The sample space of the event is

S={(R,1), (R,2), (R,3), (R,4), (R,5), (R,6), (W,1), (W,2), (W,3), (W,4), (W,5), (W,6) (B,1), (B,2), (B,3), (B,4), (B,5), (B,6)}

Q. 8 An experiment consists of recording boy–girl composition of families with 2 children.

(i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?

(ii) What is the sample space if we are interested in the number of girls in the family?

Answer:

Let B be the event a boy is born G be the event a girl is born

(i) We define the possible outcomes by an ordered set (x, y)

x denotes the event the first child is born

y denotes the event the second child is born

Sample space S= {(G, G), (G, B), (B, G), (B, B)}

(ii)When there are two children in the family then only three possible cases are possible, 2 Girl child, 1 Girl Child or no Girl Child

Sample Space S = {2, 1, 0}

Q. 9 A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

Answer:

Let R be the event a red ball is drawn and W be the event a white ball is drawn

Since all the white balls are identical so the event of drawing any one of the three-white ball is same.

Total Number of Sample space = (22 - 1) = 3 (Since both the balls cannot be red)

We define the possible outcomes by an ordered set (x, y)

x denotes drawing the first ball

y denotes drawing the second ball

Sample space S = {(W, W), (W, R), (R, W)}

Q. 10 An experiment consists of tossing a coin and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.

Answer:

Let H denote the event of a head and T denote the event of a tail,

1, 2, 3, 4, 5, 6 denote the event the respective numbers come when the die is thrown

The problem can be solved by breaking it into two cases

Case 1: Head is encountered

We define the possible outcomes by an ordered set (x, y)

x denotes the first event when the head is encountered

y denotes the second event the coin is tossed again

Sample space S_1 = {(H, T), (H, H)}

Case 2: Tail is encountered

We define the possible outcomes by an ordered set (x, y)

x denotes the first event when the tail is encountered

y denotes the second event when the die is thrown

Sample Space S_2={(T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}

The Overall Sample space

S = {(H, T), (H, H), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

Q. 11 Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non – defective(N). Write the sample space of this experiment.

Answer:

Let D denote the event the bulb is defective N denote the event the bulb is non-defective

Total Number of sample space= (2 × 2 × 2) = 8

We define the possible outcomes by an ordered set (x, y, z)

where

x denotes the event first bulb is selected

y denotes the event second bulb is selected

z denotes the event third bulb is selected

Sample space S = {(D, D, D), (D, D, N), (D, N, D), (N, D, D), (D, N, N), (N, D, N), (N, N, D), (N, N, N)}

Q. 12 A coin is tossed. If the outcome is a head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?

Answer:

Let H denote the event of a head and T denote the event of a tail,

1, 2, 3, 4, 5, 6 denote the event the respective numbers come when the die is thrown

The problem can be solved by dividing it into 3 cases

Case 1: Head is encountered and the corresponding number on the die is Odd

Total number of sample space= (1 × 3) = 3

We define the possible outcomes by an ordered set (x, y)

x denotes the first event the head is encountered

y denotes the second event the number on the die is Odd

Sample space S_1={(H,1), (H,3), (H,5)}

Case 2: Head is encountered and the corresponding number is even

Total number of sample space= $(1 \times 3 \times 6) = 18$

We define the possible outcomes by an ordered set (x, y, z)

where

x denotes the outcomes when the coin is tossed and head is encountered

y denotes the outcomes when the die is tossed and the number encountered is even

z denotes the outcomes when the die is tossed for the second time

Sample Space

S_2={(H,2,1), (H,2,2), (H,2,3), (H,2,4), (H,2,5), (H,2,6), (H,4,1), (H,4,2), (H,4,3), (H,2,4), (H,4,5), (H,4,6), (H,6,1), (H,6,2), (H,6,3), (H,6,4), (H,6,5), (H,6,6)}

Case 3: Tail is encountered

Total number of sample space=1

Sample space S_3={(T)}

The overall sample spaces

S={(H,1), (H,3), (H,5), (H,2,1), (H,2,2), (H,2,3), (H,2,4), (H,2,5), (H,2,6), (H,4,1), (H,4,2), (H,4,3), (H,2,4), (H,4,5), (H,4,6), (H,6,1), (H,6,2), (H,6,3), (H,6,4), (H,6,5), (H,6,6), (T)}

Q. 13 The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.

Answer:

Let 1, 2, 3, 4 denote the event 1, 2, 3, 4 numbered slip is drawn

When two slips are drawn without replacement the first event has 4 possible outcomes and the second event has 3 possible outcomes

So the total number of possible outcomes= $(4 \times 3) = 12$

We define the possible outcomes by an ordered event (x, y)

x denotes the first event the first slip is drawn

y denotes the second event the second slip is drawn

The sample space S = {(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)}

Q. 14 An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Answer:

Let 1, 2, 3, 4, 5, 6 denote the event the respective numbers come when the die is thrown

H denote the event of a head and T denote the event of a tail when coin is tossed

The following problem can be divided in two cases

Case 1: Even number shows up in the die

We define the possible outcomes by an ordered set (x, y)

x denotes the first event even number shows up

y denotes the second event a coin is thrown

The sample space S_1 = {(2, H), (4, H), (6, H), (2, T), (4, T), (6, T)}

Case 2: Odd number shows up in the die

We define the possible outcomes by an ordered set (x, y, z) x denotes the first event odd number shows up in the die

y denotes the second event the coin is thrown for first time

z denotes the third event the coin is thrown for second time

The sample space

S_2 = {(1, H, H), (3, H, H), (5, H, H), (1, H, T), (3, H, T), (5, H, T), (1, T, H), (3, T, H), (5, T, H), (1, T, T), (3, T, T), (5, T, T)}

Therefore, the overall sample space for the problem= S1+ S2

S = {(2, H), (4, H), (6, H), (2, T), (4, T), (6, T), (1, H, H), (3, H, H), (5, H, H), (1, H, T), (3, H, T), (5, H, T), (1, T, H), (3, T, H), (5, T, H), (1, T, T), (3, T, T), (5, T, T)}

Q. 15 A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls. If it shows head, we throw a die. Find the sample space for this experiment.

Answer:

Let H denote the event of a head and T denote the event of a tail when coin is tossed

R_1, R_2 denote the event the red balls are drawn B1, B2, B3 denote the events black ball are drawn

1, 2, 3, 4, 5, 6 denote the event the respective numbers come when the die is thrown

The following problem can be divided in two cases

Case 1: Tail turns up in the coin

We define the possible outcomes by an ordered set (x, y)

x denotes the first event the coin is tossed and tails turns up

y denotes the second event a ball is drawn

The sample space S_1 = {(T, R_1), (T, R_2), (T, B_1), (T, B_2), (T, B_3)}

Case 2: Head turns up in the coin

We define the possible outcomes by an ordered set (x, y)

x denotes the first event the coin is tossed and head turns up

y denotes the second event the die is thrown

The sample space S_2={(H,1), (H,2), (H,3), (H,4), (H,5), (H,6)}

Therefore, the overall sample space for the problem= S_1+ S_2

S= {(T, R_1), (T, R_2), (T, B_1), (T, B_2), (T, B_3), (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)}

Q. 16 A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?

Answer:

Let 1, 2, 3, 4, 5, 6 denote the event the respective numbers come when the die is thrown

Since this is a continuous event and doesn't stop until six is found so the sample space is continuous in nature.

The six may come up on the very first throw or the second or the third and this goes on continuously until six comes

The sample space when 6 comes on very first throw= {6}

The sample space when 6 comes on second throw = {(1,6), (2,6), (3,6), (4,6), (5,6)}

This event can go on for infinite times hence the sample space is infinitely defined

S = {(6), (1,6), (2,6), (3,6), (4,6), (5,6), (1,1,6), (1,2,6), (1,1,1,6)}

Exercise 16.2

Q. 1 A die is rolled. Let, E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Answer:

When the die is rolled the sample space will consist of following outcomes

S = (1, 2, 3, 4, 5, 6)

According to the given conditions

E = (4) and

(F) = (2, 4, 6)

E ∩ F = (4) ∩ (2, 4, 6)

⇒ (4) ≠ ϕ as there is a common element between E & F

Hence E and F are not mutually exclusive event.

Q. 2 A die is thrown. Describe the following events:

(i) A: a number less than 7

(ii) B: a number greater than 7

(iii) C: a multiple of 3

(iv) D: a number less than 4

(v) E: an even number greater than 4

(vi) F: a number not less than 3

Also find A B, A B, B C, E F, D E, A – C, D – E, E ∩ F', F'

Answer:

When the die is rolled the sample space will consist of following outcomes

S = (1, 2, 3, 4, 5, 6)

According to the given conditions

(i) A: a number less than 7

As every number on a dice is less than 7,

A = (1, 2, 3, 4, 5, 6)

(ii) B: a number greater than 7

B= (ϕ) as there is no number greater than 7 on the die.

(iii) C: a multiple of 3

C = (3, 6)

Only two numbers are multiple of 3 in the given sample space.

(iv) D: a number less than 4

D = (1, 2, 3)

(v) E: an even number greater than 4

E = (6)

(vi) F: a number not less than 3

F = (3, 4, 5, 6)

Now we need to find A U B, A ∩ B, B U C, E ∩ F, D ∩ E, D - E, A - C, E ∩ F', F'

From above we have the sets as follows: A = (1, 2, 3, 4, 5, 6)

B= (ϕ)

C= (3, 6)

D= (1, 2, 3)

E = (6)

F= (3, 4, 5, 6)

Therefore, A U B = (1, 2, 3, 4, 5, 6) U (ϕ) = (1, 2, 3, 4, 5, 6)

A ∩ B = (1, 2, 3, 4, 5, 6) ∩ (ϕ) = (ϕ)

B U C = (ϕ) U (3, 6) = (3, 6)

E ∩ F = (6) ∩ (3, 4, 5, 6) = (3, 4, 5, 6)

D ∩ E = (1, 2, 3) ∩ (6) = (ϕ)

D - E = (1, 2, 3) - (6) = (1, 2, 3)

A - C = (1, 2, 3, 4, 5, 6) - (3, 6) = (1, 2, 4, 5) F' = (3, 4, 5, 6))' = (1, 2) E ∩ F' = (6) ∩ (1, 2) = (ϕ)

Q. 3 An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8,

B: 2 occurs on either die

C: the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

Answer:

Since a pair of dice is thrown, all the possible outcomes or sample space (S) will be as follows

S =

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

A: the sum is greater than 8

Possible sum greater than 8 are 9, 10, 11 & 12

$\therefore$ A= $\begin{cases} (3,6)\ (4,5), & (5,4), \quad (6,3), \quad (4,6) \\ (5,5), \quad (6,4), & (5,6), \quad (6,5), \quad (6,6) \end{cases}$

B: 2 occurs on either die

Here possibilities are there that the number 2 will come on first die or second die or both the die simultaneously

B= $\begin{cases} (2,1), \quad (2,2), \quad (2,3), \quad (2,4), \quad (2,5), \quad (2,6) \\ \quad (1,2), \quad (3,2), \quad (4,2), \quad (5,2), \quad (6,2) \end{cases}$

C: The sum is at least 7 and multiple of 3

So the sum can be only 9 or 12

C= $\{(3,6),\ (4,5),\ (5,4)\ (6,3)\ (6,6)\}$

For 2 elements to be mutually exclusive, then there should not be any common element amongst them

(i) A$\cap$ B =ϕ

As there is no common element in A and B

So A& B are mutually exclusive

(ii) B $\cap$ C = ϕ

Since there is no common element between

So B and C are mutually exclusive.

(iii) A $\cap$ C

$\Rightarrow \{(3,6),\ (4,5),\ (5,4),\ (6,3),\ (6,6)\} \neq \phi$

Since A and C has common elements,

$\therefore$ A and C are mutually exclusive.

Q. 4 Three coins are tossed once. Let A denote the event ‘three heads show”, B denote the event “two heads and one tail show”, C denote the event” three tails show and D denote the event ‘a head shows on the first coin”. Which events are

(i) mutually exclusive?

(ii) simple?

(iii) Compound?

Answer:

Here, three coins are tossed once so the possible outcomes or sample space (S) consist of

S= $\{HHH\ ,\ HHT\ ,\ HTH\ ,\ THH\ ,\ HTT\ ,\ THT\ ,\ TTH,\ TTT\}$

Now,

A: ‘three heads’

A= (HHH)

B: “two heads and one tail”

B= (HHT, THH, HTH)

C: ‘three tails’

C= (TTT)

D: a head shows on the first coin

D= (HHH, HHT, HTH, HTT)

(i) Mutually exclusive

A ∩ B = (HHH) ∩ (HHT, THH, HTH)

= ϕ

Since there is no common element between A&B,

they are mutually exclusive

A ∩ C = (HHH) ∩ (TTT) = ϕ

Since here is no common element,

So, A and C are mutually exclusive.

A ∩ D = (HHH) ∩ (HHH, HHT, HTH, HTT)

= (HHH) ≠ ϕ

Since there is a common element in A and D

So they are not mutually exclusive

B ∩ C = (HHT, HTH, THH) ∩ (TTT) = ϕ

Since there is no common element in B & C, so they are mutually exclusive.

B∩ D = (HHT, THH, HTH) ∩ (HHH, HHT, HTH, HTT)

= (HHT, HTH) ≠ ϕ

Since there are common elements in B & D,

so, they not mutually exclusive.

C ∩ D = (TTT) ∩ (HHH, HHT, HTH, HTT) = ϕ

Since there is no common element in C & D,

So they are not mutually exclusive.

(ii) Simple event

An event is said to be simple if it has only one sample point in its sample space.

Here A = (HHH)

C = (TTT)

Both A & C have only one element,

so they are simple events.

(iii) Compound events

Is an event has more than one sample point of a sample space, it is called as compound event.

Here B= (HHT, HTH, THH)

D= (HHH, HHT, HTH, HTT)

Both B & D have more than one element,

So, they are compound events.

Q. 5 Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

(iii)Two events, which are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

(v) Three events which are mutually exclusive but not exhaustive.

Answer:

since 3 coins are tossed, the sample space will consist of following possibilities

S= (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)

(i) Two events which are mutually exclusive.

Let A be the event of getting only head

A= (HHH)

And let B be the event of getting only Tail

B=(TTT)

So $A \cap B = \phi$

Since there is no common element in A& B so these two are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive

Now,

Let A be the event of getting exactly two tails

A= (HTT, TTH, THT)

Let B be the event of getting at least two heads

B = (HHT, HTH, THH, HHH)

Let C be the event of getting only one tail

C= (TTT)

A ∩ B = (HTT, TTH, THT) ∩ (HHT, HTH, THH, HHH)

= ϕ

Since there is no common element in A and B hence they are mutually exclusive

B ∩ C = (HHT, HTH, THH, HHH) ∩ (TTT)

= ϕ

Since there is no common element in B and C

So they are mutually exclusive.

A ∩ C = (HTT, TTH, THT) ∩ (TTT) = ϕ

Since there is no common element in A and C,

So they mutually exclusive

Now, since A & B, B& C and A& C are mutually exclusive

∴ A, B, and C are mutually exclusive.

Also,

A ∪ B ∪ C = (HTT, TTH, THT, HHT, HTH, THH, HHH, TTT) = S

Hence A, B and C are exhaustive events.

(iii) Two events, which are not mutually exclusive

Let a be the event of getting at least two heads

A = (HHH, HHT, THH, HTH)

Let B be the event of getting only head

B= (HHH)

Now A ∩ B = (HHH) ≠ ϕ

Since there is a common element in A & B,

So they are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive

Let A be the event of getting only Head

A= (HHH)

Let b be the event of getting only tail

B = (TTT)

A ∩ B = ϕ

Since there is no common element in A & B,

These are mutually exclusive events.

But

A ∪ B = (HHH) ∪ (TTT)

$= \{(HHH), (TTT)\} \neq S$

Since A ∪ B ≠ S these are not exhaustive events.

(v) Three events which are mutually exclusive but not exhaustive

Let A be the event of getting only head

A = (HHH)

Let B be the event of getting only tail

B = (TTT)

Let C be the event of getting exactly two heads

C= (HHT, THH, HTH)

Now,

$A \cap B = (HHH) \cap (TTT) = \phi$

$A \cap C = (HHH) \cap (HHT, THH, HTH) = \phi$

$B \cap C = (TTT) \cap (HHT, THH, HTH) = \phi$

$\Rightarrow A \cap B = \phi$, $A \cap C = \phi$ and $B \cap C = \phi$

i.e. they are mutually exclusive

also

$A \cup B \cup C = (HHH\ TTT, HHT, THH, HTH) \neq S$

Since $A \cup B \cup C \neq S$,

So, A, B and C are not exhaustive.

It`s is hence proved that A, B and C are mutually exclusive but not exhaustive.

Q. 6 Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice 5.

Describe the events

(i) A’

(ii) not B

(iii) A or B

(iv) A and B

(v) A but not C

(vi) B or C

(viii) A B’ C’

Answer:

Here 2 dice are thrown

So the sample space (S) will have following events

$$S = \left.\begin{matrix} (1, 1) & (1, 2) & (1, 3) & (1, 4) & (1, 5) & (1, 6) \\ (2, 1) & (2, 2) & (2, 3) & (2, 4) & (2, 5) & (2, 6) \\ (3, 1) & (3, 2) & (3, 3) & (3, 4) & (3, 5) & (3, 6) \\ (4, 1) & (4, 2) & (4, 3) & (4, 4) & (4, 5) & (4, 6) \\ (5, 1) & (5, 2) & (5, 3) & (5, 4) & (5, 5) & (5, 6) \\ (6, 1) & (6, 2) & (6, 3) & (6, 4) & (6, 5) & (6, 6) \end{matrix}\right]$$

According to the given conditions

A: getting an even number on the first die.

A =

(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

B: getting an odd number on the first die.

B =

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)

C: getting the sum of the numbers on the dice ≤ 5

C = {(1, 1) (1, 2) (1, 3) (1, 4) (2, 1) (2, 2) (3, 1) (3, 2) (2, 3) (4, 1)}

Now

(ii) = A

(iv) A and B $(A \cap B) = \phi$

$\therefore A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$

Q. 7 Refer to question 6 above, state true or false: (give reason for your answer)

(i) A and B are mutually exclusive

(ii) A and B are mutually exclusive and exhaustive

(iii) A = B'

(iv) A and C are mutually exclusive

(v) A and B' are mutually exclusive.

(vi) A', B', C are mutually exclusive and exhaustive.

Answer:

since 2 dice are thrown the sample space will consist of following outcomes

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)

S =

(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Here A =

(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

B =

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)

And

C = {(1, 1) (1, 2) (1, 3) (1, 4) (2, 1) (2, 2) (3, 1) (3, 2) (2, 3) (4, 1)}

(i) A and B are mutually exclusive $(A \cap B) = \phi$

Since A and B has no common element,

So, A & B are mutually exclusive.

Thus, the given statement is true.

(ii) A and B are mutually exclusive and exhaustive

(1, 1)(1, 2)(1, 3)(1, 4)(1, 5)(1, 6)

(2, 1)(2, 2)(2, 3)(2, 4)(2, 5)(2, 6)

(3, 1)(3, 2)(3, 3)(3, 4)(3, 5)(3, 6) = S

(4, 1)(4, 2)(4, 3)(4, 4)(4, 5)(4, 6)

(5, 1)(5, 2)(5, 3)(5, 4)(5, 5)(5, 6)

(6, 1)(6, 2)(6, 3)(6, 4)(6, 5)(6, 6)

$\Rightarrow A \cup B = S$

And from (i) we have A and b are mutually exclusive

∴ A and B are mutually exclusive and exhaustive.

So, the statement is true.

(iii) A = B

B' = (2, 1) (2, 2)(2, 3)(2, 4)(2, 5)(2, 6)

(4, 1) (4, 2)(4, 3)(4, 4)(4, 5)(4, 6) = A

(6, 1) (6, 2)(6, 3)(6, 4)(6, 5)(6, 6)

Thus, the statement is true.

(iv) A and C are mutually exclusive

Here $A \cap C = \{(2, 1)\ (2, 2)\ (2, 3)\ (4, 1)\} \neq \phi$

Since A and C have common element,

So, they are not mutually exclusive

So, the given statement is false.

(v) A and B' are mutually exclusive.

$A \cap B' = A \cap A = A$

$\therefore A \cap B' \neq \phi$

So, A and B' not mutually exclusive.

Hence, the given statement is false.

(vi) A', B', C are mutually exclusive and exhaustive.

Here

A' = (1, 1) (1, 2)(1, 3)(1, 4)(1, 5)(1, 6)

(5, 1)(5, 2)(5, 3)(5, 4)(5, 5)(5, 6)

(3, 1)(3, 2)(3, 3)(3, 4)(3, 5)(3, 6)

B = (1, 1) (1, 2) (1, 3) (1, 4)(1, 5)(1, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5)(3, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5)(5, 6)

And C = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (3,1), (3,2), (2,3), (4,1)}

A’ ∩ B’ = ϕ

Hence there is no common element in A’ and B’

So they are mutually exclusive.

B’ ∩ C = {(2, 1) (2, 2) (2, 3) (4, 1)} ≠ ϕ

Since there is common element between B’ and C

They are not mutually exclusive.

Now, since B’ and C are not mutually exclusive,

So A’, B’ and C are not mutually exclusive and exhaustive.

Hence the given statement is false.

Exercise 16.3

Q. 1 Which of the following cannot be valid assignment of probabilities for outcomes of sample space?

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
(e)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{7}{14}$

Answer:

a) Both the conditions of axiomatic approach hold true in the given assignment, that is

1) Each of the number p (W_i) is less than zero and is positive

2) Sum of probabilities is

0.01+0.05+0.03+0.01+0.2+0.6=1

The given assignment is valid.

b) Both the conditions of axiomatic approach hold true in the given assignment, that is

1) Each of the number p (W_i) is less than zero and is positive

2) Sum of probabilities i

$\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{7}{7}=1$

The given assignment is valid.

c) Both the conditions of axiomatic approach in the given assignment are

1) Each of the number p (W_i) is less than zero and is positive

2) Sum of probabilities is

0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 = 2.8 > 1

So, the 2nd condition is not satisfied

Which states that p (W_i) ≤ 1

The given assignment is not valid.

d) The conditions of axiomatic approach don't hold true in the given assignment, that is

1) Each of the number p (W_i) is less than zero but also negative

To be true each of the number p(W_i) should be less than zero and positive

So, the assignment is not valid

e) Both the conditions of axiomatic approach in the given assignment are

1) Each of the number p (W_i) is less than zero and is positive

2) Sum of probabilities is

$$\frac{1}{14}+\frac{2}{14}+\frac{3}{14}+\frac{4}{14}+\frac{5}{14}+\frac{6}{14}+\frac{7}{14}=\frac{28}{14}\geq 1$$

The second condition doesn't hold true so the assignment is not valid.

Q. 2 A coin is tossed twice, what is the probability that at least one tail occurs?

Answer:

Here the sample space is S= (TT, HH, TH, HT)

∴ Number of possible outcomes n (S) = 4

Let, E be the event of getting at least one tail

∴ n (E) = 3

P (E) = $\frac{n(E)}{n(S)} = \frac{3}{4}$

Q. 3 A die is thrown; find the probability of following events:

(i) A prime number will appear,

(ii) A number greater than or equal to 3 will appear,

(iii) A number less than or equal to one will appear,

(iv) A number more than 6 will appear,

(v) A number less than 6 will appear.

Answer:

Here S = {1, 2, 3, 4, 5, 6}

∴n(S) = 6

(i) Let A be the event of getting a prime number,

A = {2, 3, 5} and n(A) = 3

P (A) = $\frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(ii) Let A be the event of getting a number greater than or equal to 3,

Then A = (3, 4, 5, 6) and n(A) = 4

P (A) = $\frac{n(A)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

(iii) Let A be the event of getting a number less than or equal to 1,

Then A = (1) ∴ n (A) = 1

P (A) = $\frac{n(A)}{n(S)} = \frac{1}{6}$

(iv) Let A be the event of getting a number more than 6, then

Then A = (0), ∴ n (A) = 0

P (A) = $\frac{n(A)}{n(S)} = \frac{0}{6} = 0$

(v) Let A be the event of getting a number less than 6, then

Then A= (1, 2, 3, 4, 5), ∴ n (A) = 5

P (A) = $\frac{n(A)}{n(S)} = \frac{5}{6}$

Q. 4 A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card.

Answer:

(a) Number of points\events in the sample space = 52 (number of cards in deck)

(b) Let A be the event of drawing an ace of spades.

Here A= 1, n (A) = 1

P (A) = $\frac{n(A)}{n(S)} = \frac{1}{52}$

(c) Let A be the event of drawing an ace. There are four aces.

∴ n (A)= 4

P (A) = $\frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

(d) Let A be the event of drawing a black card. There are 26 black cards.

∴ n (A) = 26

P (A) = $\frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

Q. 5 A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

Answer:

The coin and die are tossed together.

Let, S be the sample space = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

n(S) = 12

(i) Let A be the event having sum of numbers as 3.

A = {(1, 2)}, ∴ n (A) = 1

∴ P (A) $= \frac{1}{12}$

(ii) Let A be the event having sum of number as 12.

Then A = {(6, 6)}, n (A) = 1

∴ P (A) = $\frac{n(A)}{n(S)} = \frac{1}{12}$

Q. 6 There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Answer:

Here total members in the council = 4 + 6 = 10, so the sample space has 10 points

∴ n (S) = 10

Number of women are 6

Let A be the event of selecting a woman

Then n (A) = 6

∴ P (A) = $\frac{n(A)}{n(S)} = \frac{6}{10} = \frac{3}{5}$

Q. 7 A fair coin is tossed four times, and a person win Re 1 for each head and lose ₹1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Answer:

Here the sample space is,

S = (HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH,

TTTH, TTHT, THTT, HTTT, TTTT)

According to the given condition, a person will win or lose money depending up on the face of the coin so,

(i) For 4 heads = 1 + 1 + 1 + 1 = ₹4 = he wins ₹4

(ii) For 3 heads and 1 tail = 1 + 1 + 1 – 1.50 = ₹1.50 = he will be winning ₹1.50

(iii) For 2 heads and 2 tails = 1 + 1 – 1.50 – 1.50 = - ₹1 = he will be losing Re. 1

(iv)For 1 head and 3 tails = 1 – 1.50 – 1.50 – 1.50 = - ₹3.50 = he will be losing ₹3.50

(v) For 4 tails = – 1.50 – 1.50 – 1.50 – 1.50 = - ₹6 = he will be losing ₹6

Now the sample space of amounts is

S= {4, 1.50, 1.50, 1.50, 1.50, – 1, – 1, – 1, – 1, – 1, – 1, – 3.50, – 3.50, – 3.50, – 3.50, – 6}

∴ n (S) = 16

P (winning ₹4) = $\frac{1}{16}$

P (winning ₹1.50) = $\frac{4}{16} = \frac{1}{4}$

P (losing ₹1) = $\frac{6}{16} = \frac{3}{8}$

P (losing ₹3.50) = $\frac{4}{16} = \frac{1}{4}$

P (losing ₹6) = $\frac{1}{16}$

Q. 8 Three coins are tossed once. Find the probability of getting

(i) 3 heads

(ii) 2 heads

(iii) at least 2 heads

(iv) at most 2 heads

(v) no head

(vi) 3 tails

(vi) exactly two tails

(vii) (viii) no tail

(viii) (ix) at most two tails

Answer:

When three coins are tossed then S = {HHH, HHT, HTH, THH, TTH, HTT, TTT, THT}

Where s is sample space and here n(S) = 8

Let A be the event of getting 3 heads

n(A)= 1

P (getting 3 heads) = P (A) = $\frac{n(A)}{n(S)} = \frac{1}{8}$

Let A be the event of getting 2 heads

n (A) = 3

P (getting 2 heads) = P (A) =$\frac{n(A)}{n(S)} = \frac{3}{8}$

Let A be the event of getting at least 2 head

n(A) = 4

P (getting 2 heads) = P (A) = $\frac{n(A)}{n(S)} = \frac{4}{8} = 2$

- let A be the event of getting at most 2 heads

n(A) = 7

P (getting 2 heads) = P (A) =$\frac{n(A)}{n(S)} = \frac{7}{8}$

- Let A be the event of getting no heads

n(A) = 1

P (getting no heads) = P (A) = $\frac{n(A)}{n(S)} = \frac{1}{8}$

(vi) Let A be the event of getting 3 tails

n(A) = 1

P (getting 3 tails) = P (A) =$\frac{n(A)}{n(S)} = \frac{1}{8}$

(vii) Let A be the event of getting exactly 2 tails

n(A) = 3

P (getting exactly 2 tails) = P (A) = $\frac{n(A)}{n(S)} = \frac{3}{8}$

(viii) Let A be the event of getting no tails n(A) = 1

P (getting no tails) = P (A) = $\frac{n(A)}{n(S)} = \frac{1}{8}$

(ix) Let A be the event of getting at most 2 tails

n(A) = 7

P (getting at most 2 tails) = P (A) = $\frac{n(A)}{n(S)} = \frac{7}{8}$

Q. 9 If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.

Answer:

Given:

P (A) = $\frac{2}{11}$

∴ P (Not A) = 1 – P(A)

$= 1 - \frac{2}{11} = \frac{9}{11}$

Q. 10 A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is

(i) a vowel

(ii) a consonant

Answer:

There are 13 letters in the word 'ASSASSINATION' which contains 6 vowels and 7 consonants.

The sample space here will have all the letters of the word ASSASSINATION

So here n(S) = 13

(i) Let A be the event of selecting a vowel

n(A) = 6

$\therefore$ P (A) $= \frac{n(A)}{n(S)} = \frac{6}{13}$

(ii) Let A be the event of selecting the consonant

n(A)= 7

$\therefore$ P (A) $= \frac{n(A)}{n(S)} = \frac{7}{13}$

Q. 11 In a lottery, a person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers *already fixed by the lottery committee, he wins the prize. What is the* probability of winning the prize in the game?

[Hint order of the numbers is not important.

Answer:

Total numbers of numbers in the draw = 20 and numbers to be selected = 6

$\therefore$ n (S) = 20_{C_6}

Let A be the event that six numbers match with the six numbers fixed by the lottery committee.

$\therefore$ n (A) = 6_{C6} = 1

Probability of winning the prize

= P (A) $= \frac{n(A)}{n(S)} = \frac{6c_6}{20c_6} = \frac{6!14!}{20!}$

$\frac{6\times5\times4\times3\times2\times1\times14!}{20\times19\times18\times17\times16\times15\times14!} = \frac{1}{38760}$

Q. 12 Check whether the following probabilities P(A) and P(B) are consistently defined

(i) P(A) = 0.5, P(B) = 0.7, P (A B) = 0.6

(ii) P(A) = 0.5, P(B) = 0.4, P (A B) = 0.8

Answer:

(i) Here P(A) = 0.5, P(B) = 0.7 and P (A ∩ B) = 0.6

Now P (A ∩ B) > P (A)

Therefore, the given probabilities are not consistently defined.

(ii) P(A) = 0.5, P(B) = 0.4, P (A ∪ B) = 0.8

P (A ∪ B) = P(A) + P(B) - P (A ∪ B)

= 0.8 = 0.5 + 0.4 – P (A ∪ B)

= P (A ∪ B) = 0.9 – 0.8 = 0.1

∴ P (A ∪ B) < P(A) and P (A ∩ B) < P(B)

Therefore, the given probabilities are consistently defined.

Q. 13 Fill in the blanks in following table:

	P (A)	P (B)	P(A∩B)	P(A∪B)
1	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	…..
2	0.35	…..	0.25	0.6
3	0.5	0.35	…..	0.7

Answer:

(i) here P (A) = $\frac{1}{3}$, P(B) = $\frac{1}{5}$, P (A ∩ B) = $\frac{1}{15}$

P (A ∪ B) = P (A) + P(B) - P (A ∩ B)

$= \frac{1}{3} + \frac{1}{5} + \frac{1}{15} = \frac{7}{15}$

(ii) here P (A) = 0.35, P(B) =? P (A ∩ B) = 0.25, P (A ∪ B) = 0.6

P (A ∪ B) = P (A) + P(B) - P (A ∩ B)

$0.6 = 0.35 + P(B) - 0.25$

$P(B) = 0.5$

(iii) Here $P(A) = 0.5$, $P(B) = 0.35$, $P(A \cap B) = ?$ $P(A \cup B) = 0.7$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.5 + 0.35 - P(A \cap B)$

$0.7 = 0.85 - P(A \cap B)$

$P(A \cap B)\ 0.85 - 0.7 = 0.15$

Q. 14 Given P(A) = and P(B) =. Find P (A or B), if A and B are mutually exclusive events.

Answer:

Given: $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$

Since A and B are mutually exclusive events.

$\therefore P(A \cup B)$ or $P(A \text{ or } B) = P(A) + P(B)$

$= \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

Q. 15 If E and F are events such that P (E) =, P(F) = and P (E and F) =, find

(i) P (E or F)

(ii) P (not E and not F).

Answer:

$P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$

(i) Now $P(E \cup F) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$$

(ii) P (not E and not F). = P ($\overline{E} \cap \overline{F}$) = P ($\overline{E \cup F}$) = 1- P (E∪F)

$$1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{5}$$

Q. 16 Events E and F are such that P (not E or not F) = 0.25, State whether E and F are mutually exclusive.

Answer:

Given: P (not E and not F) = 0.25

P ($\overline{E} \cup \overline{F}$) = 0.25

P ($\overline{E \cap F}$) = 0.25

⇒1-P(E∩F) = 0.25

⇒P(E∩F) = 1-0.25=0.75 ≠0

∴ E and F are not mutually exclusive events.

Q. 17 An and B are events such that P(A) = 0.42, P(B) = 0.48 and P (A and B) = 0.16. Determine

(i) P (note A),

(ii) P (not B) and

(iii) P (A or B)

Answer:

Given: P(A) = 0.42, P(B)= 0.48 and P (A and B) = 0.16

(i) P (note A) = P ($\overline{A}$) = 1 – P (A) = 1 – 0.42 = 0.58

(ii) P (not B) = P ($\overline{B}$) = 1 – P (B) = 1 – 0.48 = 0.52

(iii) P (A or B) = P (A∪B) = P (A) + P (B) - P (A ∩ B)

= 0.42 + 0.48 – 0.16 = 0.74

Q. 18 In Class XI of a school 40% of the student's study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Answer:

Let A be the event that the student is studying mathematics and B be the event that the student is studying biology.

Then P (A) = $\frac{40}{100} = \frac{2}{5}$

P (B) = $\frac{30}{100} = \frac{3}{10}$

And, P (A ∩ B) = $\frac{10}{100} = \frac{1}{10}, P\ (A\ \cap\ B)$ is probability of studying both mathematics and biology.

Here, Probability of studying mathematics or biology will be given by P (A∪B)

∴ P (A∪B) = P (A)+ P (B) - P (A ∩ B)

$\frac{2}{5} + \frac{3}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$

$\frac{3}{5}$ is the probability that the student will studying mathematics or bio - logy?

Q. 19 In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Answer:

Let A be the event that the student passes the first examination and B be the event that the students passes the second examination.

P (P(A∪B) is probability of passing at least one of the examination

Then, P(A∪B) = 0.95, P(A)=0.8, P(B)=0.7

∴ P(A∪B) =P(A) + P(B)-P(A∩B)

0.95 = 0.8 + 0.7 - P(A∩B)

P(A∩B) =1.5-0.95=0.55

0.55 is the probability that student will pass both the examinations

Q. 20 The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Answer:

Let A be the event that the student passes English examination and B be the event that the students passes Hindi examination.

Here given P (A) = 0.75, P(A∩B) = 0.5, P $(\overline{A} \cap \overline{B})$ = 0.1

Now, P $(\overline{A} \cap \overline{B})$ = 1 - P(A∪B)

P(A∪B) = 1 – 0.1= 0.9

P(A∪B) = P (A) + P(B) - P(A∩B)

0.9 = 0.75 + P(B) - 0.5

P(B) = 0.9 – 0.25 = 0.65

Q. 21 In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i) The student opted for NCC or NSS.

(ii) The student has opted neither NCC nor NSS.

(iii) The student has opted NSS but not NCC.

Answer:

Given: Total number of students = 60

So the sample space - consist of n(S) = 60

Let A be the event that student opted for NCC and B be the event that the student opted for NSS.

Here n (A) = 30, n (B) = 32 and n (A∩B) = 24 being number of students who have opted for both NCC and NSS

(i) P (Student opted for NCC or NSS)

P (A) = $\frac{n(A)}{n(S)} = \frac{30}{60} = \frac{1}{2}$, P (B) = $\frac{n(A)}{n(S)} = \frac{32}{60} = \frac{8}{15}$, P (A∩B) = $\frac{n\,(A \cap B)}{N\,(S)}\ \frac{24}{100} = \frac{2}{5}$

∴ P(A∪B) = P (A) + P(B) - P(A∩B)

(i) P (student opted for NCC and NSS)

P (A or B) = P(A) + P(B) –P (A and B)

P(A∪B) = P (A) + P(B) - P(A∩B)

$$\frac{1}{2} + \frac{8}{15} - \frac{2}{5} = \frac{19}{30}$$

(ii) P (student opted neither NCC nor NSS)

P (not A and not B) = P ($\overline{A} \cap \overline{B}$)

P ($\overline{A} \cap \overline{B}$) $= P\ (\overline{A \cap B})$(by De Morgan’s law)

= 1 - P(A∪B) = 1 - $\frac{19}{30} = \frac{11}{30}$

(iii) P (student opted NSS but not NCC)

n (B - A) = n(B) – n (AB)

⇒ 32 – 24 = 8

The probability that the selected student has opted for NSS and not NCC is

$= \frac{8}{60} = \frac{2}{15}$

Miscellaneous Exercise

Q. 1 A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that

(i) all will be blue? (ii) at least one will be green?

Answer:

No. of red marbles = 10

No. of blue marbles = 20

No. of green marbles = 30

Total number of marbles = 10 + 20 + 30 = 60

Number of ways of drawing 5 marbles from 60 marbles = $60C_5$

(i) All the drawn marbles will be blue if we draw 5 marbles out of 20 blue marbles.

Number of ways of drawing 5 blue marbles from 20 blue marbles = $20C_5$

Probability that all marbles will be blue = $20C_5$ / $60C_5$

(ii) Number of ways in which the drawn marble is not green = $(20+10) C_5$

Probability that no marble is green = $30C_5$ / $60C_5$

Probability that at least one marble is green = 1 - $30C_5$ / $60C_5$

Q. 2 4 cards are drawn from a well – shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Answer:

Number of ways of drawing 4 cards from 52 cards = $52C_4$

In a deck of 52 cards, there are 13 diamonds and 13 spades.

Number of ways of drawing 3 diamonds and one spade = $13C_3 \times 13C_1$

Thus, the probability of obtaining 3 diamonds and one spade

= $(13C_3 \times 13C_1)/ 52C_4$

Q. 3 A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

(i) P (2) (ii) P (1 or 3) (iii) P (not 3)

Answer:

Total number of faces = 6

(i) Number faces with number '2' = 3

P (2) = $\frac{3}{6} = \frac{1}{2}$

(ii) P (1 or 3) = P (not 2) = 1 − P (2)

P (1 or 3) = 1 - $\frac{1}{2} = \frac{1}{2}$

(iii) Number of faces with number '3' = 1

P (3) = $\frac{1}{6}$

P (not 3) = 1 – P (3) = 1 - $\frac{1}{6} = \frac{5}{6}$

Q. 4 In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy?

(a) one ticket

(b) two tickets

(c) 10 tickets.

Answer:

Total number of tickets sold = 10,000

Number prizes awarded = 10

(i) If we buy one ticket, then

P (getting a prize) = $\frac{10}{10000} = \frac{1}{1000}$

P (not getting a prize) = 1-$\frac{1}{1000} = \frac{999}{1000}$

(ii) If we buy two tickets, then

Number of tickets not awarded = 10,000 − 10 = 9990

P (not getting a prize) = $9990C_2$ / $10000C_2$

(iii) If we buy 10 tickets, then

Number of tickets not awarded = 10,000 − 10 = 9990

P (not getting a prize) = $9990C_{10}$ / $10000C_{10}$

Q. 5 Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that

(a) you both enter the same section?

(b) you both enter the different sections?

Answer:

My friend and I are among the 100 students.

Total number of ways of selecting 2 students out of 100 students =

(a) Let S = The two of us will enter the same section if both of us are among 40 students or among 60 students.

Number of ways in which both of us enter the same section = P(S)

P(S) = $(40C_2 + 60C_2) / 100C_2$

$$P(S) = \frac{\frac{40!}{2! \times 38!} + \frac{60!}{2! \times 58!}}{\frac{100!}{2! \times 98!}} = \frac{39 \times 40 + 59 \times 60}{99 \times 100} = \frac{5100}{9900} = \frac{17}{33}$$

(b) P (we enter different sections) = 1 – P (we enter the same section)

P (we enter different sections) = $1 - \frac{17}{33} = \frac{16}{33}$

Q. 6 Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Answer:

Let L_1, L_2, L_3 be three letters and E_1, E_2, and E_3 be their corresponding envelops respectively.

Let L_1E_1 denote with letter is inserted in with envelope

∴ Sample space is

L_1E_1, L_2E_3, L_3E_2,

L_2E_2, L_1E_3, L_3E_1,

L_3E_3, L_1E_2, L_2E_1,

L_1E_1, L^2E_2, L_3E_3,

L_1E_2, L_2E_3, L_3E_1,

L_1E_3, L_2E_1, L_3E_2,

∴ there are 6 ways of inserting 3 letters in 3 envelops.

And there are 4 ways in which at least one letter is inserted in proper envelope. (first 4 rows of sample space)

Probability that at least one letter is inserted in proper envelope = $\frac{4}{6} = \frac{2}{3}$

Q. 7 An A and B are two events such that P(A) = 0.54, P(B) = 0.69 and P (A ∩ B) = 0.35.

Find (i) P (A ∪ B) (ii) P (A ∩ B) (iii) P (A ∩ B) (iv) P (B ∩ A)

Answer:

It is given that P(A) = 0.54, P(B) = 0.69, P (A ∩ B) = 0.35

(i) We know that P (A ∪ B) = P(A) + P(B) – P (A ∩ B)

⇒ P (A ∪ B) = 0.54 + 0.69 − 0.35 = 0.88

∴ P (A ∪ B) = 0.88

(ii) A′ ∩ B′ = (A ∪ B) ′ [by De Morgan's law]

So, P (A′ ∩ B′) = P (A ∪ B) = 1 – P (A ∪ B) = 1 − 0.88 = 0.12

∴ P (A' ∩ B') = 0.12

(iii) P (A ∩ B′) = P(A) – P (A ∩ B) = 0.54 − 0.35 = 0.19

∴ P (A ∩ B') = 0.19

(iv) We know that: P (B ∩ A′) = P(B) – P (A ∩ B)

⇒ P (B ∩ A′) = 0.69 – 0.35

∴ P (B ∩ A′) = 0.34

Q. 8 From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No.	Name	Sex	Age in years

1	Harish	M	30
2	Rohan	M	33
3	Sheetal	F	46
4	Alis	F	28
5	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Answer:

Given Data

S. No.	Name	Sex	Age in years
1	Harish	M	30
2	Rohan	M	33
3	Sheetal	F	46
4	Alis	F	28
5	Salim	M	41

Total no. of person = 5

No. of spokesperson who are male = 3

No. of spokesperson who are 35 years of age = 2

Let, E be the event in which the spokesperson will be a male and F be the event in which the spokesperson will be over 35 years of age.

Accordingly, P (E) = 3/5 and P (F) = 2/5

Since there is only one male who is over 35 years of age,

$\therefore P(E \cap F) = \frac{1}{5}$

We know that: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

⇒ P (E ∩ F) = $\frac{3}{5}+\frac{2}{5}-\frac{1}{5}$

∴ P (E ∪ F) = 4/5

Thus, the probability that the spokesperson will either be a male or over 35 years of age is 4/5.

Q. 9 If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when,

(i) the digits are repeated? (ii) the repetition of digits is not allowed?

Answer:

(i) When the digits are repeated

Since four-digit numbers greater than 5000 are formed,

The thousand's place digit is either 7 or 5. The remaining 3 places can be filled by any of the digits 0, 1, 3, 5, or 7 as repetition of digits is allowed.

Total number of 4-digit numbers greater than 5000 = 2 × 5 × 5 × 5 − 1

= 250 − 1 = 249

[In this case, 5000 cannot be counted; so, 1 is subtracted]

A number is divisible by 5 if the digit at its unit's place is either 0 or 5.

∴ Total number of 4-digit numbers greater than 5000 that are divisible by 5 = 2 × 5 × 5 × 2 − 1 = 100 − 1 = 99Thus, the probability of forming a number divisible by 5 when the digits are repeated is = P (number divisible by 5 when digits repeated)

⇒ P (number divisible by 5 when digits repeated) = $\frac{99}{249}=\frac{33}{83}$

(ii) When repetition of digits is not allowed

The thousands place can be filled with either of the two digits 5 or 7 i.e. by 2 ways.

The remaining 3 places can be filled with any of the remaining 4 digits.

Total number of 4-digit numbers greater than 5000 = 2 × 4 × 3 × 2 = 48

When the digit at the thousands place is 5, the unit place can be filled only with 0 and the tens and hundreds places can be filled with any two of the remaining 3 digits. Here, number of 4-digit numbers starting with 5 and divisible by 5 = 1 × 3 × 2 × 1 = 6

When the digit at the thousands place is 7, the unit place can be filled in two ways (0 or 5) and the tens and hundreds of places can be filled with any two of the remaining 3 digits. Here, number of 4-digit numbers starting with 7 and divisible by 5 = 1 × 2 × 3 × 2 = 12

Total number of 4-digit numbers greater than 5000 that are divisible by 5

= 6 + 12 = 18

Thus, the probability of forming a number divisible by 5 when the repetition of digits is not allowed

= P (number divisible by 5 when digits are not repeated)

⇒ P (number divisible by 5 when digits repeated) = $\frac{18}{48} = \frac{2}{8}$

Q. 10 The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Answer:

The number lock has 4 wheels, each labelled with ten digits i.e., from 0 to 9.

Number of ways of selecting 4 different digits out of the 10 digits = $10C_4$

Now, each combination of 4 different digits can be arranged in 4! Ways.

Number of four digits with no repetitions = 4! × $10C_4$ = 5040

There is only one number that can open the suitcase.

Thus, the required probability is $\frac{1}{5040}$