Chapter : 13. METHOD OF INTEGRATION

Exercise : 13A

Question: 1

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x + 9 = t ⇒ 2 dx = dt

$$\int t^5 (\frac{dt}{2}) = \frac{1}{2}\int t^5 dt = \frac{1}{2}\frac{t^6}{6} + c = \frac{t^6}{12} + c$$

$$= \frac{(2x+9)^6}{12} + c$$

Question: 2

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 7 – 3x = t ⇒ -3 dx = dt

$$\int t^4 (\frac{dt}{-3}) = \frac{1}{-3}\int t^4 dt = \frac{1}{-3}\frac{t^5}{5} + c = -\frac{t^5}{15} + c$$

$$= -\frac{(7-3x)^5}{15} + c$$

Question: 3

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3x - 5 = t ⇒ 3 dx = dt

$$\int t^{0.5} (\frac{dt}{3}) = \frac{1}{3}\int t^{0.5} dt = \frac{1}{3} \times \frac{t^{1.5}}{1.5} + c = \frac{2}{1} \times \frac{t^{1.5}}{9} + c$$

$$= \frac{2(3x-5)^5}{9} + c$$

Question: 4

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 4x + 3 = t ⇒ 4 dx = dt

$$\int t^{-0.5}(\frac{dt}{4}) = \frac{1}{4}\int t^{-0.5}dt = \frac{1}{4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{4} \times \frac{t^{0.5}}{1} + c$$

$$= \frac{\sqrt{4x+3}}{2} + c$$

Question: 5

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3 – 4x = t ⇒ -4 dx = dt

$$\int t^{-0.5}(\frac{dt}{-4}) = \frac{1}{-4}\int t^{-0.5}dt = \frac{1}{-4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{-4} \times \frac{t^{0.5}}{1} + c$$

$$= -\frac{\sqrt{3-4x}}{2} + c$$

Question: 6

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x – 3 = t ⇒ 2 dx = dt

$$\int t^{-\frac{3}{2}}(\frac{dt}{2}) = \frac{1}{2}\int t^{-\frac{3}{2}}dt = \frac{1}{2} \times \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + c = \frac{-2}{2} \times \frac{t^{-0.5}}{1} + c$$

$$= -\frac{1}{\sqrt{2x-3}} + c$$

Question: 7

Evaluate the foll

Solution:

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put 2x – 1 = t ⇒ 2 dx = dt

$$\int e^t(\frac{dt}{2}) = \frac{1}{2}\int e^t dt = \frac{1}{2} \times e^t + c = \frac{e^{2x-1}}{2} + c$$

$$= \frac{e^{(2x-1)}}{2} + c$$

Question: 8

Evaluate the foll

Solution:

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put 1 – 3x = t ⇒ -3 dx = dt

$$\int e^t(\frac{dt}{-3}) = \frac{1}{-3}\int e^t dt = \frac{1}{-3}\times e^t + c = \frac{e^{1-3x}}{-3} + c$$

$$= -\frac{e^{(1-3x)}}{3} + c$$

Question: 9

Evaluate the foll

Solution:

Formula = $\int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put 2 – 3x = t ⇒ -3 dx = dt

$$\int 3^t(\frac{dt}{-3}) = \frac{1}{-3}\int 3^t dt = \frac{1}{-3}\times(\frac{3^t}{\log 3}) + c = \frac{3^t}{-3\log 3} + c$$

$$= -\frac{3^{(2-3x)}}{3\log 3} + c$$

Question: 10

Evaluate the foll

Solution:

Formula = $\int \sin x\, dx = -\cos x + c$

Therefore ,

Put 3x = t ⇒ 3 dx = dt

$$\int \sin t\,(\frac{dt}{3}) = \frac{1}{3}\int \sin t\, dt = \frac{1}{3}\times(-\cos t) + c = \frac{-\cos 3x}{3} + c$$

$$= -\frac{\cos 3x}{3} + c$$

Question: 11

Evaluate the foll

Solution:

Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

Put 5 + 6x = t ⇒ 6 dx = dt

$$\int \cos t\,(\frac{dt}{6}) = \frac{1}{6}\int \cos t\, dt = \frac{1}{6}\times(\sin t) + c = \frac{\sin 5 + 6x}{6} + c$$

$$= \frac{\sin(5+6x)}{6} + c$$

Question: 12

Evaluate the foll

Solution:

Formula $\int \cos x\, dx = \sin x + c$

$1 + \cos 2x = 2cos^2\, x$

Therefore ,

$$\int \sin x\sqrt{1+\cos 2x}\,dx = \int \sin x\sqrt{2}\cos x + c$$

$$\int \sqrt{2}\sin x\cos x\ \ dx$$

Put sin x =t ⇒ cos x dx = dt

$$\int \sqrt{2}\sin x\cos x\ \ dx = \int \sqrt{2}t\,dt = \sqrt{2}\,\frac{t^2}{2} + c$$

$$= \frac{(\sin x)^2}{\sqrt{2}} + c$$

Question: 13

Evaluate the foll

Solution:

Formula $\int cosec^2 x\,dx = -\cot x + c$

Therefore ,

Put 2x + 5 =t ⇒ 2 dx = dt

$$\int cosec^2 t\,\frac{dt}{2} = -\frac{1}{2}\cot t + c = -\frac{1}{2}\cot(2x+5) + c$$

$$= -\frac{1}{2}\cot(2x+5) + c$$

Question: 14

Evaluate the foll

Solution:

Formula $\int \sin x\,dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t\,dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin x)^2}{2} + c$$

Question: 15

Evaluate the foll

Solution:

Formula $\int \sin x\,dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t^3\,dt = \frac{t^4}{4} + c$$

$$= \frac{(\sin x)^4}{4} + c$$

Question: 16

Evaluate the foll

Solution:

Formula $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put cos x =t ⇒ -sin x dx = dt

$$\int t^{0.5}(-1)dt = -\frac{t^{1.5}}{1.5} + c$$

$$= -\frac{2(\cos x)^{\frac{3}{2}}}{3} + c$$

Question: 17

Evaluate the foll

Solution:

Formula $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c \; \frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Therefore ,

Put $\sin^{-1} x$ =t ⇒ $\frac{1}{\sqrt{1-x^2}}$ dx = dt

$$\int t^1 dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin^{-1} x)^2}{2} + c$$

Question: 18

Evaluate the foll

Solution:

Formula $\int \sin t \, dx = -\cos t + c \; \frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$

Therefore ,

Put $\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$$\int \sin 2t \, dt = \frac{-\cos 2t}{2} + c$$

$$= -\frac{\cos(2\tan^{-1} x)}{2} + c$$

Question: 19

Evaluate the foll

Solution:

Formula $\int \cos t \, dx = \sin t + c \; \frac{d(\log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int \cos t \, dt = \sin t + c$$

$$= \sin(\log x) + c$$

Question: 20

Evaluate the foll

Solution:

Formula $\int cosec^2 x\, dx = -\cot x + c$ $\frac{d(\log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int cosec^2 t \frac{dt}{1} = -\cot t + c = -\cot(\log x) + c$$

$= -\cot(\log x) + c$

Question: 21

Evaluate the foll

Solution:

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int \frac{dt}{t} = \log t + c = \log(\log x) + c$$

$= \log(\log x) + c$

Question: 22

Evaluate the foll

Solution:

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

$$\int \frac{(x+1)(x+\log x)^2}{x} dx = \int \frac{x+1}{x} \times \frac{(x+\log x)^2}{1} dx$$
$$= \int (1+\frac{1}{x}) \times \frac{(x+\log x)^2}{1} dx$$

Therefore ,

Put $x + \log x = t \Rightarrow (1+\frac{1}{x}) dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{(x+\log x)^3}{3} + c$$

Question: 23

Evaluate the foll

Solution:

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c = \frac{(\log x)^3}{3} + c$$

$= \frac{(\log x)^3}{3} + c$

Question: 24

Evaluate the foll

Solution:

Formula $\int \cos t \, dx = \sin t + c \; \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

Therefore ,

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\int \cos t \;\; 2dt = 2 \sin t + c$$

$= 2 \sin(\sqrt{x}) + c$

Question: 25

Evaluate the foll

Solution:

Formula = $\int e^x dx = e^x + c \; \frac{d(\tan x)}{dx} = sec^2 x$

Therefore ,

Put tan x = t ⇒ $sec^2 x$ dx = dt

$$\int e^t dt = e^t + c$$

$= e^{\tan x} + c$

Question: 26

Evaluate the foll

Solution:

Formula = $\int e^x dx = e^x + c \; \frac{d(cos^2 x)}{dx} = 2 \cos x \, (-\sin x) = -\sin 2x$

Therefore ,

Put $cos^2 x$ = t ⇒ $-\sin 2x$ dx = dt

$$\int -e^t dt = -e^t + c$$

$= -e^{cos^2 x} + c$

Question: 27

Evaluate the foll

Solution:

Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put ax+b = t ⇒ adx = dt

$$\int \sin t \cos t \frac{dt}{a} = \frac{1}{a} \int \sin t \cos t \, dt$$

Put sin t = z � cos t dt = dz

$\frac{1}{a}\int zdz = \frac{1}{a} \times \frac{z^2}{2} + c$

$= \frac{(\sin ax + b)^2}{2a} + c$

Question: 28

Evaluate the foll

Solution:

Formula = $\int \cos x \, dx = \sin x + c$

$\cos 3x = 3\cos x - 4cos^3 x$

Therefore ,

$\int (\frac{3\cos x}{4} - \frac{\cos 3x}{4}) dx = \frac{3\sin x}{4} - \frac{\sin 3x}{4 \times 3} + c$

$= \frac{3\sin x}{4} - \frac{\sin 3x}{12} + c$

Question: 29

Evaluate the foll

Solution:

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $-\frac{1}{x}$ = t $\Rightarrow \frac{1}{x^2} dx = dt$

$\int e^t (dt) = \int e^t dt = e^t + c = e^{-\frac{1}{x}} + c$

$= e^{-\frac{1}{x}} + c$

Question: 30

Evaluate the foll

Solution:

Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put $-\frac{1}{x}$ = t $\Rightarrow \frac{1}{x^2} dx = dt$

$\int \cos t \, (dt) = \int \cos t \, dt = \sin t + c = \sin(-\frac{1}{x}) + c$

$= -\sin\frac{1}{x} + c$

Question: 31

Evaluate the foll

Solution:

Formula = $\int e^x dx = e^x + c$

Therefore ,

$\int \frac{e^x}{1 + e^{2x}} dx$

Put e^x = t ⇒ $e^x dx = dt$

$$\int \frac{1}{1+t^2}(dt) = \int \frac{1}{1+t^2} dt = \tan^{-1} t + c$$

$= \tan^{-1}(e^x) + c$

Question: 32

Evaluate the foll

Solution:

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $e^{2x} - 2$ = t ⇒ $2e^{2x} dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c$$

$= \frac{1}{2}\log(e^{2x} - 2) + c$

Question: 33

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t ⇒ $\frac{\cos x}{\sin x} dx = dt$ � $\cot x\, dx = dt$

$$\int t\, dt = \frac{t^2}{2} + c$$

$= \frac{(\log \sin x)^2}{2} + c$

Question: 34

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t ⇒ $\frac{\cos x}{\sin x} dx = dt$ � $\cot x\, dx = dt$

$$\int \frac{1}{t}\, dt = \log t + c$$

$= \log(\log \sin x) + c$

Question: 35

Evaluate the foll

Solution:

Formula = $\int \sin x\, dx = -\cos x + c$

Therefore ,

Put $x^2 + 1$= t ⇒ $2x\, dx = dt$

$$\int \sin t \; dt = -\cos t + c$$

$$= -\cos(x^2 + 1) + c$$

Question: 36

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sec x + tan x)= t

$$\frac{1}{\sec x + \tan x} \times (\sec x \tan x + sec^2 x)\; dx = dt$$

$$\frac{1}{\sec x + \tan x} \times \sec x\; (\sec x + \tan x)\; dx = dt$$

Sec x dx = dt

$$\int t\, dt = \frac{t^2}{2} + c$$

$$= \frac{(\log(\sec x + \tan x))^2}{2} + c$$

Question: 37

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\tan \sqrt{x}$= t

$$sec^2 \sqrt{x} \times \left(\frac{1}{2\sqrt{x}}\right)\; dx = dt$$

$$\int t\, dt = \frac{t^2}{2} + c$$

$$= \frac{(\tan \sqrt{x})^2}{2} + c$$

Question: 38

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\tan^{-1} x^2$ = t ⇒ $\frac{1}{1+(x^2)^2} \times 2x \times dx = dt$ � $\frac{2x}{1+x^4} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int tdt = \frac{t^2}{4} + c$$

$$= \frac{(\tan^{-1} x^2)^2}{4} + c$$

Question: 39

Evaluate the foll

Solution:

Formula $= \int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^2$ = t $\Rightarrow \frac{1}{\sqrt{1-(x^2)^2}} \times 2x \times dx = dt$ � $\frac{2x}{\sqrt{1-x^4}} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int tdt = \frac{t^2}{4} + c$$

$$= \frac{(\sin^{-1} x^2)^2}{4} + c$$

Question: 40

Evaluate the foll

Solution:

Formula $= \int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^1$ = t $\Rightarrow \frac{1}{\sqrt{1-(x^2)^1}} \times dx = dt$ � $\frac{1}{\sqrt{1-x^2}} dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{1}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log \sin^{-1} x + c$$

Question: 41

Evaluate the foll

Solution:

Formula $= \int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put 2 + log x = t $\Rightarrow \frac{1}{x} \times dx = dt$

$$\int \sqrt{t}\left(\frac{dt}{1}\right) = \int \sqrt{t}dt = \frac{2t^{1.5}}{3} + c$$

$$= \frac{2(2+\log x)^{\frac{3}{2}}}{3} + c$$

Question: 42

Evaluate the foll

Solution:

Formula $= \int \frac{1}{x} dx = \log x + c$

Therefore ,

Put 1 + tan x = t $\Rightarrow sec^2 x \times dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(1 + \tan x) + c$$

Question: 43

Evaluate the foll

Solution:

Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

Put 1 + cos x = t ⇒ $-\sin x \times dx = dt$

$$\int \left(\frac{-dt}{t}\right) = -\int \frac{1}{t} dt = -\log t + c$$

$= -\log(1 + \cos x) + c$

Question: 44

Evaluate the foll

Solution:

Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

$$\int \left(\frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}}\right) dx = \int \left(\frac{\cos x + \sin x}{\cos x - \sin x}\right) dx$$

Put cos x - sin x = t ⇒ (- cos x - sin x) dx = dt

$$\int \left(\frac{-dt}{t}\right) = -\int \frac{1}{t} dt = -\log t + c$$

$= -\log(\cos x - \sin x) + c$

Question: 45

Evaluate the foll

Solution:

(i)

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put x + log (sec x) = t ⇒$1 + \frac{1}{\sec x} \times \sec x \ \tan x\, dx = dt$

$(1 + \tan x)dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$= \log(x + \log(\sec x)) + c$

(ii)

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $x + cos^2 x$ = t ⇒$1 + 2\cos x \times (-\sin x)dx = dt$

$(1 - \sin 2x)dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$= \log(x + cos^2 x) + c$

Question: 46

Evaluate the foll

Solution:

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2 + b^2 sin^2 x = t$ � $b^2 \times 2\sin x \times \cos x \, dx = dt$

$(b^2 \sin 2x) dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{b^2}\right) = \frac{1}{b^2}\int \frac{1}{t} dt = \frac{1}{b^2}\log t + c$$

$$= \frac{1}{b^2}\log|a^2 + b^2 sin^2 x| + c$$

Question: 47

Evaluate the foll

Solution:

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2 cos^2 x + b^2 sin^2 x = t$

$(a^2 \times 2\cos x \times (-\sin x) + b^2 \times 2\sin x \times \cos x) dx = dt$

$(b^2 - a^2)\sin 2x \ \ dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{b^2 - a^2}\right) = \frac{1}{b^2 - a^2}\int \frac{1}{t} dt = \frac{1}{b^2 - a^2}\log t + c$$

$$= \frac{1}{b^2 - a^2}\log|a^2 cos^2 x + b^2 sin^2 x| + c$$

Question: 48

Evaluate the foll

Solution:

Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put 3cos x + 2sin x = t ⇒ (2cos x - 3sin x) dx = dt

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(3\cos x + 2\sin x) + c$$

Question: 49

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $2x^2 + 3 = t$ ⇒ (4x) dx = dt

$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$

$= \log(2x^2 + 3) + c$

Question: 50

Evaluate the foll

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $x^2+2x+3 = t \Rightarrow (2x+2)\, dx = dt$ � $2(x+1)dx=dt$

$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c$

$= \frac{1}{2}\log(x^2 + 2x + 3) + c$

Question: 51

Evaluate the foll

Solution:

To find: Value of $\int \frac{4x - 5}{(2x^2 - 5x + 1)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{4x - 5}{(2x^2 - 5x + 1)} dx$ ... (i)

Let $2x^2 - 5x + 1 = t$

$\Rightarrow \frac{d(2x^2 - 5x + 1)}{dx} = \frac{dt}{dx}$

$\Rightarrow 4x - 5 = \frac{dt}{dx}$

$\Rightarrow (4x - 5)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$ [$2x^2 - 5x + 1 = t$]

$I = \log|t| + c$

$I = \log|2x^2 - 5x + 1| + c$

Ans) $\log|2x^2 - 5x + 1| + c$

Question: 52

Evaluate the foll

Solution:

To find: Value of $\int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$ ... (i)

Let $3x^3 - 2x^2 + 5x + 1 = t$

$$\Rightarrow \frac{d(3x^3 - 2x^2 + 5x + 1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow 9x^2 - 4x + 5 = \frac{dt}{dx}$$

$\Rightarrow (9x^2 - 4x + 5)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t} \; [\, 3x^3 - 2x^2 + 5x + 1 = t \,]$$

$I = \log|t| + c$

$I = \log|3x^3 - 2x^2 + 5x + 1| + c$

Ans) $\log|3x^3 - 2x^2 + 5x + 1| + c$

Question: 53

Evaluate the foll

Solution:

To find: Value of $\int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$ ... (i)

Let $\log(\tan x) = t$

$$\Rightarrow \frac{d(\log(\tan x))}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(\log(\tan x))}{d\tan x} \frac{d\tan x}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{\tan x} \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \sec x \operatorname{cosec} x = \frac{dt}{dx}$$

$\Rightarrow (\sec x \operatorname{cosec} x)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t} \; [\, \log(\tan x) = t \,]$$

$I = \log|t| + c$

$I = \log|\log(\tan x)| + c$

Ans) $\log|\log(\tan x)| + c$

Question: 54

Evaluate the foll

Solution:

To find: Value of $\int \frac{(1 + \cos x)}{(x + \sin x)^3} dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{(1+\cos x)}{(x+\sin x)^3}dx$ ... (i)

Let $x + \sin x = t$

$$\Rightarrow \frac{d(x+\sin x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(x)}{dx} + \frac{d(\sin x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (1+\cos x) = \frac{dt}{dx}$$

$\Rightarrow (1 + \cos x)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^3} \; [\, x + \sin x = t \,]$$

$$\Rightarrow I = -\frac{1}{2t^2} + c$$

$$I = -\frac{1}{2(x+\sin x)^2} + c$$

$$\text{Ans)} -\frac{1}{2(x+\sin x)^2} + c$$

Question: 55

Evaluate the foll

Solution:

To find: Value of $\int \frac{\sin x}{(1+\cos x)^2}dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2}dx$ ... (i)

Let $1 + \cos x = t$

$$\Rightarrow \frac{d(1+\cos x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(1)}{dx} + \frac{d(\cos x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (0 - \sin x) = \frac{dt}{dx}$$

$\Rightarrow (-\sin x)dx = dt$

Putting this value in equation (i)

$$I = \int -\frac{dt}{t^2} \; [\, 1 + \cos x = t \,]$$

$$\Rightarrow I = \frac{1}{t} + c$$

$I = \frac{1}{1 + \cos x} + c$

Ans) $\frac{1}{1 + \cos x} + c$

Question: 56

Evaluate the foll

Solution:

To find: Value of $\int \frac{(2x + 3)}{\sqrt{x^2 + 3x - 2}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1 + \cos x)^2} dx$ ... (i)

Let $x^2 + 3x - 2 = t$

$\Rightarrow (2x + 3) = \frac{dt}{dx}$

$\Rightarrow (2x + 3)$ dx = dt

Putting this value in equation (i)

$I = \int \frac{dt}{\sqrt{t}}$ $[\, x^2 + 3x - 2 = t \,]$

$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$

$I = 2t^{\frac{1}{2}} + c$

$I = 2\sqrt{x^2 + 3x - 2} + c$

Ans) $2\sqrt{x^2 + 3x - 2} + c$

Question: 57

Evaluate the foll

Solution:

To find: Value of $\int \frac{(2x - 1)}{\sqrt{x^2 - x - 1}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1 + \cos x)^2} dx$ ... (i)

Let $x^2 - x - 1 = t$

$\Rightarrow \frac{d(x^2 - x - 1)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(x^2)}{dx} - \frac{d(x)}{dx} - \frac{d(1)}{dx} = \frac{dt}{dx}$

$\Rightarrow (2x - 1) = \frac{dt}{dx}$

$\Rightarrow (2x - 1)$ dx = dt

Putting this value in equation (i)

$$I = \int \frac{dt}{t^{\frac{1}{2}}} \; [x^2 - x - 1 = t]$$

$$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\Rightarrow I = \frac{2\sqrt{t}}{1} + c$$

$$I = \frac{2\sqrt{x^2 - x - 1}}{1} + c$$

Ans) $2\sqrt{x^2 - x - 1} + c$

Question: 58

Evaluate the foll

Solution:

To find: Value of $\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(\sqrt{x+a})^2 - (\sqrt{x+b})^2} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x + a - x - b} dx$$

$$I = \frac{1}{a-b}\left[\int \sqrt{x+a}\, dx - \int \sqrt{x+b}\, dx\right]$$

$$I = \frac{1}{a-b}\left[\int (x+a)^{\frac{1}{2}} dx - \int (x+b)^{\frac{1}{2}} dx\right]$$

$$I = \frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$I = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$

Question: 59

Evaluate the foll

Solution:

To find: Value of $\int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}}$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}} \times \frac{\sqrt{1-3x}+\sqrt{5-3x}}{\sqrt{1-3x}+\sqrt{5-3x}}$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{\left(\sqrt{1-3x}\right)^2-\left(\sqrt{5-3x}\right)^2} dx$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{(1-3x)-(5-3x)} dx$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{1-3x-5+3x} dx$$

$$I = -\frac{1}{4}\left[\int \sqrt{1-3x}\,dx + \int \sqrt{5-3x}\,dx\right]$$

$$I = -\frac{1}{4}\left[\int (1-3x)^{\frac{1}{2}}\,dx + \int (5-3x)^{\frac{1}{2}}dx\right]$$

$$I = -\frac{1}{4}\left[\frac{(1-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)} + \frac{(5-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)}\right]$$

$$I = -\frac{2}{-9\times 4}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

$$I = \frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$

Question: 60

Evaluate the foll

Solution:

To find: Value of $\int \frac{x^2}{(1+x^6)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x^2}{(1+x^6)} dx$ ... (i)

$$I = \int \frac{x^2}{1+(x^3)^2} dx$$

Let $x^3 = t$

$$\Rightarrow \frac{d(x^3)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (3x^2) = \frac{dt}{dx}$$

$$\Rightarrow (x^2)dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \frac{1}{3}\int \frac{dt}{1+t^2} \ [\ 1+\cos x = t\ ]$$

$$\Rightarrow I = \frac{1}{3}\tan^{-1}(t) + c$$

$$I = \frac{1}{3}\tan^{-1}(x^3) + c$$

Ans) $\frac{1}{3}\tan^{-1}(x^3) + c$

Question: 61

Evaluate the foll

Solution:

To find: Value of $\int \frac{x^3}{(1+x^8)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x^3}{(1+x^8)} dx$ ... (i)

$$I = \int \frac{x^3}{1+(x^4)^2} dx$$

Let $x^4 = t$

$$\Rightarrow \frac{d(x^4)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (4x^3) = \frac{dt}{dx}$$

$$\Rightarrow (x^3)dx = \frac{dt}{4}$$

Putting this value in equation (i)

$$I = \frac{1}{4}\int \frac{dt}{1+t^2} \ [\ 1+\cos x = t\ ]$$

$$\Rightarrow I = \frac{1}{4}\tan^{-1}(t) + c$$

$$I = \frac{1}{4}\tan^{-1}(x^4) + c$$

Ans) $\frac{1}{4}\tan^{-1}(x^4) + c$

Question: 62

Evaluate the foll

Solution:

To find: Value of $\int \frac{x}{(1+x^4)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x}{(1+x^4)} dx$ ... (i)

$I = \int \frac{x}{1+(x^2)^2} dx$

Let $x^2 = t$

$\Rightarrow \frac{d(x^2)}{dx} = \frac{dt}{dx}$

$\Rightarrow (2x) = \frac{dt}{dx}$

$\Rightarrow (x)dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{1+t^2}$ [$1 + \cos x = t$]

$\Rightarrow I = \frac{1}{2}\tan^{-1}(t) + c$

$I = \frac{1}{2}\tan^{-1}(x^2) + c$

Ans) $\frac{1}{2}\tan^{-1}(x^2) + c$

Question: 63

Evaluate the foll

Solution:

To find: Value of $\int \frac{x^5}{\sqrt{1+x^3}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{x^5}{\sqrt{1+x^3}} dx$ ... (i)

Let $1 + x^3 = t$

$\Rightarrow x^3 = t - 1$

$\Rightarrow \frac{d(x^3)}{dx} = \frac{d(t-1)}{dx}$

$\Rightarrow (3x^2) = \frac{dt}{dx}$

$\Rightarrow x^2$ dx $= \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \frac{x^3 x^2}{\sqrt{1 + x^3}} dx$

$I = \int \frac{(t-1)}{t^{\frac{1}{2}}} \frac{dt}{3}$ [$1 + x^3 = t$]

$\Rightarrow I = \frac{1}{3}\int \frac{t}{t^{\frac{1}{2}}} dt - \frac{1}{3}\int \frac{1}{t^{\frac{1}{2}}} dt$

$\Rightarrow I = \frac{1}{3}\left[\int t^{\frac{1}{2}}dt - \int t^{-\frac{1}{2}}dt\right]$

$\Rightarrow I = \frac{1}{3}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right]$

$\Rightarrow I = \frac{2}{3}\left[\frac{(1+x^3)^{\frac{3}{2}}}{3} - \frac{(1+x^3)^{\frac{1}{2}}}{1}\right]$

$\Rightarrow I = \frac{2(1+x^3)^{\frac{3}{2}}}{9} - \frac{2(1+x^3)^{\frac{1}{2}}}{3} + c$

Ans) $\frac{2(1+x^3)^{\frac{3}{2}}}{9} - \frac{2(1+x^3)^{\frac{1}{2}}}{3} + c$

Question: 64

Evaluate the foll

Solution:

To find: Value of $\int \frac{x}{\sqrt{1+x}}dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{x}{\sqrt{1+x}}dx$ ... (i)

Let $1 + x = t$

$\Rightarrow x = t - 1$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int \frac{t-1}{\sqrt{t}}dx\ [\ 1 + x = t\ ]$

$\Rightarrow I = \int \sqrt{t}dt - \int \frac{1}{\sqrt{t}}dt$

$\Rightarrow I = \left[\int t^{\frac{1}{2}}dt - \int t^{-\frac{1}{2}}dt\right]$

$\Rightarrow I = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right] + c$

$\Rightarrow I = 2\left[\frac{(1+x)^{\frac{3}{2}}}{3} - \frac{(1+x)^{\frac{1}{2}}}{1}\right] + c$

$\Rightarrow I = \frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$

Ans) $\frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$

Question: 65

Evaluate the foll

Solution:

To find: Value of $\int \frac{1}{x\sqrt{x^4-1}} dx$

Formula used: $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

We have, $I = \int \frac{1}{x\sqrt{x^4-1}} dx$ ... (i)

Multiplying numerator and denominator with x

$$I = \int \frac{x}{x^2\sqrt{(x^2)^2-1}} dx$$

Let $x^2 = t$

$$\Rightarrow 2x = \frac{dt}{dx}$$

$\Rightarrow$ xdx = $\frac{dt}{2}$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2-1}} \ [x^2 = t]$$

$$\Rightarrow I = \frac{1}{2}\sec^{-1} t + c$$

$$\Rightarrow I = \frac{1}{2}\sec^{-1}(x^2) + c$$

Ans) $\frac{1}{2}\sec^{-1}(x^2) + c$

Question: 66

Evaluate the foll

Solution:

To find: Value of $\int x\sqrt{x-1}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int x\sqrt{x-1}\, dx$ ... (i)

Let $x - 1 = t$

x = t + 1

$\Rightarrow dx = dt$

Putting this value in equation (i)

$$I = \int (t+1)\sqrt{t}\, dt \ [x = t+1]$$

$$\Rightarrow I = \int t\sqrt{t} dx + \int \sqrt{t}\, dx$$

$$\Rightarrow I = \int t^{\frac{3}{2}} dx + \int t^{\frac{1}{2}} dx$$

$$\Rightarrow I = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$\Rightarrow I = \frac{2}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + c$

Ans) $\frac{2}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + c$

Question: 67

Evaluate the foll

Solution:

To find: Value of $\int (1 - x)\sqrt{1 + x}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (1 - x)\sqrt{1 + x}\, dx$ … (i)

Let $1 + x = t$

x = t - 1

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int \{1 - (t - 1)\}\sqrt{t}\, dt\, [\, x = t - 1\,]$

$\Rightarrow I = \int \{1 - t + 1\}\sqrt{t}\, dt$

$\Rightarrow I = \int \{2 - t\}\sqrt{t}\, dt$

$\Rightarrow I = \int 2\sqrt{t}\, dt - \int t\sqrt{t}\, dt$

$\Rightarrow I = 2\int t^{\frac{1}{2}} dx - \int t^{\frac{3}{2}}\, dx$

$\Rightarrow I = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c$

$\Rightarrow I = \frac{4}{3}(1 + x)^{\frac{3}{2}} - \frac{2}{5}(1 + x)^{\frac{5}{2}} + c$

Ans) $\frac{4}{3}(1 + x)^{\frac{3}{2}} - \frac{2}{5}(1 + x)^{\frac{5}{2}} + c$

Question: 68

Evaluate the foll

Solution:

To find: Value of $\int x\sqrt{x^2 - 1}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{x^2 - 1}\, dx$ … (i)

Let $x^2 - 1 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow xdx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int \frac{1}{2}\sqrt{t}\,dt\ [\ x = x^2 - 1\ ]$

$\Rightarrow I = \frac{1}{2}\int t^{\frac{1}{2}}\,dx$

$\Rightarrow I = \frac{1}{2}\ \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow I = \frac{1}{3}\ t^{\frac{3}{2}} + c$

$\Rightarrow I = \frac{1}{3}\ (x^2 - 1)^{\frac{3}{2}} + c$

Ans) $\frac{1}{3}\ (x^2 - 1)^{\frac{3}{2}} + c$

Question: 69

Evaluate the foll

Solution:

To find: Value of $\int x\sqrt{3x - 2}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{3x - 2}\,dx$ ... (i)

Let $3x - 2 = t$

$\Rightarrow 3x = t + 2$

$\Rightarrow x = \frac{t+2}{3}$

$\Rightarrow 3 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int\left(\frac{t+2}{3}\right)\sqrt{t}\ \frac{dt}{3}\ [\ t = 3x - 2\ ]$

$\Rightarrow I = \frac{1}{9}\left[\int t^{\frac{3}{2}}\,dx\ + 2\int t^{\frac{1}{2}}\,dx\right]$

$\Rightarrow I = \frac{1}{9}\left[\frac{t^{\frac{5}{2}}}{\frac{5}{2}} + 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$

$\Rightarrow I = \frac{1}{9}\left[\frac{2}{5}(3x - 2)^{\frac{5}{2}} + \frac{4}{3}(3x - 2)^{\frac{3}{2}}\right] + c$

$\Rightarrow I = \frac{2}{45}(3x - 2)^{\frac{5}{2}} + \frac{4}{27}(3x - 2)^{\frac{3}{2}} + c$

$\Rightarrow I = \frac{2}{45}(3x - 2)^{\frac{5}{2}} + \frac{4}{27}(3x - 2)^{\frac{3}{2}} + c$

Ans) $\frac{2}{45}(3x - 2)^{\frac{5}{2}} + \frac{4}{27}(3x - 2)^{\frac{3}{2}} + c$

Question: 70

Evaluate the foll

Solution:

To find: Value of $\int \frac{dx}{x\cos^2(1 + \log x)}$

Formula used: $\int \sec^2 x \, dx = \tan x + c$

We have, $I = \int \frac{dx}{x\cos^2(1 + \log x)}$ ... (i)

Let $1 + \log x = t$

$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{x} dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\cos^2(t)}$ [$t = 1 + \log x$]

$\Rightarrow I = \int \sec^2 t \, dt$

$\Rightarrow I = \tan(t) + c$

$\Rightarrow I = \tan(1 + \log x) + c$

Ans) $\tan(1 + \log x) + c$

Question: 71

Evaluate the foll

Solution:

To find: Value of $\int x^2 \sin x^3 \, dx$

Formula used: $\int \sin x \, dx = -\cos x + c$

We have, $I = \int x^2 \sin x^3 \, dx$ ... (i)

Let $x^3 = t$

$\Rightarrow 3x^2 = \frac{dt}{dx}$

$\Rightarrow x^2 dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \sin t \frac{dt}{3}$ [$t = x^3$]

$\Rightarrow I = \frac{1}{3}\left[\int \sin t \, dt\right]$

$\Rightarrow I = \frac{1}{3}(-\cos t) + c$

$\Rightarrow I = \frac{1}{3}(-\cos x^3) + c$

Ans) $\frac{-\cos x^3}{3} + c$

Question: 72

Evaluate the foll

Solution:

To find: Value of $\int (2x + 4)\sqrt{x^2 + 4x + 3}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (2x + 4)\sqrt{x^2 + 4x + 3}\,dx$ ... (i)

Let $x^2 + 4x + 3 = t$

$\Rightarrow (2x + 4) = \frac{dt}{dx}$

$\Rightarrow (2x + 4)dx = dt$

Putting this value in equation (i)

$I = \int \sqrt{t}\,dt\ [\ t = (2x + 4)\ ]$

$\Rightarrow I = \int t^{\frac{1}{2}}\,dx$

$\Rightarrow I = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow I = \frac{2}{3}\left[(t)^{\frac{3}{2}}\right] + c$

$\Rightarrow I = \frac{2}{3}\left[(x^2 + 4x + 3)^{\frac{3}{2}}\right] + c$

Ans) $\frac{2}{3}\left[(x^2 + 4x + 3)^{\frac{3}{2}}\right] + c$

Question: 73

Evaluate the foll

Solution:

To find: Value of $\int \frac{\sin x}{(\sin x - \cos x)}\,dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{\sin x}{(\sin x - \cos x)}\,dx$ ... (i)

$\Rightarrow I = \frac{1}{2}\int \frac{2\sin x}{(\sin x - \cos x)}\,dx$

$$\Rightarrow I = \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x - \cos x)}\, dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}\, dx + \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}\, dx$$

Let $\sin x - \cos x = t$

$$\Rightarrow (\cos x + \sin x\,) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x + \sin x)dx = dt$$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{t} + \frac{1}{2}\int \, dx$$

$$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$$

$$\Rightarrow I = \frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$$

Ans) $\frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$

Question: 74

Evaluate the foll

Solution:

To find: Value of $\int \frac{dx}{(1 - \tan x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \tan x)}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\sin x}{\cos x}\right)}$$

$$\Rightarrow I = \int \frac{dx}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$$

$$\Rightarrow I = \frac{1}{2}\int \frac{2\cos x dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2}\int \frac{(\cos x + \sin x) + (\cos x - \sin x)dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2}\int \frac{(\cos x + \sin x)}{(\cos x - \sin x)}dx + \frac{1}{2}\int \frac{(\cos x - \sin x)}{(\cos x - \sin x)}dx$$

Let $(\cos x - \sin x\,) = t$

$$\Rightarrow (-\sin x - \cos x\,) = \frac{dt}{dx}$$

$$\Rightarrow (\sin x + \cos x)dx = -dt$$

Putting this value in equation (i)

$I = -\frac{1}{2}\int \frac{dt}{(t)}\,dx + \frac{1}{2}\int dx$

$\Rightarrow I = -\frac{1}{2}\log|\cos x - \sin x\,| + \frac{1}{2}x + c$

$\Rightarrow I = \frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x\,| + c$

Ans) $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x\,| + c$

Question: 75

Evaluate the foll

Solution:

To find: Value of $\int \frac{dx}{(1 - \cot x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \cot x)}$ ... (i)

$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\cos x}{\sin x}\right)}$

$\Rightarrow I = \int \frac{dx}{\left(\frac{\sin x - \cos x}{\sin x}\right)}$

$\Rightarrow I = \frac{1}{2}\int \frac{2\sin x dx}{(\sin x - \cos x)}$

$I = \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)dx}{(\sin x - \cos x)}$

$I = \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx + \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}dx$

Let $(\sin x - \cos x\,) = t$

$\Rightarrow (\cos x + \sin x\,) = \frac{dt}{dx}$

$\Rightarrow (\cos x + \sin x)dx = dt$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{(t)}\,dx + \frac{1}{2}\int dx$

$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x\,| + \frac{1}{2}x + c$

Ans) $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x\,| + c$

Question: 76

Evaluate the foll

Solution:

To find: Value of $\int \frac{\cos 2x}{(\sin x + \cos x)^2}dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ ... (i)

$$\Rightarrow I = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx$$

Let $(\cos x + \sin x) = t$

$$\Rightarrow (-\sin x + \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$$\Rightarrow I = \log|t| + c$$

$$\Rightarrow I = \log|\cos x + \sin x| + c$$

Ans) $\log|\cos x + \sin x| + c$

Question: 77

Evaluate the foll

Solution:

To find: Value of $\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$ ... (i)

$$\Rightarrow I = \int \frac{\cos x - \sin x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\cos x + \sin x)^2} dx$$

Let $(\sin x + \cos x) = t$

$$\Rightarrow (\cos x - \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^2}$$

$$\Rightarrow I = -\frac{1}{t} + c$$

$\Rightarrow I = -\frac{1}{\sin x + \cos x} + c$

Ans) $\frac{-1}{\sin x + \cos x} + c$

Question: 78

Evaluate the foll

Solution:

To find: Value of $\int \frac{(x+1)(x+\log x)^2}{x} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(x+1)(x+\log x)^2}{x} dx$ ... (i)

Let $(x + \log x) = t$

$\Rightarrow \left(1 + \frac{1}{x}\right) = \frac{dt}{dx}$

$\Rightarrow \left(\frac{x+1}{x}\right) = \frac{dt}{dx}$

Putting this value in equation (i)

$I = \int t^2 dt$

$\Rightarrow I = \frac{t^3}{3} + c$

$\Rightarrow I = \frac{(x+\log x)^3}{3} + c$

Ans) $\frac{(x+\log x)^3}{3} + c$

Question: 79

Evaluate the foll

Solution:

To find: Value of $\int x \sin^3 x^2 \cos x^2 dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int x \sin^3 x^2 \cos x^2 dx$ ... (i)

Let $(\sin x^2) = t$

$\Rightarrow (\sin x^2 . 2x) = \frac{dt}{dx}$

$\Rightarrow (\sin x^2 . x)dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int t^3 \frac{dt}{2}$

$I = \frac{1}{2}\int t^3 dt$

$\Rightarrow I = \frac{1}{2}\frac{t^4}{4} + c$

$\Rightarrow I = \frac{t^4}{8} + c$

$\Rightarrow I = \frac{\sin^4 x^2}{8} + c$

Ans) $\frac{\sin^4 x^2}{8} + c$

Question: 80

Evaluate the foll

Solution:

To find: Value of $\int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$

Formula used: $\int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c$

We have, $I = \int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$ ... (i)

Let $(\tan x) = t$

$\Rightarrow (\sec^2 x) = \frac{dt}{dx}$

$\Rightarrow (\sec^2 x) dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\sqrt{1 - t^2}}$

$\Rightarrow I = \sin^{-1}(t) + c$

$\Rightarrow I = \sin^{-1}(\tan x) + c$

Ans) $\sin^{-1}(\tan x) + c$

Question: 81

Evaluate the foll

Solution:

To find: Value of $\int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5)\, dx$

Formula used: $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

We have, $I = \int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5)\, dx$ ... (i)

Let $(2e^{-x} + 5) = t$

$\Rightarrow (2e^{-x}(-1)) = \frac{dt}{dx}$

$\Rightarrow (e^{-x}) dx = \frac{dt}{-2}$

Putting this value in equation (i)

$$I = \int \mathrm{cosec}^2(t)\,\frac{dt}{-2}$$

$$I = \frac{1}{-2}\int \mathrm{cosec}^2(t)\,dt$$

$$\Rightarrow I = \frac{1}{-2}(-\cot t) + c$$

$$\Rightarrow I = \frac{1}{2}\cot(2e^{-x} + 5) + c$$

Ans) $\frac{1}{2}\cot(2e^{-x} + 5) + c$

Question: 82

Evaluate the foll

Solution:

To find: Value of $\int 2x\sec^3(x^2+3)\ \tan(x^2+3)dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int 2x\sec^2(x^2+3)\sec(x^2+3)\ \tan(x^2+3)dx$ ... (i)

Let $\sec(x^2+3) = t$

$$\Rightarrow \sec(x^2+3) = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int t^2\,dt$$

$$\Rightarrow I = \frac{t^3}{3} + c$$

$$\Rightarrow I = \frac{\sec^3(x^2+3)}{3} + c$$

Ans) $\frac{\sec^3(x^2+3)}{3} + c$

Question: 83

Evaluate the foll

Solution:

To find: Value of $\int \frac{\sin 2x}{(a+b\cos x)^2}dx$

Formula used: (i) $\int \frac{1}{x}dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{\sin 2x}{(a+b\cos x)^2}dx$ ... (i)

$$I = \int \frac{2\sin x\cos x}{(a + b\cos x)^2} dx$$

Let $(a + b\cos x) = t$

$$\Rightarrow (\cos x) = \frac{t - a}{b}$$

$$\Rightarrow (\sin x)dx = \frac{dt}{-b}$$

Putting this value in equation (i)

$$I = \frac{2}{-b^2} \int \frac{t - a}{t^2} dt$$

$$I = \frac{2}{-b^2} \left[\int \frac{t}{t^2} dt - \int \frac{a}{t^2} dt \right]$$

$$I = \frac{2}{-b^2} \left[\int \frac{1}{t} dt - a \int \frac{1}{t^2} dt \right]$$

$$I = \frac{2}{-b^2} \left[\log|t| - a\left(-\frac{1}{t} \right) + c \right]$$

$$I = -\frac{2}{b^2} \left[\log|a + b\cos x| + \left(\frac{a}{a + b\cos x} \right) \right] + c$$

Ans) $-\frac{2}{b^2} \left[\log|a + b\cos x| + \left(\frac{a}{a + b\cos x} \right) \right] + c$

Question: 84

Evaluate the foll

Solution:

To find: Value of $\int \frac{dx}{(3 - 5x)}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(3 - 5x)}$ ... (i)

Let $(3 - 5x) = t$

$$\Rightarrow (-5) = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{dt}{-5}$$

Putting this value in equation (i)

$$I = \int \frac{1}{t} \frac{dt}{-5}$$

$$I = \frac{1}{-5} \int \frac{dt}{t}$$

$$\Rightarrow I = \frac{1}{-5} \log|t| + c$$

$$\Rightarrow I = -\frac{1}{5} \log|3 - 5x| + c$$

Ans) $-\frac{1}{5}\log|3 - 5x| + c$

Question: 85

Evaluate the foll

Solution:

To find: Value of $\int \sqrt{1+x}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \sqrt{1+x}\,dx$ ... (i)

Let $(1 + x) = t$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$$I = \int \sqrt{t}\,dt$$

$$I = \int t^{\frac{1}{2}}\,dt$$

$$\Rightarrow I = \frac{2}{3}(1+x)^{\frac{3}{2}} + c$$

Ans) $\frac{2}{3}(1+x)^{\frac{3}{2}} + c$

Question: 86

Evaluate the foll

Solution:

To find: Value of $\int x^2 e^{x^3} \cos(e^{x^3})dx$

Formula used: $\int \cos x\,dx = \sin x + c$

We have, $I = \int x^2 e^{x^3} \cos(e^{x^3})dx$ ... (i)

Let $e^{x^3} = t$

$$\Rightarrow e^{x^3}.\,3x^2 = \frac{dt}{dx}$$

$$\Rightarrow e^{x^3}.\,x^2.\,dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \cos(t)\,\frac{dt}{3}$$

$$I = \frac{\sin(t)}{3} + c$$

$$I = \frac{\sin\left(e^{x^3}\right)}{3} + c$$

Ans) $\frac{\sin\left(e^{x^3}\right)}{3} + c$

Question: 87

Evaluate the foll

Solution:

To find: Value of $\int \frac{e^{m\tan^{-1}x}\,dx}{(1+x^2)}$

Formula used: $\int e^t dx = e^t + c$

We have, $I = \int \frac{e^{m\tan^{-1}x}\,dx}{(1+x^2)}$ ... (i)

Let $(m\tan^{-1}x)$ = t

$$\Rightarrow m\left(\frac{1}{1+x^2}\right) = \frac{dt}{dx}$$

$$\Rightarrow \left(\frac{1}{1+x^2}\right)dx = \frac{dt}{m}$$

Putting this value in equation (i)

$$I = \int e^t \frac{dt}{m}$$

$$\Rightarrow I = \frac{e^t}{m} + c$$

$$\Rightarrow I = \frac{e^{m\tan^{-1}x}}{m} + c$$

Ans) $\frac{e^{m\tan^{-1}x}}{m} + c$

Question: 88

Evaluate the foll

Solution:

To find: Value of $\int \frac{(x+1)e^x\,dx}{\cos^2(xe^x)}$

Formula used: $\int \sec^2 x\,dx = \tan x + c$

We have, $I = \int \frac{(x+1)e^x\,dx}{\cos^2(xe^x)}$ ... (i)

Let (xe^x) = t

$$\Rightarrow xe^x + e^x.1 = \frac{dt}{dx}$$

$$\Rightarrow e^x(x+1) = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\cos^2(t)}$$

$$\Rightarrow I = \int \sec^2(t)\,dt$$

$$\Rightarrow I = \tan(t) + c$$

$$\Rightarrow I = \tan(xe^x) + c$$

Ans) $\tan(xe^x) + c$

Question: 89

Evaluate the foll

Solution:

To find: Value of $\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right) dx}{\sqrt{x}}$

Formula used: $\int \cos x \, dx = \sin x + c$

We have, $I = \int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right) dx}{\sqrt{x}}$ ... (i)

Let $\left(e^{\sqrt{x}}\right) = t$

$\Rightarrow e^{\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$

Putting this value in equation (i)

$I = \int \cos(t) \, 2dt$

$I = 2 \sin\left(e^{\sqrt{x}}\right) + c$

Ans) $2 \sin\left(e^{\sqrt{x}}\right) + c$

Question: 90

Evaluate the foll

Solution:

To find: Value of $\int \sqrt{e^x - 1} \, dx$

Formula used: $\int \frac{1}{x^2+1} dx = \tan^{-1} x + c$

We have, $I = \int \sqrt{e^x - 1} \, dx$ ... (i)

Let $(e^x - 1) = t^2$

$\Rightarrow e^x - 1 = t^2$

$\Rightarrow e^x = t^2 + 1$

$\Rightarrow e^x = \frac{2tdt}{dx}$

$\Rightarrow dx = \frac{2tdt}{e^x}$

$\Rightarrow dx = \frac{2t}{t^2+1} dt$

Putting this value in equation (i)

$I = \int \sqrt{t^2} \frac{2t}{t^2+1} dt$

$\Rightarrow I = \int \frac{2t^2}{t^2+1} dt$

$\Rightarrow I = 2\int \frac{t^2+1-1}{t^2+1} dt$

$\Rightarrow I = 2\int\left(1 - \frac{1}{t^2+1}\right)dt$

$\Rightarrow I = 2\,[\,t - \tan^{-1} t] + c$

$\Rightarrow I = 2\,[\,\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}] + c$

Ans) $2\,[\,\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}] + c$

Question: 91

Evaluate the foll

Solution:

To find: Value of $\int \frac{dx}{(x - \sqrt{x})}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(x - \sqrt{x})}$ ... (i)

$\Rightarrow I = \int \frac{dx}{\sqrt{x}(\sqrt{x} - 1)}$

Let $(\sqrt{x} - 1) = t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{\sqrt{x}} dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int \frac{1}{t}\frac{dt}{2}$

$I = \frac{1}{2}\log|t| + c$

$I = \frac{1}{2}\log|\sqrt{x} - 1| + c$

Ans) $\frac{1}{2}\log|\sqrt{x} - 1| + c$

Question: 92

Evaluate the foll

Solution:

To find: Value of $\int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)} dx$

Formula used: $\int \sec^2 x\, dx = \tan x + c$

We have, $I = \int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)} dx$ ... (i)

Let $2\tan^{-1}x = t$

$\Rightarrow \frac{2}{1+x^2} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{1+x^2}dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int \sec^2(t)\frac{dt}{2}$

$I = \frac{1}{2}\tan(t) + c$

$I = \frac{1}{2}\tan(2\tan^{-1}x) + c$

Ans) $\frac{1}{2}\tan(2\tan^{-1}x) + c$

Question: 93

Evaluate the foll

Solution:

To find: Value of $\int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$ ... (i)

Let $x + \sin^2 x = t$

$\Rightarrow 1 + 2\sin x.\cos x = \frac{dt}{dx}$

$\Rightarrow (1+\sin 2x)\,dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|t| + c$

$I = \log\left|x + \sin^2 x\right| + c$

Ans) $\log\left|x + \sin^2 x\right| + c$

Question: 94

Evaluate the foll

Solution:

To find: Value of $\int \left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$ ... (i)

Let $x + \log(\cos x) = t$

$\Rightarrow 1 + \frac{1.(-\sin x)}{\cos x} = \frac{dt}{dx}$

$\Rightarrow 1 - \tan x = \frac{dt}{dx}$

$\Rightarrow (1 - \tan x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|t| + c$

$I = \log|x + \log(\cos x)| + c$

Ans) $\log|x + \log(\cos x)| + c$

Question: 95

Evaluate the foll

Solution:

To find: Value of $\int \left(\frac{1 + \cot x}{x + \log(\sin x)}\right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \left(\frac{1 + \cot x}{x + \log(\sin x)}\right) dx$ ... (i)

Let $x + \log(\sin x) = t$

$\Rightarrow 1 + \frac{1.(\cos x)}{\sin x} = \frac{dt}{dx}$

$\Rightarrow 1 + \cot x = \frac{dt}{dx}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|x + \log(\sin x)| + c$

$I = \log|x + \log(\sin x)| + c$

Ans) $\log|x + \log(\sin x)| + c$

Question: 96

Evaluate the foll

Solution:

To find: Value of $\int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$ ... (i)

Let $1 - \tan^2 x = t$

$\Rightarrow 0 - 2.\tan x.\sec^2 x = \frac{dt}{dx}$

$\Rightarrow (\tan x.\sec^2 x)dx = \frac{dt}{-2}$

$\Rightarrow (1+\cot x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{1}{t}\frac{dt}{(-2)}$

$I = \frac{1}{2}\log|t| + c$

$I = \frac{1}{2}\log\left|1-\tan^2 x\right| + c$

Ans) $\frac{1}{2}\log\left|1-\tan^2 x\right| + c$

Question: 97

Evaluate the foll

Solution:

To find: Value of $\int \frac{\sin(2\tan^{-1}x)}{(1+x^2)}dx$

Formula used: $\int \sin x\, dx \; = \cos x + c$

We have, $I = \int \frac{\sin(2\tan^{-1}x)}{(1+x^2)}dx$ ... (i)

Let $2\tan^{-1}x = t$

$\Rightarrow 2\frac{1}{1+x^2} = \frac{dt}{dx}$

$\Rightarrow \frac{dx}{1+x^2} = \frac{dt}{2}$

$\Rightarrow (1+\cot x)dx = dt$

Putting this value in equation (i)

$I = \int \sin(t)\frac{dt}{(2)}$

$I = -\frac{1}{2}\cos(t) + c$

$I = -\frac{1}{2}\cos(2\tan^{-1}x) + c$

Ans) $-\frac{1}{2}\cos(2\tan^{-1}x) + c$

Question: 98

Evaluate the foll

Solution:

To find: Value of $\int \frac{dx}{\left(x^{\frac{1}{2}}+x^{\frac{1}{3}}\right)}$

Formula used: (i) $\int \frac{1}{x}dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$ ... (i)

Let $x = t^6$

$\Rightarrow x^{\frac{1}{6}} = t$

$\Rightarrow 6t^5\,dt = dx$

Putting this value in equation (i)

$$I = \int \frac{6t^5\,dt}{(t^3 + t^2)}$$

$$I = \int \frac{6t^5\,dt}{t^2(t+1)}$$

$$I = 6\int \frac{t^3\,dt}{(t+1)}$$

$$I = 6\int \frac{t^3 + 1 - 1}{(t+1)}dt$$

$$I = 6\int \frac{(t+1)\,(t^2 - t + 1)}{(t+1)}dt - \int \frac{1}{(t+1)}dt$$

$$I = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + c$$

$$I = [2t^3 - 3t^2 + 6t - 6\log|t+1|] + c$$

$$I = \left[2\left(x^{\frac{1}{6}}\right)^3 - 3\left(x^{\frac{1}{6}}\right)^2 + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

$$I = \left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

Ans) $\left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$

Question: 99

Evaluate the foll

Solution:

To find: Value of $\int (\sin^{-1} x)^2 dx$

Formula used: $\int \sin x\,dx = \cos x + c$

We have, $I = \int (\sin^{-1} x)^2 dx$ ... (i)

Let $\sin^{-1} x$ = t , x = sint ,

$\Rightarrow \cos t = \sqrt{1 - x^2}$

$$\Rightarrow \frac{1}{\sqrt{1 - x^2}} = \frac{dt}{dx}$$

$\Rightarrow \sqrt{1 - x^2}\, dt = dx$

$\Rightarrow \sqrt{1 - (\sin t)^2}\, dt = dx$

$\Rightarrow \sqrt{1 - \sin^2 t}\, dt = dx$

$\Rightarrow \cos t\, dt = dx$

Putting this value in equation (i)

$$I = \int t^2 \cos t\ dt$$

$$I = \int t^2 \cos t\ dt - \int \left[\frac{d(t^2)}{dt} \int \cos t\ dt\right] dt$$

$$I = t^2 \sin t - \int [2t. \sin t\,]dt$$

$$I = t^2 \sin t\ - 2\left\{\int t\,[\sin t\,]dt - \int \left[\frac{dt}{dt}\int \sin t\, dt\right]\ dt\right\}$$

$$I = t^2 \sin t\ - 2\left[-t\cos t + \int 1.\cos t\, dt\right]$$

$$I = t^2 \sin t\ + 2t\cos t - 2\sin t + c$$

$$I = (\sin^{-1} x)^2 x + 2(\sin^{-1} x)\sqrt{1 - x^2} - 2x + c$$

Ans) $(\sin^{-1} x)^2 x + 2(\sin^{-1} x)\sqrt{1 - x^2} - 2x + c$

Question: 100

Evaluate the foll

Solution:

To find: Value of $\int \frac{2x\tan^{-1}(x^2)}{(1 + x^4)} dx$

Formula used: $\int x^n dx = \frac{1}{n + 1} x^{n+1} + c$

We have, $I = \int \frac{2x\tan^{-1}(x^2)}{(1 + x^4)} dx$ ... (i)

Let $\tan^{-1}(x^2) = t$

$$\Rightarrow \frac{1}{1 + (x^2)^2} . 2x = \frac{dt}{dx}$$

$$\Rightarrow \frac{2x}{1 + x^4} dx = dt$$

Putting this value in equation (i)

$$I = \int t.\ dt$$

$$I = \frac{t^2}{2} + c$$

$$I = \frac{\{\tan^{-1}(x^2)\}^2}{2} + c$$

Ans) $\dfrac{\{\tan^{-1}(x^2)\}^2}{2} + c$

Question: 101

Evaluate the foll

Solution:

To find: Value of $\int \frac{(x^2+1)}{(x^4+1)} dx$

Formula used: $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

We have, $I = \int \frac{(x^2+1)}{(x^4+1)} dx$ ... (i)

Dividing Numerator and Denominator by x^2,

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}+2-2\right)} dx$$

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2 - 2.x.\frac{1}{x} + \left(\frac{1}{x}\right)^2 + 2\right)} dx$$

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(\left(x-\frac{1}{x}\right)^2 + (\sqrt{2})^2\right)} dx$$

Let $x - \frac{1}{x} = t$

$$\Rightarrow \left(1+\frac{1}{x^2}\right) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{1}{(t)^2 + (\sqrt{2})^2} dt$$

$$I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$$

Ans) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{x^2-1}{\sqrt{2}x}\right) + c$

Question: 102

Evaluate the foll

Solution:

To find: Value of $\int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$

Formula used: $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + c$

We have, $I = \int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}}dx$ … (i)

Let (sinx – cosx) = t

$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$

$\Rightarrow (\cos x + \sin x)\,dx = dt$

$\Rightarrow t^2 = \sin^2 x - 2\sin x.\cos x + \cos^2 x$

$\Rightarrow t^2 = 1 - 2\sin x.\cos x$

$\Rightarrow 2\sin x.\cos x = 1 - t^2$

$\Rightarrow \sin 2x = 1 - t^2$

Putting this value in equation (i)

$\Rightarrow I = \int \frac{dt}{\sqrt{1-t^2}}$

$I = \sin^{-1} t$

$I = \sin^{-1}(\sin x - \cos x)$

Let $\sin^{-1}(\sin x - \cos x) = \theta$

$\Rightarrow I = \sin^{-1}(\sin x - \cos x) = \theta$ … (ii)

$\Rightarrow \sin\theta = \sin x - \cos x$

Now if $\sin\theta = \sin x - \cos x$

Then $\cos\theta = \sqrt{1 - (\sin x - \cos x)^2}$

$\Rightarrow \cos\theta = \sqrt{1 - (\sin^2 x - 2\sin x.\cos x + \cos^2 x)}$

$\Rightarrow \cos\theta = \sqrt{1 - (1 - 2\sin x.\cos x)}$

$\Rightarrow \cos\theta = \sqrt{2\sin x.\cos x)}$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Now $\tan\theta = \frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$

Comparing the value θ from eqn. (ii)

$I = \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$

Dividing Numerator and denominator from cosx

$I = \theta = \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$

Ans.) $\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$

Exercise : OBJECTIVE QUESTIONS I

Question: 1

Mark (√) against

Solution:

Given = $\int(2x+3)^5$

Let, 2x + 3 = z

⇒ 2dx = dz

So,

$$\int(2x+3)^5 dx$$

$$=\int\frac{z^5}{2}dz$$

$$=\frac{1}{2}\frac{z^6}{6}+c$$ where c is the integrating constant.

$$=\frac{z^6}{12}+c$$

$$=\frac{(2x+3)^6}{12}+c$$

Question: 2

Mark (√) against

Solution:

Given = $\int(3-5x)^7$

Let, 3 – 5x = z

⇒ -5dx = dz

So,

$$\int(3-5x)^7 dx$$

$$=-\int\frac{z^7}{5}dz$$

$$=-\frac{1}{5}\frac{z^8}{8}+c$$ where c is the integrating constant.

$$=-\frac{z^8}{40}+c$$

$$=-\frac{(3-5x)^8}{40}+c$$

Question: 3

Mark (√) against

Solution:

Given = $\int\frac{1}{(2-3x)^4}$

Let, 2 – 3x = z

⇒ -3dx = dz

So,

$$\int \frac{1}{(2-3x)^4}dx$$

$$= \int \frac{1}{z^4}\left(\frac{dz}{-3}\right)$$

$$= -\frac{1}{3}\int \frac{dz}{z^4}$$

where c is the integrating constant.

$$= -\frac{1}{3}\int z^{-4}dz$$

$$= -\frac{1}{3}\frac{z^{-3}}{-3} + c$$

$$= \frac{1}{9(2-3x)^3} + c$$

Question: 4

Mark (√) against

Solution:

Given = $\int \sqrt{ax+b}$

Let, ax + b = z^2

⇒ adx = 2zdz

So,

$$\int \sqrt{ax+b}dx$$

$$= \int z\frac{2zdz}{a}$$

$$= \frac{2}{a}\int z^2dz$$

where c is the integrating constant.

$$= \frac{2}{a}\frac{z^3}{3} + c$$

$$= \frac{2}{3a}z^3 + c$$

$$= \frac{2(ax+b)^{3/2}}{3a} + c$$

Question: 5

Mark (√) against

Solution:

Given = $\int \sec^2(7-4x)$

Let, 7 – 4x = z

⇒ -4dx = dz

So,

$$\int \sec^2(7-4x)dx$$

$$= \int \sec^2 z \frac{dz}{-4}$$

$$= -\frac{1}{4}\int \sec^2 z dz$$ where c is the integrating constant.

$$= -\frac{1}{4}\tan z + c$$

$$= -\frac{1}{4}\tan(7-4x) + c$$

Question: 6

Mark (√) against

Solution:

Given = $\int \cos 3x$

So, $\int \cos 3x dx = \frac{\sin 3x}{3} + c$ where c is the integrating constant.

Question: 7

Mark (√) against

Solution:

Given = $\int e^{(5-3x)}$

Let, 5 – 3x = z

⇒ -3dx = dz

So,

$$\int e^{(5-3x)}dx$$

$$= \int e^z \frac{dz}{-3}$$

$$= -\frac{1}{3}\int e^z dz$$ where c is the integrating constant.

$$= -\frac{1}{3}e^z + c$$

$$= -\frac{1}{3}e^{(5-3x)} + c$$

Question: 8

Mark (√) against

Solution:

Given = $\int e^{(3x+4)}$

Let, 3x + 4 = z

⇒ 3dx = dz

So,

$\int e^{(3x+4)}dx$

$= \int e^z \frac{dz}{3}$

$= \frac{1}{3}\int e^z dz$

$= \frac{1}{3}e^z + c$

$= \frac{1}{3}e^{(3x+4)} + c$

where c is the integrating constant.

Question: 9

Mark (√) against

Solution:

Given = $\int \tan^2 \frac{x}{2}$

Let, $\frac{x}{2} = z$

⇒ dx = 2dz

So,

$\int \tan^2 \frac{x}{2} dx$

$= 2\int \tan^2 z dz$

$= 2\int \frac{\sin^2 z}{\cos^2 z} dz$

$= 2\int \frac{1-\cos^2 z}{\cos^2 z} dz$

$= 2\int (\sec^2 z - 1) dz$

$= 2\left[\tan z - z\right] + c$

$= 2[\tan \frac{x}{2} - \frac{x}{2}] + c$ where c is the integrating constant.

Question: 10

Mark (√) against

Solution:

Given = $\int \sqrt{1-\cos x}$

So,

$$\int \sqrt{1-\cos x}dx$$

$$= \int \sqrt{1-\cos x}\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}dx$$

$$= \int \frac{\sqrt{1-\cos^2 x}}{\sqrt{1+\cos x}}dx$$

$$= \int \frac{\sin x}{\sqrt{1+\cos x}}dx$$

Let 1 + cosx = u^2

So, -sinxdx = 2udu

$$-\int \frac{2u}{u}du = -2\int du = -2u + c = -2\sqrt{1+\cos x} + c$$

where c is the integrating constant.

Question: 11

Mark (√) against

Solution:

Given = $\int \sqrt{1+\sin x}$

So,

$$\int \sqrt{1+\sin x}dx$$

$$= \int \sqrt{1+\sin x}\frac{\sqrt{1-\sin x}}{\sqrt{1-\sin x}}dx$$

$$= \int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\sin x}}dx$$

$$= \int \frac{\cos x}{\sqrt{1-\sin x}}dx$$

Let 1 - sinx = u^2

So, -cosxdx = 2udu

$$-\int \frac{2u}{u}du = -2\int du = -2u + c = -2\sqrt{1-\sin x} + c$$

where c is the integrating constant.

Question: 12

Mark (√) against

Solution:

Given = $\int \sin^3 x dx$

So,

$$\int \sin^3 x dx$$

$$= \int \sin^2 x \sin x dx$$

$$= \int \left(1-\cos^2 x\right)\sin x dx$$

Let cosx = u

So, -sinxdx = du

$$-\int\left(1-u^2\right)du$$

$$= -\int du + \int u^2 du$$

$$= -u + \frac{u^3}{3} + c$$

$$= -\cos x + \frac{\cos^3 x}{3} + c$$

where c is the integrating constant.

Question: 13

Mark (√) against

Solution:

Given = $\int \frac{\log x}{x}$

Let, logx = u

So, $\frac{1}{x}dx = du$

So,

$$\int \frac{\log x}{x}dx$$

$$= \int u du$$

$$= \frac{u^2}{2} + c$$

$$= \frac{(\log x)^2}{2} + c$$

where c is the integrating constant.

Question: 14

Mark (√) against

Solution:

Given = $\int \frac{\sec^2(\log x)}{x}$

Let, logx = z

⇒ $\frac{dx}{x} = dz$

So,

$$\int \frac{\sec^2(\log x)}{x} dx$$
$$= \int \sec^2 z dz$$
$$= \tan z + c$$
$$= \tan(\log x) + c$$

where c is the integrating constant.

Question: 15

Mark (√) against

Solution:

Given = $\int \frac{1}{x(\log x)}$

Let, logx = z

$$\Rightarrow \frac{dx}{x} = dz$$

So,

$$\int \frac{1}{x(\log x)} dx$$
$$= \int \frac{1}{z} dz$$
$$= \log z + c$$
$$= \log(\log x) + c$$

where c is the integrating constant.

Question: 16

Mark (√) against

Solution:

Given = $\int e^{x^3} x^2$

Let, $x^3 = z$

$\Rightarrow 3x^2dx = dz$

$$\Rightarrow x^2 dx = \frac{dz}{3}$$

So,

$$\int e^{x^3} x^2 dx$$
$$= \frac{1}{3}\int e^z dz$$
$$= \frac{1}{3}e^z + c$$
$$= \frac{1}{3}e^{x^3} + c$$

where c is the integrating constant.

Question: 17

Mark (√) against

Solution:

Given = $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}$

Let, x = z^2

⇒ dx = 2zdz

So,

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

$= \int \frac{e^{z}}{z} 2zdz$

$= 2\int e^{z} dz$

$= 2e^{z} + c$

$= 2e^{\sqrt{x}} + c$

where c is the integrating constant.

Question: 18

Mark (√) against

Solution:

Given = $\int \frac{e^{\tan^{-1} x}}{\left(1+x^2\right)}$

Let, $\tan^{-1}x$ = z

⇒ $\frac{1}{1+x^2} dx = dz$

So,

$\int \frac{e^{\tan^{-1} x}}{\left(1+x^2\right)} dx$

$= \int e^{z} dz$

$= e^{z} + c$

$= e^{\tan^{-1} x} + c$

where c is the integrating constant.

Question: 19

Mark (√) against

Solution:

Given = $\int \frac{\sin \sqrt{x}}{\sqrt{x}}$

Let, $x = z^2$

⇒ $dx = 2zdz$

So,

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$$
$$= \int \frac{\sin z}{z} 2zdz$$
$$= 2\int \sin z dz$$
$$= -2\cos z + c$$
$$= -2\cos\sqrt{x} + c$$

where c is the integrating constant.

Question: 20

Mark (√) against

Solution:

Given = $\int \left(\sqrt{\sin x}\right)\cos x$

Let, $\sin x = z^2$

⇒ $\cos x dx = 2zdz$

So,

$$\int \left(\sqrt{\sin x}\right)\cos x dx$$
$$= 2\int z^2 dz$$
$$= 2\frac{z^3}{3} + c$$
$$= \frac{2}{3}\sin^{3/2} x + c$$

where c is the integrating constant.

Question: 21

Mark (√) against

Solution:

Given = $\int \frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}$

Let, $\tan^{-1}x = z^2$

$$\Rightarrow \frac{1}{1+x^2} dx = 2zdz$$

So,

$$\int \frac{1}{(1+x^2)\sqrt{\tan^{-1}x}}dx$$

$$= \int \frac{2z}{z}dz$$

$$= 2\int dz$$

$$= 2z + c$$

$$= 2\sqrt{\tan^{-1}x} + c$$

where c is the integrating constant.

Question: 22

Mark (√) against

Solution:

Given = $\int \frac{\cot x}{\log(\sin x)}$

Let, sinx = z

⇒ cosxdx = dz

So,

$$\int \frac{\cot x}{\log(\sin x)}dx$$

$$= \int \frac{\cos x}{\sin x \log(\sin x)}dx$$

$$= \int \frac{dz}{z \log z}$$

Let, logz = u

⇒ $\frac{1}{z}dz = du$

So,

$$\int \frac{dz}{z \log z}$$

$$= \int \frac{du}{u}$$

$$= \log u + c$$

$$= \log|\log z| + c$$

where c is the integrating constant.

Question: 23

Mark (√) against

Solution:

Given = $\int \frac{1}{x \cos^2(1+\log x)}$

Let, 1 + logx = z

$$\Rightarrow \frac{1}{x}dx = dz$$

So,

$$\int \frac{1}{x\cos^2(1+\log x)}dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(1+\log x) + c$$

where c is the integrating constant.

Question: 24

Mark (√) against

Solution:

Given = $\int \frac{x^2 \tan^{-1} x^3}{(1+x^6)}dx$

Let, $\tan^{-1}x^3 = z$

$$\Rightarrow \frac{1}{1+x^6} \times 3x^2 dx = dz$$

$$\Rightarrow \frac{x^2}{1+x^6}dx = \frac{dz}{3}$$

So,

$$\frac{1}{3}\int z dz$$

$$= \frac{1}{3}\frac{z^2}{2} + c$$

$$= \frac{z^2}{6} + c$$

$$= \frac{(\tan^{-1} x^3)^2}{6} + c$$

where c is the integrating constant.

Question: 25

Mark (√) against

Solution:

Given = $\int \sec^5 x \tan x$

So, $\int \sec^5 \tan x dx = \int \sec^4 x(\sec x \tan x)dx$

Let, secx = z

⇒ secxtanxdx = dz

$$\int \sec^4 x(\sec x \tan x)dx$$

$$= \int z^4 dz$$

$$= \frac{z^5}{5} + c$$

$$= \frac{\sec^5 x}{5} + c$$

where c is the integrating constant.

Question: 26

Mark (√) against

Solution:

Given = $\int \operatorname{cosec}^3(2x+1)\cot(2x+1)$

So,

$$\int \operatorname{cosec}^3(2x+1)\cot(2x+1)dx$$

$$= \int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$

Let, cosec(2x + 1) = z

⇒ -2cosec(2x + 1)cot(2x + 1)dx = dz

$$\int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$

$$= \int z^2 \frac{dz}{-2} =$$

$$= -\frac{1}{2}\frac{z^3}{3} + c$$

$$= -\frac{\operatorname{cosec}^6(2x+1)}{6} + c$$

where c is the integrating constant.

Question: 27

Mark (√) against

Solution:

Given = $\int \frac{\tan(\sin^{-1} x)}{\sqrt{1-x^2}}$

Let, $\sin^{-1}x = z$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dz$$

So,

$$\int \frac{\tan\left(\sin^{-1} x\right)}{\sqrt{1-x^2}} dx$$

$$= \int \tan z dz$$

$$= \log\left|\sec z\right| + c$$

$$= \log\left|\sec\left(\sin^{-1} x\right)\right| + c$$

where c is the integrating constant.

Question: 28

Mark (√) against

Solution:

Given = $\int \frac{\tan(\log x)}{x}$

Let, logx = z

$\Rightarrow \frac{1}{x} dx = dz$

So,

$$\int \frac{\tan(\log x)}{x} dx$$

$$= \int \tan z dz$$

$$= \log\left|\sec z\right| + c$$

$$= \log\left|\sec(\log x)\right| + c$$

$$= -\log\left|\cos(\log x)\right| + c$$

where c is the integrating constant.

Question: 29

Mark (√) against

Solution:

Given = $\int e^x \cot\left(e^x\right) dx$

Let, $e^x = z$

$\Rightarrow e^x dx = dz$

So,

$$\int e^x \cot\left(e^x\right) dx$$

$$= \int \cot z dz$$

$$= \log\left|\sin z\right| + c$$

$$= \log\left|\sin\left(e^x\right)\right| + c$$

where c is the integrating constant.

Question: 30

Mark (√) against

Solution:

Given = $\int \frac{e^x}{\sqrt{1+e^x}}$

Let, 1 + e^x = z^2

⇒ $e^x dx$ = 2zdz

So,

$$\int \frac{e^x}{\sqrt{1+e^x}} dx$$

$$= \int \frac{2zdz}{z}$$

$$= 2\int dz$$

$$= 2z + c$$

$$= 2\sqrt{1+e^x} + c$$

where c is the integrating constant.

Question: 31

Mark (√) against

Solution:

Given = $\int \frac{x}{\sqrt{1-x^2}} dx$

Let, 1 – x^2 = z^2

⇒ -2xdx = 2zdz

So,

$$\int \frac{x}{\sqrt{1-x^2}} dx$$

$$= -\int \frac{zdz}{z}$$

$$= -\int dz$$

$$= -z + c$$

$$= -\sqrt{1-x^2} + c$$

where c is the integrating constant.

Question: 32

Mark (√) against

Solution:

Given = $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$

Let, xe^x = z

$\Rightarrow e^x(1 + x)dx = dz$

So,

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(xe^x) + c$$

where c is the integrating constant.

Question: 33

Mark (√) against

Solution:

Given =

$$\int \frac{dx}{(e^x + e^{-x})}$$

$$= \int \frac{e^x}{(e^x + 1)}dx$$

Let, $e^x + 1 = z$

$\Rightarrow e^x dx = dz$

So,

$$\int \frac{e^x dx}{(e^x + 1)}$$

$$= \int \frac{dz}{z}$$

$$= \log|z| + c$$

$$= \tan|e^x + 1| + c$$

where c is the integrating constant.

Question: 34

Mark (√) against

Solution:

Given =

$$\int \frac{2^x dx}{1 - 4^x}$$

$$= \int \frac{2^x}{1-(2^x)^2}dx$$

Let, $2^x = z$

$\Rightarrow 2^x(\log 2)dx = dz$

So,

$$\int \frac{2^x dx}{1-\left(2^x\right)^2}$$

$$= \frac{1}{\log 2}\int \frac{dz}{1-z^2}$$

$$= \frac{1}{\log 2}\sin^{-1} z + c$$

$$= \frac{\sin^{-1} 2x}{\log 2} + c$$

where c is the integrating constant.

Question: 35

Mark (√) against

Solution:

Given =

$$\int \frac{dx}{e^x - 1}$$

$$= -\int \frac{-1 + e^x - e^x}{e^x - 1} dx$$

$$= -\int \frac{e^x - 1}{e^x - 1} dx + \int \frac{e^x}{e^x - 1} dx$$

$$= -\int dx + \int \frac{e^x}{e^x - 1} dx$$

Let, $e^x - 1 = z$

$\Rightarrow e^x dx = dz$

So,

$$-\int dx + \int \frac{e^x}{e^x - 1} dx$$

$$= -x + \int \frac{dz}{z}$$

$$= -x + \log z + c$$

$$= -x + \log\left|e^x - 1\right| + c$$

where c is the integrating constant.

Question: 36

Mark (√) against

Solution:

Given =

$$\int \frac{dx}{\left(\sqrt{x}+x\right)}$$

$$= \int \frac{1}{\sqrt{x}} \frac{1}{\left(1+\sqrt{x}\right)} dx$$

Let, $1+\sqrt{x} = z$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dz$$

So,

$$\int \frac{1}{\sqrt{x}} \frac{1}{\left(1+\sqrt{x}\right)} dx$$

$$= 2\int \frac{dz}{z}$$

$$= 2\log|z| + c$$

$$= 2\tan\left|1+\sqrt{x}\right| + c$$

where c is the integrating constant.

Question: 37

Mark (√) against

Solution:

Given

$$\int \frac{dx}{1+\sin x}$$

$$= \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \int \frac{dx}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= \int \frac{\sec^2\frac{x}{2} dx}{\left(\tan\frac{x}{2} + 1\right)^2}$$

Let, $\tan\frac{x}{2} + 1 = z$

$$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2} dx = dz$$

So,

$$\int \frac{2dz}{z^2}$$

$$= -\frac{2}{z} + c$$

$$= -\frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

Question: 38

Mark (√) against

Solution:

Given

$$\int \frac{\sin x}{1 + \sin x} dx$$

$$= \int dx - \int \frac{dx}{1 + \sin x}$$

$$= x - \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= x - \int \frac{dx}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= x - \int \frac{\sec^2\frac{x}{2} dx}{\left(\tan\frac{x}{2} + 1\right)^2}$$

Let, $\tan\frac{x}{2} + 1 = z$

$$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2} dx = dz$$

So,

$$x - \int \frac{2dz}{z^2}$$

$$= x + \frac{2}{z} + c$$

$$= x + \frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

Question: 39

Mark (√) against

Solution:

Given

$\int \frac{\sin x}{1-\sin x} dx$

$= -\int dx + \int \frac{dx}{1-\sin x}$

$= -x + \int \frac{dx}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} - 2\sin \frac{x}{2} \cos \frac{x}{2}}$

$= -x + \int \frac{dx}{\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)^2}$

$= -x + \int \frac{\sec^2 \frac{x}{2} dx}{\left(\tan \frac{x}{2} - 1\right)^2}$

Let, $\tan \frac{x}{2} - 1 = z$

$\Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dz$

So,

$-x + \int \frac{2dz}{z^2}$

$= -x - \frac{2}{z} + c$

$= -x - \frac{2}{\tan \frac{x}{2} + 1} + c$

where c is the integrating constant.

Question: 40

Mark (√) against

Solution:

Given

$$\int \frac{dx}{1+\cos x}$$

$$= \int \frac{dx}{1+2\cos^2 \frac{x}{2} - 1}$$

$$= \frac{1}{2}\int \frac{dx}{\cos^2 \frac{x}{2}}$$

$$= \frac{1}{2}\int \sec^2 \frac{x}{2} dx$$

$$= \frac{1}{2} 2\tan \frac{x}{2} + c$$

$$= \tan \frac{x}{2} + c$$

where c is the integrating constant.

Question: 41

Mark (√) against

Solution:

Given

$$\int \frac{dx}{1-\cos x}$$

$$= \int \frac{dx}{1-1+2\sin^2 \frac{x}{2}}$$

$$= \frac{1}{2}\int \frac{dx}{\sin^2 \frac{x}{2}}$$

$$= \frac{1}{2}\int \operatorname{cosec}^2 \frac{x}{2} dx$$

$$= -\frac{1}{2} 2\cot \frac{x}{2} + c$$

$$= -\cot \frac{x}{2} + c$$

where c is the integrating constant.

Question: 42

Mark (√) against

Solution:

Given

$$\int \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}dx$$

$$= \int \frac{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}dx$$

$$= \int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}dx$$

Let, $\cos\frac{x}{2}+\sin\frac{x}{2}=z$

$$\Rightarrow \left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx = dz$$

So,

$$\int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}dx$$

$$= \int \frac{dz}{z}$$

$$= \log z + c$$

$$= \log\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+c$$

where c is the integrating constant.

Question: 43

Mark (√) against

Solution:

Given

$$\int \sqrt{e^x}\,dx$$

$$= \int \left(e^x\right)^{\frac{1}{2}}dx$$

$$= \int e^{\frac{1}{2}x}dx$$

$$= 2e^{\frac{1}{2}x} + c$$

$$= 2\sqrt{e^x} + c$$

where c is the integrating constant.

Question: 44

Mark (√) against

Solution:

Given

$$\int \frac{\cos x dx}{1+\cos x}$$

$$= \int \frac{1+\cos x - 1}{1+\cos x} dx$$

$$= \int dx - \int \frac{dx}{1+\cos x}$$

$$= x - \tan \frac{x}{2} + c$$

[From Question no. 40] where c is the integrating constant.

Question: 45

Mark (√) against

Solution:

Given

$$\int \sec^2 x \operatorname{cosec}^2 x dx$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

Question: 46

Mark (√) against

Solution:

Given

$$\int \frac{(1-\cos 2x)}{(1+\cos 2x)} dx$$

$$= \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$$

$$= \int \tan^2 \frac{x}{2} dx$$

$$= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= 2\tan \frac{x}{2} - x + c$$

where c is the integrating constant.

Question: 47

Mark (√) against

Solution:

Given

$$\int \frac{(1+\cos 2x)}{(1-\cos 2x)} dx$$

$$= \int \frac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}} dx$$

$$= \int \cot^2 \frac{x}{2} dx$$

$$= \int \left(\operatorname{cosec}^2 \frac{x}{2} - 1 \right) dx$$

$$= -2\cot \frac{x}{2} - x + c$$

where c is the integrating constant.

Question: 48

Mark (√) against

Solution:

Given

$$\int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

Question: 49

Mark (√) against

Solution:

Given

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx$$

$$= \int \operatorname{cosec}^2 x dx - \int \sec^2 x dx$$

$$= -\tan x - \cot x + c$$

where c is the integrating constant.

Question: 50

Mark (√) against

Solution:

Given

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx$$

$$= \int \frac{-2\sin\left(\frac{2x+2\alpha}{2}\right)\sin\left(\frac{2x-2\alpha}{2}\right)}{-2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right) \times 2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 2\int 2\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\int \cos\left(\frac{x+\alpha}{2} + \frac{x-\alpha}{2}\right) + \cos\left(\frac{x+\alpha}{2} - \frac{x-\alpha}{2}\right)$$

$$= 2\int (\cos x + \cos \alpha) dx$$

$$= 2[\sin x + x\cos \alpha] + c$$

where c is the integrating constant.

Question: 51

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \cos 2x = 2\cos^2 x$; $1 - \cos 2x = 2\sin^2 x$

Therefore ,

$\Rightarrow \int \tan^{-1} \sqrt{\frac{1-\cos 2x}{1+\cos 2x}} dx = \int \tan^{-1} \sqrt{\frac{2\sin^2 x}{2\cos^2 x}} dx = \int \tan^{-1} \tan x \, dx$

$\Rightarrow \int x \, dx = \frac{x^2}{2} + c$

Question: 52

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \sin x = (\cos \frac{x}{2} + \sin \frac{x}{2})^2$

$\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$; $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

Therefore ,

$\Rightarrow \int \tan^{-1}(\sec x + \tan x) \, dx = \int \tan^{-1} \left(\frac{1+\sin x}{\cos x}\right) dx$

$\Rightarrow \int \tan^{-1} \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})^2}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} dx = \int \tan^{-1} \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})^2}{(\cos \frac{x}{2} + \sin \frac{x}{2})(\cos \frac{x}{2} - \sin \frac{x}{2})} dx$

$\Rightarrow \int \tan^{-1} \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})^1}{(\cos \frac{x}{2} - \sin \frac{x}{2})} dx = \int \tan^{-1} \frac{1+\tan \frac{x}{2}}{1-\tan \frac{x}{2}} dx$

(Multiply by $\sec \frac{x}{2}$ in numerator and denominator)

$\Rightarrow \int \tan^{-1} \frac{1+\tan \frac{x}{2}}{1-\tan \frac{x}{2}} dx = \int \tan^{-1} \frac{\tan \frac{\pi}{4} + \tan \frac{x}{2}}{\tan \frac{\pi}{4} - \tan \frac{\pi}{4} \tan \frac{x}{2}} dx = \int \tan^{-1} \tan(\frac{\pi}{4} + \frac{x}{2}) \, dx$

$\Rightarrow \int (\frac{\pi}{4} + \frac{x}{2}) dx = \frac{\pi x}{4} + \frac{x^2}{4} + c$

Question: 53

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$\Rightarrow \int \frac{1+\sin x(1+\sin x)}{1-\sin x(1+\sin x)} dx$

$\Rightarrow \int \frac{(1+\sin x)^2}{1-\sin^2 x} dx = \int \frac{1+\sin^2 x + 2\sin x}{\cos^2 x} dx$

$\Rightarrow \int \frac{1}{\cos^2 x} dx + 2\int \frac{\sin x}{\cos^2 x} dx + \int \frac{\sin^2 x}{\cos^2 x} dx$

$\Rightarrow \int \sec^2 x \, dx + 2\int \frac{\sin x}{\cos^2 x} dx + \int \tan^2 x \, dx$

$\Rightarrow \int \sec^2 x \, dx + 2\int \frac{\sin x}{\cos^2 x} dx + \int (-1 + \sec^2 x) \, dx$

$\Rightarrow 2\int \sec^2 x \, dx + 2\int \frac{\sin x}{\cos^2 x} dx - \int 1 \, dx$

Put cos x = t

Therefore -> sin x dx = - dt

$\Rightarrow 2\tan x - 2\int \frac{dt}{t^2} - x + c$

$\Rightarrow 2\tan x + 2\frac{1}{t} - x + c$

$\Rightarrow 2\tan x + 2\sec x - x + c$

Question: 54

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\, x$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{x^4+1-1}{1+x^2} dx$

$\Rightarrow \int \frac{x^4-1}{1+x^2} dx + \int \frac{1}{1+x^2} dx = \int \frac{(1+x^2)(x^2-1)}{1+x^2} dx + \int \frac{1}{1+x^2} dx$

$\Rightarrow \int x^2 - 1\, dx + \int \frac{1}{1+x^2} dx$

$\Rightarrow \frac{x^3}{3} - x + \tan^{-1} x + c$

Question: 55

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int cot\, x = log\, (sin\, x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin(x+\propto-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \frac{\sin(x+\propto)\cos(-2\propto)+\cos(x+\propto)\sin(-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \cos(2\propto)\, dx - \sin 2\propto \int \cot(x+\propto)\, dx$

$\Rightarrow \cos(2\propto)\, x - \sin 2\propto \log|\sin(x+\propto)| + c$

Question: 56

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int cot\, x = log\, (sin\, x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{x+3}+\sqrt{x+2})}{(\sqrt{x+3}-\sqrt{x+2})(\sqrt{x+3}+\sqrt{x+2})} dx$ (Rationalizing the denominator)

$\Rightarrow \int (\sqrt{x+3} + \sqrt{x+2}) dx$

$\Rightarrow \int \sqrt{x+3}\, dx + \int \sqrt{x+2}\, dx$

$\Rightarrow \frac{2(x+3)^{\frac{3}{2}}}{3} + \frac{2(x+2)^{\frac{3}{2}}}{3} + c$

Question: 57

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$

Put cos x - sin x = t

(- sin x - cos x) dx = dt

(sin x + cos x) dx = -dt

$\Rightarrow \int \frac{-dt}{t} = -\log t + c$

$\Rightarrow -\log|\cos x - \sin x| + c$

Question: 59

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^3 = t$ $3x^2 dx = dt$

$\Rightarrow \int \frac{dt}{1+t^2}$

$\Rightarrow \tan^{-1} t + c$

$\Rightarrow \tan^{-1} x^3 + c$

Question: 59

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Therefore ,

Put $x^3 = t$,$3x^2 dx = dt$

$\Rightarrow \int \frac{dt}{x \times 3x^2 \sqrt{t^2-1}} = \int \frac{dt}{3t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3} \int \frac{dt}{t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3} \sec^{-1} t + c$

$\Rightarrow \frac{1}{3}\sec^{-1}x^3 + c$

Question: 60

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}x + c$

Therefore ,

Put $x^2 + x + 1 = t$, $(2x + 1)dx = dt$

$\Rightarrow \int \sqrt{t}dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3}t^{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3}(x^2 + x + 1)^{\frac{3}{2}} + c$

Question: 61

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a + b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{2x+3}-\sqrt{2x-3})}{(\sqrt{2x+3}+\sqrt{2x-3})(\sqrt{2x+3}-\sqrt{2x-3})} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{6} dx$

$\Rightarrow \frac{1}{6}\int \sqrt{2x+3}\, dx - \frac{1}{6}\int \sqrt{2x-3}\, dx$

$\Rightarrow \frac{2(2x+3)^{\frac{3}{2}}}{3\times 6\times 2} - \frac{2(2x-3)^{\frac{3}{2}}}{3\times 6\times 2} + c$

$\Rightarrow \frac{(2x+3)^{\frac{3}{2}}}{18} - \frac{(2x-3)^{\frac{3}{2}}}{18} + c$

Question: 62

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a + b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin x}{\cos x} dx$

Put cos x = t -sin x dx = dt

$\Rightarrow \int \frac{-dt}{t}$

$\Rightarrow -\log t + c$

$\Rightarrow -\log|\cos x| + c$

Question: 63

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$

$\Rightarrow \int \frac{sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$

Put $\sec x + \tan x = t$, $(sec^2 x + \sec x \tan x)$dx = dt

$\Rightarrow \int \frac{dt}{t}$

$\Rightarrow \log t + c$

$\Rightarrow \log|\sec x + \tan x| + c$

Question: 64

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \operatorname{cosec} x \frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x - \cot x} dx$

$$\Rightarrow \int \frac{cosec^2 x - \operatorname{cosec} x \cot x}{\operatorname{cosec} x - \cot x} dx$$

Put $\operatorname{cosec} x - \cot x = t$, $(cosec^2 x - \operatorname{cosec} x \cot x)$dx = dt

$$\Rightarrow \int \frac{dt}{t}$$

$\Rightarrow \log t + c$

$\Rightarrow \log|\operatorname{cosec} x - \cot x| + c$

Question: 65

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = \tan x$

Therefore ,

$\Rightarrow \int \frac{1+\sin x}{2cos^2\frac{x}{2}} dx$

$\Rightarrow \int \frac{1}{2cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2cos^2\frac{x}{2}} dx = \frac{1}{2}\int sec^2\frac{x}{2} dx + \int \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} dx$

$\Rightarrow \frac{1}{2}\tan\frac{x}{2} \times 2 + \int tan\ \frac{x}{2} dx$

$\Rightarrow \tan\frac{x}{2} + 2\left(-\log\cos\frac{x}{2}\right) + c$

$\Rightarrow \tan\frac{x}{2} - 2\log|\cos\frac{x}{2}| + c$

Question: 66

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ; \int sec^2 x dx = tan\ x$

Therefore ,

$\Rightarrow \int \frac{\sec x \tan x}{sec^2 x+1} dx$

Put sec x = t (sec x tan x) dx = dt

$\Rightarrow \int \frac{dt}{1+t^2} = \tan^{-1} t + c$

$\Rightarrow \tan^{-1} \sec x + c$

$\Rightarrow -\tan^{-1}(\cos x) + c$

Question: 67

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ; \int sec^2 x dx = tan\ x$

Therefore ,

$\Rightarrow \int \sqrt{\frac{(1+x)^2}{(1+x)(1-x)}} dx$

$\Rightarrow \int \frac{1+x}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx$

Put $1 - x^2 = t$ -2x dx = dt

$\Rightarrow \sin^{-1} x - \frac{1}{2}\int \frac{1}{\sqrt{t}} dt + c$

$\Rightarrow \sin^{-1} x - \frac{1}{2}\frac{\sqrt{t}}{\frac{1}{2}} + c$

$\Rightarrow \sin^{-1} x - \sqrt{t} + c \quad = \quad \sin^{-1} x - \sqrt{1-x^2} + c$

Question: 68

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ; \int sec^2 x dx = tan\ x$

Therefore ,

Put $-\frac{1}{x} = t \; \frac{1}{x^2} dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c$

$\Rightarrow e^{-\frac{1}{x}} + c$

Question: 69

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \; \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^4 = t \; 4x^3 dx = dt$

$\Rightarrow \frac{1}{4} \int \frac{1}{1+t^2} dt$

$\Rightarrow \frac{1}{4} \tan^{-1} t + c$

$\Rightarrow \frac{1}{4} \tan^{-1} x^4 + c$

Question: 70

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \; \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^1 + \log x = t \; (1 + \frac{1}{x}) dx = dt \Rightarrow (\frac{x+1}{x}) dx = dt$

$\Rightarrow \int t^2 dt$

$\Rightarrow \frac{t^3}{3} + c$

$\Rightarrow \frac{(x+\log x)^3}{3} + c$

Question: 71

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \; \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $\tan^{-1} x^2 = t \; (\frac{1}{1+(x^2)^2} \times 2x) dx = dt \Rightarrow (\frac{2x}{1+x^4}) dx = dt$

$\Rightarrow \int t^1 dt$

$\Rightarrow \frac{t^2}{2} + c$

$\Rightarrow \frac{(\tan^{-1} x^2)^2}{2} + c$

Question: 72

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $2 - 3x = t\ -3dx = dt$

$\Rightarrow -\frac{1}{3} \int \frac{1}{t} dt$

$\Rightarrow -\frac{1}{3} \log t + c$

$\Rightarrow -\frac{1}{3} \log |2 - 3x| + c$

Question: 73

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $x^2 - 1 = t\ 2xdx = dt$

$\Rightarrow \int \sqrt{t} dt$

$\Rightarrow \frac{1}{2} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \Rightarrow \frac{t^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{(x^2 - 1)^{\frac{3}{2}}}{3} + c$

Question: 74

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put $5 - 3x = t\ -3dx = dt$

$\Rightarrow -\frac{1}{3} \int 3^t dt$

$\Rightarrow -\frac{1}{3} \times \frac{3^t}{\log 3} + c \Rightarrow -\frac{1}{3} \times \frac{3^{(5-3x)}}{\log 3} + c$

$\Rightarrow -\frac{3^{(5-3x)}}{3 \log 3} + c$

Question: 75

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int e^x dx = e^x + c$

Therefore ,

Put $\tan x = t\ sec^2 x dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c \Rightarrow e^{\tan x} + c$

Question: 76

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int e^x dx = e^x + c$

Therefore ,

Put $cos^2 x = t \Rightarrow 2\cos x\,(-\sin x)dx = dt \Rightarrow -\sin 2x \;\; dx = dt$

$\Rightarrow -\int e^t dt$

$\Rightarrow -e^t + c \Rightarrow -e^{cos^2 x} + c$

Question: 77

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int e^x dx = e^x + c$

Therefore ,

Put $\sin x^2 = t \Rightarrow 2x \cos x^2 \, dx = dt$

$\Rightarrow \frac{1}{2}\int t^3 dt$

$\Rightarrow \frac{1}{2}\frac{t^4}{4} + c \Rightarrow \frac{t^4}{8} + c$

$\Rightarrow \frac{(\sin x^2)^4}{8} + c$

Question: 78

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int e^x dx = e^x + c$

Therefore ,

Put $\sin e^{\sqrt{x}} = t \Rightarrow (\cos e^{\sqrt{x}}) \times (e^{\sqrt{x}}) \times (\frac{1}{2\sqrt{x}})dx = dt$

$\Rightarrow \int 2dt$

$\Rightarrow 2t + c \Rightarrow 2\sin e^{\sqrt{x}} + c$

Question: 79

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int e^x dx = e^x + c$

Therefore ,

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$\Rightarrow \frac{1}{3}\int \sin t \, dt$

$\Rightarrow -\frac{1}{3}\cos t + c \Rightarrow -\frac{1}{3}\cos x^3 + c$

Question: 80

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $xe^x = t \Rightarrow (e^x + xe^x)dx = dt \Rightarrow e^x(1+x)dx = dt$

$\Rightarrow \int \frac{dt}{cos^2 t} \Rightarrow \int sec^2 t\, dt = \tan t + c$

$\Rightarrow \tan(xe^x) + c$

Question: 81

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x^2 = t \Rightarrow 2xdx = dt$

$\Rightarrow \int \frac{1}{x\sqrt{t^2-1}} \times \frac{dt}{2x} \Rightarrow \frac{1}{2} \int \frac{1}{t\sqrt{t^2-1}} dt$

$\Rightarrow \frac{1}{2}\sec^{-1} t + c \Rightarrow \frac{1}{2}\sec^{-1} x^2 + c$

Question: 82

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x - 1 = t \Rightarrow x = t + 1 \Rightarrow dx = dt$

$\Rightarrow \int (t+1) \times \sqrt{t} dt \Rightarrow \int t^{\frac{3}{2}} dt + \int t^{\frac{1}{2}} dt$

$\Rightarrow \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \Rightarrow \frac{2t^{\frac{5}{2}}}{5} + \frac{2t^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{2(x-1)^{\frac{5}{2}}}{5} + \frac{2(x-1)^{\frac{3}{2}}}{3} + c$

Question: 83

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

$\Rightarrow \int x\sqrt{x^2 - 1} dx$

Put $x^2 - 1 = t \Rightarrow 2xdx = dt$

$\Rightarrow \int \sqrt{t}\frac{dt}{2} \Rightarrow \frac{1}{2}\int \frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt$

$\Rightarrow \frac{t^{\frac{3}{2}}}{3} + c \Rightarrow \frac{(x^2-1)^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{1}{3}(x^2-1)^{\frac{3}{2}} + c$

Question: 84

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \; \int \frac{1}{t\sqrt{t^2-1}}dt = \sec^{-1} t + c$

Therefore ,

$\Rightarrow \int \frac{1}{1+\sqrt{x}}dx$

Put $x = t^2 \Rightarrow dx = 2tdt$

$\Rightarrow \int \frac{2t}{1+t}dt \Rightarrow 2\int \frac{t}{1+t}dt \Rightarrow 2\int \frac{t+1-1}{1+t}dt \Rightarrow 2\int dt - 2\int \frac{1}{1+t}dt$

$\Rightarrow 2t - 2\log(1+t) + c \Rightarrow 2\sqrt{x} - 2\log(1+\sqrt{x}) + c$

Question: 85

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\Rightarrow \int \sqrt{e^x - 1}dx$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$\Rightarrow \int \sqrt{t}\frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t}dt$

Put $t = z^2$ dt = 2z dz

$\Rightarrow \int \frac{2z^2}{1+z^2}dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2}dz \Rightarrow 2\int \frac{1+z^2}{1+z^2}dz - 2\int \frac{1}{1+z^2}dz$

$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2}dz \Rightarrow 2z - 2\tan^{-1} z + c$

$\Rightarrow 2\sqrt{t} - 2\tan^{-1}\sqrt{t} + c \Rightarrow 2\sqrt{e^x-1} - 2\tan^{-1}\sqrt{e^x-1} + c$

Question: 86

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \; \int e^x dx = e^x + c$

Therefore ,

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x)+(\sin x + \cos x)]}{(\sin x - \cos x)}dx$

$\Rightarrow \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$

Put $(\sin x - \cos x) = t$ $(\sin x + \cos x)\, dx = dt$

$\Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{1}{t} dt \Rightarrow \frac{x}{2} + \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

Question: 87

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\,;\ \int e^x dx = e^x + c$

Therefore ,

$\Rightarrow \int \frac{1}{1-\frac{\sin x}{\cos x}} dx \Rightarrow \int \frac{\cos x}{\cos x - \sin x} dx$

We can write $\cos x = \frac{1}{2}[(\cos x - \sin x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\cos x - \sin x) + (\sin x + \cos x)]}{(\cos x - \sin x)} dx$

$\Rightarrow \frac{1}{2}\int \frac{(\cos x - \sin x)}{\cos x - \sin x} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$

Put $(\cos x - \sin x) = t$ $(\sin x + \cos x)\, dx = -dt$

$\Rightarrow \frac{x}{2} - \frac{1}{2}\int \frac{1}{t} dt \Rightarrow \frac{x}{2} - \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x - \frac{1}{2}\log|\cos x - \sin x| + c$

Question: 88

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\,;\ \int e^x dx = e^x + c$

Therefore ,

$\Rightarrow \int \frac{1}{1-\frac{\cos x}{\sin x}} dx \Rightarrow \int \frac{\sin x}{\sin x - \cos x} dx$

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]}{(\sin x - \cos x)} dx$

$\Rightarrow \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$

Put $(\sin x - \cos x) = t$ $(\sin x + \cos x)\, dx = dt$

$\Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{1}{t} dt \Rightarrow \frac{x}{2} + \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

Question: 89

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $\tan x = t \Rightarrow sec^2 x\, dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{1-t^2}} dt \Rightarrow \sin^{-1} t + c$

$\Rightarrow \sin^{-1}(\tan x) + c$

Question: 90

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

Therefore ,

$\Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2} dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+2} dx$

Put $x - \frac{1}{x} = t \Rightarrow (1 + \frac{1}{x^2})\, dx = dt$

$\Rightarrow \int \frac{1}{t^2+2} dt \Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + c$

$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}[\frac{1}{\sqrt{2}}(x - \frac{1}{x})] + c$

Question: 91

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{sin^6 x}{cos^6 x\, cos^2 x} dx \Rightarrow \int \frac{tan^6 x}{cos^2 x} dx \Rightarrow \int tan^6 x\, sec^2 x dx$

Put $\tan x = t \Rightarrow sec^2 x\, dx = dt$

$\Rightarrow \int t^6 dt \Rightarrow \frac{t^7}{7} + c$

$\Rightarrow \frac{(\tan x)^7}{7} + c$

Question: 92

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^4 x \sec x \tan x\, dx$

Put $\sec x = t \Rightarrow \sec x \tan x\, dx = dt$

$\Rightarrow \int t^4 dt \Rightarrow \frac{t^5}{5} + c$

$\Rightarrow \frac{(\sec x)^5}{5} + c$

Question: 93

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int tan^3 x\, tan^2 x dx \Rightarrow \int tan^3 x(sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int tan^3 x dx \Rightarrow \int tan^3 x sec^2 x dx - \int tan^1 x\, tan^2 x dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x\,(sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x\, sec^2 x dx + \int \tan x\, dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^3 dt - \int t^1 dt + \log|\sec x| \Rightarrow \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + c$

$\Rightarrow \frac{(\tan x)^4}{4} - \frac{(\tan x)^2}{2} + \log|\sec x| + c$

Question: 94

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \cos x\,(cos^2 x\, sin^3 x)dx \Rightarrow \int \cos x\,((1 - sin^2 x)\, sin^3 x)dx$

$\Rightarrow \int \cos x\,(sin^3 x - sin^5 x)dx \Rightarrow \int sin^3 x cos\, x dx - \int sin^5 x \cos x\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

$\Rightarrow \int t^3 dt - \int t^5 dt \Rightarrow \frac{t^4}{4} - \frac{t^6}{6} + c$

$\Rightarrow \frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} + c$

Question: 95

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x\, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x\, dx$

$\Rightarrow \int sec^2 x \tan x\, dx + \int tan^3 x sec^2 x dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^1 dt + \int t^3 dt \Rightarrow \frac{t^2}{2} + \frac{t^4}{4} + c$

$\Rightarrow \frac{(\tan x)^2}{2} + \frac{(\tan x)^4}{4} + c$

Question: 96

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x\, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x\, dx$

$\Rightarrow \int sec^2 x \tan x\, dx + \int tan^3 x sec^2 x dx$

Put $\log(\tan x) = t \Rightarrow \frac{1}{\tan x} sec^2 x dx = dt \Rightarrow \frac{1}{\sin x \cos x} dx = dt$

$\Rightarrow \int t^1 dt \Rightarrow \frac{t^2}{2} + c$

$\Rightarrow \frac{(\log|\tan x|)^2}{2} + c$

Question: 97

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sin^2(2x+1) \sin(2x+1)\, dx \Rightarrow \int (1 - cos^2(2x+1)) \sin(2x+1)\, dx$

$\Rightarrow \int \sin(2x+1)\, dx - \int cos^2(2x+1) \sin(2x+1)\, dx$

Put $\cos(2x+1) = t \Rightarrow -2\sin(2x+1)\, dx = dt$

$\Rightarrow -\int \frac{dt}{2} - (-\frac{1}{2}) \int t^2 dt \Rightarrow -\frac{1}{2}\int dt + \frac{1}{2}\int t^2 dt$

$\Rightarrow -\frac{1}{2} t + \frac{1}{2}\frac{t^3}{3} + c \Rightarrow -\frac{1}{2} t + \frac{t^3}{6} + c$

$\Rightarrow -\frac{1}{2}\cos(2x+1) + \frac{[\cos(2x+1)]^3}{6} + c$

Question: 98

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{\sqrt{\tan x}}{\sin x \times \cos x} dx \Rightarrow \int \frac{\sqrt{\tan x}}{\frac{\tan x}{\sec x} \times \frac{1}{\sec x}} dx \Rightarrow \int \frac{sec^2 x}{\sqrt{\tan x}} dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int \frac{dt}{\sqrt{t}} \Rightarrow \frac{\sqrt{t}}{\frac{1}{2}} + c \Rightarrow 2\sqrt{t} + c$

$\Rightarrow 2\sqrt{\tan x} + c$

Question: 99

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{\cos x+\sin x}{\cos^2 x+\sin^2 x-\sin 2x} dx \Rightarrow \int \frac{\cos x+\sin x}{(\cos x-\sin x)^2} dx$

Put $\cos x - \sin x = t \Rightarrow (\cos x + \sin x) dx = -dt$

$\Rightarrow \int \frac{-dt}{t^2} \Rightarrow \frac{1}{t} + c \Rightarrow \frac{1}{\cos x-\sin x} + c$

Question: 100

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\Rightarrow \int \sqrt{e^x - 1} dx$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$\Rightarrow \int \sqrt{t} \frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t} dt$

Put $t = z^2$ dt = 2z dz

$\Rightarrow \int \frac{2z^2}{1+z^2} dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2} dz \Rightarrow 2\int \frac{1+z^2}{1+z^2} dz - 2\int \frac{1}{1+z^2} dz$

$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2} dz \Rightarrow 2z - 2\tan^{-1} z + c$

$\Rightarrow 2\sqrt{t} - 2\tan^{-1}\sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

Question: 101

Mark (√) against

Solution:

Let $I = \int \frac{dx}{\sqrt{\sin^3 x \cos x}}$

Now multiplying and dividing by $\cos^2 x$, we get,

$$I = \int \frac{dx}{\sqrt{\sin^3 x \times \cos x}} \times \frac{1}{\cos^2 x} \times \cos^2 x$$

$$I = \int \frac{(\sec^2 x)}{\sqrt{\frac{\sin^3 x}{\cos^3 x}}} dx$$

$$I = \int \frac{\sec^2 x}{\sqrt{\tan^3 x}} dx$$

Let tan x = t

Differentiating both sides, we get,

$\sec^2 x$ dx = dt

Therefore,

$$I = \int \frac{dt}{t^{3/2}}$$

Integrating, we get,

$$I = \frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$$

$$I = \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$I = -\frac{2}{\sqrt{t}} + C$$

$$I = -\frac{2}{\sqrt{\tan x}} + C$$

Exercise : 13B

Question: 1

Evaluate the foll

Solution:

i) $\int sin^2 x dx$

$\Rightarrow \int sin^2 x dx$

Now, we know that 1-cos2x=2sin^{2}x

So, applying this identity in the given integral, we get,

$$\int sin^2 x dx = \int \frac{(1 - cos2x)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx - \int cos2x dx)$

$$\Rightarrow \frac{x}{2} - \frac{sin2x}{2 \times 2} + c$$

$$\Rightarrow \frac{x}{2} - \frac{sin2x}{4} + c$$

Ans: $\int sin^2 x dx = \frac{x}{2} - \frac{sin2x}{4} + c$

ii) $\int cos^2 x \, dx$

$\Rightarrow \int cos^2 x \, dx$

Now, we know that 1+cos2x=2cos^{2}x

So, applying this identity in the given integral, we get,

$$\int cos^2 x \, dx = \int \frac{(1 + cos2x)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx + \int cos2x dx)$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2 \times 2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{4} + c$$

Ans: $\int cos^2 x \, dx = \frac{x}{2} + \frac{sin2x}{4} + c$

Question: 2

Evaluate the foll

Solution:

(i) $\int \cos^2 (x / 2) dx$

$\Rightarrow \int cos^2 \left(\frac{x}{2}\right) dx$

Now, we know that 1+cosx=$2\cos^2$ (x/2)

So, applying this identity in the given integral, we get,

$$\int cos^2(\frac{x}{2})\,dx = \int \frac{(1+cosx)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx + \int cosxdx)$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2} + c$$

Ans: $\frac{x}{2} + \frac{sin2x}{2} + c$

ii) $\int \cot^2 (x/2)dx$

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx$

Now, we know that $cosec^2$x-cot^2 x=1

So, applying this identity in the given integral we get,

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx = \int (cosec^2\left(\frac{x}{2}\right) - 1)\,dx$

$\Rightarrow \int (cosec^2\left(\frac{x}{2}\right) - 1)\,dx = \int cosec^2\left(\frac{x}{2}\right)dx - \int 1dx$

$\Rightarrow \int cosec^2\left(\frac{x}{2}\right)dx - \int 1dx = \frac{-cotx}{\frac{1}{2}} - x + c$

⇒-2cotx-x+c

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx$ =-2cotx-x+c

Ans: -2cotx-x+c

Question: 3

Evaluate the foll

Solution:

i)$\int sin^2nxdx$

$\Rightarrow \int sin^2nxdx$

Now, we know that 1-cos2nx=$2\sin^2$nx

So, applying this identity in the given integral, we get,

$$\int sin^2nxdx = \int \frac{(1-cos2nx)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx - \int cos2nxdx)$

$$\Rightarrow \frac{x}{2} - \frac{sin2nx}{2n \times 2} + c$$

$$\Rightarrow \frac{x}{2} - \frac{sin2x}{4n} + c$$

Ans: $\int sin^2nxdx = \frac{x}{2} - \frac{sin2nx}{4n} + c$

(ii) $\int \sin^5 x\,dx$

We know that $1-\cos^2x=\sin^2x$

$\Rightarrow \int sin^5 x dx = \int (1-cos^2 x)^2 sinx dx$

⇒Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int (1-cos^2 x)^2 sinx dx = -\int (1-t^2)^2 dt$

$\Rightarrow -\int (1-t^2)^2 dt = -\int (1+t^4-2t^2) dt$

$\Rightarrow -\int dt + \int 2t^2 dt - \int t^4 dt$

$\Rightarrow -t + \frac{2t^3}{3} - \frac{t^5}{5} + c$

Resubstituting the value of t=cosx we get,

$\Rightarrow -cosx + \frac{2cos^3 x}{3} - \frac{cos^5 x}{5} + c$

Ans: $-cosx + \frac{2cos^3 x}{3} - \frac{cos^5 x}{5} + c$

Question: 4

Evaluate the foll

Solution:

Substitute 3x+5=u

⇒3dx=du

⇒dx=du/3

$\Rightarrow \int cos^3(3x+5) dx = \frac{1}{3}\int cos^3(u) du$

Now We know that $1-\cos^2x=\sin^2x$,

$\Rightarrow \frac{1}{3}\int cos^3(u) du = \frac{1}{3}\int (1-sin^2(u)) cosu\, du$

⇒Substitute sinu=t

⇒cosu du=dt

$\Rightarrow \frac{1}{3}\int (1-sin^2(u)) cosu\, du = \frac{1}{3}\int (1-t^2)\, dt$

$\Rightarrow \frac{1}{3}\int dt - \frac{1}{3}\int t^2 dt$

$\Rightarrow \frac{t}{3} - \frac{t^3}{3\times 3} + c$

$\Rightarrow \frac{t}{3} - \frac{t^3}{9} + c$

Resubstituting the value of t=sinu and u=3x+5 we get,

$\Rightarrow \frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

Ans: $\frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

Question: 5

Evaluate the foll

Solution:

$\Rightarrow -\int sin^7(2x-3) dx$

Substitute 2x-3=u

⇒ 2dx=du

⇒dx=du/2

⇒$-\left(\frac{1}{2}\right)\int sin^7(u)du$

⇒ We know that 1-cos^{2}x=sin^{2}x

⇒$-\left(\frac{1}{2}\right)\int\left(1-cos^2(u)\right)^3 sinu\ du$

⇒Put cosu=t

⇒-sinxdu=dt

⇒$\left(\frac{1}{2}\right)\int(1-t^2)^3dt$

⇒$\left(\frac{1}{2}\right)\int(1-t^6-3t^2+3t^4)dt$

⇒$\left(\frac{1}{2}\right)[\int dt-\int t^6dt-\int 3t^2dt+\int 3t^4dt]$

⇒$\left(\frac{1}{2}\right)\left[t-\frac{t^7}{7}-\frac{3t^3}{3}+\frac{3t^5}{5}\right]+c$

⇒$\left(\frac{1}{2}\right)\left[t-\frac{t^7}{7}-t^3+\frac{3t^5}{5}\right]+c$

Resubstituting the value of t=cosu and u=2x-3 we get

⇒$\left(\frac{1}{2}\right)\left[\cos(2x-3)-\frac{cos^7(2x-3)}{7}-cos^3(2x-3)+\frac{3cos^5(2x-3)}{5}\right]+c$

⇒$\frac{\cos(2x-3)}{2}-\frac{cos^7(2x-3)}{14}-\frac{cos^3(2x-3)}{2}+\frac{3cos^5(2x-3)}{10}+c$

Now as we know cos(-x)=cosx

⇒$\frac{\cos(2x-3)}{2}-\frac{cos^7(2x-3)}{14}-\frac{cos^3(2x-3)}{2}+\frac{3cos^5(2x-3)}{10}+c$

=$\frac{\cos(3-2x)}{2}-\frac{cos^7(3-2x)}{14}-\frac{cos^3(3-2x)}{2}+\frac{3cos^5(3-2x)}{10}+c$

Ans: $\frac{\cos(3-2x)}{2}-\frac{cos^7(3-2x)}{14}-\frac{cos^3(3-2x)}{2}+\frac{3cos^5(3-2x)}{10}+c$

Question: 6

Evaluate the foll

Solution:

(i) $\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$

⇒$\int\frac{1-cos2x}{1+cos2x}dx$

1-cos2x=2sin^{2}x and 1+cos2x=2cos^{2}x

⇒$\int\frac{1-cos2x}{1+cos2x}dx=\int\frac{2sin^2x}{2cos^2x}dx$

⇒$\int tan^2x\ dx$

Now sec^{2}x-1=tan^{2}x

⇒$\int(sec^2x-1)dx$

⇒$\int sec^2xdx-\int dx$

⇒tanx-x+c

Ans: tanx-x+c

(ii) $\left(\frac{1+\cos 2x}{1-\cos 2x}\right)dx$

$\Rightarrow\int \frac{1+cos2x}{1-cos2x}dx$

1-cos2x=2sin^{2}x and 1+cos2x=2cos^{2}x

$\Rightarrow\int \frac{1+cos2x}{1-cos2x}dx = \int \frac{2cos^2x}{2sin^2x}dx$

$\Rightarrow\int cot^2x\,dx$

Now cosec^{2}x-1=cot^{2}x

$\Rightarrow\int (cosec^2x - 1)dx$

$\Rightarrow\int cosec^2x dx - \int dx$

⇒-cotx-x+c

Ans: -cotx-x+c

Question: 7

Evaluate the foll

Solution:

i) $\int \frac{1-cosx}{1+cosx}dx$

$\Rightarrow\int \frac{1-cosx}{1+cosx}dx$

1-cosx=2sin^{2}x/2 and 1+cosx=2cos^{2}x/2

$\Rightarrow\int \frac{1-cosx}{1+cosx}dx = \int \frac{2sin^2(\frac{x}{2})}{2cos^2(\frac{x}{2})}dx$

$\Rightarrow\int tan^2(\frac{x}{2})dx$

Now sec^2(x/2)-1=tan2(x/2)

$\Rightarrow\int \left(sec^2(\frac{x}{2}) - 1\right)dx$

$\Rightarrow\int sec^2(\frac{x}{2})dx - \int dx$

⇒2tan(x/2)-x+c

Ans: 2tan(x/2)-x+c

(ii) $\int \frac{1+cosx}{1-cosx}dx$

$\Rightarrow\int \frac{1+cosx}{1-cosx}dx$

1-cosx=2sin^{2}x/2 and 1+cosx=2cos^{2}x/2

$\Rightarrow\int \frac{1+cosx}{1-cosx}dx = \int \frac{2cos^2(\frac{x}{2})}{2sin^2(\frac{x}{2})}dx$

$\Rightarrow\int cot^2(\frac{x}{2})dx$

Now cosec^2 (x/2)-1=cot^2 (x/2)

$\Rightarrow\int \left(cosec^2(\frac{x}{2}) - 1\right)dx$

$\Rightarrow\int cosec^2(\frac{x}{2})dx - \int dx$

⇒-2cot(x/2)-x+c

Ans: ⇒-2cot(x/2)-x+c

Question: 8

Evaluate the foll

Solution:

$\Rightarrow\int sin3x\ cos4x\ dx$

Applying the formula: sinx×cosy=1/2(sin(x+y)-sin(y-x))

$\Rightarrow\frac{1}{2}\int(sin7x - sinx)dx$

$\Rightarrow\frac{1}{2}\int sin7x\ dx - \frac{1}{2}\int sinx\ dx$

$\Rightarrow\frac{-cos7x}{14} + \frac{cosx}{2} + c$

Ans: $\frac{-cos7x}{14} + \frac{cosx}{2} + c$

Question: 9

Evaluate the foll

Solution:

$\Rightarrow\int cos4x\ cos3x\ dx$

Applying the formula: cosx×cosy=1/2(cos(x+y)+cos(x-y))

$\Rightarrow\frac{1}{2}\int(cos7x + cosx)dx$

$\Rightarrow\frac{1}{2}\int cos7x\ dx + \frac{1}{2}\int cosx\ dx$

$\Rightarrow\frac{sin7x}{14} + \frac{sinx}{2} + c$

Ans: $\frac{sin7x}{14} + \frac{sinx}{2} + c$

Question: 10

Evaluate the foll

Solution:

$\Rightarrow\int sin4x\ sin8x\ dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

$\Rightarrow\frac{1}{2}\int(cos4x - cos12x)dx$

$\Rightarrow\frac{1}{2}\int cos4x\ dx - \frac{1}{2}\int cos12x\ dx$

$\Rightarrow\frac{sin4x}{8} - \frac{sin12x}{24} + c$

Ans: $\frac{sin4x}{8} - \frac{sin12x}{24} + c$

Question: 11

Evaluate the foll

Solution:

$\Rightarrow\int sin6x\ cosx\ dx$

Applying the formula: sinx×cosy=1/2(sin(y+x)-sin(y-x))

$\Rightarrow\frac{1}{2}\int(sin7x - \sin(-5x))dx$

$\Rightarrow \frac{1}{2}\int sin7x\ dx + \frac{1}{2}\int sin5x\ dx$

$\Rightarrow \frac{-cos7x}{14} - \frac{cosx}{10} + c$

Ans: $\frac{-cos7x}{14} - \frac{cosx}{10} + c$

Question: 12

Evaluate the foll

Solution:

we know that $1+cos2x=2cos^2x$

So, applying this identity in the given integral we get,

$\Rightarrow \int sinx\sqrt{1+cos2x}dx$

$\Rightarrow \int sinx\sqrt{(2cos^2x)}dx$

$\Rightarrow \sqrt{2}\int sinxcosxdx$

Let sinx =t

$\Rightarrow$ cosx dx=dt

$\Rightarrow \sqrt{2}\int tdt$

$\Rightarrow \sqrt{2}\frac{t^2}{2} + c = \frac{t^2}{\sqrt{2}} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{sin^2x}{\sqrt{2}} + c$

Ans: $\frac{sin^2x}{\sqrt{2}} + c$

Question: 13

Evaluate the foll

Solution:

$\Rightarrow \int cos^2xcos^2xdx$

$\Rightarrow \int (\frac{1+cos2x}{2})(\frac{1+cos2x}{2})dx \ldots (\frac{1+cos2x}{2} = cos^2x)$

$\Rightarrow \frac{1}{4}\int (1+cos2x)^2dx$

$\Rightarrow \frac{1}{4}\int (1+cos^22x+2cos2x)dx$

$\Rightarrow \frac{1}{4}[\int 1dx\ + \int cos^2 2xdx + \int 2cos2x\ dx$

$\Rightarrow \frac{1}{4}[x + \int \frac{(1+cos4x)dx}{2} + 2\frac{sin2x}{2}]$ …($1+cos4x=2cos^2x$)

$\Rightarrow \frac{1}{4}[x + \frac{1}{2}(\int dx + \int cos4xdx) + sin2x] + c$

$\Rightarrow [\frac{x}{4} + \frac{1}{2}\times\frac{1}{4}\left(\int dx + \int cos4xdx\right) + \frac{sin2x}{4}\right] + c$

$\Rightarrow [\frac{x}{4} + \left(\frac{x}{8} + \frac{sin4x}{32}\right) + \frac{sin2x}{4}\Big] + c$

$\Rightarrow \frac{3x}{8} + \frac{sin4x}{32} + \frac{sin2x}{4} + c$

Ans: $\frac{3x}{8} + \frac{sin4x}{32} + \frac{sin2x}{4} + c$

Question: 14

Evaluate the foll

Solution:

$\Rightarrow\int cos2x\, cos4x\, \cos 6x dx$

$\Rightarrow\frac{1}{2}\int(cos6x+cos2x)cos6xdx$

$\Rightarrow\frac{1}{2}\int cos^2 6xdx+\frac{1}{2}\int cos2xcos6xdx$

$$\Rightarrow \frac{1}{2}\int cos^2 6xdx+\frac{1}{4}\int(cos8x+cos4x)dx$$

$$\Rightarrow \frac{1}{2}\int cos^2 6xdx+\frac{1}{4}\int cos8x\, dx+\frac{1}{4}\int cos4x\, dx$$

$$\Rightarrow \frac{1}{2}\int\frac{(1+cos12x)dx}{2}+\frac{1}{4}\frac{sin8x}{8}+\frac{1}{4}\frac{sin4x}{4}+c$$

$$\Rightarrow \frac{1}{4}\left(x+\frac{sin12x}{12}\right)+\frac{sin8x}{32}+\frac{sin4x}{16}+c$$

$$\Rightarrow \frac{x}{4}+\frac{sin12x}{48}+\frac{sin8x}{32}+\frac{sin4x}{16}+c$$

Ans: $\frac{x}{4}+\frac{sin12x}{48}+\frac{sin8x}{32}+\frac{sin4x}{16}+c$

Question: 15

Evaluate the foll

Solution:

Let sinx =t

⇒ cosx dx =dt

$\Rightarrow\int sin^3 x\, cosx\, dx = \int t^3 dt$

$\Rightarrow\frac{t^4}{4}+c$

Resubstituting the value of t=sinx we get

$$\Rightarrow \frac{sin^4 x}{4}+c$$

Ans: $\frac{sin^4 x}{4}+c$

Question: 16

Evaluate the foll

Solution:

$\Rightarrow\int sec^4 dx=\int sec^2 xsec^2 xdx$

$\Rightarrow\int sec^2 x(1+tan^2 x)dx$

⇒Put tanx=t ⇒sec^{2}dx=dt

$\Rightarrow\int(1+t^2)dt$

$\Rightarrow t+\frac{t^3}{3}+c$

Resubstituting the value of t=tanx we get

$\Rightarrow tanx+\frac{tan^3 x}{3}+c$

Ans: $tanx + \frac{tan^3 x}{3} + c$

Question: 17

Evaluate the foll

Solution:

$\Rightarrow \int cos^3 x sin^4 x\, dx$

$\Rightarrow \int cosx\, sin^4 x cos^2 x dx$

$\Rightarrow \int cosx\, sin^4 x(1 - sin^2 x)dx$

Put sinx=t

⇒cosxdx=dt

$\Rightarrow \int t^4(1 - t^2)dt$

$\Rightarrow \int t^4 dt - \int t^6\, dt$

$\Rightarrow \frac{t^5}{5} - \frac{t^7}{7} + c$

Resubstituting the value of t=sinx we get,

$\Rightarrow \frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

Ans: $\frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

Question: 18

Evaluate the foll

Solution:

$\Rightarrow \int cos^4 x sin^3 x\, dx$

$\Rightarrow \int sinx\, sin^2 x cos^4 x dx$

$\Rightarrow \int sinx\, cos^4 x(1 - cos^2 x)dx$

Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int t^4(t^2 - 1)dt$

$\Rightarrow \int t^6 dt - \int t^4\, dt$

$\Rightarrow \frac{t^7}{7} - \frac{t^5}{5} + c$

Resubstituting the value of t=sinx we get,

$\Rightarrow \frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

Ans: $\frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

Question: 19

Evaluate the foll

Solution:

$\Rightarrow \int cos^3 x sin^{\frac{2}{3}} x\, dx$

$\Rightarrow \int cosx\, cos^2 x sin^{\frac{2}{3}} x dx$

$\Rightarrow \int cosx\,(1-sin^2x)sin^{\frac{2}{3}}xdx$

Put sinx=t

⇒cosxdx=dt

$\Rightarrow \int t^{\frac{2}{3}}(1-t^2)dt$

$\Rightarrow \int t^{\frac{2}{3}}dt - \int t^{\frac{8}{3}}dt$

$$\Rightarrow \frac{t^{\frac{5}{3}}}{\frac{5}{3}} - \frac{t^{\frac{11}{3}}}{\frac{11}{3}} + c$$

Resubstituting the value of t=sinx we get

$$\Rightarrow \frac{3sin^{\frac{5}{3}}x}{5} - \frac{3sin^{\frac{11}{3}}x}{11} + c$$

Ans: $\frac{3sin^{\frac{5}{3}}x}{5} - \frac{3sin^{\frac{11}{3}}x}{11} + c$

Question: 20

Evaluate the foll

Solution:

$\Rightarrow \int sin^3xcos^{\frac{3}{5}}x\,dx$

$\Rightarrow \int sinx\,sin^2xcos^{\frac{3}{5}}xdx$

$\Rightarrow \int sinx\,(1-cos^2x)cos^{\frac{3}{5}}xdx$

Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int t^{\frac{3}{5}}(t^2-1)dt$

$\Rightarrow \int t^{\frac{13}{5}}dt - \int t^{\frac{3}{5}}dt$

$$\Rightarrow \frac{t^{\frac{18}{5}}}{\frac{18}{5}} - \frac{t^{\frac{8}{5}}}{\frac{8}{5}} + c$$

Resubstituting the value of t=cosx we get

$$\Rightarrow \frac{5cos^{\frac{18}{5}}x}{18} - \frac{5cos^{\frac{8}{5}}x}{8} + c$$

Ans: $\frac{5cos^{\frac{18}{5}}x}{18} - \frac{5cos^{\frac{8}{5}}x}{8} + c$

Question: 21

Evaluate the foll

Solution:

$$\Rightarrow \int cosec^4 2xdx$$

$$\Rightarrow \int cosec^2 2xcosec^2 2xdx$$

$\Rightarrow \int cosec^2 2x\,(1 + cot^2 2x)dx$

$\Rightarrow$cot2x=t $\Rightarrow$-2cosec^2 2xdx=dt

$\Rightarrow -1/2 \int (1 + t^2)dt$

$\Rightarrow -1/2 \int dt - 1/2 \int t^2 dt$

$\Rightarrow -(\frac{1}{2})\mathrm{t} - \frac{t^3}{6} + c$

Resubstituting the value of t=cotx we get

$\Rightarrow -\frac{\mathrm{cotx}}{2} - \frac{cot^3 x}{6} + c$

Ans: $-\frac{\mathrm{cotx}}{2} - \frac{cot^3 x}{6} + c$

Question: 22

Evaluate the foll

Solution:

$\Rightarrow \int \frac{cos2x}{cosx} dx = \int \frac{2cos^2 x - 1}{cosx} dx$

$\Rightarrow \int \frac{2cos^2 x}{cosx} dx - \int \frac{1}{cosx} dx$

$\Rightarrow \int 2cosxdx - \int secxdx$

$\Rightarrow 2\mathrm{sinx} - \log|\mathrm{secx} + \mathrm{tanx}| + \mathrm{c}$

Ans: 2sinx-log|secx+tanx|+c

Question: 23

Evaluate the foll

Solution:

$\Rightarrow \int \frac{cosx}{\cos(x+\alpha)} dx = \int \frac{\cos\big((x+\alpha)-\alpha\big)}{\cos(x+\alpha)} dx$

$\Rightarrow \int \frac{\cos(x+\alpha)cos\alpha + \sin(x+\alpha)\, sin\alpha}{\cos(x+\alpha)} dx$

$\Rightarrow \int cos\alpha dx + \int \tan(x+\alpha)\, sin\alpha\, dx$

Now α is a constant

$\Rightarrow \mathrm{x}\cos\alpha - \sin\alpha\log|\cos(\mathrm{x}+\alpha)| + \mathrm{c}$

Ans:xcos α-sin αlog|cos(x+ α)|+c

Question: 24

Evaluate the foll

Solution:

$\Rightarrow \int sin2x\, cos^3 xdx$

$\Rightarrow \int 2sinxcosxcos^3 xdx$

$\Rightarrow \int 2sinx\,cos^4 x dx$

Now put cosx=t

⇒-sinxdx=dt

$\Rightarrow -2\int t^4 dt$

$\Rightarrow -2\times\frac{t^5}{5}+c$

Resubstituting the value of t= cosx we get,

$\Rightarrow \frac{-2cos^5 x}{5}+c$

Ans: $\frac{-2cos^5 x}{5}+c$

Question: 25

Evaluate the foll

Solution:

$\Rightarrow \int \frac{cos^9 x}{sinx}dx$

$\Rightarrow \int \frac{cos^9 x}{sin^2 x}sinxdx$

$\Rightarrow \int \frac{cos^9 x}{1-cos^2 x}sinxdx$

Put cosx =t

⇒ -sinxdx=dt

$\Rightarrow \int \frac{t^9}{t^2-1}dt$

Now put t^2-1=a

⇒2tdt=da

And $t^8=(a+1)^4$

$\Rightarrow \frac{1}{2}\int \frac{(a+1)^4}{a}da$

$\Rightarrow \frac{1}{2}\int (a^3+4a^2+6a+\frac{1}{a}+4)da$

$\Rightarrow \frac{1}{2}\left(\frac{a^4}{4}+\frac{4a^3}{3}+\frac{6a^2}{2}+lna+4a\right)+c$

$\Rightarrow \left(\frac{a^4}{8}+\frac{2a^3}{3}+\frac{3a^2}{2}+\frac{lna}{2}+2a\right)+c$

Resubstituting the value of $a=t^2-1$ and t=cosx ⇒$a=cos^2x-1=-sin^2x$ we get

$\Rightarrow \left(\frac{(-sin^2 x)^4}{8}+\frac{2(-sin^2 x)^3}{3}+\frac{3(-sin^2 x)^2}{2}+\frac{\ln|(-sin^2 x)|}{2}+2(-sin^2 x)\right)+c$

$\Rightarrow \left(\frac{sin^8 x}{8}-\frac{2sin^6 x}{3}+\frac{3sin^4 x}{2}+\frac{2\ln|(-sinx)|}{2}-2sin^2 x\right)+c$

$\Rightarrow\left(\frac{sin^8x}{8}-\frac{2sin^6x}{3}+\frac{3sin^4x}{2}+\ln(sinx)-2sin^2x\right)+c$

Ans: $\left(\frac{sin^8x}{8}-\frac{2sin^6x}{3}+\frac{3sin^4x}{2}+\ln(sinx)-2sin^2x\right)+c$

Question: 26

Evaluate the foll

Solution:

$\Rightarrow\int cos^2 2xcos^2 2xdx$

$\Rightarrow\int (\frac{1+cos4x}{2})(\frac{1+cos4x}{2})dx$ ...$(\frac{1+cos4x}{2}=cos^2 2x)$

$\Rightarrow\frac{1}{4}\int(1+cos4x)^2dx$

$\Rightarrow\frac{1}{4}\int(1+cos^2 4x+2cos4x)dx$

$\Rightarrow\frac{1}{4}[\int 1dx\ +\int cos^2 4xdx+\int 2cos4x\, dx$

$\Rightarrow\frac{1}{4}[x+\int\frac{(1+cos8x)dx}{2}+2\frac{sin4x}{4}]$ ...(1+cos8x=2cos^2 4x)

$\Rightarrow\frac{1}{4}[x+\frac{1}{2}\left(\int dx+\int cos8xdx\right)+(\frac{sin4x}{2})\Big]+c$

$\Rightarrow[\frac{x}{4}+\frac{1}{2}\times\frac{1}{4}\left(\int dx+\int cos8xdx\right)+\frac{sin4x}{8}\Big]+c$

$\Rightarrow[\frac{x}{4}+\left(\frac{x}{8}+\frac{sin8x}{64}\right)+\frac{sin4x}{8}\Big]+c$

$\Rightarrow\frac{3x}{8}+\frac{sin8x}{64}+\frac{sin4x}{8}+c$

Ans: $\frac{3x}{8}+\frac{sin8x}{64}+\frac{sin4x}{8}+c$

Question: 27

Evaluate the foll

Solution:

Doing tangent half angle substitution we get,

$$\Rightarrow\int\frac{sin^2x}{(1+cos^2x)}dx=\int\frac{\left(\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)^2}{[1+(\frac{1-tan^2\frac{x}{2}}{1-tan^2\frac{x}{2}})]^2}$$

Substitute u=tan(x/2)

$\Rightarrow$2du=sec^2(x/2)dx

$\Rightarrow$dx=$\frac{2du}{u^2+1}$

$\Rightarrow 2\int\frac{u^2}{1+u^2}du$

$\Rightarrow 2\int\frac{1+u^2}{1+u^2}du-2\int\frac{1}{1+u^2}du$

$\Rightarrow 2\int du-\tan^{-1}u+c$

$\Rightarrow 2u-\tan^{-1}u+c$

Resubstituting the values we get,

$\Rightarrow 2\tan\frac{x}{2} - \tan^{-1}\tan\frac{x}{2} + c$

$\Rightarrow 2\tan\frac{x}{2} - \frac{x}{2} + c$

Ans: $2\tan\frac{x}{2} - \frac{x}{2} + c$

Question: 28

Evaluate the foll

Solution:

$$\int \frac{dx}{3cosx + 4sinx} = \int \frac{dx}{3\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) + 4\left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3 + 8tan\frac{x}{2} - 3tan^2\frac{x}{2}}$$

Let $\tan\frac{x}{2}$=t

$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$

$$\Rightarrow \int \frac{2dt}{3 + 8t - 3t^2} = \frac{2}{3}\int \frac{dt}{1 + \frac{8}{3}t - t^2} = \frac{2}{3}\int \frac{dt}{1 - \left(t - \frac{4}{3}\right)^2 + \frac{16}{9}}$$

$$\Rightarrow \frac{2}{3}\int \frac{dt}{\frac{25}{9} - \left(t - \frac{4}{3}\right)^2} = \frac{2}{3}\int \frac{dt}{\left(\frac{5}{3}\right)^2 - \left(t - \frac{4}{3}\right)^2}$$

$$\Rightarrow \frac{2}{3} \times \frac{1}{2 \times \frac{5}{3}} \ln\left|\frac{\frac{5}{3} + \left(t - \frac{4}{3}\right)}{\frac{5}{3} - \left(t - \frac{4}{3}\right)}\right| + c = \frac{1}{5}\ln\left|\frac{1 + 3t}{9 - 3t}\right| + c$$

Resubstituting the value of t we get

$$\Rightarrow \frac{1}{5}\ln\left|\frac{1 + 3tan\frac{x}{2}}{9 - 3tan\frac{x}{2}}\right| + c$$

Ans: $\frac{1}{5}\ln\left|\frac{1+3tan\frac{x}{2}}{9-3tan\frac{x}{2}}\right| + c$

Question: 29

Evaluate the foll

Solution:

$$\int \frac{dx}{(acosx + bsinx)^2}$$

Taking bcosx common from the denominator we get,

$$\int \frac{dx}{b^2cos^2x(\frac{a}{b} + tanx)^2}$$

$$\Rightarrow \frac{1}{b^2}\int \frac{sec^2xdx}{(\frac{a}{b} + tanx)^2}$$

Let (a/b)+tanx=t

$\therefore sec^2xdx = dt$

$$\Rightarrow \frac{1}{b^2}\int \frac{dt}{t^2} = \frac{-1}{b^2} \times \frac{1}{t} = \frac{-1}{b^2 t} + c$$

Resubstituting the value of t = (a/b)+tanx we get

$$\Rightarrow \frac{-1}{b^2(\frac{a}{b} + tanx)} + c = \frac{-1}{ab + b^2 tanx} + c$$

Ans: $\frac{-1}{ab+b^2 tanx} + c$

Question: 30

Evaluate the foll

Solution:

$$\int \frac{dx}{cosx - sinx} = \int \frac{dx}{\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) - \left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{1 - 2tan\frac{x}{2} - tan^2\frac{x}{2}}$$

Let tan$\frac{x}{2}$=t

$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$

$$\Rightarrow \int \frac{2dt}{1 - 2t - t^2} = -2\int \frac{dt}{t^2 + 2t - 1} = -2\int \frac{dt}{(t+1)^2 - 2}$$
$$= -2\int \frac{dt}{(t+1)^2 - (\sqrt{2})^2}$$

$\Rightarrow -2 \times \frac{1}{2\times\sqrt{2}}\ln\left|\frac{t+1-\sqrt{2}}{t+1+\sqrt{2}}\right| + c$ resubstituting the value of t we get

$$\Rightarrow \frac{-1}{\sqrt{2}}\ln\left|\frac{tan\frac{x}{2} + 1 - \sqrt{2}}{tan\frac{x}{2} + 1 + \sqrt{2}}\right| + c = \frac{-1}{\sqrt{2}}\ln\left|\tan\left(\frac{\pi}{8} - \frac{x}{2}\right)\right| + c$$

Ans: $\frac{-1}{\sqrt{2}}\ln\left|\tan\left(\frac{\pi}{8} - \frac{x}{2}\right)\right| + c$

Question: 31

Evaluate the foll

Solution:

$$\int (2tanx - 3cotx)^2 dx$$

$$\Rightarrow \int (4tan^2x + 9cot^2x - 12tanxcotx)dx$$

$$\Rightarrow \int (4(sec^2x - 1) + 9(cosec^2x - 1) - 12)dx$$

$$\Rightarrow \int 4sec^2x\,dx + \int 9cosec^2x dx - \int 25dx$$

$\Rightarrow$ 4tanx − 9cotx − 25x + c

Ans: 4tanx-9cotx-25x+c

Question: 32

Evaluate the foll

Solution:

$\Rightarrow \int sinx\ sin2x sin3x\ dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

$\Rightarrow \frac{1}{2}\int (cos2x - cos4x)sin2x dx$

$\Rightarrow \frac{1}{2}\int sin2x cos2x\ dx - \frac{1}{2}\int sin2x cos4x\ dx$

$\Rightarrow \frac{1}{4}\int sin4x\ dx - \frac{1}{4}\int (sin6x - sin2x)dx$

$\Rightarrow \frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

Ans: $\frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

Question: 33

Evaluate the foll

Solution:

$$\Rightarrow \int \frac{1 - cotx}{1 + cotx} dx = \int \frac{1 - \frac{cosx}{sinx}}{1 + \frac{cosx}{sinx}} dx$$

$$\Rightarrow \int \frac{sinx - cosx}{sinx + cosx} dx = - \int \frac{cosx - sinx}{sinx + cosx} dx$$

$$\Rightarrow - \int \frac{d(sinx + cosx)}{sinx + cosx}$$

$\Rightarrow -\log|\sin x + \cos x| + c$

Ans: -log(sinx+cosx)+c

Question: 34

Evaluate the foll

Solution:

$$\int \frac{dx}{cosx + 2sinx + 3} = \int \frac{dx}{\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) + 2\left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) + 3}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3 + 1 + 3tan^2\frac{x}{2} + 4tan\frac{x}{2} - tan^2\frac{x}{2}}$$

Let $\tan\frac{x}{2}$=t

$\therefore \frac{1}{2} sec^2\frac{x}{2} dx = dt$

$$\Rightarrow \int \frac{2dt}{4 + 4t + 2t^2} = \int \frac{dt}{2 + 2t + t^2} = \frac{2}{3}\int \frac{dt}{(t+1)^2 + 2 - 1}$$

$$\Rightarrow \int \frac{dt}{(t+1)^2 + 1} = \int \frac{dt}{(1)^2 + (t+1)^2}$$

$\Rightarrow \tan^{-1}(t + 1) + c$

Resubstituting the value of t we get

$$\Rightarrow \tan^{-1}(tan\frac{x}{2} + 1) + c$$

Ans: $\tan^{-1}(tan\frac{x}{2} + 1) + c$

Exercise : 13C

Question: 1

Evaluate the foll

Solution:

Using BY PART METHOD.

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x.e^x dx = x\int e^x - \int\frac{dx}{dx}.\int e^x dx$$

$$= xe^x - \int 1.e^x dx$$

$$= xe^x - e^x + c$$

$$= e^x(x-1)+c$$

Question: 2

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos xdx = x\int \cos x - \int\left[\frac{dx}{dx}.\int \cos xdx\right]dx$$

$$= x\sin x - \int 1.\sin xdx$$

$$= x\sin x + \cos x + c$$

Question: 3

Evaluate the foll

Solution:

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^{2x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int xe^{2x}dx = x\int e^{2x}dx - \int\left[\frac{dx}{dx}.\int e^{2x}dx\right]dx$$

$$= x\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{2\times 2} + c$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + c$$

Question: 4

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Sin 3x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\sin 3xdx = x\int \sin 3xdx - \int\left[\frac{dx}{dx}.\int \sin 3xdx\right]dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1.\left(\frac{-\cos 3x}{3}\right)dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{3\times 3}\right) + c$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{9}\right) + c$$

Question: 5

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos 2x dx = x\int \cos 2x dx - \int\left[\frac{dx}{dx}.\int \cos 2x dx\right]dx$$

$$= x\left(\frac{\sin 2x}{2}\right) - \int 1.\left(\frac{\sin 2x}{2}\right)dx$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{2\times 2}\right) + c$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{4}\right) + c$$

Question: 6

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log 2x is the first function, and x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\log 2x dx = \log 2x\int xdx - \int\left[\frac{d\log 2x}{dx}.\int xdx\right]dx$$

$$= \log 2x.\frac{x^2}{2} - \int\left[\frac{1\times 2}{2x}\frac{x^2}{2}\right]dx$$

$$= \frac{x^2}{2}\log 2x - \int\frac{x}{2}dx$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{2\times 2} + c$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{4} + c$$

Question: 7

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and $\text{cosec}^2 x$ is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\,\mathrm{cosec}^2 x dx = x\int \mathrm{cosec}^2 x - \int\left[\frac{dx}{dx}.\int \mathrm{cosec}^2 x dx\right]dx$$

$$= x(-\cot x) - \int 1.(-\cot x)dx$$

$$= -x\cot x + \int \cot x dx$$

$$= -x\cot x + \ln|\sin x| + c$$

Question: 8

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x^2 \cos x dx = x^2\int \cos x dx - \int\left[\frac{dx^2}{dx}.\int \cos x dx\right]dx$$

$$= x^2 \sin x - \int[2x \times \sin x]dx$$

$$= x^2 \sin x - 2\left[\int x \sin x dx\right]$$

Again applying by the part method in the second half, we get

$$x^2 \sin x - 2\int x \sin x dx$$

$$= x^2 \sin x - 2\left[x\int \sin x dx - \int\left(\frac{dx}{dx}.\int \sin x dx\right)dx\right]$$

$$= x^2 \sin x - 2\left[x(-\cos x) - \int 1.(-\cos x)dx\right]$$

$$= x^2 \sin x - 2[-x\cos x + \sin x] + c$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + c$$

Question: 9

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

Writing $\mathrm{Sin}^2 x = \dfrac{1+\cos 2x}{2}$

We have

$$\int x\sin^2 x dx = \int x\left(\frac{1-\cos 2x}{2}\right)dx$$

$$= \int\left(\frac{x}{2} - \frac{x\cos 2x}{2}\right)dx$$

$$= \int \frac{x}{2}dx - \int \frac{x\cos 2x}{2}dx$$

$$= \frac{x^2}{2\times 2} - \frac{1}{2}\int x\cos 2x dx$$

Taking X as first function and Cos 2x as the second function.

$$= \frac{x^2}{4} - \frac{1}{2}\left\{x\int \cos 2x dx - \int\left(\frac{dx}{dx}.\int \cos 2x dx\right)dx\right\}$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{x.\frac{\sin 2x}{2} - \int\left(1.\frac{\sin 2x}{2}\right)dx\right\}$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{\frac{x\sin 2x}{2} - \left(\frac{-\cos 2x}{2\times 2}\right)\right\} + c$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right\} + c$$

$$= \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c$$

Question: 10

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

Writing $\tan^2 x = \sec^2 x - 1$

We have

$$\int x\tan^2 x dx = \int x\left(\sec^2 x - 1\right)dx$$

$$= \int x\sec^2 x dx - \int x dx$$

Using x as the first function and $\text{Sec}^2 x$ as the second function

$$\int x\sec^2 x dx - \int x dx$$

$$= \left\{x\int \sec^2 x dx - \int\left(\frac{dx}{dx}.\int \sec^2 x dx\right)dx\right\} - \frac{x^2}{2}$$

$$= \left\{x.\tan x - \int 1.\tan x dx\right\} - \frac{x^2}{2}$$

$$= x\tan x - \ln|\sec x| - \frac{x^2}{2} + c$$

$$= x\tan x - \ln\left|\frac{1}{\cos x}\right| - \frac{x^2}{2} + c$$

$$x\tan x + \ln|\cos x| - \frac{x^2}{2} + c$$

Question: 11

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\int x^2 e^x dx = \left[x^2\int e^x dx - \int\left(\frac{dx^2}{dx}.\int e^x dx\right)dx\right]$$

$$= x^2 e^x - \int 2x.e^x dx$$

$$= x^2 e^x - 2\int xe^x dx$$

$$= x^2 e^x - 2\left[x\int e^x dx - \int\left(\frac{dx}{dx}.\int e^x dx\right)dx\right]$$

$$= x^2 e^x - 2\left[xe^x - \int 1.e^x dx\right]$$

$$= x^2 e^x - 2\left[xe^x - e^x\right] + c$$

$$= x^2 e^x - 2xe^x + 2e^x + c$$

$$= e^x\left(x^2 - 2x + 2\right) + c$$

Question: 12

Evaluate the foll

Solution:

We know that Cos3x = 4Cos^{3}x - 3Cosx

$$\text{Cos}^3\text{x} = \frac{\cos 3x + 3\cos x}{4}$$

$$\int x^2\cos^3 x dx=\int x^2\left(\frac{\cos 3x+3\cos x}{4}\right)dx$$

$$=\frac{1}{4}\left(\int x^2\cos 3x dx+3\int x^2\cos x dx\right)$$

Taking X^2 as the first function and cos 3x and cos x as the second function and applying By part method.

$$\frac{1}{4}\left(\int x^2\cos 3x dx+3\int x^2\cos x dx\right)$$

$$=\frac{1}{4}\left\{\left(x^2\int\cos 3x dx-\int\left[\frac{dx^2}{dx}.\int\cos 3x dx\right]dx\right)+3\left(x^2\int\cos x dx-\int\left[\frac{dx^2}{dx}.\int\cos x dx\right]dx\right)\right\}$$

$$=\frac{1}{4}\left\{\left(\frac{x^2.\sin 3x}{3}-\int 2x.\frac{\sin 3x}{3}dx\right)+3\left(x^2\sin x-\int 2x.\sin x dx\right)\right\}$$

$$=\frac{1}{4}\left\{\left(\frac{x^2\sin 3x}{3}-\frac{2}{3}\int x\sin 3x dx\right)+3\left(x^2\sin x-2\int x\sin x dx\right)\right\}$$

$$=\frac{1}{4}\left\{\left(\frac{x^2\sin 3x}{3}-\frac{2}{3}\left[x\int\sin 3x dx-\int\left(\frac{dx}{dx}.\int\sin 3x dx\right)dx\right]\right)+3\left(x^2\sin x-2\left[x\int\sin x dx-\int\left(\frac{dx}{dx}.\int\sin x dx\right)dx\right]\right)\right\}$$

$$=\frac{1}{4}\left\{\left(\frac{x^2\sin 3x}{3}-\frac{2}{3}\left[x\frac{-\cos 3x}{3}-\int 1.\frac{-\cos 3x}{3}dx\right]\right)+3\left(x^2\sin x-2\left[-x\cos x-\int-\cos x dx\right]\right)\right\}$$

$$=\frac{1}{4}\left\{\left(\frac{x^2\sin 3x}{3}-\frac{2}{3}\left[\frac{-x\cos 3x}{3}+\frac{\sin 3x}{9}\right]\right)+3\left(x^2\sin x+2x\cos x-2\sin x\right)\right\}+c$$

$$=\frac{1}{4}\left\{\frac{x^2\sin 3x}{3}+\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+3x^2\sin x+6x\cos x-6\sin x\right\}+c$$

$$=\frac{x^2\sin 3x}{12}+\frac{x\cos 3x}{18}-\frac{\sin 3x}{54}+\frac{3x^2\sin x}{4}+\frac{3x\cos x}{2}-\frac{3}{2}\sin x+c$$

Question: 13

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^{3x} is the second function.

Using Integration by part

$$\int a.b.dx=a\int bdx-\int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x^2e^{3x}dx = x^2\int e^{3x}dx - \int\left(\frac{dx^2}{dx}.\int e^{3x}dx\right)dx$$

$$= x^2\frac{e^{3x}}{3} - \int 2x.\frac{e^{3x}}{3}dx$$

$$= x^2\frac{e^{3x}}{3} - \frac{2}{3}\int xe^{3x}dx$$

$$= x^2\frac{e^{3x}}{3} - \frac{2}{3}\left(x\int e^{3x}dx - \int\left[\frac{dx}{dx}.\int e^{3x}dx\right]dx\right)$$

$$= x^2\frac{e^{3x}}{3} - \frac{2}{3}\left(x\frac{e^{3x}}{3} - \int\frac{e^{3x}}{3}dx\right)$$

$$= x^2\frac{e^{3x}}{3} - \frac{2}{3}\left(x\frac{e^{3x}}{3} - \frac{e^{3x}}{9}\right) + c$$

$$= x^2\frac{e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + c$$

$$= e^{3x}\left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27}\right) + c$$

Question: 14

Evaluate the foll

Solution:

We can write $\sin^2 x = \frac{1-\cos 2x}{2}$

We have

$$\int x^2\left(\frac{1-\cos 2x}{2}\right)dx = \int\frac{x^2}{2} - \frac{x^2\cos 2x}{2}dx$$

$$= \int\frac{x^2}{2}dx - \int\frac{x^2\cos 2x}{2}dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$=\frac{x^3}{3\times 2}-\frac{1}{2}\int x^2\cos 2x dx$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2\int \cos 2x dx-\int\left[\frac{dx^2}{dx}.\int \cos 2x dx\right]dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\int 2x.\frac{\sin 2x}{2}dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\int x.\sin 2x dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\left[x\int \sin 2x dx-\int\left(\frac{dx}{dx}.\int \sin 2x dx\right)dx\right]\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\left[x\frac{-\cos 2x}{2}-\int 1.\frac{-\cos 2x}{2}dx\right]\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}+\frac{x\cos 2x}{2}-\frac{\sin 2x}{4}\right)+c$$

$$=\frac{x^3}{6}-\frac{x^2\sin 2x}{4}-\frac{x\cos 2x}{4}+\frac{\sin 2x}{8}+c$$

Question: 15

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log2x is the first function, and x^3 is the second function.

Using Integration by part

$$\int a.b.dx = a\int b dx-\int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\int x^3\log 2x dx = \log 2x\int x^3 dx-\int\left(\frac{d\log 2x}{dx}.\int x^3 dx\right)dx$$

$$=\log 2x\frac{x^4}{4}-\int\frac{1.2}{2x}.\frac{x^4}{4}dx$$

$$=\log 2x\frac{x^4}{4}-\frac{1}{4}\int x^3 dx$$

$$=\log 2x\frac{x^4}{4}-\frac{1}{4}.\frac{x^4}{4}+c$$

$$=\log 2x\frac{x^4}{4}-\frac{x^4}{16}+c$$

Question: 16

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(x + 1) is first function and x is second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x\log(x+1) = \log(x+1)\int xdx - \int\left(\frac{d\log(x+1)}{dx}.\int xdx\right)dx$$

$$= \log(x+1)\frac{x^2}{2} - \int\frac{1}{x+1}\times\frac{x^2}{2}dx$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\int\frac{x^2-1+1}{x+1}dx$$

Adding and subtracting 1 in the numerator,

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int\frac{x^2-1}{x+1} + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int\frac{(x+1)(x-1)}{x+1} + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int(x-1) + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\frac{x^2}{2} - x + \log(x+1)\right] + c$$

$$= \log(x+1)\frac{x^2}{2} - \frac{x^2}{4} + \frac{x}{2} - \frac{\log(x+1)}{2} + c$$

$$= \log(x+1)\frac{x^2-1}{2} - \frac{x^2}{4} + \frac{x}{2} + c$$

Question: 17

Evaluate the foll

Solution:

We can write it as $\int x^{-n}.\log xdx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logx is the first function, and x^{-n} is the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x^{-n} \log x dx = \log x \int x^{-n} dx - \int \left(\frac{d \log x}{dx} . \int x^{-n} dx \right) dx$$

$$= \log x \left(\frac{x^{-n+1}}{-n+1} \right) - \int \frac{1}{x} . \frac{x^{-n+1}}{-n+1} dx$$

$$= \frac{x^{-n+1} \log x}{1-n} + \frac{1}{1-n} \int \frac{x^{-n} . x}{x} dx$$

$$= \frac{x^{-n+1} \log x}{1-n} + \frac{1}{1-n} \times \frac{x^{-n+1}}{-n+1} + c$$

$$= \frac{x^{-n+1} \log x}{1-n} - \frac{x^{-n+1}}{(1-n)^2} + c$$

Question: 18

Evaluate the foll

Solution:

We can write it as $\int 2.x.x^2.e^{x^2} dx$

Let $x^2 = t$

$2xdx = dt$

Using the relation in the above condition, we get

$$\int 2x.x^2.e^{x^2} dx = \int t.e^t dt$$

Integrating with respect to t

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function, and e^t is the second function.

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} . \int b dx \right] dx$$

$$\int te^t dt = t \int e^t dt - \int \left(\frac{dt}{dt} . \int e^t dt \right) dt$$

$$= te^t - \int 1.e^t dt$$

$$= te^t - e^t + c$$

Replacing t with x^2,we get

$$x^2 e^{x^2} - e^{x^2} + c$$

$$= e^{x^2} \left(x^2 - 1 \right) + c$$

Question: 19

Evaluate the foll

Solution:

We know that $\text{Sin}3x = 3\text{Sin}x - 4\text{Sin}^3x$

$\text{Sin}^3x = (3\text{Sin}x - \text{Sin}3x)/4$

$$\int x\sin^3 x dx = \int x\left(\frac{3\sin x - \sin 3x}{4}\right)dx$$

$$= \frac{1}{4}\int 3x\sin x - x\sin 3x dx$$

$$= \frac{3}{4}\int x\sin x dx - \frac{1}{4}\int x\sin 3x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sinx and sin3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{3}{4}\int x\sin x dx - \frac{1}{4}\int x\sin 3x dx$$

$$= \frac{3}{4}\left(x\int \sin x dx - \int\left[\frac{dx}{dx}.\int \sin x dx\right]dx\right) - \frac{1}{4}\left(s\int \sin 3x dx - \int\left[\frac{dx}{dx}.\int \sin 3x dx\right]dx\right)$$

$$= \frac{3}{4}\left(-x\cos x + \int \cos x dx\right) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \int\frac{\cos 3x}{3}dx\right)$$

$$= \frac{3}{4}\left(-x\cos x + \sin x\right) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}\right) + c$$

$$= \frac{-3x\cos x}{4} + \frac{3\sin x}{4} + \frac{x\cos 3x}{12} - \frac{\sin 3x}{36} + c$$

Question: 20

Evaluate the foll

Solution:

We can write $\cos^3 x = (\cos 3x + 3\cos x)/4$, we have

$$\int x\cos^3 x dx = \int x\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\int x\cos 3x dx + \frac{3}{4}\int x\cos x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and cosx and cos3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{1}{4}\left(x\int \cos 3x dx - \int\left[\frac{dx}{dx}.\int \cos 3x dx\right]dx\right) + \frac{3}{4}\left(x\int \cos x dx - \int\left[\frac{dx}{dx}.\int \cos x dx\right]dx\right)$$

$$= \frac{1}{4}\left(x\frac{\sin 3x}{3} - \int\frac{\sin 3x}{3}dx\right) + \frac{3}{4}\left(x\sin x - \int \sin x dx\right)$$

$$= \frac{1}{4}\left(\frac{x\sin 3x}{3} + \frac{\cos 3x}{9}\right) + \frac{3}{4}\left(x\sin x + \cos x\right) + c$$

$$= \frac{x\sin 3x}{12} + \frac{\cos 3x}{36} + \frac{3x\sin x}{4} + \frac{3\cos x}{4} + c$$

Question: 21

Evaluate the foll

Solution:

We can write it as

$$\int x.x^2 \cos x^2 dx$$

Now let $x^2 = t$

2xdx = dt

Xdx = dt/2

Now

$$\frac{1}{2}\int t\cos tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and cost as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int t\cos tdt = \frac{1}{2}\left(t\int \cos tdt - \int\left[\frac{dt}{dt}.\int \cos tdt\right]dt\right)$$

$$= \frac{1}{2}\left(t\sin t - \int \sin tdt\right)$$

$$= \frac{1}{2}\left(t\sin t + \cos t\right) + c$$

Replacing t with x^2

$$= \frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + c$$

Question: 22

Evaluate the foll

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(cosx) is the first function and sinx as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \sin x \log(\cos x)dx = \log(\cos x)\int \sin xdx - \int\left(\frac{d\log(\cos x)}{dx}.\int \sin xdx\right)dx$$

$$= -\cos x\log(\cos x) + \int \frac{-\sin x}{\cos x}.\cos xdx$$

$$= -\cos x\log(\cos x) - \int \sin xdx$$

$$= -\cos x\log(\cos x) + \cos x + c$$

Question: 23

Evaluate the foll

Solution:

We know that Sin2x = 2Sinxcosx

$$\int x\sin x\cos x dx = \frac{1}{2}\int x\sin 2x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sin2x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int x\sin 2x dx = \frac{1}{2}\left(x\int \sin 2x dx - \int\left[\frac{dx}{dx}.\int \sin 2x dx\right]dx\right)$$

$$= \frac{1}{2}\left(x\frac{-\cos 2x}{2} + \int\frac{\cos 2x}{2}dx\right)$$

$$= \frac{1}{2}\left(\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4}\right) + c$$

$$= \frac{-x\cos 2x}{4} + \frac{\sin 2x}{8} + c$$

Question: 24

Evaluate the foll

Solution:

Let √x = t

$$\frac{1}{2\sqrt{x}}dx = dt$$

$$\Rightarrow dx = 2\sqrt{x}dt$$

$$\Rightarrow dx = 2tdt$$

We can write it as

$$\int \cos\sqrt{x}dx = 2\int t\cos tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is first function and cos t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow 2\int t\cos tdt = 2\left(t\int \cos tdt - \int\left[\frac{dt}{dt}\right]\int \cos tdt\right)dt$$

$$= 2\left(t\sin t - \int \sin tdt\right)$$

$$= 2t\sin t + 2\cos t + c$$

Replacing t with √x

= 2√xsin√x + 2cos√x + c

= 2(cos√x + √xsin√x) + c

Question: 25

Evaluate the foll

Solution:

We can write it as $\int \operatorname{cosec}^3 dx = \int \operatorname{cosec}x.\operatorname{cosec}^2 x dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cosecx is first function and $\operatorname{cosec}^2 x$ as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \operatorname{cosec}x.\operatorname{cosec}^2 xdx = \operatorname{cosec}x\int .\operatorname{cosec}^2 xdx - \int\left(\frac{d\operatorname{cosec}x}{dx}.\int .\operatorname{cosec}^2 xdx\right)dx$$

$$= \operatorname{cosec}x(-\cot x) - \int(-\operatorname{cosec}x.\cot x)(-\cot x)dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x.\cot^2 xdx$$

We know that $\operatorname{Cot}^2 x = \operatorname{Cosec}^2 x - 1$

$$-\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x\left(\operatorname{cosec}^2 x - 1\right)dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 xdx + \int \operatorname{cosec}xdx$$

We can write $\int \operatorname{cosec}^3 xdx = I$

$$\Rightarrow \int \operatorname{cosec}^3 xdx - \operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 xdx + \int \operatorname{cosec}xdx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 xdx = -\operatorname{cosec}x.\cot x + \int \operatorname{cosec}xdx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 xdx = -\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x| + c_1$$

$$\Rightarrow \int \operatorname{cosec}^3 xdx = \frac{-\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x|}{2} + c$$

Question: 26

Evaluate the foll

Solution:

We can write it as $\int x\sin^2 x\sin x\cos xdx$

We also know that 2sinx.cosx = sin2x

$$\int x\sin^2 x\sin x\cos xdx = \frac{1}{2}\int x\sin^2 x\sin 2xdx$$

We also know that $\sin^2 x = \frac{1-\cos 2x}{2}$

$$\frac{1}{2}\int x\sin^2 x\sin 2x dx = \frac{1}{2}\int x.\left(\frac{1-\cos 2x}{2}\right)\sin 2x dx$$

$$= \frac{1}{2}\left[\left(\int \frac{x\sin 2x}{2}dx - \int \frac{x\cos 2x\sin 2x}{2}dx\right)\right]$$

Here Sin4x = 2sin2x.cos2x

$$= \frac{1}{2}\left[\left(\int \frac{x\sin 2x}{2}dx - \frac{1}{4}\int x\sin 4x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and Sin2x and sin4x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{x\int \sin 2x dx - \int\left(\frac{dx}{dx}.\int \sin 2x dx\right)dx\right\}\right) - \left(\frac{1}{4}\left\{x\int \sin 4x - \int\left(\frac{dx}{dx}.\int \sin 4x dx\right)dx\right\}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2} + \int \frac{\cos 2x}{2}dx\right\}\right) - \left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4} + \int \frac{\cos 4x}{4}dx\right\}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2} + \frac{\sin 2x}{4}\right\}\right) - \left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4} + \frac{\sin 4x}{16}\right\}\right)\right] + c$$

$$= \frac{-x\cos 2x}{8} + \frac{\sin 2x}{16} + \frac{x\cos 4x}{32} - \frac{\sin 4x}{128} + c$$

Question: 27

Evaluate the foll

Solution:

Let cosx = t

- sinxdx = dt

Now the integral we have is

$$\int \sin x\log(\cos x)dx = -\int \log t dt$$

$$= -\int 1.\log t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$-\int 1.\log t dt = \log t \int 1 dt - \int\left(\frac{d \log t}{dt}.\int 1.dt\right)dt$$

$$= -\log t.t + \int \frac{1}{t}.tdt$$

$$= -t \log t + t + c$$

Replacing t with cosx

$$t(-\log t + 1) + c$$

$$= \cos x(1 - \log(\cos x)) + c$$

Question: 28

Evaluate the foll

Solution:

Let logx = t

1/x dx = dt

$$\int \frac{\log(\log x)}{x} dx = \int \log t dt = \int 1.\log t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\int 1.\log t dt = \log t \int 1 dt - \int\left(\frac{d \log t}{dt}.\int 1.dt\right)dt$$

$$= t.\log t - \int \frac{1}{t} t dt$$

$$= t \log t - t + c$$

Now replacing t with logx

$$\log x.\log(\log x) - \log x + c$$

$$= \log x(\log(\log x) - 1) + c$$

Question: 29

Evaluate the foll

Solution:

$$= \int 1.\log(2 + x^2) dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(2 + x^2) is the first function and 1 as the second function.

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\int 1.\log\left(2+x^2\right)dx = \log\left(2+x^2\right)\int 1dx - \int\left(\frac{d\log\left(2+x^2\right)}{dx}.\int 1dx\right)dx$$

$$= \log\left(2+x^2\right).x - \int\frac{1.2x}{2+x^2}.xdx$$

$$= x\log\left(2+x^2\right) - \int\frac{2x^2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\int\frac{x^2+2-2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\left[\left(\int 1dx\right) - \int\frac{2}{2+x^2}dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - \left(2\int\frac{1}{2+x}\right)dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - 2\left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\right)\right] + c$$

$$= x\log\left(2+x^2\right) - 2x + 2\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}} + c$$

Question: 30

Evaluate the foll

Solution:

$$\int\frac{x}{1+\sin x}dx = \int\frac{x(1-\sin x)}{(1+\sin x).(1-\sin x)}dx$$

We can write it as $= \int\frac{x(1-\sin x)}{1-\sin^2 x}dx$

$$= \int\frac{x(1-\sin x)}{\cos^2 x}dx$$

$$= \int x\sec^2 xdx - \int x\tan x\sec xdx$$

Using by part and ILATE

Taking x as first function and $\sec^2 x$ and secxtanx as the second function, we have

$$\int x\sec^2 xdx - \int x\sec x\tan xdx = \left(x\int\sec^2 xdx - \int\left(\frac{dx}{dx}.\int\sec^2 xdx\right)dx\right)$$

$$-\left(x\int\sec x\tan xdx - \int\left(\frac{dx}{dx}.\int\sec x\tan xdx\right)dx\right)$$

$$= \left(x\tan x - \int 1.\tan xdx\right) - \left(x.\sec x - \int 1.\sec xdx\right)$$

$$= x\tan x - \ln|\sec x| - x\sec x + \ln|\sec x + \tan x| + c$$

$$= x(\tan x - \sec x) + \ln\left|\frac{\sec x + \tan x}{\sec x}\right| + c$$

$$= x(\tan x - \sec x) + \ln\left|1+\sin x\right| + c$$

Question: 31

Evaluate the foll

Solution:

Let us assume logx = t

X = e^t

dx = $e^t dt$

Now we have

$$\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx=\int\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$$

Considering f(x) = 1/t ; $f'(x)$ = - 1/t^2

$$\frac{d}{dt}\left(\frac{1}{t}\right)=-\frac{1}{t^2}$$

By the integral property of $\int\{f(x)+f'(x)\}e^x dx=e^x.f(x)+c$

So the solution of the integral is

$$\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx=e^t\times\frac{1}{t}+c$$

Substituting the value of t as logx

$$=e^{\log x}\times\frac{1}{\log x}+c$$

$$=\frac{x}{\log x}+c$$

Question: 32

Evaluate the foll

Solution:

$$\cos A.\cos B=\frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$

We know that $\Rightarrow \cos 4x.\cos 2x=\frac{1}{2}\left[\cos(4x+2x)+\cos(4x-2x)\right]$

$$=\frac{1}{2}\left[\cos 6x+\cos 2x\right]$$

Putting in the original equation

$$\int e^{-x}\cos 2x.\cos 4x dx=\int e^{-x}\left(\frac{1}{2}\left[\cos 6x+\cos 2x\right]\right)$$

$$=\frac{1}{2}\left[\left(\int e^{-x}\cos 6x dx\right)+\left(\int e^{-x}\cos 2x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cos6x and cos2x is first function and e^{-x} as the second function.

$$\int a.b.dx=a\int b dx-\int\left[\frac{da}{dx}.\int b dx\right]dx$$

Solving both parts individually

$$I=\int e^{-x}\cos 6x dx=\cos 6x\int e^{-x}dx-\int\left(\frac{d\cos 6x}{dx}.\int e^{-x}dx\right)dx$$

$$I=\cos 6x.\left(-e^{-x}\right)-\int\left(-6\sin 6x\right).\left(-e^{-x}\right)dt$$

$$I=-\cos 6x.e^{-x}-6\int\sin 6x.e^{-x}dx$$

$$I=-e^{-x}\cos 6x-6\left[\sin 6x\int e^{-x}dx-\int\left(\frac{d\sin 6x}{dx}.\int e^{-x}dx\right)dx\right]$$

$$I=-e^{-x}\cos 6x-6\left[\sin 6x\left(-e^{-x}\right)-\int\left(6\cos 6x\right).\left(-e^{-x}\right)dt\right]$$

$$I=-e^{-x}\cos 6x-6\left[-e^{-x}\sin 6x+6\int e^{-x}\cos 6x dx\right]$$

$$I=-e^{-x}\cos 6x-6\left[-e^{-x}\sin 6x+6I\right]$$

$$I=-e^{-x}\cos 6x+6e^{-x}\sin 6x-36I$$

$$37I=e^{-x}\left(6\sin 6x-\cos 6x\right)$$

$$I=\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{37}$$

Solving the second part,

$$I=\int e^{-x}\cos 2x dx=\cos 2x\int e^{-x}dx-\int\left(\frac{d\cos 2x}{dx}.\int e^{-x}dx\right)dx$$

$$J=\cos 2x.\left(-e^{-x}\right)-\int\left(-2\sin 2x\right).\left(-e^{-x}\right)dt$$

$$J=-\cos 2x.e^{-x}-2\int\sin 2x.e^{-x}dx$$

$$J=-e^{-x}\cos 2x-2\left[\sin 2x\int e^{-x}dx-\int\left(\frac{d\sin 2x}{dx}.\int e^{-x}dx\right)dx\right]$$

$$J=-e^{-x}\cos 2x-2\left[\sin 2x\left(-e^{-x}\right)-\int\left(2\cos 2x\right).\left(-e^{-x}\right)dt\right]$$

$$J=-e^{-x}\cos 2x-2\left[-e^{-x}\sin 2x+2\int e^{-x}\cos 2x dx\right]$$

$$J=-e^{-x}\cos 2x-2\left[-e^{-x}\sin 2x+2J\right]$$

$$J=-e^{-x}\cos 2x+2e^{-x}\sin 2x-4J$$

$$5J=e^{-x}\left(2\sin 2x-\cos 2x\right)$$

$$J=\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{5}$$

Putting in the obtained equation

$$=\frac{1}{2}\left[\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{37}+\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{5}\right]+c$$

$$=\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{74}+\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{10}+c$$

$$=e^{-x}\left(\frac{\left(6\sin 6x-\cos 6x\right)}{74}+\frac{\left(2\sin 2x-\cos 2x\right)}{10}\right)+c$$

Question: 33

Evaluate the foll

Solution:

Let √x = t

$$\frac{1}{2\sqrt{x}}dx = dt$$

$$dx = 2\sqrt{x}dt$$

$$\Rightarrow dx = 2tdt$$

Replacing in the original equation , we get

$$\int e^{\sqrt{x}}dx = \int e^{t}.2tdt$$

$$= 2\int te^{t}dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int te^{t}dt = 2\left[t\int e^{t}dt - \int\left(\frac{dt}{dt}.\int e^{t}dt\right)dt\right]$$

$$= 2\left[te^{t} - \int 1.e^{t}dt\right]$$

$$= 2\left[te^{t} - e^{t}\right] + c$$

$$= 2e^{t}(t-1) + c$$

Replacing t with √x

= $2e^{\sqrt{x}}(\sqrt{x} - 1) + c$

Question: 34

Evaluate the foll

Solution:

We can write Sin2x = 2sinx.cosx

$$\int e^{\sin x}\sin 2xdx = 2\int e^{\sin x}.\sin x\cos xdx$$

Let Sinx = t

Cosxdx = dt

$$2\int e^{\sin x}\sin x\cos xdx = 2\int e^{t}.t.dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int e^t.tdt = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt\right]$$

$$= 2\left[t.e^t - \int 1.e^t dt\right]$$

$$= 2\left[t.e^t - e^t\right] + c$$

$$= 2e^t(t-1) + c$$

Replacing t with sin x

$= 2e^{\sin x}(\sin x - 1) + c$

Question: 35

Evaluate the foll

Solution:

Let $\sin^{-1}x = t$

$X = \sin t$

$$\frac{1}{\sqrt{1-x^2}}dx = dt$$

Putting this in the original equation, we get

$$\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t.\sin t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t.\sin tdt = t\int \sin tdt - \int\left(\frac{dt}{dt}.\int \sin tdt\right)dt$$

$$= t(-\cos t) - \int 1.(-\cos t)dt$$

$$= -t\cos t + \sin t + c$$

We can write $\cos t = \sqrt{1 - \sin^2 t}$

$= -t(\sqrt{1 - \sin^2 t}) + \sin t + c$

Now replacing $\sin^{-1}x = t$

$= -\sin^{-1}x(\sqrt{1 - x^2}) + x + c$

Question: 36

Evaluate the foll

Solution:

Let $\tan^{-1} x = t$ and $x = \tan t$

Differentiating both sides, we get

$$\frac{1}{1+x^2}dx = dt$$

Now we have

$$\int \frac{x^2 \tan^{-1} x}{\left(1+x^2\right)} dx = \int \tan^2 t.tdt$$

$$\int t.\tan^2 tdt = \int t\left(\sec^2 t - 1\right)dt$$

$$= \int t\sec^2 tdt - \int tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and $\sec^2 t$ as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t\sec^2 tdt - \int tdt = t\int \sec^2 tdt - \int\left(\frac{dt}{dt}.\int \sec^2 tdt\right)dt - \frac{t^2}{2}$$

$$= t.\tan t - \int \tan tdt - \frac{t^2}{2}$$

$$= t.\tan t - \ln|\sec t| - \frac{t^2}{2} + c$$

We know that $\sec t = \sqrt{\tan^2 t + 1}$

$$= \tan^{-1} x.x - \ln\left|\sqrt{\tan^2 t + 1}\right| - \frac{\tan^2 x}{2} + c$$

$$= x\tan^{-1} x - \ln\left|\sqrt{x^2 + 1}\right| - \frac{\tan^2 x}{2} + c$$

Question: 37

Evaluate the foll

Solution:

We can write it as $\int \log(x+2).\frac{1}{(x+2)^2}dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(x + 2) is first function and $(x + 2)^{-2}$ as second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \log(x+2).\frac{1}{(x+2)^2}dx = \log(x+2)$$

$$\int \frac{1}{(x+2)^2}dx - \int\left(\frac{d\log(x+2)}{dx}.\int\frac{1}{(x+2)^2}dx\right)dx$$

$$= \log(x+2).\frac{-1}{(x+2)} - \int\frac{1}{x+2}.\frac{-1}{(x+2)}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} + \int\frac{1}{(x+2)^2}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} - \frac{1}{(x+2)} + c$$

Question: 38

Evaluate the foll

Solution:

Let x = sin t ; t = $\sin^{-1}x$

dx = cos t dt

$$\Rightarrow \int x\sin^{-1}x dx = \int \sin t.\sin^{-1}(\sin t)\cos t dt$$

$$= \int \sin t.t.\cos t dt$$

We know that sin 2t = 2 sint×cost

We have $\int t\cos t\sin t dt = \frac{1}{2}\int t\sin 2t dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin 2t as the second function.

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\frac{1}{2}\int t\sin 2t dt = \frac{1}{2}\left(t\int \sin 2t dt - \int\left[\frac{dt}{dt}.\int \sin 2t dt\right]dt\right)$$

$$= \frac{1}{2}\left(t.\frac{-\cos 2t}{2} + \int\frac{\cos 2t}{2}dt\right)$$

$$= \frac{1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$

$$= \frac{-t\cos 2t}{4} + \frac{\sin 2t}{8} + c$$

We know that cos2t = 1 - 2$\sin^2$t , sin2t = 2sint×cost and cos t = $\sqrt{1} - \sin^2$t

Replacing in above equation

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{2\sin t \times \cos t}{8} + c$$

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{\sqrt{1-\sin^2 t}}{4}.\sin t + c$$

$$= \frac{-\sin^{-1} x\left(1-2x^2\right)}{4} + \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \frac{1}{2}x^2 \sin^{-1} x - \frac{\sin^{-1} x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

$$= \frac{1}{2}x^2 \sin^{-1} x - \frac{\sin^{-1} x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

Question: 39

Evaluate the foll

Solution:

Let x = cos t ; t = $\cos^{-1}x$

dx = - sin t dt

$$\int x\cos^{-1} x dx = -\int \cos t.\cos^{-1}(\cos t)\sin t dt$$

$$= -\int \cos t.t.\sin t.dt$$

We know that sin 2t = 2 sint×cost

We have $-\int t\cos t\sin t dt = \frac{-1}{2}\int t\sin 2t dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking first function to the one which comes first in the list.

Here t is first function and sin 2t as second function.

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\frac{-1}{2}\int t\sin 2t dt = \frac{-1}{2}\left(t\int \sin 2t dt - \int\left[\frac{dt}{dt}.\int \sin 2t dt\right]dt\right)$$

$$= \frac{-1}{2}\left(t.\frac{-\cos 2t}{2} + \int \frac{\cos 2t}{2}dt\right)$$

$$= \frac{-1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$

$$= \frac{t\cos 2t}{4} - \frac{\sin 2t}{8} + c$$

We know that cos2t = $2\cos^2 t$ - 1 and sin2t = 2sint×cost and sint = √1 - $\cos^2 t$

Replacing in above equation

$$= \frac{t\left(2\cos^2 t-1\right)}{4} - \frac{2\sin t \times \cos t}{8} + c$$

$$= \frac{t\left(2\cos^2 t-1\right)}{4} - \frac{\sqrt{1-\cos^2 t}}{4}.\cos t + c$$

$$= \frac{\cos^{-1} x\left(2x^2-1\right)}{4} - \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \frac{1}{2}x^2\cos^{-1}x - \frac{\cos^{-1}x}{4} - \frac{1}{4}x\sqrt{1-x^2} + c$$

$$= \frac{1}{2}x^2\cos^{-1}x + \frac{\sin^{-1}x}{4} - \frac{1}{4}x\sqrt{1-x^2} + c$$

Question: 40

Evaluate the foll

Solution:

We can write it as $\int \cot^{-1} x.1dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\cot^{-1}x$ is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \cot^{-1}x.1dx = \cot^{-1}x\int 1dx - \int\left(\frac{d\cot^{-1}x}{dx}.\int 1dx\right)dx$$

$$= \cot^{-1}x.x - \int \frac{-1}{1+x^2}.x.dx$$

$$= x\cot^{-1}x + \int \frac{x}{1+x^2}dx$$

Let $1 + x^2 = t$

2xdx = dt

Xdx = dt/2

$$\Rightarrow \int \cot^{-1}xdx = x\cot^{-1}x + \int \frac{dt}{2t}$$

$$= x\cot^{-1}x + \frac{\log t}{2} + c$$

Now replacing t with $1 + x^2$

$= x\cot^{-1}x + \log(1 + x^2)/2 + c$

Question: 41

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x$,

$$\therefore \int x\cot^{-1}x\,dx$$

$$= \cot^{-1}x\int xdx - \int\left\{\frac{d}{dx}(\cot^{-1}x)\int xdx\right\}dx$$

$$= \frac{x^2\cot^{-1}x}{2} - \int\frac{1}{(1+x^2)}\times\frac{x^2}{2}dx$$

$$= \frac{x^2\cot^{-1}x}{2} - \frac{1}{2}\int\frac{x^2}{(1+x^2)}dx$$

$$= \frac{x^2\cot^{-1}x}{2} - \frac{1}{2}\int\frac{1+x^2-x^2}{(1+x^2)}dx$$

$$= \frac{x^2\cot^{-1}x}{2} - \frac{1}{2}\int 1-\frac{1}{(1+x^2)}dx$$

$= \frac{x^2\cot^{-1}x}{2} - \frac{1}{2}[x-\tan^{-1}x]+c$, where c is the integrating constant

Question: 42

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x^2$,

$$\therefore \int x^2\cot^{-1}x\,dx$$

$$= \cot^{-1}x\int x^2dx - \int\left\{\frac{d}{dx}(\cot^{-1}x)\int x^2dx\right\}dx$$

$$= \frac{x^3\cot^{-1}x}{3} - \int\frac{1}{(1+x^2)}\times\frac{x^3}{3}dx$$

$$= \frac{x^3\cot^{-1}x}{3} - \frac{1}{3}\int\frac{x^3}{(1+x^2)}dx$$

Taking $(1+x^2)=a$,

$2xdx=da$ i.e. $xdx=da/2$

Again, $x^2=a-1$

$$\therefore \frac{1}{3}\int\frac{x^2\times xdx}{(1+x^2)}$$

$$= \frac{1}{3}\int\frac{(a-1)da}{2a}$$

$$= \frac{1}{6}\int\left(1-\frac{1}{a}\right)da$$

$$= \frac{1}{6}(a-\ln a)$$

Replacing the value of a, we get,

$$\therefore \frac{1}{6}(a - \ln a)$$

$$= \frac{1}{6}[(1 + x^2) - \ln|x^2 + 1| + c_1$$

$$= \frac{x^2}{6} - \frac{\ln|x^2 + 1|}{6} + \left(c_1 + \frac{1}{6}\right)$$

$$= \frac{x^2}{6} - \frac{\ln|x^2 + 1|}{6} + c$$

The total integration yields as

$= \frac{\mathbf{x^3 \cot^{-1} x}}{\mathbf{3}} + \frac{\mathbf{x^2}}{\mathbf{6}} - \frac{\mathbf{\ln|x^2+1|}}{\mathbf{6}} + \mathbf{c}$, where c is the integrating constant

Question: 43

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \sin^{-1}\sqrt{x}\, dx$$

$$= \sin^{-1}\sqrt{x} \int dx - \int \left\{\frac{d}{dx}(\sin^{-1}\sqrt{x}) \int dx\right\} dx$$

$$= x \sin^{-1}\sqrt{x} - \int \frac{1}{2\sqrt{x}\sqrt{1-x}} \times x\, dx$$

$$= x \sin^{-1}\sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

$$= \frac{1}{2} \int \frac{\sqrt{1-a^2}}{a} (-2ada)$$

$$= -\int \sqrt{1-a^2}\, da$$

$$= -\left[\frac{1}{2} a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2} a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

$$= -\left[\frac{1}{2} x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= \mathbf{x\sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c}$, where c is the integrating constant

Question: 44

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \cos^{-1}\sqrt{x}\,dx$$

$$= \cos^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$$

$$= x\cos^{-1}\sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times x\,dx$$

$$= x\cos^{-1}\sqrt{x} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= \mathbf{x\cos^{-1}\sqrt{x} - \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c}$, where c is the integrating constant

Question: 45

Evaluate the foll

Solution:

Formula to be used – We know , $\cos 3x = 4\cos^3 x - 3\cos x$

$$\therefore \int \cos^{-1}(4x^3 - 3x)\,dx$$

Assuming x = cosa, $4\cos^3 a-3\cos a=\cos 3a$

And, dx = -sinada

Hence, $a=\cos^{-1}x$

Again, sina=$\sqrt{(1-x^2)}$

$$\therefore \int \cos^{-1}(4x^3 - 3x)\,dx$$

$$= \int \cos^{-1}(\cos 3a)\{-\sin a da\}$$

$$= -3 \int a \sin a da$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = a and $f_2(x)$ = sina,

$$\therefore -3 \int a \sin a da$$

$$= -3\left[a \int \sin a da - \int \left\{\frac{d}{dx} a \int \sin a da\right\} da\right]$$

$$= 3a\cos a - \int \cos a da$$

$$= 3a\cos a - \sin a + c$$

Replacing the value of a we get,

$$\therefore 3a\cos a - \sin a + c$$

$= \mathbf{3x\cos^{-1}x - \sqrt{1-x^2} + c}$, where c is the integrating constant

Question: 46

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and $f_2(x)$ = 1,

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$$

$$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \int dx - \int \left[\frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\} \int dx\right] dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int \left[\frac{\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}}\right] dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int \frac{-4x^2dx}{(1+x^2)^2 \times \frac{1}{1+x^2} \times 2x}$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) - \int \frac{2xdx}{1+x^2}$$

Now,

$$\int \frac{2xdx}{1+x^2}$$

$$= \int \frac{d(1+x^2)}{1+x^2}$$

$$= \ln(1+x^2) + c$$

Again, we know,

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

$$\Rightarrow 2x = \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$$

Replacing x by tanx, it is obtained that,

$$2\tan x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

So, the final integral yielded is

$\mathbf{2x\tan x - \ln(1+x^2) + c}$, where c is the integrating constant

Question: 47

Evaluate the foll

Solution:

Formula to be used - We know, $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

Assuming x = tana,

$$\frac{2\tan a}{1-\tan^2 a} = \tan 2a$$

And, dx = $\sec^2 a da$

Hence, a=$\tan^{-1}x$

Now, $\sec^2 a$-$\tan^2 a$=1 , so,seca=$\sqrt{(1+x^2)}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

$$= \int \tan^{-1}(\tan 2a)\{\sec^2 a da\}$$

$$= 2\int a\sec^2 a da$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = a$ and $f_2(x) = \sec^2 a$,

$$\therefore 2\int a\sec^2 a\,da$$

$$= 2\left[a\int \sec^2 a\,da - \int\left\{\frac{d}{dx}a\int \sec^2 a\,da\right\}da\right]$$

$$= 2a\tan a - \int \tan a\,da$$

$$= 2a\tan a - \ln|\sec a| + c$$

Replacing the value of a we get,

$$\therefore 2a\tan a - \ln|\sec a| + c$$

$= \mathbf{2x\tan^{-1}x - \ln\sqrt{1+x^2} + c}$, where c is the integrating constant

Question: 48

Evaluate the foll

Solution:

Formula to be used - We know, $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)dx$$

Assuming $x = \tan a$,

$$\frac{3\tan a - \tan^3 a}{1 - 3\tan^2 a} = \tan 3a$$

And, $dx = \sec^2 a\,da$

Hence, $a = \tan^{-1}x$

Now, $\sec^2 a - \tan^2 a = 1$, so, $\sec a = \sqrt{(1+x^2)}$

$$\therefore \int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)dx$$

$$= \int \tan^{-1}(\tan 3a)\{\sec^2 a\,da\}$$

$$= 3\int a\sec^2 a\,da$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = a$ and $f_2(x) = \sec^2 a$,

$$\therefore 3\int a\sec^2 a\,da$$

$$= 3\left[a\int \sec^2 a\,da - \int\left\{\frac{d}{dx}a\int \sec^2 a\,da\right\}da\right]$$

$$= 3a\tan a - \frac{3}{2}\int \tan a\,da$$

$$= 3a\tan a - \frac{3}{2}\ln|\sec a| + c$$

Replacing the value of a we get,

$\therefore 3a\tan a - \frac{3}{2}\ln|\sec a| + c$

$= \mathbf{3x\tan^{-1}x - \frac{3}{2}\ln\sqrt{1+x^2} + c}$, where c is the integrating constant

Question: 49

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = 1/x^2$,

$$\therefore \int \frac{\sin^{-1}x}{x^2}dx$$

$$= \sin^{-1}x\int\frac{1}{x^2}dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int\frac{1}{x^2}dx\right\}dx$$

$$= \frac{-\sin^{-1}x}{x} - \int\frac{1}{\sqrt{1-x^2}}\times\left(-\frac{1}{x}\right)dx$$

$$= \frac{-\sin^{-1}x}{x} + \int\frac{1}{x\sqrt{1-x^2}}dx$$

Taking x= sina, dx = cosada

Hence, coseca=1/x

Now, $\mathrm{cosec}^2a-\cot^2a = 1$ so $\cot a=\sqrt{(1-x^2)}/x$

$$\therefore \int\frac{1}{x\sqrt{1-x^2}}dx$$

$$= \int\frac{1}{\sin a\cos a}(\cos a\,da)$$

$$= \int \mathrm{cosec}\,a\,da$$

$$= \ln|\mathrm{cosec}\,a - \cot a| + c$$

Replacing the value of a, we get,

$$\therefore \ln|\mathrm{cosec}\,a - \cot a| + c$$

$$= \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c$$

The total integration yields as

$= \mathbf{\frac{-\sin^{-1}x}{x} + \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c}$, where c is the integrating constant

Question: 50

Evaluate the foll

Solution:

Say, tanx = a

Hence, $\sec^2 x dx = da$

$$\therefore \int \frac{\tan x \sec^2 x}{1 - \tan^2 x} dx$$

$$= \int \frac{a da}{1 - a^2}$$

Now, taking $1-a^2 = k$, $-2ada=dk$ i.e. $ada=-dk/2$

$$\therefore \int \frac{a da}{1 - a^2}$$

$$= \int \frac{-dk}{2k}$$

$$= -\frac{1}{2} \ln|k| + c$$

Replacing the value of k,

$$-\frac{1}{2} \ln|k| + c$$

$$= -\frac{1}{2} \ln|1 - a^2| + c$$

Replacing the value of a,

$$-\frac{1}{2} \ln|1 - a^2| + c$$

$= -\frac{1}{2} \mathbf{\ln|1 - \tan^2 x| + c}$, where c is the integrating constant

Question: 51

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin 4x$ and $f_2(x) = e^{3x}$,

$$\therefore \int e^{3x} \sin 4x dx$$

$$= \sin 4x \int e^{3x} dx - \int \left\{ \frac{d}{dx} (\sin 4x) \int e^{3x} dx \right\} dx$$

$$= \frac{e^{3x} \sin 4x}{3} - \int 4\cos 4x \times \frac{e^{3x}}{3} dx$$

$$= \frac{e^{3x} \sin 4x}{3} - \frac{4}{3} \int e^{3x} \cos 4x dx$$

$$= \frac{e^{3x} \sin 4x}{3} - \frac{4}{3} \left[\cos 4x \int e^{3x} dx - \int \left\{ \frac{d}{dx} (\cos 4x) \int e^{3x} dx \right\} dx \right]$$

$$= \frac{e^{3x} \sin 4x}{3} - \frac{4e^{3x} \cos 4x}{9} - \frac{4}{3} \int 4\sin 4x \times \frac{e^{3x}}{3} dx$$

$$= \frac{e^{3x} \sin 4x}{3} - \frac{4e^{3x} \cos 4x}{9} - \frac{16}{9} \int e^{3x} \sin 4x dx$$

$$\therefore \left(1+\frac{16}{9}\right)\int e^{3x}\sin 4x dx = \frac{e^{3x}\sin 4x}{3} - \frac{4e^{3x}\cos 4x}{9} + c_1$$

$$\Rightarrow \frac{25}{9}\int e^{3x}\sin 4x dx = \frac{3e^{3x}\sin 4x - 4e^{3x}\cos 4x}{9} + c_1$$

$\Rightarrow \int e^{3x}\sin 4x dx = \frac{e^{3x}}{25}(\mathbf{3\sin 4x - 4\cos 4x}) + \mathbf{c}$, where c is the integrating constant

Question: 52

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\sin x dx$$

$$= \sin x\int e^{2x}dx - \int\left\{\frac{d}{dx}(\sin x)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\sin x}{2} - \int \cos x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x\int e^{2x}dx - \int\left\{\frac{d}{dx}(\cos x)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{2}\int \sin x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}\int e^{2x}\sin x dx$$

$$\therefore \left(1+\frac{1}{4}\right)\int e^{2x}\sin x dx = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} + c_1$$

$$\Rightarrow \frac{5}{4}\int e^{2x}\sin x dx = \frac{2e^{2x}\sin x - e^{2x}\cos x}{4} + c_1$$

$\Rightarrow \int e^{2x}\sin x dx = \frac{e^{2x}}{5}(\mathbf{2\sin x - \cos x}) + \mathbf{c}$, where c is the integrating constant

Question: 53

Evaluate the foll

Solution:

$$\int e^{2x}\sin x\cos x dx$$

$$= \frac{1}{2}\int e^{2x} \times 2\sin x\cos x dx$$

$$= \frac{1}{2}\int e^{2x}\sin 2x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = sin2x and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\sin 2x dx$$

$$= \sin 2x \int e^{2x}dx - \int\left\{\frac{d}{dx}(\sin 2x)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \int 2\cos 2x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \int e^{2x}\cos 2x dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \left[\cos 2x\int e^{2x}dx - \int\left\{\frac{d}{dx}(\cos 2x)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} - \int 2\sin 2x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} - \int e^{2x}\sin x dx$$

$$\therefore (1+1)\int e^{2x}\sin 2x dx = \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} + c_1$$

$$\Rightarrow 2\int e^{2x}\sin 2x dx = \frac{e^{2x}\sin 2x - e^{2x}\cos 2x}{2} + c_1$$

$$\Rightarrow \int e^{2x}\sin 2x dx = \frac{e^{2x}}{4}(\sin 2x - \cos 2x) + c'$$

$$\therefore \frac{1}{2}\int e^{2x}\sin 2x dx$$

$$= \frac{1}{2} \times \left[\frac{e^{2x}}{4}(\sin 2x - \cos 2x) + c'\right]$$

$= \frac{e^{2x}}{8}(\mathbf{\sin 2x - \cos 2x}) + \mathbf{c}$, where c is the integrating constant

Question: 54

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = cos(3x+4) and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\cos(3x+4)\,dx$$

$$= \cos(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\cos(3x+4)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \int 3\sin(3x+4) \times \frac{e^{2x}}{2}dx$$

$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\int e^{2x}\sin(3x+4)dx$

$= \frac{e^{2x}\cos(3x+4)}{2}$

$+ \frac{3}{2}\left[\sin(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\sin(3x+4)\int e^{2x}dx\right\}dx\right]$

$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{3}{2}\int 3\cos(3x+4) \times \frac{e^{2x}}{2}dx$

$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{9}{4}\int e^{2x}\cos(3x+4)dx$

$\therefore \left(1+\frac{9}{4}\right)\int e^{2x}\cos(3x+4)dx = \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} + c_1$

$\Rightarrow \frac{13}{4}\int e^{2x}\cos(3x+4)\,dx = \frac{2e^{2x}\cos(3x+4) + 3e^{2x}\sin(3x+4)}{4} + c_1$

$\Rightarrow \int e^{2x}\cos(3x+4)\,dx = \frac{e^{2x}}{13}\left(\mathbf{2\cos(3x+4) + 3\sin(3x+4)}\right) + \mathbf{c}$, where c is the integrating constant

Question: 55

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = cosx and $f_2(x) = e^{-x}$,

$\therefore \int e^{-x}\cos x\,dx$

$= \cos x\int e^{-x}dx - \int\left\{\frac{d}{dx}\cos x\int e^{-x}dx\right\}dx$

$= -e^{-x}\cos x - \int e^{-x}\sin x\,dx$

$= -e^{-x}\cos x - \left[\sin x\int e^{-x}dx - \int\left\{\frac{d}{dx}\sin x\int e^{-x}dx\right\}dx\right]$

$= -e^{-x}\cos x - \left[-e^{-x}\sin x + \int e^{-x}\cos x dx\right]$

$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x dx$

$\therefore (1+1)\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$

$\Rightarrow 2\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$

$\Rightarrow \int e^{-x}\cos x dx = \frac{e^{-x}}{2}(\mathbf{\sin x - \cos x}) + \mathbf{c}$, where c is the integrating constant

Question: 56

Evaluate the foll

Solution:

$\int e^x(\sin x + \cos x)dx$

$= \int e^x \sin x dx + \int e^x \cos x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \sin x dx + \int e^x \cos x dx$

$= \sin x \int e^x dx - \int \left[\frac{d}{dx}(\sin x) \int e^x dx\right] dx + \int e^x \cos x dx$

$= e^x \sin x - \int e^x \cos x dx + \int e^x \cos x dx + c$

$= \mathbf{e^x \sin x + c}$, where c is the integrating constant

Question: 57

Evaluate the foll

Solution:

$\int e^x(\cot x - \mathrm{cosec}^2 x)dx$

$= \int e^x \cot x dx + \int e^x \mathrm{cosec}^2 x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \cot x dx + \int e^x \mathrm{cosec}^2 x dx$

$= \cot x \int e^x dx - \int \left[\frac{d}{dx}(\cot x) \int e^x dx\right] dx + \int e^x \mathrm{cosec}^2 x dx$

$= e^x \cot x - \int e^x \mathrm{cosec}^2 x dx + \int e^x \mathrm{cosec}^2 x dx + c$

$= \mathbf{e^x \cot x + c}$, where c is the integrating constant

Question: 58

Evaluate the foll

Solution:

$\int e^x \sec x(1 + \tan x)dx$

$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = secx and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \sec x dx + \int e^x \sec x \tan x dx$$

$$= \sec x \int e^x dx - \int\left[\frac{d}{dx}(\sec x)\int e^x dx\right]dx + \int e^x \sec x \tan x dx$$

$$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c$$

$= \mathbf{e^x \sec x + c}$, where c is the integrating constant

Question: 59

Evaluate the foll

Solution:

$$\int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$$

$$= \int e^x \tan^{-1}x\, dx + \int \frac{e^x}{1+x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan^{-1}x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \tan^{-1}x\, dx + \int \frac{e^x}{1+x^2}dx$$

$$= \tan^{-1}x \int e^x dx - \int\left[\frac{d}{dx}(\tan^{-1}x)\int e^x dx\right]dx + \int \frac{e^x}{1+x^2}dx$$

$$= e^x \tan^{-1}x - \int \frac{e^x}{1+x^2}dx + \int \frac{e^x}{1+x^2}dx + c$$

$= \mathbf{e^x \tan^{-1}x + c}$, where c is the integrating constant

Question: 60

Evaluate the foll

Solution:

$$\int e^x(\cot x + \log\sin x)dx$$

$$= \int e^x \cot x\, dx + \int e^x \log\sin x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logsinx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \cot x\, dx + \int e^x \log\sin x dx$$

$$= \int e^x \cot x\,dx + \log\sin x \int e^x dx - \int \left[\frac{d}{dx}(\log\sin x)\int e^x dx\right]$$

$$= \int e^x \cot x\,dx + e^x \log\sin x - \int e^x \cot x\,dx + c$$

$= \mathbf{e^x \log|\sin x| + c}$, where c is the integrating constant

Question: 61

Evaluate the foll

Solution:

$$\int e^x(\tan x + \log\cos x)dx$$

$$= \int e^x \tan x\,dx + \int e^x \log\cos x\,dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \tan x\,dx - \int e^x \log\cos x\,dx$$

$$= \int e^x \tan x\,dx - \log\cos x \int e^x dx + \int \left[\frac{d}{dx}(\log\cos x)\int e^x dx\right]$$

$$= \int e^x \tan x\,dx - e^x \log\cos x - \int e^x \tan x\,dx + c$$

$= \mathbf{e^x \log|\sec x| + c}$, where c is the integrating constant

Question: 62

Evaluate the foll

Solution:

$$\int e^x[\sec x + \log(\sec x + \tan x)]dx$$

$$= \int e^x \sec x\,dx + \int e^x \log(\sec x + \tan x)dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \sec x\,dx + \int e^x \log(\sec x + \tan x)dx$$

$$= \int e^x \sec x\,dx + \log(\sec x + \tan x)\int e^x dx - \int \left[\frac{d}{dx}(\log(\sec x + \tan x))\int e^x dx\right]$$

$$= \int e^x \sec x\, dx + e^x \log(\sec x + \tan x) - \int \frac{e^x \tan x \times (\sec^2 x + \sec x \tan x)\, dx}{\sec x + \tan x} + c$$

$$= \int e^x \sec x\, dx + e^x \log(\sec x + \tan x) - \int e^x \sec x\, dx + c$$

$= \mathbf{e^x \log|\sec x + \tan x| + c}$, where c is the integrating constant

Question: 63

Evaluate the foll

Solution:

$$\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x}\right) dx$$

$$= \int e^x (\sec^2 x + \tan x) dx$$

$$= \int e^x \sec^2 x dx + \int e^x \tan x dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \sec^2 x dx + \int e^x \tan x dx$$

$$= \int e^x \sec^2 x dx + \tan x \int e^x dx - \int \left[\frac{d}{dx}(\tan x) \int e^x dx\right]$$

$$= \int e^x \sec^2 x\, dx + e^x \tan x - \int e^x \sec^2 x\, dx + c$$

$= \mathbf{e^x \tan x + c}$, where c is the integrating constant

Question: 64

Evaluate the foll

Solution:

$$\int e^x \left(\frac{\sin x \cos x - 1}{\sin^2 x}\right) dx$$

$$= \int e^x (\cot x - \mathrm{cosec}^2 x) dx$$

$$= \int e^x \cot x dx - \int e^x \mathrm{cosec}^2 x dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \cot x dx - \int e^x \mathrm{cosec}^2 x dx$$

$= \cot x \int e^x dx - \int \left\{\frac{d}{dx}(\cot x) \int e^x dx\right\} dx - \int e^x \operatorname{cosec}^2 x dx$

$= e^x \cot x + \int e^x \operatorname{cosec}^2 x dx - \int e^x \operatorname{cosec}^2 x dx + c$

$= \mathbf{e^x \cot x + c}$, where c is the integrating constant

Question: 65

Evaluate the foll

Solution:

$\int e^x \left(\frac{\cos x + \sin x}{\cos^2 x}\right) dx$

$= \int e^x (\sec x + \sec x \tan x) dx$

$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \sec x dx + \int e^x \sec x \tan x dx$

$= \sec x \int e^x dx - \int \left[\frac{d}{dx}(\sec x) \int e^x dx\right] dx + \int e^x \sec x \tan x dx$

$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c$

$= \mathbf{e^x \sec x + c}$, where c is the integrating constant

Question: 66

Evaluate the foll

Solution:

$\int e^x \left(\frac{2 - \sin 2x}{1 - \cos 2x}\right) dx$

$= \int e^x \left(\frac{1 - \sin x \cos x}{\sin^2 x}\right) dx$

$= \int e^x (\operatorname{cosec}^2 x - \cot x) dx$

$= \int e^x \operatorname{cosec}^2 x dx - \int e^x \cot x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int e^x \operatorname{cosec}^2 x dx - \int e^x \cot x dx$

$$= \int e^x \operatorname{cosec}^2 x dx - \cot x \int e^x dx + \int \left\{\frac{d}{dx}(\cot x) \int e^x dx\right\} dx$$

$$= \int e^x \operatorname{cosec}^2 x dx - e^x \cot x - \int e^x \operatorname{cosec}^2 x dx$$

$= \mathbf{-e^x \cot x + c}$, where c is the integrating constant

Question: 67

Evaluate the foll

Solution:

$$\left(\frac{1 + \sin x}{1 + \cos x}\right)$$

$$= \left(\frac{1 + \frac{2\tan {}^x/_2}{1 + \tan^2 ({}^x/_2)}}{1 + \frac{1 - \tan^2 ({}^x/_2)}{1 + \tan^2 ({}^x/_2)}}\right)$$

$$= \frac{\left(1 + \tan {}^x/_2\right)^2}{2}$$

$$\therefore \int e^x \left(\frac{1 + \sin x}{1 + \cos x}\right) dx$$

$$= \int e^x \times \frac{\left(1 + \tan {}^x/_2\right)^2}{2}$$

$$= \int \frac{e^x\left(1 + \tan^2 {}^x/_2 + 2\tan {}^x/_2\right)}{2} dx$$

$$= \int \frac{e^x\left(\sec^2 {}^x/_2 + 2\tan {}^x/_2\right)}{2} dx$$

$$= \int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + \int e^x \tan {}^x/_2 \, dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan(x/2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + \int e^x \tan {}^x/_2 \, dx$$

$$= \int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + \tan {}^x/_2 \int e^x dx - \int \left[\frac{d}{dx}\left(\tan {}^x/_2\right) \int e^x dx\right] dx$$

$$= \int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + e^x \tan {}^x/_2 - \int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + c$$

$= \mathbf{e^x \tan {}^x/_2 + c}$, where c is the integrating constant

Question: 68

Evaluate the foll

Solution:

$$\int e^x \left(\frac{\sin 4x - 1}{1 - \cos 4x}\right) dx$$

$= \int e^x\left(\frac{2\sin 2x\cos 2x - 4}{2\sin^2 2x}\right)dx$

$= \int e^x(\cot 2x - 2\mathrm{cosec}^2 2x)dx$

$= \int e^x\cot 2x dx - \int 2e^x\mathrm{cosec}^2 2x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot 2x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x\cot 2x dx - \int 2e^x\mathrm{cosec}^2 2x dx$

$= \cot 2x\int e^x dx - \int\left\{\frac{d}{dx}(\cot 2x)\int e^x dx\right\}dx - \int 2e^x\mathrm{cosec}^2 2x dx$

$= e^x\cot 2x + \int 2e^x\mathrm{cosec}^2 2x dx - \int 2e^x\mathrm{cosec}^2 2x dx + c$

$= \mathbf{e^x\cot 2x + c}$, where c is the integrating constant

Question: 69

Evaluate the foll

Solution:

$\int \frac{e^x\left[\sqrt{1-x^2}\sin^{-1}x + 1\right]}{\sqrt{1-x^2}}dx$

$= \int e^x\left(\sin^{-1}x + \frac{1}{\sqrt{1-x^2}}\right)dx$

$= \int e^x\sin^{-1}x\,dx + \int \frac{e^x}{\sqrt{1-x^2}}dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x\sin^{-1}x\,dx + \int \frac{e^x}{\sqrt{1-x^2}}dx$

$= \sin^{-1}x\int e^x dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int e^x dx\right\}dx + \int \frac{e^x}{\sqrt{1-x^2}}dx$

$= e^x\sin^{-1}x - \int \frac{e^x}{\sqrt{1-x^2}}dx + \int \frac{e^x}{\sqrt{1-x^2}}dx + c$

$= \mathbf{e^x\sin^{-1}x + c}$, where c is the integrating constant

Question: 70

Evaluate the foll

Solution:

$$\int e^x\left(\frac{1+x\log x}{x}\right)dx$$

$$=\int e^x\left(\frac{1}{x}+\log x\right)dx$$

$$=\int \frac{e^x}{x}dx+\int e^x\log x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{x}dx+\int e^x\log x dx$$

$$=\int \frac{e^x}{x}dx+\log x\int e^x dx-\int\left[\frac{d}{dx}(\log x)\int e^x dx\right]dx$$

$$=\int \frac{e^x}{x}dx+e^x\log x-\int \frac{e^x}{x}dx+c$$

$= \mathbf{e^x\log x + c}$, where c is the integrating constant

Question: 71

Evaluate the foll

Solution:

$$\frac{x}{(1+x)^2}=\frac{A}{(1+x)}+\frac{B}{(1+x)^2}$$

$\Rightarrow x = A(1+x)+B$

For x=-1, equation: -1 = B i.e. B = -1

For x=0, equation: 0 = A-1 i.e. A = 1

$$\therefore \frac{x}{(1+x)^2}$$

$$=\frac{1}{(1+x)}-\frac{1}{(1+x)^2}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(1+x)}-\frac{1}{(1+x)^2}\right]dx$$

$$=\int e^x\times\frac{1}{(1+x)}dx-\int e^x\times\frac{1}{(1+x)^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(1+x)}dx-\int \frac{e^x}{(1+x)^2}dx$$

$$=\frac{1}{(1+x)}\int e^x dx-\int\left[\frac{d}{dx}\left(\frac{1}{1+x}\right)\int e^x dx\right]dx-\int \frac{e^x}{(1+x)^2}dx$$

$= \frac{e^x}{(1+x)} + \int \frac{e^x}{(1+x)^2}dx - \int \frac{e^x}{(1+x)^2}dx + c$

$= \frac{\mathbf{e^x}}{\mathbf{(1+x)}} + \mathbf{c}$, where c is the integrating constant

Question: 72

Evaluate the foll

Solution:

$\frac{x-1}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$

$\Rightarrow x - 1 = A(x+1)^2 + B(x+1) + C$

For x=-1, equation: -2 = C i.e. C = -2

For x=0, equation: -1 = A+B-2 i.e. A+B = 1

For x=1, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇨1+A = 1

⇨A = 0

And, B = 1

$\therefore \frac{x-1}{(x+1)^3}$

$= \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}$

The given equation becomes

$\int e^x \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right]dx$

$= \int e^x \times \frac{1}{(x+1)^2}dx - \int e^x \times \frac{2}{(x+1)^3}dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int \frac{e^x}{(x+1)^2}dx - \int \frac{2e^x}{(x+1)^3}dx$

$= \frac{1}{(x+1)^2}\int e^x dx - \int\left[\frac{d}{dx}\left(\frac{1}{(x+1)^2}\right)\int e^x dx\right]dx - \int \frac{2e^x}{(x+1)^3}dx$

$= \frac{e^x}{(x+1)^2} + \int \frac{2e^x}{(x+1)^3}dx - \int \frac{2e^x}{(x+1)^3}dx + c$

$= \frac{\mathbf{e^x}}{\mathbf{(x+1)^2}} + \mathbf{c}$, where c is the integrating constant

Question: 73

Evaluate the foll

Solution:

$$\frac{2-x}{(1-x)^2}=\frac{A}{(1-x)}+\frac{B}{(1-x)^2}$$

$\Rightarrow 2-x=A(1-x)+B$

For x=1, equation: 1 = B i.e. B = 1

For x=2, equation: 0 = -A+1 i.e. A = 1

$\therefore \frac{2-x}{(1-x)^2}$

$=\frac{1}{(1-x)}+\frac{1}{(1-x)^2}$

The given equation becomes

$$\int e^x\left[\frac{1}{(1-x)}+\frac{1}{(1-x)^2}\right]dx$$

$$=\int e^x\times\frac{1}{(1-x)^2}dx+\int e^x\times\frac{1}{1-x}dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(1-x) and $f_2(x)$ = e^x in the second integral and keeping the first integral intact,

$$\int\frac{e^x}{(1-x)^2}dx+\int\frac{e^x}{1-x}dx$$

$$=\int\frac{e^x}{(1-x)^2}dx+\frac{1}{1-x}\int e^x dx-\int\left[\frac{d}{dx}\left(\frac{1}{1-x}\right)\int e^x dx\right]dx$$

$$=\int\frac{e^x}{(1-x)^2}dx+\frac{e^x}{1-x}-\int\frac{e^x}{(1-x)^2}dx+c$$

$=\frac{e^x}{1-x}+c$, where c is the integrating constant

Question: 74

Evaluate the foll

Solution:

$$\frac{x-3}{(x-1)^3}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$$

$\Rightarrow x-3=A(x-1)^2+B(x-1)+C$

For x=1, equation: -2 = C i.e. C = -2

For x=0, equation: -3 = A-B-2 i.e. B = A+1

For x=3, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇨1+3A = 1

⇨A = 0

And, B = 1

$\therefore \frac{x-3}{(x-1)^3}$

$$= \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right] dx$$

$$= \int e^x \times \frac{1}{(x-1)^2} dx - \int e^x \times \frac{2}{(x-1)^3} dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(x-1)^2} dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{1}{(x-1)^2} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right) \int e^x dx\right] dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{e^x}{(x-1)^2} + \int \frac{2e^x}{(x-1)^3} dx - \int \frac{2e^x}{(x-1)^3} dx + c$$

$= \frac{e^x}{(x-1)^2} + c$, where c is the integrating constant

Question: 75

Evaluate the foll

Solution:

$$\int e^{3x} \left(\frac{3x-1}{9x^2}\right) dx$$

$$= \int \frac{e^{3x}}{3x} dx - \int \frac{e^{3x}}{9x^2} dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/3x$and $f_2(x) = e^{3x}$ in the first integral and keeping the second integral intact,

$$\int \frac{e^{3x}}{3x} dx - \int \frac{e^{3x}}{9x^2} dx$$

$$= \frac{1}{3x} \int e^{3x} dx - \int \left[\frac{d}{dx}\left(\frac{1}{3x}\right) \int e^{3x} dx\right] dx - \int \frac{e^{3x}}{9x^2} dx$$

$$= \frac{e^{3x}}{9x} + \int \frac{e^{3x}}{9x^2} dx - \int \frac{e^{3x}}{9x^2} dx + c$$

$= \frac{e^{3x}}{9x} + c$, where c is the integrating constant

Question: 76

Evaluate the foll

Solution:

$$\frac{x+1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$$

$\Rightarrow x + 1 = A(x+2) + B$

For x=-2, equation: -1 = B i.e. B = -1

For x=-1, equation: 0 = A-1 i.e. A = 1

$$\therefore \frac{x+1}{(x+2)^2}$$

$$= \frac{1}{(x+2)} - \frac{1}{(x+2)^2}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(x+2)} - \frac{1}{(x+2)^2}\right] dx$$

$$= \int e^x \times \frac{1}{x+2} dx - \int e^x \times \frac{1}{(x+2)^2} dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(x+2) and $f_2(x)$ = e^x in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{x+2} dx - \int \frac{e^x}{(x+2)^2} dx$$

$$= \frac{1}{x+2} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{x+2}\right) \int e^x dx\right] dx - \int \frac{e^x}{(x+2)^2} dx$$

$$= \frac{e^x}{x+2} + \int \frac{e^x}{(x+2)^2} dx - \int \frac{e^x}{(x+2)^2} dx + c$$

$= \frac{e^x}{x+2} + \mathbf{c}$, where c is the integrating constant

Question: 77

Evaluate the foll

Solution:

$$\frac{x}{(1+2x)^2} = \frac{A}{(1+2x)} + \frac{B}{(1+2x)^2}$$

$\Rightarrow x = A(1+2x) + B$

For x=-1/2, equation: -1/2 = B i.e. B = -1/2

For x=0, equation: 0 = A-1/2 i.e. A = 1/2

$$\therefore \frac{x}{(1+2x)^2}$$

$$= \frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}$$

The given equation becomes

$$\int e^{2x} \left[\frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}\right] dx$$

$$= \int e^{2x} \times \frac{1}{2(1+2x)} dx - \int e^{2x} \times \frac{1}{2(1+2x)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+2x)$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int e^{2x}\times\frac{1}{2(1+2x)}dx - \int e^{2x}\times\frac{1}{2(1+2x)^2}dx$$

$$=\frac{1}{2}\left[\frac{1}{1+2x}\int e^{2x}dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+2x}\right)\int e^{2x}dx\right]dx - \int\frac{e^{2x}}{(1+2x)^2}dx\right]$$

$$=\frac{1}{2}\left[\frac{e^{2x}}{2(2x+1)} + \int\frac{e^{2x}}{(2x+1)^2}dx - \int\frac{e^{2x}}{(2x+1)^2}dx\right]$$

$=\frac{e^{2x}}{4(2x+1)} + c$, where c is the integrating constant

Question: 78

Evaluate the foll

Solution:

$$\int e^{2x}\left(\frac{2x-1}{4x^2}\right)dx$$

$$=\int\frac{e^{2x}}{2x}dx - \int\frac{e^{2x}}{4x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/2x$ and $f_2(x) = e^{2x}$ in the first integral and keeping the second integral intact,

$$\int\frac{e^{2x}}{2x}dx - \int\frac{e^{2x}}{4x^2}dx$$

$$=\frac{1}{2x}\int e^{2x}dx - \int\left[\frac{d}{dx}\left(\frac{1}{2x}\right)\int e^{2x}dx\right]dx - \int\frac{e^{2x}}{4x^2}dx$$

$$=\frac{e^{2x}}{4x} + \int\frac{e^{2x}}{4x^2}dx - \int\frac{e^{2x}}{4x^2}dx + c$$

$=\frac{e^{2x}}{4x} + c$, where c is the integrating constant

Question: 79

Evaluate the foll

Solution:

$$\int e^x\left(\log x + \frac{1}{x^2}\right)dx$$

$$=\int e^x\log x\,dx - \int\frac{e^x}{x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \log x dx - \int \frac{e^x}{x^2} dx$$

$$= \log x \int e^x dx - \int \left[\frac{d}{dx}(\log x) \int e^x dx\right] dx - \int \frac{e^x}{x^2} dx$$

$$= e^x \log x - \int \frac{e^x}{x} dx - \int \frac{e^x}{x^2} dx$$

$$= e^x \log x - \left[\frac{1}{x} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{x}\right) \int e^x dx\right] dx\right] - \int \frac{e^x}{x^2} dx$$

$$= e^x \log x - \frac{e^x}{x} + \int \frac{e^x}{x^2} dx - \int \frac{e^x}{x^2} dx + c$$

$= \mathbf{e^x \left(\log x - \frac{1}{x}\right) + c}$, where c is the integrating constant

Question: 80

Evaluate the foll

Solution:

$$\frac{\log x}{(1 + \log x)^2} = \frac{A}{(1 + \log x)} + \frac{B}{(1 + \log x)^2}$$

$\Rightarrow \log x = A(1 + \log x) + B$

For x=1, equation: 0 = A+B

For x=1/e, equation: -1 = B i.e. B = -1

So, A = 1

$$\therefore \frac{\log x}{(1 + \log x)^2}$$

$$= \frac{1}{(1 + \log x)} - \frac{1}{(1 + \log x)^2}$$

The given equation becomes

$$\int \left[\frac{1}{(1 + \log x)} - \frac{1}{(1 + \log x)^2}\right] dx$$

$$= \int \frac{1}{(1 + \log x)} dx - \int \frac{1}{(1 + \log x)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+\log x)$ and $f_2(x) = 1$in the second integral and keeping the first integral intact,

$$\int \frac{1}{(1 + \log x)} dx - \int \frac{1}{(1 + \log x)^2} dx$$

$$= \frac{1}{(1 + \log x)} \int dx - \int \left[\frac{d}{dx}\left(\frac{1}{(1 + \log x)}\right) \int dx\right] dx - \int \frac{1}{(1 + \log x)^2} dx$$

$$= \frac{x}{(1 + \log x)} + \int \frac{1}{(1 + \log x)^2} dx - \int \frac{1}{(1 + \log x)^2} dx + c$$

$= \mathbf{\frac{x}{(1 + \log x)} + c}$, where c is the integrating constant

Question: 81

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = sin(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \sin(\log x)dx + \int \cos(\log x)dx$$

$$= \sin(\log x)\int dx - \int\left[\frac{d}{dx}(\sin(\log x))\int dx\right]dx + \int \cos(\log x)dx$$

$$= x\sin(\log x) - \int \cos(\log x)\,dx + \int \cos(\log x)dx + c$$

$= \mathbf{e^{\log x}\sin(\log x) + c}$, where c is the integrating constant

Question: 82

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \frac{1}{\log x}dx - \int \frac{1}{(\log x)^2}dx$$

$$= \frac{1}{\log x}\int dx - \int\left[\frac{d}{dx}\left(\frac{1}{\log x}\right)\int dx\right]dx - \int \frac{1}{(\log x)^2}dx$$

$$= \frac{x}{\log x} + \int \frac{1}{(\log x)^2}dx - \int \frac{1}{(\log x)^2}dx + c$$

$= \mathbf{\frac{x}{\log x} + c}$, where c is the integrating constant

Question: 83

Evaluate the foll

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = log(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \log(\log x)dx + \int \frac{1}{(\log x)^2}dx$$

$$= \log(\log x)\int dx - \int\left[\frac{d}{dx}(\log(\log x))\int dx\right]dx + \int \frac{1}{(\log x)^2}dx$$

$= x\log(\log x) - \int \frac{1}{\log x}dx + \int \frac{1}{(\log x)^2}dx$

$= x\log(\log x) - \left[\frac{1}{\log x}\int dx - \int \left[\frac{d}{dx}\left(\frac{1}{\log x}\right)\int dx\right]dx\right] + \int \frac{1}{(\log x)^2}dx$

$= x\log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2}dx + \int \frac{1}{(\log x)^2}dx + c$

$= \mathbf{x}\left[\mathbf{\log(\log x)} - \frac{\mathbf{1}}{\mathbf{\log x}}\right] + \mathbf{c}$, where c is the integrating constant

Question: 84

Evaluate the foll

Solution:

It is know that $\sin^{-1}x+\cos^{-1}x = \pi/2$

$\therefore \left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right)$

$= \frac{2}{\pi}\left(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\right)$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Now, for the first term,

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$\therefore \int \sin^{-1}\sqrt{x}\,dx$

$= \sin^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right)\int dx\right\}dx$

$= x\sin^{-1}\sqrt{x} - \int \frac{1}{2\sqrt{x}\sqrt{1-x}} \times x\,dx$

$= x\sin^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$

$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$

$= -\int \sqrt{1-a^2}\,da$

$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$

Replacing the value of a, we get,

$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$

$= -\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c$

The total integration yields as

$= x\sin^{-1}\sqrt{x}+\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c'$, where c' is the integrating constant

For the second term,

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$\therefore \int \cos^{-1}\sqrt{x}\,dx$

$= \cos^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$

$= x\cos^{-1}\sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}}\times x\,dx$

$= x\cos^{-1}\sqrt{x}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$

$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$

$= -\int \sqrt{1-a^2}\,da$

$= -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$

Replacing the value of a, we get,

$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$

$= -\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c$

The total integration yields as

$= x\cos^{-1}\sqrt{x}-\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c''$, where c'' is the integrating constant

$\therefore \int\left(\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}\right)dx$

$= \frac{2}{\pi}\int\left(\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}\right)dx$

$$= \frac{2}{\pi}\Bigg[x\sin^{-1}\sqrt{x}+\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]-x\cos^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]\Bigg]+c$$

$= \frac{2}{\pi}\left[\sqrt{x - x^2} + x\left(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\right) + \sin^{-1}\sqrt{1 - x}\right] + c$ where c is the integrating constant

Question: 85

Evaluate the foll

Solution:

Tip – 5^x is to be replaced by a

$\therefore 5^x = a$

$\Rightarrow 5^x \log 5 dx = da$

$\Rightarrow 5^x dx = \frac{da}{\log 5}$

The equation becomes as follows:

$$\int 5^{5^a} \times 5^a \times \frac{1}{\log 5} da$$

Tip – 5^a is to be replaced by k

$\therefore 5^a = k$

$\Rightarrow 5^a \log 5 da = dk$

$\Rightarrow 5^a da = \frac{dk}{\log 5}$

The equation becomes as follows:

$$\int 5^k \times \frac{1}{(\log 5)^2} dk$$

$$= \frac{1}{(\log 5)^2} \int 5^k dk$$

$$= \frac{5^k}{(\log 5)^3} + c$$

Re-replacing the value of k,

$$\frac{5^{5^a}}{(\log 5)^3} + c$$

Re-replacing the value of a,

$\frac{5^{5^{5^x}}}{(\log 5)^3} + c$, where c is the integrating constant

Question: 86

Evaluate the foll

Solution:

$$\left(\frac{1 + \sin 2x}{1 + \cos 2x}\right)$$

$$= \left(\frac{1 + \frac{2\tan x}{1 + \tan^2 x}}{1 + \frac{1 - \tan^2 x}{1 + \tan^2 x}}\right)$$

$$= \frac{(1 + \tan x)^2}{2}$$

$$\therefore \int e^{2x}\left(\frac{1 + \sin 2x}{1 + \cos 2x}\right) dx$$

$= \int e^{2x} \times \frac{(1+\tan x)^2}{2}$

$= \int \frac{e^{2x}(1+\tan^2 x + 2\tan x)}{2} dx$

$= \int \frac{e^{2x}(\sec^2 x + 2\tan x)}{2} dx$

$= \int \frac{e^{2x}\sec^2 x dx}{2} + \int e^{2x}\tan x\, dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$\int \frac{e^{2x}\sec^2 x dx}{2} + \int e^{2x}\tan x\, dx$

$= \int \frac{e^{2x}\sec^2 x dx}{2} + \tan x \int e^{2x} dx - \int \left[\frac{d}{dx}(\tan x) \int e^{2x} dx\right] dx$

$= \int \frac{e^{2x}\sec^2 x dx}{2} + \frac{1}{2} e^{2x}\tan x - \int \frac{e^{2x}\sec^2 x dx}{2} + c$

$= \mathbf{\frac{1}{2} e^x \tan {}^x\!/_2 + c}$, where c is the integrating constant

Question: 87

Evaluate the foll

Solution:

$\left(\frac{1-\sin 2x}{1-\cos 2x}\right)$

$= \left(\frac{1 - \frac{2\tan x}{1+\tan^2 x}}{1 - \frac{1-\tan^2 x}{1+\tan^2 x}}\right)$

$= \frac{(1-\tan x)^2}{2}$

$\therefore \int e^{2x} \left(\frac{1-\sin 2x}{1-\cos 2x}\right) dx$

$= \int e^{2x} \times \frac{(1-\tan x)^2}{2}$

$= \int \frac{e^{2x}(1+\tan^2 x - 2\tan x)}{2} dx$

$= \int \frac{e^{2x}(\sec^2 x - 2\tan x)}{2} dx$

$= \int \frac{e^{2x}\sec^2 x dx}{2} - \int e^{2x}\tan x\, dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int \frac{e^{2x}\sec^2 x dx}{2} - \int e^{2x}\tan x dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} - \tan x \int e^{2x}dx + \int \left[\frac{d}{dx}(\tan x) \int e^{2x}dx\right] dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} - \frac{1}{2}e^{2x}\tan x + \int \frac{e^{2x}\sec^2 x dx}{2} + c$$

$= -\frac{1}{2}e^x \tan {}^x/_2 + c$, where c is the integrating constant

Exercise : OBJECTIVE QUESTIONS II

Question: 1

Mark (√) against

Solution:

To find: Value of $\int x\, e^x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int x\, e^x dx$ ... (i)

$$I = \int x\, e^x dx$$

$$\Rightarrow x\int e^x dx - \int \left[\frac{d(x)}{x} \int e^x dx\right]dx$$

$$\Rightarrow I = x\, e^x - \int 1.e^x dx$$

$$\Rightarrow I = x\, e^x - e^x + c$$

$$\therefore I = e^x (x-1) + c$$

Ans) c $e^x (x-1) + c$

Question: 2

Mark (√) against

Solution:

To find: Value of $\int x\, e^{2x} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int x\, e^{2x} dx$ ... (i)

$$I = \int x\, e^{2x} dx$$

$$\Rightarrow x\int e^{2x} dx - \int \left[\frac{d(x)}{x} \int e^{2x} dx\right]dx$$

$\Rightarrow I = x\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx$

$\Rightarrow I = x\frac{e^{2x}}{2} - \frac{1}{2}\int \frac{e^{2x}}{2}dx$

$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx$

$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{1}{2}\frac{e^{2x}}{2} + c$

$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

$\therefore I = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

Ans) B $\frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

Question: 3

Mark (√) against

Solution:

To find: Value of $\int x\cos 2x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cos 2x dx$ ... (i)

Let 2x = t

$\Rightarrow x = \frac{t}{2}$

$\Rightarrow 2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2}$

$I = \int \frac{t}{2}\cos t \frac{dt}{2}$

$I = \frac{1}{4}\int t\cos t\, dt$

Taking 1st function as t and second function as $\cos t$

$\Rightarrow I = \frac{1}{4}\left[t\int \cos t\, dt - \int\left(\frac{dt}{dt}\int \cos t\, dt\right)dt\right]$

$\Rightarrow I = \frac{1}{4}\left[t(\sin t) - \int(1(\sin t))dt\right]$

$\Rightarrow I = \frac{1}{4}[t(\sin t) - (-\cos t)] + c$

$\Rightarrow I = \frac{1}{4}[t\sin t + \cos t] + c$

$\Rightarrow I = \frac{1}{4}[2x\sin 2x + \cos 2x] + c$

$\Rightarrow I = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

Ans) A $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

Question: 4

Mark (√) against

Solution:

To find: Value of $\int x\sec^2 x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\sec^2 x\,dx$ ... (i)

Taking 1st function as x and second function as $\sec^2 x$

$\Rightarrow I = \left[x\int \sec^2 x\ dx - \int\left(\frac{dx}{dx}\int \sec^2 x\ dx\right)dx\right]$

$\Rightarrow I = \left[x\tan x - \int (1\tan x)dx\right]$

$\Rightarrow I = \left[x\tan x - \int \tan x dx\right]$

$\Rightarrow I = [x\tan x - (-\log|\cos x|)] + c$

$\Rightarrow I = x\tan x + \log|\cos x| + c$

Ans) B $x\tan x + \log|\cos x| + c$

Question: 5

Mark (√) against

Solution:

To find: Value of $\int x\sin 2x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\sin 2x dx$ ... (i)

Let 2x = t

$\Rightarrow x = \frac{t}{2}$

$\Rightarrow 2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2}$

$I = \int \frac{t}{2}\sin t\frac{dt}{2}$

$I = \frac{1}{4}\int t\sin t\, dt$

Taking 1^{st} function as t and second function as sint

$\Rightarrow I = \frac{1}{4}\left[t\int \sin t\, dt - \int\left(\frac{dt}{dt}\int \sin t\, dt\right)dt\right]$

$\Rightarrow I = \frac{1}{4}\left[t(-\cos t) - \int(1(-\cos t))\,dt\right]$

$\Rightarrow I = \frac{1}{4}\left[-t\cos t - \int -\cos t\, dt\right]$

$\Rightarrow I = \frac{1}{4}[-t\cos t + \sin t]+c$

$\Rightarrow I = \frac{1}{4}[-2x\cos 2x + \sin 2x]+c$

$\Rightarrow I = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x+c$

Ans) C $-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x+c$

Question: 6

Mark (√) against

Solution:

To find: Value of $\int x\log x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\log x\, dx$ ... (i)

Taking 1^{st} function as logx and second function as x

$\Rightarrow I = \left[\log x\int x\, dx - \int\left(\frac{d\log x}{dx}\int x\, dx\right)dx\right]$

$\Rightarrow I = \left[\log x\frac{x^2}{2} - \int\left(\frac{1}{x}\frac{x^2}{2}\right)dx\right]$

$\Rightarrow I = \left[\log x\frac{x^2}{2} - \int\left(\frac{x}{2}\right)dx\right]$

$\Rightarrow I = \left[\log x\frac{x^2}{2} - \frac{1}{2}\int x dx\right]$

$\Rightarrow I = \left[\log x\frac{x^2}{2} - \frac{1}{2}\frac{x^2}{2}\right]+c$

$\Rightarrow I = \frac{1}{2}x^2\log x - \frac{1}{4}x^2+c$

Ans) C $\frac{1}{2}x^2\log x - \frac{1}{4}x^2+c$

Question: 7

Mark (√) against

Solution:

To find: Value of $\int x\, cosec^2 x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\, cosec^2 x dx$ … (i)

$I = \int x\, cosec^2 x dx$

$\Rightarrow x\int cosec^2 x dx - \int\left[\frac{d(x)}{x}\int cosec^2 x dx\right]dx$

$\Rightarrow I = x(-cotx) - \int 1.(-cotx)dx$

$\Rightarrow I = -x(cotx) + \log|sinx| + c$

Ans) D None of these

Question: 8

Mark (√) against

Solution:

To find: Value of $\int x\, sinx\, cosx dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\, sinx\, cosxdx$ … (i)

$I = \frac{1}{2}\int x\, 2sinx\, cosxdx$

$I = \frac{1}{2}\int x\, sin2xdx$

$\Rightarrow \frac{1}{2}\left[x\int sin2xdx - \int\left[\frac{d(x)}{x}\int sin2xdx\right]dx\right]$

$\Rightarrow \frac{1}{2}\left[\frac{-x\, cos2x}{2} - \int\left[1\ \frac{-cos2x}{2}\right]dx\right]$

$\Rightarrow \frac{1}{2}\left[\frac{-x\, cos2x}{2} + \frac{sin2x}{4}\right] + c$

$\Rightarrow \frac{-x\, cos2x}{4} + \frac{sin2x}{8} + c$

Ans) D $\frac{-x\, cos2x}{4} + \frac{sin2x}{8} + c$

Question: 9

Mark (√) against

Solution:

To find: Value of $\int x \cos^2 x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cos^2 x\,dx$ ... (i)

$$I = \int x\frac{1}{2}(1+\cos2x)dx$$

$$I = \frac{1}{2}\int x\,dx + \frac{1}{2}\int x\cos2xdx$$

$$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[x\int \cos2xdx - \int\left[\frac{d(x)}{x}\int \cos2xdx\right]dx\right]$$

$$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[x\frac{\sin2x}{2} - \int 1.\frac{\sin2x}{2}dx\right]$$

$$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[x\frac{\sin2x}{2} - \frac{1}{2}\int \sin2xdx\right]$$

$$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[x\frac{\sin2x}{2} - \frac{1}{2}\left(-\frac{\cos2x}{2}\right) + c\right]$$

$$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[\frac{x\sin2x}{2} + \frac{\cos2x}{4} + c\right]$$

$$I = \frac{x^2}{4} + \frac{x\sin2x}{4} + \frac{\cos2x}{8} + c$$

Ans) D $\frac{x^2}{4} + \frac{x\sin2x}{4} + \frac{\cos2x}{8} + c$

Question: 10

Mark (√) against

Solution:

To find: Value of $\int \frac{\log x}{x^2}dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \frac{\log x}{x^2}dx$ ... (i)

$$I = \int x^{-2}\log xdx$$

$$\Rightarrow \log x\int x^{-2}dx - \int\left[\frac{d(\log x)}{x}\int x^{-2}dx\right]dx$$

$$\Rightarrow \log x\frac{x^{-1}}{-1} - \int\left(\frac{1}{-x^2}\right)dx$$

$$\Rightarrow -\frac{\log x}{x} + \left(-\frac{1}{x}\right) + c$$

$$\Rightarrow -\frac{1}{x}(\log x+1) + c$$

Ans) A $-\frac{1}{x}(\log x+1) + c$

Question: 11

Mark (√) against

Solution:

To find: Value of $\int \log x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \log x \,.\, 1.\, dx$ ... (i)

Taking 1st function as $\log x$ and second function as 1

$$\Rightarrow I = \left[\log x \int 1\, dx - \int\left(\frac{d\log x}{dx}\int 1\, dx\right)dx\right]$$

$$\Rightarrow I = \left[\log x \,.\, x - \int\left(\frac{1}{x}\int 1\, dx\right)dx\right]$$

$$\Rightarrow I = \left[\log x \,.\, x - \int\left(\frac{1}{x}\, x\right)dx\right]$$

$$\Rightarrow I = \left[\log x \,.\, x - \int 1 dx\right]$$

$$\Rightarrow I = [\log x \,.\, x - x] + c$$

$$\Rightarrow I = x(\log x - 1) + c$$

Ans) D $x(\log x-1)+c$

Question: 12

Mark (√) against

Solution:

To find: Value of $\int \log_{10} x\, dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \log_{10} x\, dx$ ... (i)

$$I = \int \log_{10} x\, dx = \int \frac{\log x}{\log 10}\, dx$$

$$I = \frac{1}{\log_e 10}\int \log x \,.\, 1\, dx$$

Taking 1st function as $\log x$ and second function as 1

$$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \int 1\, dx - \int\left(\frac{d\log x}{dx}\int 1\, dx\right)dx\right]$$

$$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int\left(\frac{1}{x}\int 1\, dx\right)dx\right]$$

$$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int\left(\frac{1}{x}\, x\right)dx\right]$$

$$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int 1 dx\right]$$

$\Rightarrow I = \frac{1}{\log_e 10}[\log x \cdot x - x] + c$

$\Rightarrow I = x(\log x - 1)\log_{10} e + c$

Ans) D $x(\log x - 1)\log_{10} e + c$

Question: 13

Mark (√) against

Solution:

To find: Value of $\int (\log x)^2\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int (\log x)^2 \cdot 1 \cdot dx$ ... (i)

Taking 1st function as $(\log x)^2$ and second function as 1

$$\Rightarrow I = \left[(\log x)^2\int 1\,dx - \int\left(\frac{d(\log x)^2}{dx}\int 1\,dx\right)dx\right]$$

$$\Rightarrow I = \left[(\log x)^2\int 1\,dx - \int\left(\frac{2(\log x)}{x}\int 1\,dx\right)dx\right]$$

$$\Rightarrow I = \left[(\log x)^2 . x - 2\int \log x dx\right]$$

$\Rightarrow I = [(\log x)^2 . x - 2(x\log x - x)] + c$

$\Rightarrow I = [(\log x)^2 . x - 2x\log x + 2x] + c$

$\Rightarrow I = x(\log x)^2 - 2x\log x + 2x + c$

Ans) C $x(\log x)^2 - 2x\log x + 2x + c$

Question: 14

Mark (√) against

Solution:

To find: Value of $\int e^{\sqrt{x}}dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int e^{\sqrt{x}}dx$ ... (i)

Putting $\sqrt{x} = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$\Rightarrow dx = 2\sqrt{x}\, dt$

$\Rightarrow$ dx = 2t dt

$$\Rightarrow I = 2\int t.e^t\, dt$$

$$\Rightarrow I = 2\left[t\int e^t\, dt - \int\left[\frac{d(t)}{dt}\int e^t dt\right]dt\right]$$

$\Rightarrow I = 2\left[te^t - \int [1\ e^t]dt\right]$

$\Rightarrow I = 2[te^t - e^t]$

$\Rightarrow I = e^t.\ 2(t-1) + c$

$\therefore I = 2\ e^{\sqrt{x}}\ (\sqrt{x}-1) + c$

Ans) C $2\ e^{\sqrt{x}}\ (\sqrt{x}-1) + c$

Question: 15

Mark (√) against

Solution:

To find: Value of $\int \cos\sqrt{x}\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int \cos\sqrt{x}\, dx$ ... (i)

Putting $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx = 2\sqrt{x}\ dt$

⇒ dx = 2t dt

$\Rightarrow\ I = \int \cos t.\ 2t\, dt$

$\Rightarrow I = 2\int t.\ \cos t\ dt$

$\Rightarrow I = 2\left[t\int \cos t\ \ dt - \int \left[\frac{d(t)}{dt}\ \int \cos t\, dt\right]dt\right]$

$\Rightarrow I = 2\left[te^t - \int [1\ e^t]dt\right]$

$\Rightarrow I = 2[te^t - e^t]$

$\Rightarrow I = e^t.\ 2(t-1) + c$

$\therefore I = 2\ e^{\sqrt{x}}\ (\sqrt{x}-1) + c$

Ans) C $2\ e^{\sqrt{x}}\ (\sqrt{x}-1) + c$

Question: 16

Mark (√) against

Solution:

To find: Value of $\int \cos(logx)\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int \cos(\log x)\, dx$ ... (i)

$I = \int 1.\ \cos(\log x)\, dx$

Taking cos(logx) as first function and 1 as second function.

$$\Rightarrow I = \left[\cos\log x \int 1\, dx - \int \left[\frac{d[\cos(\log x)]}{dx} \int 1 dx\right] dx\right]$$

$$\Rightarrow I = \left[x.\cos(\log x) - \int \left[-\sin(\log x)\frac{1}{x}\, x\right] dx\right]$$

$$\Rightarrow I = \left[x.\cos(\log x) + \int [\sin(\log x)]\, dx\right]$$

$$\Rightarrow I = \left[x.\cos(\log x) + \int [1.\ \sin(\log x)]\, dx\right]$$

$$\Rightarrow I = \left[x.\cos(\log x) + \left\{\sin(\log x) \int 1\, dx - \left(\frac{d\sin(\log x)}{dx} \int 1.dx\right) dx\right\}\right]$$

$$\Rightarrow I = \left[x.\cos(\log x) + \left\{x.\ \sin(\log x) - \left(\cos(\log x)\frac{1}{x}\, x\right) dx\right\}\right]$$

$$\Rightarrow I = \left[x.\cos(\log x) + \left\{x.\ \sin(\log x) - \left(\cos(\log x)\frac{1}{x}\, x\right) dx\right\}\right]$$

$$\Rightarrow I = [x.\cos(\log x) + \{x.\ \sin(\log x) - (\cos(\log x))dx\}]$$

$$\Rightarrow I = [x.\cos(\log x) + x.\ \sin(\log x) - I]$$

$$\Rightarrow 2I = [x.\cos(\log x) + x.\ \sin(\log x)]$$

$$\Rightarrow I = \frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$$

Ans) B $\frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$

Question: 17

Mark (√) against

Solution:

To find: Value of $\int \sec^3 x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int \sec^3 x\, dx$ ... (i)

$I = \int \sec x \sec^2 x\, dx$

Taking secx as first function and $\sec^2$x as second function.

$$\Rightarrow I = \left[\sec x \int \sec^2 x\, dx - \int \left[\frac{d[\sec x]}{dx} \int \sec^2 x\, dx\right] dx\right]$$

$$\Rightarrow I = \left[\sec x \tan x - \int [\sec x \tan x \tan x] dx\right]$$

$$\Rightarrow I = \left[\sec x \tan x - \int [\sec x \tan^2 x] dx\right]$$

$\Rightarrow I = \left[\sec x \tan x - \int [\sec x(\sec^2 x - 1)]dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int (\sec^3 x - \sec x)\, dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int \sec^3 x\, dx + \int \sec x dx\right]$

$\Rightarrow I = [\sec x \tan x - I + \log|\sec x + \tan x| + c]$

$\Rightarrow 2I = [\sec x \tan x + \log|\sec x + \tan x| + c]$

$\Rightarrow I = \frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x| + c]$

Ans) B $\frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x| + c]$

Question: 18

Mark (√) against

Solution:

To find: Value of $\int \left\{\frac{1}{(logx)} - \frac{1}{(logx)^2}\right\} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int \left\{\frac{1}{(\log x)} - \frac{1}{(\log x)^2}\right\} dx$ ... (i)

Put $t = \log x$

$e^t = e^{\log x} = x$

$\frac{dx}{dt} = e^t$

$\Rightarrow dx = e^t\, dt$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx$

We know $\int e^x \left(f(x) + f'(x)\right) dx = e^x f(x)$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx = e^t \frac{1}{t}$

$\Rightarrow \frac{x}{\log x} + c$

Ans) B $\frac{x}{\log x} + c$

Question: 19

Mark (√) against

Solution:

To find: Value of $\int \left\{\frac{1}{(logx)} - \frac{1}{(logx)^2}\right\} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \left\{\frac{1}{(\log x)} - \frac{1}{(\log x)^2}\right\}dx$ ... (i)

Put $t = \log x$

$e^t = e^{\log x} = x$

$\frac{dx}{dt} = e^t$

$\Rightarrow dx = e^t\,dt$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\}dx$

We know $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\}dx = e^t\frac{1}{t}$

$\Rightarrow \frac{x}{\log x} + c$

Ans) B $\frac{x}{\log x} + c$

Question: 20

Mark (√) against

Solution:

To find: Value of $\int (x2^x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int (x2^x)dx$ ... (i)

$\Rightarrow I = x\int 2^x dx - \int\left(\frac{dx}{dx}\int 2^x dx\right)dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \int\left(\frac{2^x}{\log 2}\right)dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \frac{1}{\log 2}\int 2^x dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \frac{1}{\log 2}\frac{2^x}{\log 2}$

$\Rightarrow I = \frac{x.\,2^x}{\log 2} - \frac{2^x}{(\log 2)^2} + c$

$\Rightarrow I = \frac{2^x}{(\log 2)^2}(x\log 2 - 1) + c$

Ans) D

Question: 21

Mark (√) against

Solution:

To find: Value of $\int x\cot^2 x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cot^2 x\,dx$ ... (i)

$\Rightarrow I = x\int \cot^2 x\,dx - \int\left(\frac{dx}{dx}\int \cot^2 x\,dx\right)dx$

$\Rightarrow I = x\int(\operatorname{cosec}^2 x-1)\,dx - \int\left(1.\int(\operatorname{cosec}^2 x-1)dx\right)dx$

$\Rightarrow I = x(-\cot x-x)-\int(-\cot x-x)dx$

$\Rightarrow I = -x\cot x-x^2+\log|\sin x|+\frac{x^2}{2}$

$\Rightarrow I = -x\cot x-\frac{x^2}{2}+\log|\sin x|+c$

Ans) B $-x\cot x-\frac{x^2}{2}+\log|\sin x|+c$

Question: 22

Mark (√) against

Solution:

To find: Value of $\int \sin\sqrt{x}\,dx$

Formula used: $\int \frac{1}{x}dx = \log|x|+c$

We have, $I = \int \sin\sqrt{x}\,dx$ ... (i)

$\sqrt{x}=t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx=2\sqrt{x}dt$

$\Rightarrow dx=2tdt$

$I = \int \sin t.\,2t\,dt$

$I = 2\int t.\,\sin t\,dt$

$\Rightarrow I = 2t\int \sin t\,dt - \int\left(\frac{dt}{dt}\int \sin t\,dt\right)dt$

$\Rightarrow I = 2t(-\cos t)-\int 1(-\cos t)dt$

$\Rightarrow I = 2t(-\cos t)+\int \cos t\,dt$

$\Rightarrow I = 2t(-\cos t)+\sin t+c$

$\Rightarrow I = -2\sqrt{x}\cos\sqrt{x}+\sin\sqrt{x}+c$

Ans) C $2\sqrt{x}\cos\sqrt{x}+\sin\sqrt{x}+c$

Question: 23

Mark (√) against

Solution:

To find: Value of $\int e^{sinx} \sin 2x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^{\sin x} \sin 2x\, dx$ ... (i)

$I = \int e^{\sin x} 2\sin x\cos x\, dx$

Put sinx = t

$\cos x = \frac{dt}{dx}$

$\Rightarrow \cos x\, dx = dt$

$I = 2\int e^t . t . dt$

$\Rightarrow I = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}\int e^t dt\right)dt\right]$

$\Rightarrow I = 2\left[t e^t - \int 1\, e^t dt\right]$

$\Rightarrow I = 2t e^t - 2e^t + c$

$\Rightarrow I = 2 e^t (t-1) + c$

$\Rightarrow I = 2 e^{\sin x} (\sin x - 1) + c$

Ans) D $2 e^{\sin x} (\sin x - 1) + c$

Question: 24

Mark (√) against

Solution:

To find: Value of $\int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}} dx$ ... (i)

$I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}} dx$

$$I = \int \frac{\sin^{-1} x}{\sqrt{1-x^2}\,(1-x^2)} dx$$

Putting $\sin^{-1}x = t$, $x = \sin t$

$$\Rightarrow \cos t = \sqrt{1-x^2}$$

$$\Rightarrow \tan t = \frac{x}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$I = \int \frac{t}{\left(1-\sin^2 t\right)} dt$$

$$I = \int \frac{t}{\cos^2 t} dt$$

$$I = \int t.\sec^2 t\ dt$$

$$\Rightarrow I = \left[t \int \sec^2 t\ \ dt - \int \left(\frac{dt}{dt} \int \sec^2 t\ \ dt\right) dt\right]$$

$$\Rightarrow I = \left[t \tan t - \int 1 \tan t\, dt\right]$$

$$\Rightarrow I = \left[\sin^{-1}x \frac{x}{\sqrt{1-x^2}} - \log\left|\sqrt{1-x^2}\right| + c\right]$$

$$\Rightarrow I = [t \tan t - \log|\cos t| + c\,]$$

$$\Rightarrow I = 2t\, e^t - 2e^t + c$$

$$\Rightarrow I = 2\, e^t\, (t-1) + c$$

$$\Rightarrow I = 2\, e^{\sin x}\, (\sin x - 1) + c$$

Ans) D $2\, e^{\sin x}\, (\sin x - 1) + c$

Question: 25

Mark (√) against

Solution:

To find: Value of $\int \frac{x \tan^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx$ ... (i)

$$I = \int \frac{x \tan^{-1} x}{\sqrt{1+x^2}\,(1+x^2)} dx$$

Putting $\tan^{-1}x = t$, $x = \tan t$

$dx = \sec^2 t\ dt$

When $x = \tan t$

$\Rightarrow 1+x^2 = 1+\tan^2 t$

$\Rightarrow 1+x^2 = \sec^2 t$

$\Rightarrow \sqrt{1+x^2} = \sec t$

$\Rightarrow \sqrt{1+x^2} = \sec t$

$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \cos t$

$\Rightarrow \frac{1}{1+x^2} = \cos^2 t$

$\Rightarrow 1 - \frac{1}{1+x^2} = 1-\cos^2 t$

$\Rightarrow \frac{1+x^2-1}{1+x^2} = \sin^2 t$

$\Rightarrow \frac{x}{\sqrt{1+x^2}} = \sin t$

$I = \int \frac{\tan t\, t}{\sec t \sec^2 t} \sec^2 t\, dt$

$I = \int t \sin t dt$

Taking 1st function as t and second function as $\sin t$

$\Rightarrow I = \left[t \int \sin t\, dt - \int \left(\frac{dt}{dt} \int \sin t\, dt\right) dt\right]$

$\Rightarrow I = \left[t(-\cos t) - \int (1\,(-\cos t))dt\right]$

$\Rightarrow I = \left[t(-\cos t) + \int \cos t dt\right]$

$\Rightarrow I = -t\cos t + \sin t + c$

$\Rightarrow I = -\tan^{-1} x \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$

$\Rightarrow I = \frac{-\tan^{-1} x\, 1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$

Ans) B $\frac{-\tan^{-1} x\, 1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$

Question: 26

Mark (√) against

Solution:

To find: Value of $\int x \tan^{-1} x \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int x \tan^{-1} x\, dx$ ... (i)

Taking 1st function as $\tan^{-1}x$ and second function as x

$$\Rightarrow I = \left[\tan^{-1}x\int x\,dx - \int\left(\frac{d(\tan^{-1}x)}{dx}\int x\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\tan^{-1}x\frac{x^2}{2} - \int\left(\frac{1}{1+x^2}\ \frac{x^2}{2}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(\frac{x^2+1-1}{1+x^2}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[\int 1dx - \int\frac{1}{1+x^2}dx\right]\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[x - \tan^{-1}x\right]\right] + c$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right] + c$$

$$\Rightarrow I = \frac{1}{2}(1+x^2)\tan^{-1}x - \frac{1}{2}x + c$$

Ans) C $\frac{1}{2}(1+x^2)\tan^{-1}x - \frac{1}{2}x + c$

Question: 27

Mark (√) against

Solution:

To find: Value of $\int \tan^{-1}\sqrt{x}\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \tan^{-1}\sqrt{x}\ dx$ ... (i)

Let $\sqrt{x} = t$,

$$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$$

$\Rightarrow dx = 2t\ dt$

$$I = \int \tan^{-1}\sqrt{x}\ dx$$

$$\Rightarrow I = \int \tan^{-1}t\ 2t\,dt$$

$$\Rightarrow I = 2\int \tan^{-1}t\ t\,dt$$

Taking 1st function as $\tan^{-1}t$ and second function as t

$$\Rightarrow I = 2\left[\tan^{-1}t\int t\,dt - \int\left(\frac{d(\tan^{-1}t)}{dt}\int t\,dt\right)dt\right]$$

$$\Rightarrow I = 2\left[\tan^{-1} t\frac{t^2}{2} - \int\left(\frac{1}{1+t^2}\ \frac{t^2}{2}\right)dt\right]$$

$$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\int\left(\frac{t^2+1-1}{1+t^2}\right)dt\right]$$

$$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\left[\int 1dt - \int\frac{1}{1+t^2}dt\right]\right]$$

$$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}[t - \tan^{-1} t]\right] + c$$

$$\Rightarrow I = 2\left[\frac{x}{2}\tan^{-1}\sqrt{x} - \frac{1}{2}\sqrt{x} + \frac{1}{2}\tan^{-1}\sqrt{x}\right] + c$$

$$\Rightarrow I = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + c$$

$$\Rightarrow I = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$$

Ans) B $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$

Question: 28

Mark (√) against

Solution:

To find: Value of $\int \cos^{-1} x\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos^{-1} x\ dx$ ... (i)

Let $\cos^{-1} x = \theta$, ⇒ x = cosθ

⇒ dx = -sinθ dθ

If x = cosθ ,

Then $\sqrt{1-x^2}$ = sinθ

$$I = \int \cos^{-1} x\ dx$$

$$\Rightarrow I = -\int \theta\sin\theta\ d\theta$$

Taking 1st function as θ and second function as $\sin\theta$

$$\Rightarrow I = -\left[\theta\int \sin\theta\ d\theta - \int\left(\frac{d\theta}{d\theta}\int \sin\theta\ d\theta\right)d\theta\right]$$

$$\Rightarrow I = -\left[\theta(-\cos\theta) - \int(-\cos\theta)d\theta\right] + c$$

$$\Rightarrow I = -[\theta(-\cos\theta) - (-\sin\theta)] + c$$

$$\Rightarrow I = -[\theta(-\cos\theta) + \sin\theta] + c$$

$$\Rightarrow I = \theta\cos\theta - \sin\theta + c$$

$\Rightarrow I = x \cdot \cos^{-1} x - \sqrt{1-x^2} + c$

Ans) A $x \cdot \cos^{-1} x - \sqrt{1-x^2} + c$

Question: 29

Mark (√) against

Solution:

To find: Value of $\int \tan^{-1} x\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \tan^{-1} x\ dx$ ... (i)

Let $\tan^{-1} x = \theta$, $\Rightarrow x = \tan\theta$

$\Rightarrow dx = \sec^2\theta\ d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$I = \int \tan^{-1} x\ dx$

$\Rightarrow I = \int \theta \sec^2 \theta\ d\theta$

Taking 1st function as θ and second function as $\sec^2 \theta$

$\Rightarrow I = \left[\theta \int \sec^2 \theta\ d\theta - \int\left(\frac{d\theta}{d\theta} \int \sec^2 \theta\ d\theta\right) d\theta\right]$

$\Rightarrow I = \left[\theta(\tan\theta) - \int (1\ (\tan\theta))\, d\theta\right] + c$

$\Rightarrow I = [\theta(\tan\theta) - (\ \log|\sec\theta|)] + c$

$\Rightarrow I = \left[\tan^{-1} x\ (x) - \log\left|\sec\left(\sec^{-1}\sqrt{1+x^2}\right)\right|\right] + c$

$\Rightarrow I = \left[x \cdot \tan^{-1} x - (\log\left|\sqrt{1+x^2}\right|)\right] + c$

$\Rightarrow I = x \cdot \tan^{-1} x - \frac{1}{2}\log|1+x^2| + c$

Ans) B $x \cdot \tan^{-1} x - \frac{1}{2}\log|1+x^2| + c$

Question: 30

Mark (√) against

Solution:

To find: Value of $\int \sec^{-1} x\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sec^{-1} x\, dx$ … (i)

Let $\sec^{-1} x = \theta$, ⇒ $x = \sec\theta$

⇒ $dx = \sec\theta \tan\theta\, d\theta$

If $x = \sec\theta$,

Then $\sqrt{x^2-1} = \tan\theta$

$$I = \int \sec^{-1} x\, dx$$

$$\Rightarrow I = \int \theta \sec\theta \tan\theta\, d\theta$$

Taking 1st function as θ and second function as $\sec\theta \tan\theta$

$$\Rightarrow I = \left[\theta \int \sec\theta \tan\theta\, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec\theta \tan\theta\, d\theta\right) d\theta\right]$$

$$\Rightarrow I = \left[\theta(\sec\theta) - \int (1\,(\sec\theta))\, d\theta\right] + c$$

$$\Rightarrow I = [\theta(\sec\theta) - (\log|\sec\theta + \tan\theta|)] + c$$

$$\Rightarrow I = \left[\sec^{-1} x\,(x) - \left(\log\left|x + \sqrt{x^2-1}\right|\right)\right] + c$$

$$\Rightarrow I = x \cdot \sec^{-1} x - \log\left|x + \sqrt{x^2-1}\right| + c$$

Ans) B $x \cdot \sec^{-1} x - \log\left|x + \sqrt{x^2-1}\right| + c$

Question: 31

Mark (√) against

Solution:

To find: Value of $\int \sin^{-1}(3x - 4x^3)\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int \sin^{-1}(3x-4x^3)\, dx$ … (i)

Let $x = \sin\theta$, ⇒ $\theta = \sin^{-1}x$

⇒ $dx = \cos\theta\, d\theta$

If $x = \sin\theta$,

Then $\sqrt{1-x^2} = \cos\theta$

$$I = \int \sin^{-1}(3x-4x^3)\, dx$$

$$\Rightarrow I = \int \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right) \cos\theta\, d\theta$$

$$\Rightarrow I = \int \sin^{-1}(\sin 3\theta) \cos\theta\, d\theta$$

$$\Rightarrow I = \int 3\theta \cos\theta\, d\theta$$

$\Rightarrow I = 3\int \theta\cos\theta\, d\theta$

Taking 1st function as θ and second function as $\cos\theta$

$\Rightarrow I = 3\left[\theta\int \cos\theta\, d\theta - \int\left(\frac{d\theta}{d\theta}\int \cos\theta\, d\theta\right)d\theta\right]$

$\Rightarrow I = 3\left[\theta(\sin\theta) - \int(1\,(\sin\theta))\,d\theta\right]$

$\Rightarrow I = 3[\theta(\sin\theta)-(-\cos\theta)]+c$

$\Rightarrow I = 3[\theta(\sin\theta)+\cos\theta]+c$

$\Rightarrow I = 3\sin^{-1}x\,(x)+3\sqrt{1-x^2}+c$

$\Rightarrow I = 3\,x\sin^{-1}x+3\sqrt{1-x^2}+c$

$\Rightarrow I = 3\left[x\sin^{-1}x+\sqrt{1-x^2}\right]+c$

Ans) A $3\left[x\sin^{-1}x+\sqrt{1-x^2}\right]+c$

Question: 32

Mark (√) against

Solution:

To find: Value of $\int \sin^{-1}\frac{2x}{1+x^2}\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin^{-1}\frac{2x}{1+x^2}\,dx$ ... (i)

Let $x = \tan\theta$, $\Rightarrow \theta = \tan^{-1}x$

$\Rightarrow dx = \sec^2\theta\, d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$$I = \int \sin^{-1}\frac{2x}{1+x^2}\,dx$$

$$\Rightarrow I = \int \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta\, d\theta$$

$$\Rightarrow I = \int \sin^{-1}(\sin 2\theta)\sec^2\theta\, d\theta$$

$$\Rightarrow I = \int 2\theta\sec^2\theta\, d\theta$$

$$\Rightarrow I = 2\int \theta\sec^2\theta\, d\theta$$

Taking 1st function as θ and second function as $\sec^2\theta$

$\Rightarrow I = 2\left[\theta \int \sec^2\theta\ d\theta - \int\left(\frac{d\theta}{d\theta}\int \sec^2\theta\ d\theta\right)d\theta\right]$

$\Rightarrow I = 2\left[\theta(\tan\theta) - \int(1\,(\tan\theta))d\theta\right]$

$\Rightarrow I = 2[\theta(\tan\theta)-(\log(\sec\theta)]+c$

$\Rightarrow I = 2\left[\tan^{-1}x\,(x)-(\log(\sec\left(\sec^{-1}\sqrt{1+x^2}\right))\right]+c$

$\Rightarrow I = 2\left[\tan^{-1}x\,(x)-(\log\sqrt{1+x^2})\right]+c$

$\Rightarrow I = 2\left[x\,.\,\tan^{-1}x - \frac{1}{2}(\log 1+x^2)\right]+c$

$\Rightarrow I = 2x\,.\,\tan^{-1}x-(\log 1+x^2)+c$

Ans) B $2x\,.\,\tan^{-1}x-(\log 1+x^2)+c$

Question: 33

Mark (√) against

Solution:

To find: Value of $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$

Formula used: $\int \frac{1}{x}dx = \log|x|+c$

We have, $I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$ ... (i)

Let x = cosθ , ⇒ θ = $\cos^{-1}$x

⇒ dx = -sinθ dθ

If x = cosθ ,

Then $\sqrt{1-x^2}$= sinθ

$$I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$$

$$\Rightarrow I = \int \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\ .\ -\sin\theta\ d\theta$$

$$\Rightarrow I = \int \tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\ .\ -\sin\theta\ d\theta$$

$$\Rightarrow I = \int \tan^{-1}\sqrt{\tan^2\frac{\theta}{2}}\ .\ -\sin\theta\ d\theta$$

$$\Rightarrow I = \int \tan^{-1}\left(\tan\frac{\theta}{2}\right)\ .\ -\sin\theta\ d\theta$$

$$\Rightarrow I = \int \frac{\theta}{2} . -\sin\theta\, d\theta$$

$$\Rightarrow I = -\frac{1}{2}\int \theta. \sin\theta\, d\theta$$

Taking 1^{st} function as θ and second function as $\sin\theta$

$$\Rightarrow I = -\frac{1}{2}\left[\theta \int \sin\theta\, d\theta - \int\left(\frac{d\theta}{d\theta} \int \sin\theta\, d\theta\right)d\theta\right]$$

$$\Rightarrow I = -\frac{1}{2}\left[\theta(-\cos\theta) - \int(1\,(-\cos\theta))d\theta\right]$$

$$\Rightarrow I = -\frac{1}{2}\left[\theta(-\cos\theta) + \int(\cos\theta)d\theta\right]$$

$$\Rightarrow I = -\frac{1}{2}[\theta(-\cos\theta)+\sin\theta]+c$$

$$\Rightarrow I = \frac{1}{2}\cos^{-1}x\,(x) - \frac{1}{2}\sqrt{1-x^2}+c$$

$$\Rightarrow I = \frac{1}{2}x.\ \cos^{-1}x - \frac{1}{2}\sqrt{1-x^2}+c$$

Ans) C $\frac{1}{2}x.\ \cos^{-1}x - \frac{1}{2}\sqrt{1-x^2}+c$

Question: 34

Mark (√) against

Solution:

To find: Value of $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x|+c$

We have, $I = \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$ ... (i)

Let $x = \tan\theta$, $\Rightarrow \theta = \tan^{-1}x$

$\Rightarrow dx = \sec^2\theta\, d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$$I = \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$$

$$\Rightarrow I = \int \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)\sec^2\theta\, d\theta$$

$$\Rightarrow I = \int \tan^{-1}(\tan 3\theta)\sec^2\theta\ d\theta$$

$$\Rightarrow I = \int 3\theta \sec^2\theta\ d\theta$$

$\Rightarrow I = 3\int \theta \sec^2\theta \, d\theta$

Taking 1st function as θ and second function as $\sec^2\theta$

$\Rightarrow I = 3\left[\theta\int \sec^2\theta \, d\theta - \int\left(\frac{d\theta}{d\theta}\int \sec^2\theta \, d\theta\right)d\theta\right]$

$\Rightarrow I = 3\left[\theta\tan\theta - \int(\tan\theta)d\theta\right]$

$\Rightarrow I = 3[\theta\tan\theta-(\log\sec\theta)]+c$

$\Rightarrow I = 3\theta\tan\theta-3\log(\sec\theta)+c$

$\Rightarrow I = 3\tan^{-1}x\tan(\tan^{-1}x)-3\log\left\{\sec\left(\sec^{-1}\sqrt{1+x^2}\right)\right\}+c$

$\Rightarrow I = 3x.\tan^{-1}x-3\log\left\{\sqrt{1+x^2}\right\}+c$

$\Rightarrow I = 3x.\tan^{-1}x-\frac{3}{2}\log\{1+x^2\}+c$

Ans) B $3x.\tan^{-1}x-\frac{3}{2}\log\{1+x^2\}+c$

Question: 35

Mark (√) against

Solution:

To find: Value of $\int x^2\cos x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x^2\cos x\,dx$ … (i)

Taking 1st function as x^2 and second function as $\cos x$

$\Rightarrow I = \left[x^2\int \cos x\,dx - \int\left(\frac{dx^2}{dx}\int \cos x\,dx\right)dx\right]$

$\Rightarrow I = \left[x^2\sin x - \int(2x\sin x)dx\right]$

$\Rightarrow I = \left[x^2\sin x - 2\int(x\sin x)dx\right]$

Taking 1st function as x and second function as $\sin x$

$\Rightarrow I = x^2\sin x - 2\left[x\int \sin x\,dx - \int\left(\frac{dx}{dx}\int \sin x\,dx\right)dx\right]$

$\Rightarrow I = x^2\sin x - 2\left[x(-\cos x) - \int(1(-\cos x))dx\right]$

$\Rightarrow I = x^2\sin x - 2[x(-\cos x)-(-\sin x)]+c$

$\Rightarrow I = x^2\sin x - 2[x(-\cos x)+\sin x]+c$

$\Rightarrow I = x^2\sin x + 2x\cos x - 2\sin x + c$

Ans) A $x^2\sin x + 2x\cos x - 2\sin x + c$

Question: 36

Mark (√) against

Solution:

To find: Value of $\int sinx\log(cosx)\ dx$

Formula used:

$$\textbf{(i)}\ \int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$$

We have, $I = \int sinx \log(cosx)\ dx$ ... (i)

Let cosx = t

-sinx dx = dt

$$I = \int sinx \log(cosx)\ dx$$

$$I = -\int \log t\, dt$$

$$I = -\int \log t\,.\,1\,.\,dt$$

Taking 1st function as $\log t$ and second function as 1

$$\Rightarrow I = -\left[\log t \int 1\, dt - \int\left(\frac{d\log t}{dt}\int 1\, dt\right)dt\right]$$

$$\Rightarrow I = -\left[\log t\,.\,t - \int\left(\frac{1}{t}\,t\right)dt\right]$$

$$\Rightarrow I = -\left[\log t\,.\,t - \int 1dt\right]$$

$$\Rightarrow I = -[\log t\,.\,t - t] + c$$

$$\Rightarrow I = -\log t\,.\,t + t + c$$

$$\Rightarrow I = -cosx\,.\,\log(cosx) + cosx + c$$

Ans) B $-cosx\,.\,\log(cosx) + cosx + c$

Question: 37

Mark (√) against

Solution:

To find: Value of $\int x\ sinx\ cosx\ dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int x\ sinx\ cosx\ dx$ ... (i)

$$I = \frac{1}{2}\int x\ 2sinx\ cosx\ dx$$

$$I = \frac{1}{2}\int x\ sin2x\ dx$$

Let 2x = t

2dx = dt

$dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{t}{2}\ sint\ \frac{dt}{2}$

$I = \frac{1}{8}\int t\, sint\, dt$

Taking 1st function as t and second function as sint

$\Rightarrow I = \frac{1}{8}\left[t\int sin t\ \ dt - \int\left(\frac{dt}{dt}\int sin t\ \ dt\right)dt\right]$

$\Rightarrow I = \frac{1}{8}\left[t.(-cost) - \int (-cost)\ dt\right]$

$\Rightarrow I = \frac{1}{8}[-t.cost-(-sint)]+c$

$\Rightarrow I = \frac{1}{8}[-t.cost+sint]+c$

$\Rightarrow I = -\frac{1}{8}2x.cos2x+\frac{1}{8}sin2x+c$

$\Rightarrow I = -\frac{1}{4}x.cos2x+\frac{1}{8}sin2x+c$

Ans) A $-\frac{1}{4}x.cos2x+\frac{1}{8}sin2x+c$

Question: 38

Mark (√) against

Solution:

To find: Value of $\int x^3 cosx^2 dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x^3cosx^2dx$ ... (i)

Let $x^2 = t$

$\Rightarrow xdx = \frac{1}{2}dt$

$I = \int x^3cosx^2dx$

$I = \int x.\ x^2cosx^2dx$

$I = \int t\cos t\ \frac{1}{2}\ dt$

$I = \frac{1}{2}\int t\cos t\, dt$

Taking 1st function as t and second function as $\cos t$

$$\Rightarrow I = \frac{1}{2}\left[t\int \cos t\ dt - \int\left(\frac{dt}{dt}\int \cos t\ dt\right)dt\right]$$

$$\Rightarrow I = \frac{1}{2}\left[t \,.\, \sin t - \int \sin t\, dt\right]$$

$$\Rightarrow I = \frac{1}{2}[t \,.\, \sin t - (-\cos t) + c]$$

$$\Rightarrow I = \frac{1}{2}[t \,.\, \sin t + \cos t + c]$$

$$\Rightarrow I = \frac{1}{2}x^2 \,.\, \sin x^2 + \frac{1}{2}\cos x^2 + c$$

Ans) B $\frac{1}{2}x^2 \,.\, \sin x^2 + \frac{1}{2}\cos x^2 + c$

Question: 39

Mark (√) against

Solution:

To find: Value of $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$ ... (i)

Let x = tant , t = $\tan^{-1}$x

⇒ dx = $\sec^2$t dt

If tant = x ,

sec t = 1 + x ^{2}

$$I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

$$I = \int \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right)\sec^2 t\, dt$$

$$I = \int \cos^{-1}(\cos 2t)\sec^2 t\ dt$$

$$I = \int 2t\sec^2 t\ dt$$

$$I = 2\int t\sec^2 t\ dt$$

Taking 1st function as t and second function as $\sec^2 t$

$$\Rightarrow I = 2\left[t\int \sec^2 t\ dt - \int\left(\frac{dt}{dt}\int \sec^2 t\ dt\right)dt\right]$$

$$\Rightarrow I = 2\left[t\tan t - \int \tan t\, dt\right]$$

$$\Rightarrow I = 2[t\tan t - \log|\sec t| + c]$$

$\Rightarrow I = 2[\tan^{-1}x\ x - \log|1+x^2| + c]$

$\Rightarrow I = 2x\tan^{-1}x - 2\log|1+x^2| + c$

Ans) D None of these

Question: 40

Mark (√) against

Solution:

To find: Value of $\int x\tan^{-1}x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\tan^{-1}x\,dx$ ... (i)

Taking 1^{st} function as $\tan^{-1}x$ and second function as x

$$\Rightarrow I = \left[\tan^{-1}x\int x\,dx - \int\left(\frac{d\tan^{-1}x}{dx}\int x\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\tan^{-1}x\frac{x^2}{2} - \int\left(\frac{1}{(1+x^2)}\frac{x^2}{2}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(\frac{x^2}{(1+x^2)}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(1-\frac{1}{(1+x^2)}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int 1.\,dx + \frac{1}{2}\int\frac{1}{(1+x^2)}.\,dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right] + c$$

$$\Rightarrow I = \left[\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x\right] + c$$

$$\Rightarrow I = \frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c$$

Ans) A $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c$

Question: 41

Mark (√) against

Solution:

To find: Value of $\int \sin(\log x)\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin(\log x)\,dx$ ... (i)

$I = \int \sin(\log x) \,.\, 1 \,.\, dx$

Taking 1^{st} function as sin(logx) and second function as 1

$\Rightarrow I = \left[\sin(\log x)\int 1\,dx - \int\left(\frac{d\sin(\log x)}{dx}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \left[\sin(\log x)\,.\,x - \int \frac{\cos(\log x).\,x}{x}dx\right]$

$\Rightarrow I = \left[\sin(\log x)\,.\,x - \int \cos(\log x)\,dx\right]$

Taking 1^{st} function as cos(logx) and second function as 1

$\Rightarrow I = \sin(\log x)\,.x - \left[\cos(\log x)\int 1\,dx - \int\left(\frac{d\cos(\log x)}{dx}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \sin(\log x)\,.\,x - \left[\cos(\log x)\,.x - \int -\frac{\sin(\log x).\,x}{x}dx\right]$

$\Rightarrow I = \sin(\log x)\,.\,x - \left[\cos(\log x)\,.x + \int \sin(\log x)\,dx\right]$

$\Rightarrow I = \sin(\log x)\,.\,x - [\cos(\log x)\,.x + I] + c$

$\Rightarrow I = \sin(\log x)\,.\,x - \cos(\log x)\,.x - I + c$

$\Rightarrow 2I = \sin(\log x)\,.\,x - \cos(\log x)\,.x + c$

$\Rightarrow I = \frac{\sin(\log x)\,.\,x - \cos(\log x)\,.x}{2} + c$

$\Rightarrow I = \frac{1}{2}x.\,\sin(\log x) - x\,.\,\frac{1}{2}\cos(\log x) + c$

Ans) B $\frac{1}{2}x.\,\sin(\log x) - x\,.\,\frac{1}{2}\cos(\log x) + c$

Question: 42

Mark (√) against

Solution:

To find: Value of $\int(\sin^{-1}x)^2dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int(\sin^{-1}x)^2dx$ ... (i)

Putting sint = x, ⇒ t = $\sin^{-1}x$

⇒ dx = cost dt

When x = sint then $\sqrt{1-x^2} = \cos t$

$I = \int(\sin^{-1}x)^2dx$

$\Rightarrow I = \int(\sin^{-1}(\sin t))^2\cos t\,dt$

$\Rightarrow I = \int t^2 \cos t\, dt$

Taking 1st function as t^2 and second function as cost

$\Rightarrow I = \left[t^2 \int \cos t\, dt - \int \left(\frac{dt^2}{dt} \int \cos t\, dt\right) dt\right]$

$\Rightarrow I = \left[t^2 \sin t - \int (2t \sin t) dt\right]$

$\Rightarrow I = \left[t^2 \sin t - 2 \int (t \sin t) dt\right]$

Taking 1st function as t and second function as sint

$\Rightarrow I = t^2 \sin t - 2\left[\int (t \sin t) dt\right]$

$\Rightarrow I = t^2 \sin t - 2\left[t \int \sin t \;\; dt - \int \left(\frac{dt}{dt} \int \sin t \;\; dt\right) dt\right]$

$\Rightarrow I = t^2 \sin t - 2\left[t(-\cos t) - \int (-\cos t) dt\right]$

$\Rightarrow I = t^2 \sin t - 2[-t\cos t - (-\sin t) + c]$

$\Rightarrow I = t^2 \sin t + 2t\cos t - 2\sin t + c$

$\Rightarrow I = x\left(\sin^{-1} x\right)^2 + 2 \sin^{-1} x \sqrt{1-x^2} - 2x + c$

Ans) D $x\left(\sin^{-1} x\right)^2 + 2 \sin^{-1} x \sqrt{1-x^2} - 2x + c$

Question: 43

Mark (√) against

Solution:

To find: Value of $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ ... (i)

Here $f(x) = \frac{1}{x}$

$\Rightarrow f'(x) = -\frac{1}{x^2}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \frac{1}{x} + c$

Ans) C $e^x \frac{1}{x} + c$

Question: 44

Mark (√) against

Solution:

To find: Value of $\int e^x\left(\frac{1}{x^2}-\frac{2}{x^3}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1}{x^2}-\frac{2}{x^3}\right)dx$ ... (i)

Here $f(x) = \frac{1}{x^2}$

$\Rightarrow f'(x) = -\frac{2}{x^3}$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x\frac{1}{x^2}+c$

Ans) B $e^x\frac{1}{x^2}+c$

Question: 45

Mark (√) against

Solution:

To find: Value of $\int e^x\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right)dx$ ... (i)

Here $f(x) = \sin^{-1}x$

$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x\sin^{-1}x+c$

Ans) B $e^x\sin^{-1}x+c$

Question: 46

Mark (√) against

Solution:

To find: Value of $\int e^x(\tan x+\log(\sec x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\tan x + \log(\sec x))dx$ ... (i)

$\Rightarrow I = \int e^x(\tan x - \log(\cos x))dx$

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) A $e^x \log(\sec x) + c$

Question: 47

Mark (√) against

Solution:

To find: Value of $\int e^x(\tan x + \log(\sec x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\tan x + \log(\sec x))dx$ ... (i)

$\Rightarrow I = \int e^x(\tan x - \log(\cos x))dx$

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) A $e^x \log(\sec x) + c$

Question: 48

Mark (√) against

Solution:

To find: Value of $\int e^x(\cot x + \log(\sin x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\cot x + \log(\sin x))dx$ ... (i)

Here $f(x) = \log(\sin x)$

$\Rightarrow f'(x) = \cot x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \log(\sin x) + c$

Ans) D None of these

Question: 49

Mark (√) against

Solution:

To find: Value of $\int e^x\left(\tan^{-1}x + \frac{1}{(1+x)^2}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\tan^{-1}x + \frac{1}{(1+x)^2}\right)dx$ ... (i)

Here $f(x) = \tan^{-1}x$

$\Rightarrow f'(x) = \frac{1}{(1+x)^2}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x(\tan^{-1}x) + c$

Ans) B $e^x(\tan^{-1}x) + c$

Question: 50

Mark (√) against

Solution:

To find: Value of $\int e^x(\tan x - \log(\cos x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\tan x - \log(\cos x))dx$ ... (i)

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) C $e^x \log(\sec x) + c$

Question: 51

Mark (√) against

Solution:

To find: Value of $\int e^x(\cot x - \operatorname{cosec}^2 x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\cot x - \operatorname{cosec}^2 x)dx$ ... (i)

Here $f(x) = \cot x$

$\Rightarrow f'(x) = -\operatorname{cosec}^2 x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \cot x + c$

Ans) B $e^x \cot x + c$

Question: 52

Mark (√) against

Solution:

To find: Value of $\int e^x(\sin x + \cos x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\sin x + \cos x)dx$ ... (i)

Here $f(x) = \sin x$

$\Rightarrow f'(x) = \cos x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \sin x + c$

Ans) A $e^x \sin x + c$

Question: 53

Mark (√) against

Solution:

To find: Value of $\int e^x \sec x\,(1 + \tan x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x \sec x\,(1+\tan x)dx$ ... (i)

$I = \int e^x(\sec x+\sec x\tan x)dx$

Here $f(x) = \sec x$

$\Rightarrow f'(x) = \sec x\tan x$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x \sec x+c$

Ans) B $e^x \sec x+c$

Question: 54

Mark (√) against

Solution:

To find: Value of $\int e^x\left(\frac{1+x\log x}{x}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1+x\log x}{x}\right)dx$ ... (i)

$I = \int e^x\left(\frac{1}{x}+\log x\right)dx$

Here $f(x) = \log x$

$\Rightarrow f'(x) = \frac{1}{x}$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x \log x+c$

Ans) B $e^x \log x+c$

Question: 55

Mark (√) against

Solution:

To find: Value of $\int e^x \frac{x}{(1+x)^2}dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x \frac{x}{(1+x)^2}dx$ ... (i)

$$I = \int e^x \left(\frac{x+1-1}{(1+x)^2}\right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right) dx$$

Here $f(x) = \frac{1}{(1+x)}$

$$\Rightarrow f'(x) = -\frac{1}{(1+x)^2}$$

$$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \frac{1}{(1+x)} + c$$

Ans) A $e^x \frac{1}{(1+x)} + c$

Question: 56

Mark (√) against

Solution:

To find: Value of $\int e^x \left(\frac{1+sinx}{1+cosx}\right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$ ... (i)

$$I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{1+\cos x} + \frac{\sin x}{1+\cos x}\right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right) dx$$

Here $f(x) = \tan\frac{x}{2}$

$$\Rightarrow f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$$

$$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \tan\frac{x}{2} + c$$

Ans) C $e^x \tan\frac{x}{2} + c$