3. Indices and Cube roots

- **Terminology of index form**

  In the index form:

  - The number, which is multiplied repeatedly, is considered as **base**.
  - The number of times by which the base is multiplied is considered **index** or **exponent** of base.
  - The repeated multiplication is written as "$\textbf{base}^{\textbf{index}}$".

  For example,

  $a \times a \times a \times a \times a \times a \times ...n$ times $= a^n$

  In a^n, a is base and n is index.

  Also, if the index form is given, we can write it in multiplication form.

  For example, a^n means that a is to be multiplied n times.

  $a^n = a \times a \times a \times a \times a \times a \times ...n$ times

- **Reading index form**

  a^n can be read as "a raised to the power n" or "a to the power n" or "a raised to n" or "n^{th} power of a".

- **When the base is given, we can represent the required number in index form.**

  For example, if number 625 is to be written in index form with base 5 then we just need to factorize 625 using 5 as the only factor to get the multiplication form which can easily be represented as index form.

5	625
5	125
5	25
5	5
	1

  So, $625 = 5 \times 5 \times 5 \times 5 = 5^4$

- **Laws of rational exponents of real numbers:**

Let a and b be two real numbers and m and n be two rational numbers then

  - $a^p \cdot a^q = a^{p+q}$
  - $(a^p)^q = a^{pq}$
  - $\frac{a^p}{a^q} = a^{p-q}$

- $a^p b^p = (ab)^p$
- $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
- $a^{-p} = \frac{1}{a^p}$

Example:

$\sqrt[3]{(512)^{-2}}$

$= \left[(512)^{-2}\right]^{\frac{1}{3}}$

$= (512)^{\frac{-2}{3}}$ $[(a^m)^n = a^{mn}]$

$= (8^3)^{\frac{-2}{3}}$

$= (8)^{3\times\frac{-2}{3}}$ $[(a^m)^n = a^{mn}]$

$= (8)^{-2}$

$= \frac{1}{8^2}$ $\left[a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{64}$

- A number is said to be a **perfect cube** if each of its prime factors appears in group of three.
- Prime factorization method can be used to check whether a number is a perfect cube or not.

  For example, 5832 = 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3

  Here, each of the prime factors occurs in groups of three. Hence, 5832 is a perfect cube.

- Cube root is the inverse operation of finding a cube. The symbol $\sqrt[3]{\ }$ denotes cube-root.

  Example:

  $\sqrt[3]{64} = 4$since 4 × 4 × 4 = 64

- The cube root of a perfect cube can be found by prime factorization method.

  Example:

  Cube root of 287496 is 66.

  Prime factorization of 287496

$$\begin{array}{r|l} 2 & 287496 \\ 2 & 143748 \\ 2 & 71874 \\ 3 & 35937 \\ 3 & 11979 \\ 3 & 3993 \\ 11 & 1331 \\ 11 & 121 \\ 11 & 11 \\ & 1 \end{array}$$

Thus, the number 287496 can be expressed as a product of its prime factors as

$$287496 = \underline{2\times2\times2}\times\underline{3\times3\times3}\times\underline{11\times11\times11} = 2^3\times3^3\times11^3 = (2\times3\times11)^3$$

$$\therefore\ \sqrt[3]{287496} = 2\times3\times11 = 66$$