6.7 ELEMENTARY PARTICLES

6.291 The formula is

$$T = \sqrt{c^2 p^2 + m_0^2 c^4} - m_0 c^2$$

Thus $\quad T = 5{\cdot}3\,\text{MeV} \quad \text{for } p = 0{\cdot}10\,\frac{GeV}{c} = 5{\cdot}3 \times 10^{-3}\,\text{GeV}$

$$T = 0{\cdot}433\,\text{GeV} \quad \text{for } p = 1{\cdot}0\,\frac{GeV}{c}$$

$$T = 9{\cdot}106\,\text{GeV} \quad \text{for } p = 10\,\frac{GeV}{c}$$

Here we have used $m_0 c^2 = 0{\cdot}938\,\text{GeV}$

6.292 Energy of pions is $(1+\eta)\, m_0 c^2$ so

$$(1+\eta)\, m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-\beta^2}}$$

Hence $\quad \frac{1}{\sqrt{1-\beta^2}} = 1+\eta \quad \text{or} \quad \beta = \frac{\sqrt{\eta\,(2+\eta)}}{1+\eta}$

Here $\beta = \frac{v}{c}$ of pion. Hence time dilation factor is $1+\eta$ and the distance traversed by the pion in its lifetime will be

$$\frac{c\,\beta\,\tau_0}{\sqrt{1-\beta^2}} = c\,\tau_0 \sqrt{\eta\;(2+\eta)} = 15{\cdot}0\ \text{metres}$$

on substituting the values of various quantities. (Note. The factor $\frac{1}{\sqrt{1-\beta^2}}$ can be looked at as a time dilation effect in the laboratory frame or as length contraction factor brought to the other side in the proper frame of the pion).

6.293 From the previous problem $\quad l = c\,\tau_0 \sqrt{\eta\,(\eta+2)}$

where $\eta = \frac{T}{m_\pi c^2}$, m_π is the rest mass of pions.

substitution gives $\quad \tau_0 = \frac{l}{c\sqrt{\eta\,(2+\eta)}} = 2{\cdot}63\ \text{ns}$

$$= \frac{l\, m_\pi c}{\sqrt{T\,(T + 2\, m_\pi c^2)}}$$

where we have used $\eta = \frac{100}{139{\cdot}6} = 0{\cdot}716$

6.294 Here $\eta = \frac{T}{m\,c^2} = 1$ so the life time of the pion in the laboratory frame is

$$\eta = (1+\eta)\,\tau_0 = 2\,\tau_0$$

The law of radioactive decay implies that the flux decrease by the factor.

$$\frac{J}{J_0} = e^{-t/\tau} = e^{-l/v\tau} = e^{-l/c\,\tau_0\sqrt{\eta\,(2+\eta)}}$$

$$= \exp\left(-\frac{m\,c\,l}{\tau_0\sqrt{T\,(T+2\,m\,c^2)}}\right) = 0{\cdot}221$$

6.295 Energy-momentum conservation implies

$$O = \vec{p_\mu} + \vec{p_\nu}$$

$$m_\pi c^2 = E_\mu + E_\nu \quad \text{or} \quad m_\pi c^2 - E_\nu = E_\mu$$

But $\quad E_\nu = c\,|\vec{p_\nu}| = c\,|p_\mu|$. Thus

$$m_\pi^2 c^4 - 2\,m_\pi c^2 \cdot c\,|\vec{p_\mu}| + c^2 p_\mu^2 = E_\mu^2 = c^2 p_\mu^2 + m_\mu^2 c^4$$

Hence $\quad c\,|\vec{p_\mu}| = \frac{m_\pi^2 - m_\mu^2}{2\,m_\pi} \cdot c^2$

So $\quad T_\mu = \sqrt{c^2 p_\mu^2 + m_\mu^2 c^4} - m_\mu c^2 = \sqrt{\frac{(m_\pi^2 - m_\mu^2)^2}{4\,m_\pi^2} + m_\mu^2} \cdot c^2 - m_\mu c^2$

$$= \frac{m_\pi^2 + m_\mu^2}{2\,m_\pi} c^2 - m_\mu c^2 = \frac{(m_\pi - m_\mu)^2}{2\,m_\pi} \cdot c^2$$

Substituting $\quad m_\pi c^2 = 139{\cdot}6\ \text{MeV}$

$m_\mu c^2 = 105{\cdot}7\ \text{MeV}$ we get

$$T_\mu = 4{\cdot}12\ \text{MeV}$$

Also $\quad E_\nu = \frac{m_\pi^2 - m_\mu^2}{2\,m_\pi} c^2 = 29{\cdot}8\ \text{MeV}$

6.296 We have

$$O = \vec{p_n} + \vec{p_\pi} \tag{1}$$

$$m_\Sigma c^2 = E_n + E_\pi$$

or $\quad (m_\Sigma c^2 - E_n)^2 = E_\pi^2$

or $\quad m_\Sigma^2 c^4 - 2\,m_\Sigma c^2 E_n = E_\pi^2 - E_n^2 = c^4 m_\pi^2 - c^4 m_n^2$

because (1) implies $\quad E_\pi^2 - E_n^2 = m_\pi^2 c^4 - m_n^2 c^4$

Hence $\quad E_n = \frac{m_\Sigma^2 + m_n^2 - m_\pi^2}{2\,m_\Sigma} c^2$

and $$T_n = \left(\frac{m_\Sigma^2 + m_n^2 - m_\pi^2}{2\, m_\Sigma} - m_n\right) c^2 = \frac{(m_\Sigma - m_n)^2 - m_\pi^2}{2\, m_\Sigma} c^2 .$$

Substitution gives $$T_n = 19{\cdot}55 \text{ MeV}$$

6.297 The reaction is

$$\mu^+ \to e^+ + \bar{\nu}_e + \bar{\nu}_\mu$$

The neutrinoes are massless. The positron will carry largest momentum if both neutriones (ν_e & $\bar{\nu}_\mu$) move in the same direction in the rest frame of the nuon. Then the final product is effectively a two body system and we get from problem (295)

$$(T_{e^+})_{\max} = \frac{(m_\mu - m_e)^2}{2\, m_\mu} c^2$$

Substitution gives $$(T_{e^+})_{\max} = 52{\cdot}35 \text{ MeV}$$

6.298 By conservation of energy-momentum

$$M c^2 = E_p + E_\pi$$
$$O = \vec{p}_p + \vec{p}_\pi$$

Then $$m_\pi^2 c^4 = E_\pi^2 - \vec{p}_\pi^{\,2} c^2 = (M c^2 - E_p)^2 - c^2 \vec{p}_p^{\,2}$$
$$= M^2 c^4 - 2 M c^2 E_p + m_p^2 c^4$$

This is a quadratic equation in M

$$M^2 - 2 \frac{E_p}{c^2} M + m_p^2 - m_\pi^2 = 0$$

or using $E_p = m_p c^2 + T$ and solving

$$\left(M - \frac{E_p}{c^2}\right)^2 = \frac{E_p^2}{c^4} - m_p^2 + m_\pi^2$$

Hence, $$M = \frac{E_p}{c^2} + \sqrt{\frac{E_p^2}{c^4} - m_p^2 + m_\pi^2}$$

taking the positive sign. Thus

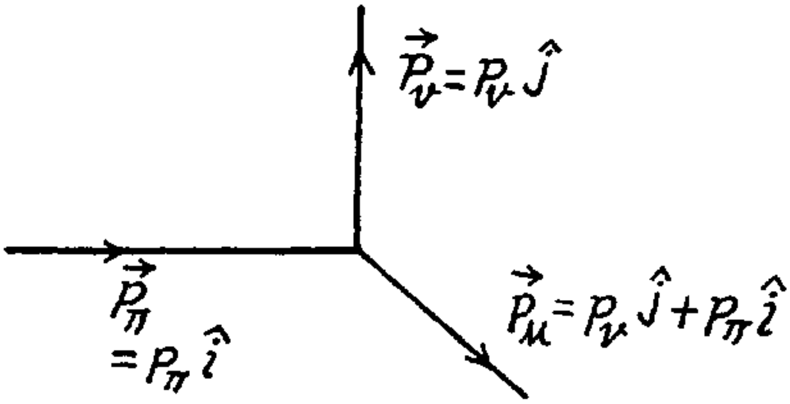
$$M = m_p + \frac{T}{c^2} + \sqrt{m_\pi^2 + \frac{T}{c^2}\left(2\, m_p + \frac{T}{c^2}\right)}$$

Substitution gives

$$M = 1115{\cdot}4 \frac{\text{MeV}}{c^2}$$

From the table of masses we identify the particle as a Λ particle

6.299



See the diagram. By conservation of energy

$$\sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} = c\, p_\nu + \sqrt{m_\mu^2 c^4 + p_\pi^2 c^2 + c^2 p_\nu^2}$$

or $$\left(\sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} - c\, p_\nu\right)^2 = m_\mu^2 c^4 + c^2 p_\pi^2 + c^2 p_\nu^2$$

or $$m_\pi^2 c^4 - 2\, c\, p_\nu \sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} = m_\mu^2 c^4$$

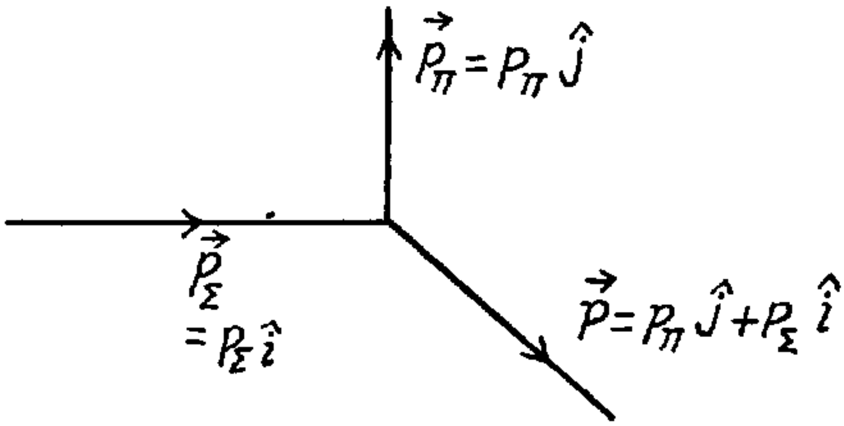
Hence the energy of the neutrino is

$$E_\nu = c\, p_\nu = \frac{m_\pi^2 c^4 - m_\mu^2 c^4}{2\,(m_\pi c^2 + T)}$$

on writing $$\sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} = m_\pi c^2 + T$$

Substitution gives $$E_\nu = 21{\cdot}93 \text{ MeV}$$

6.300



By energy conservation

$$\sqrt{m_\Sigma^2 c^4 + c^2 p_\Sigma^2} = \sqrt{m_\pi^2 c^4 + c^2 p_\pi^2} + \sqrt{m_n^2 c^4 + c^2 p_\pi^2 + c^2 p_\Sigma^2}$$

or $$\left(\sqrt{m_\Sigma^2 c^4 + c^2 p_\Sigma^2} - \sqrt{m_\pi^2 c^4 + c^2 p_\pi^2}\right)^2 = m_n^2 c^4 + c^2 p_\pi^2 + c^2 p_\Sigma^2$$

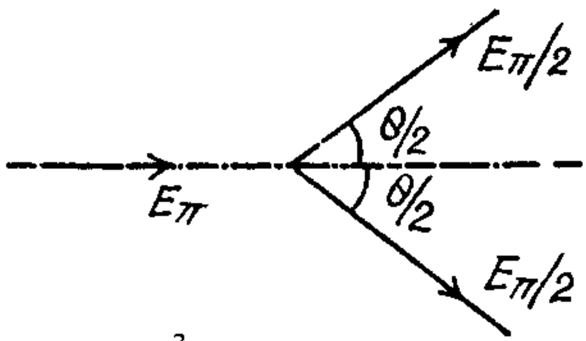
or $$m_\Sigma^2 c^4 + c^2 p_\Sigma^2 + m_\pi^2 c^4 + c^2 p_\pi^2 - 2\sqrt{m_\pi^2 c^4 + c^2 p_\Sigma^2}\sqrt{m_\pi^2 c^2 + c^2 p_\pi^2}$$
$$= m_n^2 c^2 + c^2 p_\pi^2 + c^2 p_\Sigma^2$$

or using the K.E. of Σ & π

$$m_n^2 = m_\Sigma^2 + m_\pi^2 - 2\left(m_\Sigma + \frac{T_\Sigma}{c^2}\right)\left(m_\pi + \frac{T_\pi}{c^2}\right)$$

and $$m_n = \sqrt{m_\Sigma^2 + m_\pi^2 - 2\left(m_\Sigma + \frac{T_\Sigma}{c^2}\right)\left(m_\pi + \frac{T_\pi}{c^2}\right)} = 0{\cdot}949\,\frac{\text{GeV}}{c^2}$$

6.301 Here by conservation of momentum



$$p_\pi = 2 \times \frac{E_\pi}{2c} \times \cos\frac{\theta}{2}$$

or $$c\,p_\pi = E_\pi \cos\frac{\theta}{2}$$

Thus $$E_\pi^2 \cos^2\frac{\theta}{2} = E_n^2 - m_\pi^2 c^4$$

or $$E_\pi = \frac{m_\pi c^2}{\sin\frac{\theta}{2}}$$

and $$T_\pi = m_\pi c^2\left(\operatorname{cosec}\frac{\theta}{2} - 1\right)$$

substitution gives $T_\pi = m_\pi c^2 = 135$ MeV for $\theta = 60°$.

Also $$E_\gamma = \frac{m_\pi c^2 + T_\pi}{2} = \frac{m_\pi c^2}{2}\operatorname{cosec}\frac{\theta}{2}$$

$$= m_\pi c^2 \text{ in this case } (\theta = 60°)$$

6.302 With particle masses standing for the names of the particles, the reaction is

$$m + M \rightarrow m_1 + m_2 + \dots$$

On R.H.S. let the energy momenta be $(E_1,\ c\vec{p_1})$, $(E_2,\ c\vec{p_2})$ etc. On the left the energy momentum of the particle m is $(E,\ c\vec{p})$ and that of the other particle is $(Mc^2,\ \vec{O})$, where, ofcourse, the usual relations

$$E^2 - c^2\vec{p}^{\,2} = m^2 c^4 \text{ etc}$$

hold. From the conservation of energy momentum we see that

$$(E + Mc^2)^2 - c^2\vec{p}^{\,2} = (\Sigma E_i)^2 - (\Sigma c\vec{p_i})^2$$

Left hand side is

$$m^2c^4 + M^2c^4 + 2Mc^2E$$

We evaluate the R.H.S. in the frame where $\Sigma\vec{p_i} = 0$ (CM frame of the decay product).

Then $$R.H.S. = (\Sigma E_i)^2 \geq (\Sigma m_i c^2)^2$$

because all energies are +ve. Therefore we have the result

$$E \geq \frac{(\Sigma m_i)^2 - m^2 - M^2}{2M} c^2$$

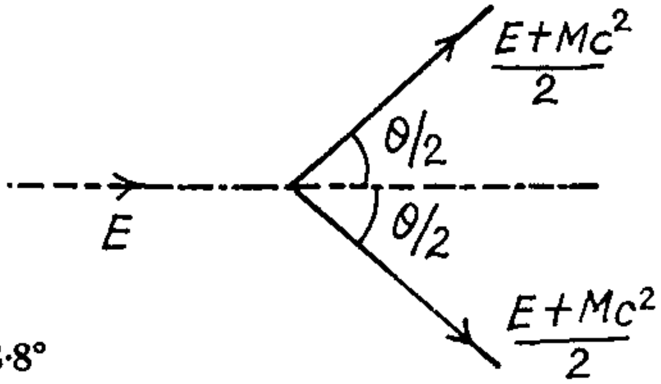
or since $E = mc^2 + T$, we see that $T \geq T_{th}$ where

$$T_{th} = \frac{(\Sigma m_i)^2 - (m+M)^2}{2M} c^2$$

6.303 By momentum conservation

$$\sqrt{E^2 - m_e^2 c^4} = 2\,\frac{E + m_e c^2}{2} \cos\frac{\theta}{2}$$

or $\cos\frac{\theta}{2} = \sqrt{\frac{E - m_e c^2}{E + m_e c^2}} = \sqrt{\frac{T}{T + 2 m_e c^2}}$



Substitution gives

$$\theta = 98{\cdot}8°$$

6.304 The formula of problem 3.02 gives

$$E_{th} = \frac{(\Sigma m_i)^2 - M^2}{2M} c^2$$

when the projectile is a photon

(a) For $\gamma + e^- \to e^- + e^- + e^+$

$$E_{th} = \frac{9 m_e^2 - m_e^2}{2 m_e} c^2 = 4 m_e c^2 = 2{\cdot}04\,\text{MeV}$$

(b) For $\gamma + p \to p + \pi^+ \pi^-$

$$E_{th} = \frac{(M_p + 2 m_\pi)^2 - M_p^2}{2 M_p} c^2 = \frac{4 m_\pi M_p + 4 m_\pi^2}{2 M_p} c^2 = 2\left(m_\pi + \frac{m_\pi^2}{M_p}\right) c^2 = 320{\cdot}8\,\text{MeV}$$

6.305 (a) For $p + p \to p + p + p + \bar{p}$

$$T \geq T_{th} = \frac{16 m_p^2 - 4 m_p^2}{2 m_p} c^2 = 6 m_p c^2 = 5{\cdot}63\,\text{GeV}$$

(b) For $p + p \to p + p + \pi^0$

$$T \geq T_{th} = \frac{(2 m_p + m_{\pi^0})^2 - 4 m_p^2}{2 m_p} c^2$$

$$= \left(2 m_\pi + \frac{m_{\pi^0}^2}{2 m_p}\right) c^2 = 0{\cdot}280\,\text{GeV}$$

6.306 (a) Here

$$T_{th} = \frac{(m_K + m_\Sigma)^2 - (m_\pi + m_p)^2}{2\, m_p} c^2$$

Substitution gives $T_{th} = 0{\cdot}904$ GeV

(b) $$T_{th} = \frac{(m_{K^0} + m_\Lambda)^2 - (m_{\pi^0} + m_p)^2}{2\, m_p} c^2$$

Substitution gives $T_{th} = 0{\cdot}77$ GeV.

6.307 From the Gell-Mann Nishijima formula

$$Q = T_Z + \frac{Y}{2}$$

we get $$O = \frac{1}{2} + \frac{Y}{2} \text{ or } Y = -1$$

Also $Y = B + S \Rightarrow S = -2$. Thus the particle is Ξ^0 0.

6.308 (1) The process $n \to p + e^- + \nu_e$ cannot occur as there are 2 more leptons (e^-, ν_e) on the right comopared to zero on the left.

(2) The process $\pi^+ \to \mu^+ + e^- + e^+$ is forbidden because this corresponds to a change of lepton number by, (0 on the left - 1 on the right)

(3) The process $\pi^- \to \mu^- + \nu_\mu$ is forbidden because μ^-, ν_μ being both leptons $\Delta L = 2$ hre.

(4), (5), (6) are allowed (except that one must distinguish between muon neutrinoes and electron neutrinoes). The correct names would be

(4) $p + e^- \to n + \nu_e$

(5) $\mu^+ \to e^+ + \nu_e + \tilde{\nu}_\mu$

(6) $K^- \to \mu^- + \tilde{\nu}_\mu$.

6.309 (1) $\pi^- + p \to \Sigma^- + K^+$

$$0 \quad 0 \quad -1 \quad 1$$

so $$\Delta S = 0\text{, allowed}$$

(2) $\pi^- + p \to \Sigma^+ + K^-$

$$0 \quad 0 \quad -1 \quad 1$$

so $$\Delta S = -2\text{. forbidden}$$

(3) $\pi^- + p \to K^- + K^+ + n$

$$0 \quad 0 \to -1 \quad 1 \quad 0$$

so $$\Delta S = 0\text{, allowed.}$$

(4) $n + p \rightarrow \Lambda^{\circ} + \Sigma^{+}$

$$0 \quad 0 \quad -1 \quad -1$$

so $\Delta S = -2$. forbidden

(5) $\pi^{-} + n \rightarrow \Xi^{-} + K^{+} + K^{-}$

$$0 \quad 0 \rightarrow -2 \quad 1 \quad -1$$

so $\Delta S = -2$. forbidden.

(6) $K^{-} + p \rightarrow \Omega^{-} + K^{+} K^{\circ}$

$$-1 \quad 0 \quad -3 \quad +1 \quad +1$$

so $\Delta S = 0$. allowed.

6.310 (1) $\Sigma^{-} \rightarrow \Lambda^{\circ} + \pi^{-}$
is forbidden by energy conservation. The mass difference

$$M_{\Sigma^-} - M_{\Lambda^\circ} = 82 \frac{\text{MeV}}{c^2} < m_{\pi^-}$$

(The process $1 \rightarrow 2 + 3$ will be allowed only if $m_1 > m_2 + m_3$.)

(2) $\pi^{-} + p \rightarrow K^{+} + K^{-}$
is disallowed by conservation of baryon number.

(3) $K^{-} + n \rightarrow \Omega^{-} + K^{+} + K^{\circ}$
is forbidden by conservation of charge

(4) $n + p \rightarrow \Sigma^{+} + \Lambda^{\circ}$
is forbidden by strangeness conservation.

(5) $\pi^{-} \rightarrow \mu^{-} + e^{-} + e^{+}$
is forbidden by conservation of muon number (or lepton number).

(6) $\mu^{-} \rightarrow e^{-} + \nu_e + \tilde{\nu}_{\mu}$
is forbidden by the separate conservation of muon number as well as lepton number.

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