MIND MAP : LEARNING MADE SIMPLE

CHAPTER - 6

Application of Derivatives

Rate of change of quantities

If a quantity if 'y' varies with another quantity x so that $y = f(x)$, then $\frac{dy}{dx}\left[f'(x)\right]$ represents the rate of change of y w.r.t x and $\left.\frac{dy}{dx}\right|_{x=x_o}\left(f'(x_0)\right)$ represents the rate of change of y w.r.t. x at $x = x_o$

If 'x' and 'y' varies with another variable 't' i.e., if $x = f(t)$ and $y = g(t)$, then by chain rule $\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt}$, if $\frac{dx}{dt} \neq 0$.

For eg: if the radius of a circle, r = 5 cm, then the rate of change of the area of a circle per second w.r.t 'r' is – $\left.\frac{da}{dr}\right|_{r=5} = \left.\frac{d}{dr}(\pi r^2)\right|_{r=5} = \left.2\pi r\right|_{r=5} = 10\pi$

Approximations

Let $y=f(x)\Delta x$ be a small increment in 'x' and Δy be the small increment in y corresponding to the increment in 'x', i.e. $\Delta y = f(x+\Delta x) - f(x)$. Then, Δy is given by $dy=f'(x)dx$ or $dy = \left(\frac{dy}{dx}\right)\Delta x$, is a good approximation of Δy when $dx=\Delta x$ is relatively small and denote by $dy \approx \Delta y$. For eg: Let us approximate $\sqrt{36.6}$. To do this, we take $y=\sqrt{x}$, $x = 36$, $\Delta x = 0.6$ then $\Delta y = \sqrt{x+dx} - \sqrt{x}$

$= \sqrt{36.6} - \sqrt{36}$

$= \sqrt{36.6} - 6 \quad \Rightarrow \sqrt{36.6} = 6 + dy$

Now, dy is approximately Δy and is given by dy

$= \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(0.6) = \frac{1}{2\sqrt{36}}(0.6) = 0.05.$ So, $\sqrt{36.6} \approx 6 + 0.05 = 6.05.$

Maxima and Minima

A point C in the domain of 'f' at which either $f'(c)=0$ or is not differentiable is called a critical point of f.

First derivative test

Let f be continuous at a critical point C in open I. Then **(i)** if $f'(x) > 0$ at every point left of C and $f'(x) < 0$ at every point right of C, then 'C' is a point of local maxima. **(ii)** If $f'(x) < 0$ at every point left of C and $f'(x) > 0$ at every point right of C, then 'C' is a point of local minima.
(iii) If $f'(x)$ does not change sign as 'x' increases through C, then 'C' is called the point of inflection.

Second derivative test

Let f be a function defined on I and C∈I, f is twice differentiable at C. Then
(i) $x=C$ is a point of local max. If $f'(C)=0$ and $f''(C) < 0$, $f(C)$ is local max. of f.
(ii) $x=C$ is a point of local min if $f'(C)=0$ and $f''(C)>0$. $f(C)$ is local min of f. **(iii)** The test fails if $f'(C)=0$ and $f''(C)=0$

Increasing and decreasing functions

A function f is said to be (i) increasing on (a,b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) \leq f(x_2) \forall x_1 x_2 \in (a,b)$, and (ii) decreasing on (a, b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) > f(x_2) \forall x_1 x_2 \in (a,b)$

If $f'(x) \geq 0 \forall x \in (a,b)$ then f is increasing in (a,b) and if $f'(x) \leq 0 \forall x \in (a,b)$, then f is decreasing in (a,b) For eg: Let $f(x) = x^3 - 3x^2 + 4x, x \in R$, then $f'(x) = 3x^2 - 6x + 4 = 3(x-1)^2 + 1 > 0 \forall x \in R$. So, the function f is strictly increasing on R.

Tangents and Normals

The equation of the tangent at (x_0, y_0), to the curve $y = f(x)$ is given by $(y - y_0) = \left.\frac{dy}{dx}\right]_{(x_0, y_0)}(x - x_0)$ if $\frac{dy}{dx}$ does not exists at (x_0, y_0), then the tangent at (x_0, y_0) is parallel to the y–axis and its equation is $x = x_0$. If tangent to a curve $y = f(x)$ at $x = x_0$ is parallel to x–axis, then $\left.\frac{dy}{dx}\right|_{x=x_0} = 0$.

Equation of the normal to the curve

$y = f(x)$ at (x_0, y_0) is $y - y_0 = -\frac{1}{\left.\frac{dy}{dx}\right|_{(x_0, y_0)}}(x - x_0)$ if $\frac{dy}{dx}$ at (x_0, y_0) is zero, then equation of the normal is $x = x_0$. If $\frac{dy}{dx}$ at (x_0, y_0) does not exist, then the normal is parallel to x-axis and its equation is $y=y_0$ For eg: Let $y=x^3-x$ be a curve, then the slope of the tangent to $y=x^3-x$ at $x=2$ is $\left.\frac{dy}{dx}\right|_{x=2} = 3x^2 - 1 = 3.2^2 - 1 = 11$