

Exercise: 7.3

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1. $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P , show that

- (i) $\triangle ABD \cong \triangle ACD$
- (ii) $\triangle ABP \cong \triangle ACP$
- (iii) AP bisects $\angle A$ as well as $\angle D$.
- (iv) AP is the perpendicular bisector of BC .

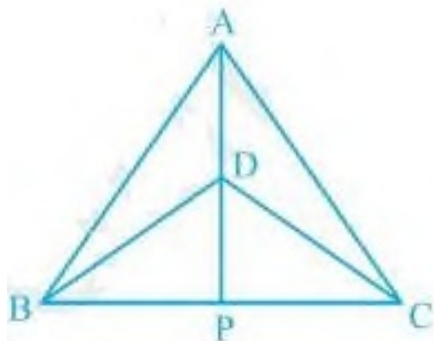


Fig. 7.39

Solution:

In the above question, it is given that $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles.

(i) $\triangle ABD$ and $\triangle ACD$ are similar by SSS congruency because:

$AD = AD$ (It is the common arm)

$AB = AC$ (Since $\triangle ABC$ is isosceles)

$BD = CD$ (Since $\triangle DBC$ is isosceles)

$\therefore \triangle ABD \cong \triangle ACD$.

(ii) $\triangle ABP$ and $\triangle ACP$ are similar as:

$AP = AP$ (It is the common side)

$\angle PAB = \angle PAC$ (by CPCT since $\triangle ABD \cong \triangle ACD$)

$AB = AC$ (Since $\triangle ABC$ is isosceles)

So, $\triangle ABP \cong \triangle ACP$ by SAS congruency condition.

(iii) $\angle PAB = \angle PAC$ by CPCT as $\triangle ABD \cong \triangle ACD$.

AP bisects $\angle A$ (i)

Also, $\triangle BPD$ and $\triangle CPD$ are similar by SSS congruency as

$PD = PD$ (It is the common side)

$BD = CD$ (Since $\triangle DBC$ is isosceles.)

$BP = CP$ (by CPCT as $\triangle ABP \cong \triangle ACP$)

So, $\triangle BPD \cong \triangle CPD$.

Thus, $\angle BDP = \angle CDP$ by CPCT. ... (ii)

Now by comparing (i) and (ii) it can be said that AP bisects $\angle A$ as well as $\angle D$.

(iv) $\angle BPD = \angle CPD$ (by CPCT as $\triangle BPD \cong \triangle CPD$)

and $BP = CP$... (i)

also,

$\angle BPD + \angle CPD = 180^\circ$ (Since BC is a straight line.)

$$\Rightarrow 2\text{BPD} = 180^\circ$$

$$\Rightarrow \text{BPD} = 90^\circ \quad \dots \text{(ii)}$$

Now, from equations (i) and (ii), it can be said that

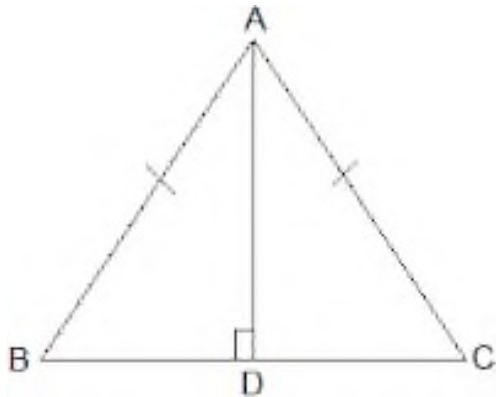
AP is the perpendicular bisector of BC.

**2. AD is an altitude of an isosceles triangle ABC in which $AB = AC$.
Show that**

(i) AD bisects BC (ii) AD bisects A.

Solution:

It is given that AD is an altitude and $AB = AC$. The diagram is as follows:



(i) In $\triangle ABD$ and $\triangle ACD$,

$$\angle ADB = \angle ADC = 90^\circ$$

$AB = AC$ (It is given in the question)

$AD = AD$ (Common arm)

$\therefore \triangle ABD \cong \triangle ACD$ by RHS congruence condition.

Now, by the rule of CPCT,

$$BD = CD.$$

So, AD bisects BC

(ii) Again, by the rule of CPCT, $\angle BAD = \angle CAD$

Hence, AD bisects $\angle A$.

3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of $\triangle PQR$ (see Fig. 7.40). Show that:

(i) $\triangle ABM \cong \triangle PQN$

(ii) $\triangle ABC \cong \triangle PQR$

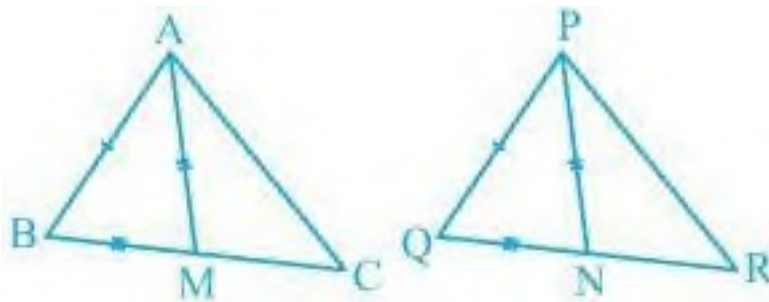


Fig. 7.40

Solution:

Given parameters are:

$$AB = PQ,$$

$$BC = QR \text{ and}$$

$$AM = PN$$

$$(i) \frac{1}{2} BC = BM \text{ and } \frac{1}{2} QR = QN \quad (\text{Since AM and PN are medians})$$

$$\text{Also, } BC = QR$$

$$\text{So, } \frac{1}{2} BC = \frac{1}{2} QR$$

$$\Rightarrow BM = QN$$

In $\triangle ABM$ and $\triangle PQN$,

$AM = PN$ and $AB = PQ$ (As given in the question)

$BM = QN$ (Already proved)

$\therefore \triangle ABM \cong \triangle PQN$ by SSS congruency.

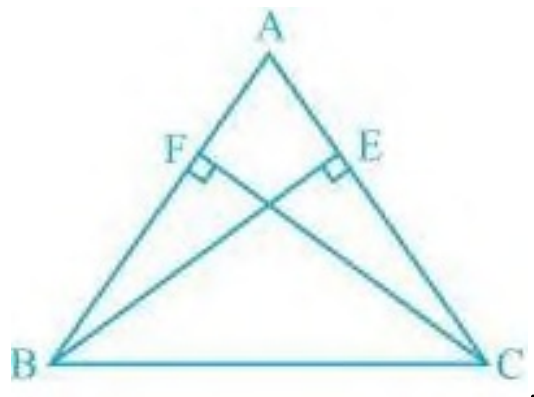
(ii) In $\triangle ABC$ and $\triangle PQR$,

$AB = PQ$ and $BC = QR$ (As given in the question)

$\angle ABC = \angle PQR$ (by CPCT)

So, $\triangle ABC \cong \triangle PQR$ by SAS congruency.

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



Solution:

It is known that BE and CF are two equal altitudes.

Now, in $\triangle BEC$ and $\triangle CFB$,

$\angle BEC = \angle CFB = 90^\circ$ (Same Altitudes)

$BC = CB$ (Common side)

$BE = CF$ (Common side)

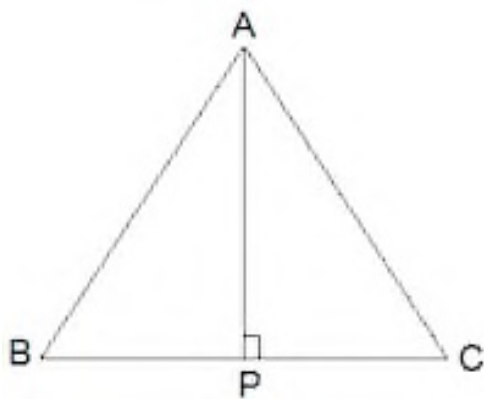
So, $\triangle BEC \cong \triangle CFB$ by RHS congruence criterion.

Also, $\angle C = \angle B$ (by CPCT)

Therefore, $AB = AC$ as sides opposite to the equal angles is always equal.

5. ABC is an isosceles triangle with $AB = AC$. Draw $AP \perp BC$ to show that $\angle B = \angle C$.

Solution:



In the question, it is given that $AB = AC$

Now, $\triangle ABP$ and $\triangle ACP$ are similar by RHS congruency as

$\angle APB = \angle APC = 90^\circ$ (AP is altitude)

$AB = AC$ (Given in the question)

$AP = AP$ (Common side)

So, $\triangle ABP \cong \triangle ACP$.

$\therefore \angle B = \angle C$ (by CPCT)