Chapter 2

Flywheels and Gear Trains

CHAPTER HIGHLIGHTS

- *Flywheels*
- *Turning Moment Diagrams and Flywheel*
- *Flywheel*
- *Fluctuation of Energy*
- *Coefficient of Steadiness*
- *Energy Stored in Flywheel*
- *Flywheel Rim Dimensions*
- *Flywheel in Punching Press*
- *Gear Trains*
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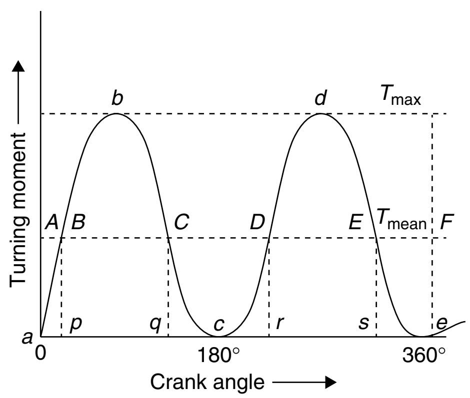
Flywheels

Turning Moment Diagrams and Flywheel

Turning moment diagram is the graphical representation of turning moment or crank effort for various positions of the crank. It is also known as **crank effort diagram**. In plotting the diagram, the turning moment is taken as the ordinate and the crank angle as abscissa.

The turning moment diagram for a single cylinder, double acting steam engine is as shown below.



As work done is the product of the turning moment and the angle turned, **the area of the turning moment diagram represents the work done per revolution.**

The mean torque (T_{mean}) against which the engine works is given by

$$T_{mean} = \frac{\text{Area of turning moment diagram for one cycle}}{\text{Angle turned by crank (in radian) per cycle}}$$

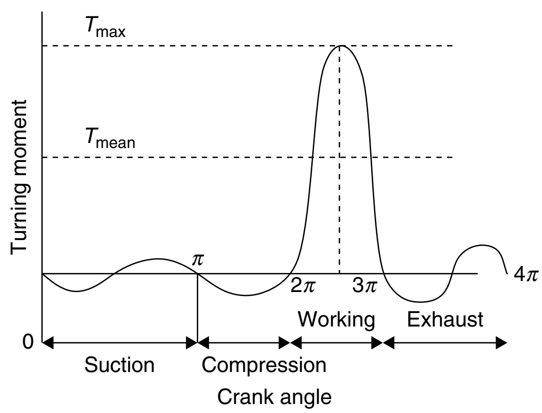
For steam engine and 2-stroke IC engines, angle turned by crank/cycle = 2π rad while it is 4π rad for 4 stroke IC engines.

It is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque. The area of the rectangle *aAFe* in the figure is proportional to the work done against the mean resisting torque.

When the engine torque is more than the mean resisting torque, the crank shaft accelerates and the work is done by the steam. This is represented in areas between *B* and *C* or *D* and *E*.

When the engine torque is less than the mean resisting torque, the crank shaft rotates and the work is done on the steam.

The turning moment diagram for a 4-stroke cycle IC engine is as shown below.



During suction stroke, the pressure inside the cylinder is less than the atmospheric and a part negative loop is formed. During compression stroke, the work is done on the gases and a higher negative loop is formed. During working stroke, the fuel burns and the gases expand, and a large positive loop is obtained. During exhaust stroke, the work is done on the gases and a part negative loop is formed. The effect of inertia forces on the piston is also taken into account. The loops above and below abscissa are positive and negative, respectively.

Flywheel

The flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than the supply.

The flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation for constant load.

In the case of an engine, the energy is produced only during the power stroke; therefore, the input torque varies but the load on the crank shaft is constant. The flywheel **does not control the speed variations** caused by **varying loads** on crank shaft. The speed control in that case is done by **governors**.

Therefore, there is energy fluctuations and as a result there is speed fluctuations. The function of the flywheel is to control the speed fluctuations per cycle. It can also be used to perform the said function **when the input torque is constant and the load varies** during the cycle **as in the case of punching press or rivetting machines**.

Fluctuation of Energy

When the engine torque is more than the mean torque, the flywheel is accelerated and excess energy is stored as kinetic energy. When the engine torque is less than the mean torque, the flywheel releases the stored kinetic energy. The variations of energy above and below the mean resisting torque line are called fluctuations of energy. The speed will be maximum at the end of a positive loop (Positions q or s in figure) and minimum at the end a negative loop (Position p or r in figure). The difference between the maximum and minimum energies is known as **maximum fluctuation of energy.**

Coefficient of fluctuation of energy is defined as the ratio of the maximum fluctuation of energy to the work done per cycle.

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

Work done per cycle = $T_{\text{mean}} \times \theta$

Where T_{mean} = Mean torque and

θ = Angle turned in radian during a cycle

= 2π for steam engine and two stroke IC Engine

= 4π for 4 stroke IC Engine

$$T_{\text{mean}} = \frac{P \times 60}{2\pi N}$$

Where P = Power transmitted in watt

N = Speed in rpm

The work done per cycle may also be obtained using the relation

$$\text{Work done per cycle} = \frac{P \times 60}{n}$$

Where n = Number of working strokes per Minute

Coefficient of Fluctuation of Speed (C_s)

The difference between the maximum and the minimum speeds during a cycle is called the maximum fluctuation of speed.

N_1 = maximum speed

N_2 = Minimum speed

$$N = \text{Mean speed} = \frac{N_1 + N_2}{2}$$

Coefficient of Fluctuation of Speed C_s

$$= \frac{\text{Maximum fluctuation of speed}}{\text{Mean speed}}$$

$$= \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} = \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2}$$

(ω = angular velocity

v = linear speed)

Coefficient of Steadiness (m)

$$M = \frac{1}{C_s} = \frac{N}{N_1 - N_2}$$

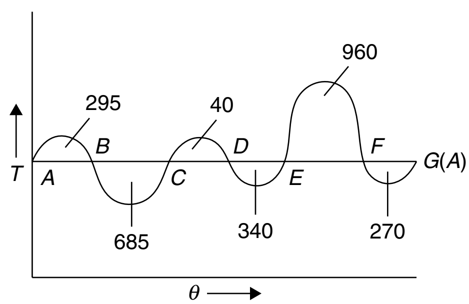
Solved Examples

Example 1: The turning moment diagram of a petrol engine is drawn to the following scales. Turning moment, 1 mm = 5 Nm Crank angle, 1 mm = 1°

The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm^2. The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150 mm. Coefficient of fluctuation of speed when the engine runs at 1800 rpm is ________

Solution:

m = 36 kg; k = 150 mm = 0.15 m; N = 1800 rpm



1 mm^2 of turning moment diagram

$$= 5 \times \frac{\pi}{180} = \frac{\pi}{36} \text{ N m}$$

Let the total energy at $A = E$
Energy at $B = E + 295$
Energy at $C = E + 295 - 685 = E - 390$
Energy at $D = E - 390 + 40 = E - 350$
Energy at $E = E - 350 - 340 = E - 690$
Energy at $F = E - 690 + 960 = E + 270$
Energy at $G = E + 270 - 270 = E$
It is seen that the maximum energy is at B and the minimum energy is at E
Maximum fluctuation of energy

$$\Delta E = (E + 295) - (E - 690)$$

$$= 985 \text{ mm}^2 = 985 \times \frac{\pi}{36} = 86 \text{ N m}$$

$$\Delta E = mk^2\omega^2 C_s \left(\because \omega = \frac{2\pi N}{60} \text{ rad/s}\right)$$

$$= \frac{\pi^2}{900} mk^2 N^2 C_s,$$

where

C_s = coefficient of fluctuation of speed

$$\therefore 86 = \frac{\pi^2}{900} \times 36 \times (0.15)^2 \times (1800)^2 C_s$$

$$C_s = 0.00299$$
$$= 0.3\%$$

Energy Stored in Flywheel

The mean kinetic energy of the flywheel

$$E = \frac{1}{2} I\omega^2 = \frac{1}{2} mk^2\omega^2$$

Where I = mass moment of inertia in kg m^2
m = mass in kg
ω = Mean angular speed in radian/s
k = radius of gyration in metre
The maximum fluctuation of energy
ΔE = Maximum kinetic energy – Minimum kinetic energy

$$= \frac{1}{2} I\left(\omega_1^2 - \omega_2^2\right)$$

$$= \frac{1}{2} I(\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= I\omega(\omega_1 - \omega_2)\left[\because \frac{\omega_1 + \omega_2}{2} = \omega\right]$$

$$= I\omega^2 \frac{(\omega_1 - \omega_2)}{\omega}$$

$$= I\omega^2 . C_s \left[\because \frac{\omega_1 - \omega_2}{\omega} = C_s\right]$$

$= m v^2 C_s$ (v = mean peripheral speed of flywheel)

$$= 2 E C_s \left(\because E = \frac{1}{2} I\omega^2 = \frac{1}{2} mv^2\right)$$

Since $\omega = \dfrac{2\pi N}{60}$

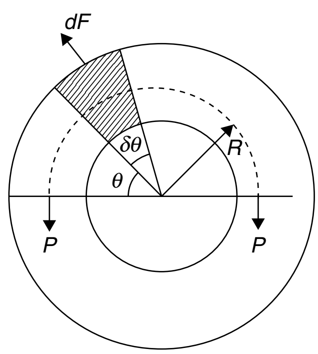
$$\Delta E = \frac{\pi^2}{900} IN(N_1 - N_2)$$

$$= \frac{\pi^2}{900} IN^2 C_s$$

For finding out the mass moment of inertia

$I = mk^2$, the radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. The mass moment of inertia of the hub and arms is neglected as these are nearer to the axis of rotation and the value of the moment of inertia for these is very small compared to the moment of inertia of the rim.

Flywheel Rim Dimensions



Consider a small element of rim as shown in figure.
Volume of the element = $A \times R\, \delta\theta$
where A = area of cross section
Mass of the element
$dm = \rho A R\, \delta\theta$
where ρ = density, R = means radius of rim
Centrifugal force $dF = dm\, \omega^2 R$

$$= \rho A R^2 \delta\theta\, \omega^2$$

Vertical component of centrifugal force

$$= dF \sin\theta$$
$$= \rho A R^2 \omega^2 \delta\theta \sin\theta$$

Total vertical upward force = $\rho A R^2 \omega^2$
This vertical upward force will produce tensile stress or hoop stress and it is resisted by $2P$ such that

$$2P = 2\sigma A$$

where σ = tensile or hoop stress
From the above we get,

$$2\,\rho\,A\,R^2\,\omega^2 = 2\,\sigma\,A$$

$$\sigma = \rho\,\omega^2\,R^2 = \rho\,v^2,$$

where v = mean speed of flywheel (m/s), measured at the mean radius of rim = ωR
Mass of the rim m = Volume × density

$$= \pi\,D\,A \times \rho,$$

D = mean diameter of rim

$$\therefore A = \frac{m}{\pi D \rho}$$

Example 2: From the turning moment diagram of a multi cylinder engine running at 800 rpm, the maximum fluctuation of energy is found to be 23500 N m.

The engine has a stroke of 300 mm. The fluctuation of speed is not to exceed ± 2% of the mean speed. If safe centrifugal stress is not to exceed 7 MPa and density of material is 7200 kg/m^3, the mean diameter and cross-sectional area of the rim of the flywheel are _____.

Solution:

$$N = 800 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = 83.8 \text{ rad/s}$$

Stroke = 300 mm

$$\sigma = 7 \text{ MPa}$$
$$= 7 \times 10^6 \text{ N/m}^2$$
$$\rho = 7200 \text{ kg/m}^3$$

Fluctuation of speed

$$\omega_1 - \omega_2 = 4\% \text{ of } \omega$$
$$= 0.04\,\omega$$

$$\therefore C_s = \frac{\omega_1 - \omega_2}{\omega}$$

$$= \frac{0.04\,\omega}{\omega} = 0.04$$

$\sigma = \rho v^2$
$\therefore 7 \times 10^6 = 7200\,v^2$
or $v = 31.2$ m/s
Let D be the mean diameter of the flywheel

$$v = \frac{\pi D N}{60}$$

$$\therefore \frac{\pi \times D \times 800}{60} = 31.2$$

$D = 0.745$ m
Maximum fluctuation of energy

$$= I\,\omega^2 C_s$$

$$= mR^2 \left(\frac{v}{R}\right)^2 C_s$$

$$= m\,v^2 C_s$$

$$= 23500 \text{ N m } (\because \text{ data})$$

$\therefore m \times (31.2)^2 \times 0.04 = 23500$

$\Rightarrow m = 603.53$ kg
But $\pi\,D\,A\,\rho = m$
Where A = cross sectional area of rim
$\therefore \pi \times 0.745 \times A \times 7200 = 603.53$
$\Rightarrow A = 0.0358$ m^2.

Flywheel in Punching Press

In the case where engine load is constant and the input torque varies during a cycle. The flywheel is used to reduce fluctuations of speed. But in the case of a punching press or a rivetting machine, the input torque is constant and the load during cycle varies. Here also a flywheel can be used to reduce the fluctuation of speed.

Work done or energy required for punching a hole,

$$E_1 = \frac{1}{2} F_s \times t,$$

where
F_s = Maximum shear force
$= \pi\,d_1\,t_1\,\tau_u$
Where d_1 = diameter of hole to be punched
t_1 = thickness of plate
τ_u = ultimate shear stress of plate material
(It is assumed that as the hole is punched, the shear force decreases uniformly from maximum to zero)

Assuming one punching operation per revolution, the energy supplied to shaft by motor per revolution also should be equal to E_1

Let the punching operation take place during the crank angle positions θ_1 to θ_2. The energy supplied by the motor during punching operation.

$$E_2 = E_1 \left(\frac{\theta_2 - \theta_1}{2\pi}\right)$$

The energy supplied by the flywheel,
(Balance energy required for punching)

$$= E_1 - E_2$$

$$= E_1 - E_1 \left(\frac{\theta_2 - \theta_1}{2\pi}\right)$$

$$= E_1 \left[1 - \left(\frac{\theta_2 - \theta_1}{2\pi}\right)\right]$$

The energy is supplied by the flywheel by a decrease in its kinetic energy when the speed falls from maximum to minimum.
Thus, the maximum fluctuation of energy

$$\Delta E = E_1 - E_2$$

$$= E_1 \left(1 - \frac{(\theta_2 - \theta_1)}{2\pi}\right)$$

Example 3: A punching press is driven by a constant torque electric motor. The flywheel of the punching press rotates at a maximum speed of 220 rpm. The radius of gyration of the flywheel is 0.5 m. The press punches 12 holes per minute. Each punching operation requires 15 kN m of energy and takes 2 s. If the minimum speed of the flywheel is limited to 200 rpm, the minimum mass of the flywheel is _______.

Solution:

$N_1 = 220$ rpm; $N_2 = 200$ rpm; $k = 0.5$ m; $E_1 = 15$ kN m;

Holes/minute = 12

Required power of the motor

$$= 15000 \times \frac{12}{60}\frac{\text{Nm}}{s} = 3000 \text{ W}$$
$$= 3 \text{ kW}$$

Energy supplied by the motor during the punching operation of 2 s

$$= 3000 \times 2 \text{ J} = 6000 \text{ J}$$

Energy supplied by the flywheel during punching = maximum fluctuation of energy

$$\Delta E = 15000 - 6000 = 9000 \text{ J}$$

$$\text{Mean speed} = \frac{N_1 + N_2}{2}$$
$$= \frac{220 + 200}{2} = 210 \text{ rpm}$$

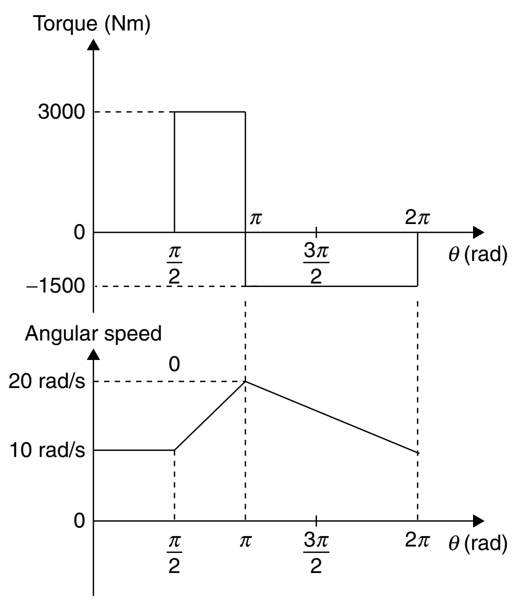
Let m be the mass of flywheel

$$\Delta E = \frac{\pi^2}{900} mk^2 N (N_1 - N_2)$$

i.e., $9000 = \frac{\pi^2}{900} \times m \times 0.5^2 \times 210(220 - 200)$

$\Rightarrow m = 781.62$ kg, is the minimum mass needed for the flywheel

Example 4:



The torque and the angular speed data over one cycle for a shaft carrying a flywheel are as shown in the above figures. The moment of inertia (in kg m^2) of the flywheel is _______________.

Solution:

During the angular displacement from $\theta_1 = \frac{\pi}{2}$ rad to $\theta_2 = \pi$ rad, the energy absorbed by the flywheel is equal to the area of turning moment diagram for this interval.

$$\therefore \Delta E = \text{Area of rectangle}$$
$$= \left(\pi - \frac{\pi}{2}\right) \times 3000$$
$$= 1500\pi \text{ Nm}$$

During the time, ω varies from

$\omega_1 = 10$ rad/s to $\omega_2 = 20$ rad/s (from graph)

We have $\Delta E = \frac{1}{2} I \left(\omega_2^2 - \omega_1^2\right)$

$$\Rightarrow I = \frac{2\Delta E}{\left(\omega_2^2 - \omega_1^2\right)}$$
$$= \frac{2 \times 1500\pi}{\left(20^2 - 10^2\right)}$$
$$= 10\pi \text{ kg m}^2$$
$$= 31.42 \text{ kg m}^2$$

Hence, the moment of inertia of the flywheel is 31.42 kg m^2.

Example 5: If C_f is the coefficient of speed fluctuation of a flywheel, then ratio of $\omega_{max}/\omega_{min}$ will be

(A) $\frac{1 - 2C_f}{1 + 2C_f}$ (B) $\frac{2 - C_f}{2 + C_f}$

(C) $\frac{1 + 2C_f}{1 - 2C_f}$ (D) $\frac{2 + C_f}{2 - C_f}$

Solution:

$$C_f = \frac{\omega_{max} - \omega_{min}}{\omega}$$
$$= \frac{2(\omega_{max} - \omega_{min})}{(\omega_{max} + \omega_{min})}$$
$$= \frac{2\omega_{min}\left(\frac{\omega_{max}}{\omega_{min}} - 1\right)}{\omega_{min}\left(\frac{\omega_{max}}{\omega_{min}} + 1\right)}$$
$$= \frac{2\left(\frac{\omega_{max}}{\omega_{min}} - 1\right)}{\left(\frac{\omega_{max}}{\omega_{min}} + 1\right)}$$

$$C_f\left(\frac{\omega_{\max}}{\omega_{\min}}\right)+C_f = 2\frac{\omega_{\max}}{\omega_{\min}}-2$$

$$\Rightarrow \left(\frac{\omega_{\max}}{\omega_{\min}}\right)=\left(\frac{2+C_f}{2-C_f}\right)$$

Example 6: The maximum fluctuation of kinetic energy in an engine has been calculated to be 2600 J. Assuming that the engine runs at an average speed of 200 rpm, the polar mass moment of inertia (in kg m^2) of the flywheel to keep the speed fluctuation within ± 0.5% of the average speed is ________.

Solution:

Given $N = 200$ rpm: $\Delta E = 2600$ J

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times 200}{60}\text{ rad s}^{-1}$$

$$= 20.944\text{ rad s}^{-1}$$

$$C_s = 0.01\ (\because\ \pm 0.5\% = 1\%)$$

$$\Delta E = I\omega^2 C_s$$

$$\Rightarrow I = \frac{\Delta E}{\omega^2 C_s}$$

$$= \frac{2600}{(20.944)^2\times 0.01}$$

$$= 592.73\text{ kg m}^2$$

Hence, the polar moment of inertia of the flywheel is 592.73 kg m^2.

Example 7: Consider the following statements.

1. Flywheel reduces speed fluctuations during a cycle for a constant load but flywheel does not control the mean speed of an engine, if the load changes.
2. Flywheel can be used to control speed fluctuations during a cycle when input torque is constant and load varies during the cycle.
3. Governor controls the speed fluctuations during a cycle for constant load but governor does not control the mean speed of the engine if the load changes. The correct statements are

(A) 2 and 3 only (B) 1 and 3 only
(C) 1 only (D) 1 and 2 only

Solution:

Statement 3 is not correct. Other statements are correct.

Example 8: The moment of inertia of a flywheel is 1000 kg m^2. It starts from rest and rotates with a uniform angular acceleration of 0.5 rad/s^2. Its kinetic energy (in J) after 5 s from start is

(A) 2500 J (B) 3125 J
(C) 12500 J (D) 25000 J

Solution:

Given $I = 1000$ kg m^2

$\alpha = 0.5$ rad/s^2

$t = 5$ s

$\omega_0 = 0$

$\omega = \omega_0 + \alpha t = 0 + 0.5\times 5 = 2.5$ rad/s

$$\text{KE} = \frac{1}{2}I\,\omega^2 = \frac{1}{2}\times 1000\times(2.5)^2$$

$= 3125$ J Choice (B)

Example 9: A flywheel is fitted to the crankshaft of an engine having indicated work per revolution E. If the permissible limits of the coefficients of fluctuation of energy and speed are k_E and k_S respectively, then the kinetic energy of the flywheel is equal to

(A) $\frac{k_E E}{2k_S}$ (B) $\frac{2k_E E}{k_S}$

(C) $\frac{k_S E}{2k_E}$ (D) $\frac{k_E E}{k_S}$

Solution:

$$\Delta E = k_E E$$

$$\Delta E = \frac{1}{2}I\left(\omega_{\max}{}^2-\omega_{\min}{}^2\right)$$

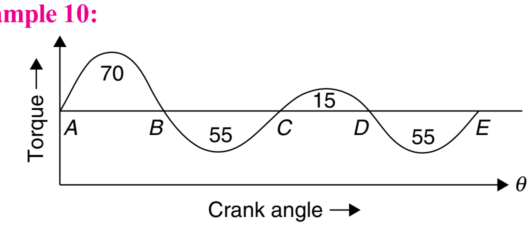
$$= \frac{1}{2}I(\omega_{\max}+\omega_{\min})(\omega_{\max}-\omega_{\min})$$

$$= I\omega\left(\frac{\omega_{\max}-\omega_{\min}}{\omega}\right)\omega$$

$$= I\omega^2 K_s = \left(\frac{1}{2}I\omega^2\right)2K_s$$

$$\Rightarrow \text{kinetic energy, } \left(\frac{1}{2}I\omega^2\right) = \frac{\Delta E}{2k_s} = \frac{k_E E}{2k_s}$$

Example 10:



The crank-effort diagram per cycle of an engine running a machine is shown in the areas above and below the mean line (in J). The maximum fluctuation of energy per cycle as per this diagram is

(A) 50 J (B) 85 J (C) 95 J (D) 45 J

Solution:

Let the energy at A be E_0

$\therefore$ Energy at $B = (E_0 + 70)$

Energy at $C = E_0 + 70 - 55 = (E_0 + 15)$

Energy at $D = E_0 + 15 + 15 = (E_0 + 30)$

Energy at $E = E_0 + 30 - 55 = E_0 - 25$

$\therefore E_{\max}$ (at B) $= E_0 + 70$

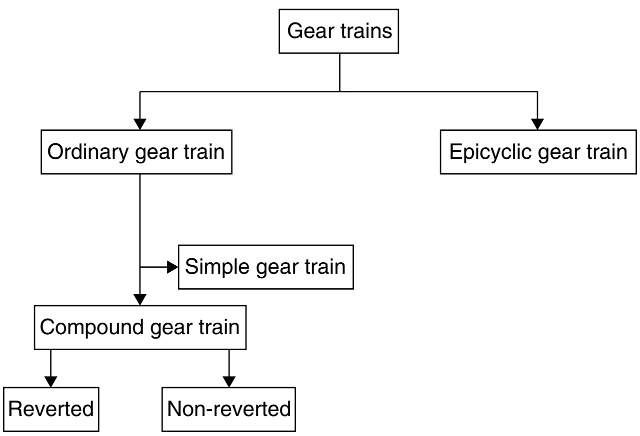
$E_{\min}$ (at E) $= E_0$

$\therefore \Delta E = (E_0 + 70) - (E_0 - 25) = 95$ J

Gear Trains

This section deals exclusively with the Gear Trains. For all other related concepts on gears, the section 'Gears' in 'Machine Design' of this book shall be referred to. However, some important concepts that are needed for study of gear trains are reiterated here.

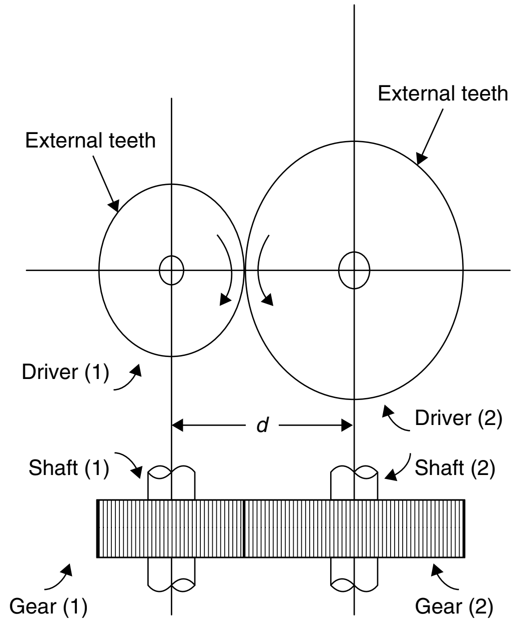
1. Module (m) is the ratio of pitch circle diameter (in mm) to the number of teeth. The module of two mating gears must be the same. $m = \dfrac{D_m}{T}$
2. Diametral pitch (P_d) is the number of teeth per unit length of pitch circle diameter (in mm) $P_d = \dfrac{T}{D_m}$,
   where
   T = no of teeth
   D_m = pitch circle diameter in (mm)
3. Circular pitch (P_c) is the distance along a pitch circle from one point on a tooth to the corresponding point on the next tooth.
   $$P_c = \frac{\pi D}{T} = \pi m$$
4. Gear ratio (G) is the ratio of number of teeth on gear to the number of teeth on pinion $G = \dfrac{T_G}{T_P}$
   where
   T_G = number of teeth on gear and
   T_P = number of teeth on pinion
5. Velocity ratio (VR) is the ratio of angular velocity of follower (ω_2) to the angular velocity of driver (ω_1)
   $$V_R = \frac{\omega_2}{\omega_1} = \frac{T_1}{T_2},$$
   where
   T_1 = number of teeth on driver and
   T_2 = number of teeth on driven
   A combination of two or more gears used for transmitting power from a driving shaft to a driven shaft is known as gear train. These are usually used when **a large speed reduction** is to be carried out **in a small available space**. The classification of gear trains is as follows.



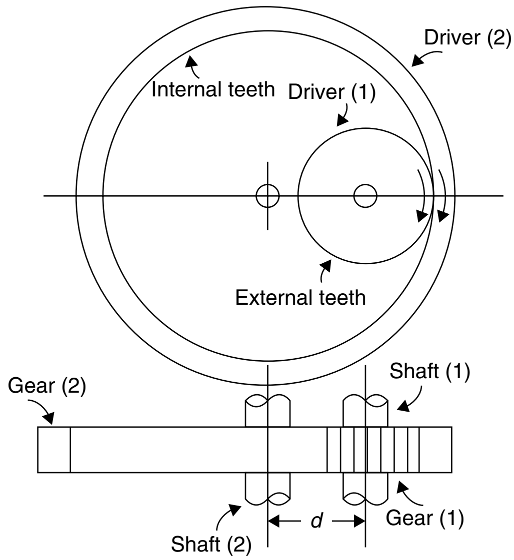
The mating gears in gear trains can have **external meshing**, in which case the **mating gears rotate in opposite sense** or they can have **internal meshing**, in which case the **mating gears rotate in the same sense**.

In **ordinary gear trains**, the **axes of the shafts** on which the gears are mounted **remain fixed** relative to each other.

Simple Gear Train



Simple Gear Train (External Meshing)

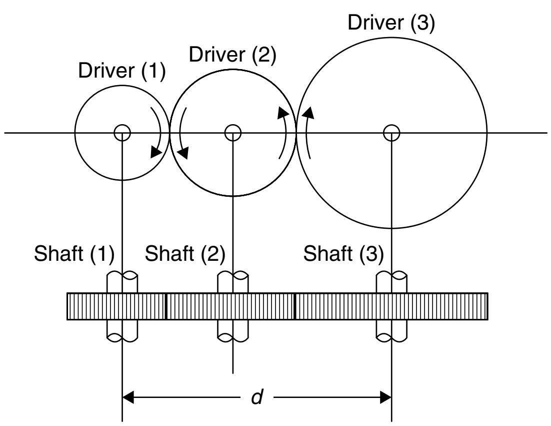


Simple Gear Train (Internal Meshing)

In simple gear trains,

1. Each gear is mounted on a separate shaft i.e. the number of gears is equal to the number of shafts. In the simplest form, there are only two gears.

2. The axes of the shafts on which the gears are mounted are fixed relative to each other.
3. When power is given to gear 1, it is called **the driver** and the other gear (gear 2) is called **the follower (or driven)**.
4. **Gears having external mesh rotate in opposite sense** while **gears having internal mesh rotate in same sense**.
5. If R_1 = pitch circle radius of gear 1 and R_2 = pitch circle radius of gear 2, then the distance between the axes of gears (d) is given by (when shafts are parallel) $d = R_1 + R_2$, for external meshing and $d = |R_1 - R_2|$ for internal meshing.
6. When the distance between the axes of the shafts is to be increased without changing the gear sizes, intermediate gears (mounted as separate axes) can be used. These intermediate gears are called idlers. For gears in external mesh, the driver and driven gears will rotate in the same sense, if the number of idlers (or total number of gears in train) is odd and they will rotate in the opposite sense, if the number of idlers (or total number of gears in train) is even.



$d = R_1 + 2R_2 + R_3$, with one idler. This formula holds good only when all the three shafts are parallel and centres of gears are on same straight line.

7. **Speed ratio (or velocity ratio)** of a gear train is defined as the ratio of angular speed of driver (ω_1) to the angular speed of driven (or follower)ω_2.

$\therefore$ speed ratio = $\dfrac{\omega_1}{\omega_2} = \dfrac{N_1}{N_2}$, where N_1 and N_2 are the speeds in rpm of the driver and follower.

The peripheral velocity V of any point on the pitch circle must be the same for all meshing gears; otherwise there will be slipping.

$$\therefore V = \omega_1 \frac{D_1}{2} = \omega_2 \frac{D_2}{2}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{D_2}{D_1} = \frac{mT_2}{mT_1} = \frac{T_2}{T_1}$$

(Here, D_1, D_2 = pitch circle diameters of gear 1 and 2, T_1, T_2 = number of teeth on gears 1 and 2
m = module of gears

$$\text{Speed ratio} = \frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

$$= \frac{\text{Speed of driver}}{\text{Speed of follower}}$$

$$= \frac{\text{No. of teeth on driven (or follower)}}{\text{No. of teeth on driver}}$$

8. The inverse of the speed ratio is known as the **train value**.

$$\text{Train value} = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

$$= \frac{\text{Speed of follower}}{\text{Speed of driver}}$$

$$= \frac{\text{No. of teeth on driver}}{\text{No. of teeth on follower}}$$

9. In an ideal gear train, the input and output powers are the same

$$\therefore \text{Power,} \quad P = \frac{2\pi N_1 \tau_1}{60} = \frac{2\pi N_2 \tau_2}{60}$$

where τ_1, τ_2 = torque on driver 1 and follower 2 respectively

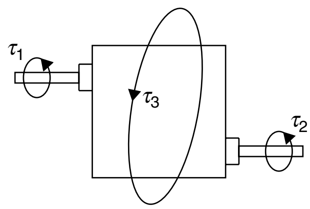
$$\therefore \frac{\tau_2}{\tau_1} = \frac{N_1}{N_2} = \text{speed ratio} = \frac{1}{\text{Train value}}$$

If mechanical efficiency (η) is given,

$$\text{then} \quad \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\left(\dfrac{2\pi N_2 \tau_2}{60}\right)}{\left(\dfrac{2\pi N_1 \tau_1}{60}\right)}$$

$$= \frac{N_2 \tau_2}{N_1 \tau_1}$$

As the output and input torque are different, in order to prevent the body of gear box from rotating, the gear box has to be clamped (i.e. a holding torque τ_3 must be applied to the body of gear box through the clamps), so that the total toque on system is zero.



$$\therefore \tau_1 + \tau_2 + \tau_3 = 0$$

Conventionally anti-clockwise torques are taken as positive and clockwise torques negative.

NOTES

1. The intermediate gears (or idlers) have no effect on speed ratio or train value of a simple gear train. They are only used for adjusting the distance between the axes of driver and follower, and also to change the direction of rotation of follower (clockwise or anticlockwise).
2. When **bevel gears are used in simple gear trains**, the formula for distance between the shafts given earlier cannot be used.
3. If the driver and the follower rotate in opposite sense, the speed ratio (or train value) will be negative; if the driver and the follower rotate in same sense, the speed ratio (or train value) will be positive.
4. The degree of freedom of a simple gear train is one.

Direction for questions (Example 11 and 12): Two parallel shafts are connected with the help of two gears A and B, with one gear on each shaft. The number of teeth on gear A is 41 and it is mounted on a shaft which is rotating at 540 rpm. Given the speed ratio is equal to 5 and circular pitch of gears is 22 mm. A is the driver and B is the driven gear.

Example 11: The number of teeth on gear B and the speed of its shaft (in rpm) are respectively

(A) 205, 2700 rpm (B) 205, 108 rpm
(C) 108, 205 rpm (D) 108, 2700 rpm

Solution:

Given $T_A = 41$; $N_A = 540$ rpm; speed ratio = 5 and circular pitch $P_c = 22$ mm

$$\text{Speed ratio} = \frac{N_A}{N_B}\left(=\frac{\text{Speed of driver}}{\text{Speed of driven}}\right)$$

$$\Rightarrow 5 = \frac{540}{N_B} \Rightarrow N_B = \frac{540}{5} = 108 \text{ rpm}$$

∴ Hence, speed of shaft of gear B is 108 rpm.

$$\text{Also, the speed ratio} = \frac{T_B}{T_A}$$

$$\Rightarrow 5 = \frac{T_B}{41} \Rightarrow T_B = 41 \times 5 = 205$$

Hence, number of teeth on B is 205 and speed of shaft of B is 108 rpm.

Example 12: The centre distance between the two shafts (in mm) is

(A) 861.35 mm (B) 633.72 mm
(C) 781.45 mm (D) 821.36 mm

Solution:

$$\text{We know } P_C = \frac{\pi D}{T} \Rightarrow D = \frac{P_C T}{\pi}$$

$$\therefore D_A = \frac{P_C T_A}{\pi} = \frac{22 \times 41}{\pi} = 287.12 \text{ mm}$$

$$D_B = \frac{P_C T_B}{\pi} = \frac{22 \times 205}{\pi} = 1435.58 \text{ mm}$$

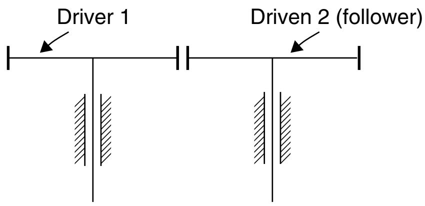
d = Distance between parallel shafts

$$= R_A + R_B$$

$$= \frac{D_A + D_B}{2} = \frac{287.12 + 1435.58}{2}$$

$$= \frac{1722.7}{2} = 861.35 \text{ mm.}$$

Example 13:



In the simple gear train shown, the shafts are parallel and carry one gear on each. The number of teeth on the driver is 25 and on the driven is 65 respectively. The train value of the gear train and the speed of the driver shaft (in rpm), if the driven shaft rotates at 250 rpm is

(A) 2.6, 650 rpm (B) 0.3846, 96.2 rpm
(C) 0.3846, 650 rpm (D) 0.6312, 396 rpm

Solution:

$$\text{Train value} = \frac{\text{Speed of follower}}{\text{Speed of driver}}$$

$$= \frac{\text{No. of teeth on driver}}{\text{No. of teeth on follower}}$$

$$= \frac{25}{65} = 0.3846$$

$$\therefore \text{Speed ratio} = \frac{1}{\text{Train value}}$$

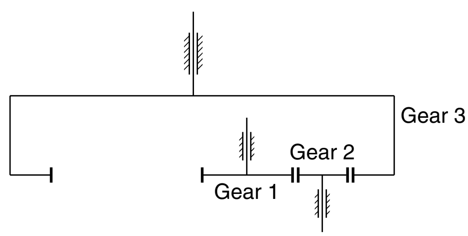
$$= \frac{65}{25} = 2.6$$

$$\text{But speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of follower}}$$

⇒ speed of driver = Speed ratio × Speed of follower

$$= 2.6 \times 250 = 650 \text{ rpm}$$

Example 14:



Gear 1, gear 2 and gear 3 form a simple gear train with parallel shafts. The number of teeth in gear 1, gear 2 and gear 3 are 45, 25 and 135 respectively. Gear 1 is the driver and rotates clockwise at 300 rpm. The speed ratio of the gear train and direction of rotation of the follower (gear 3) are respectively

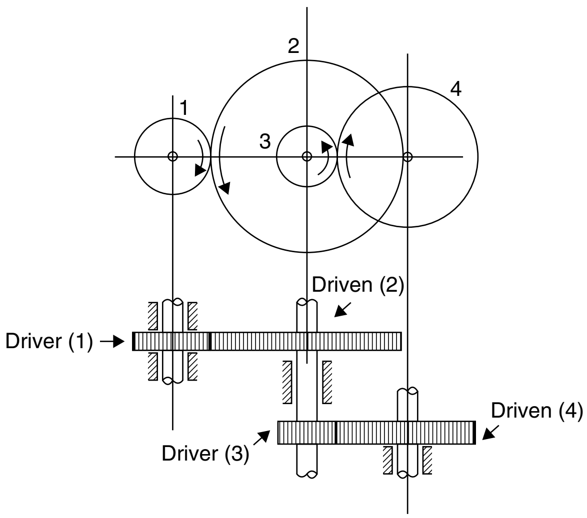
(A) 100, CCW (B) 1.67 , CCW
(C) 3, CW (D) 3, CCW

Solution:
Gear 2 is an idler and it does not affect the speed ratio of simple gear train. Gear 1 and Gear 2 are having external mesh and so they rotate in opposite directions → gear 2 rotates anticlockwise (∵ 1 rotates CW). Gear 2 and Gear 3 are having internal mesh → gear 2 and gear 3 rotate in same sense.
∴ Gear 3 rotates in anticlockwise direction (i.e. counter-clockwise CCW)

$$\text{Speed ratio} = \frac{N_1}{N_3} = \frac{T_3}{T_1} = \frac{135}{45} = 3$$

Compound Gear Trains

1. In compound gear trains, at least one shaft carries more than one gear. This shaft is usually the intermediate shaft and all the gears on this shaft rotate about the same axis, in the same sense, with the same angular velocity



A compound gear train is shown in figure. Gears 2 and 3 are mounted on the same shaft (intermediate shaft). Hence, gear 2 and gear 3 rotate with the same angular velocity, in the same sense of rotation.

The number of teeth on gears 1, 2, 3 and 4 are T_1, T_2, T_3 and T_4 respectively. The driver is gear 1 which rotates at a speed N_1 in clockwise (say) direction. This drives gear 2 which rotates in counter clockwise (∵ external mesh) direction at a speed N_2. As gear 3 mounted on same shaft as gear 2, speed of 3 is also N_2 (in CCW). Gear 3 drives gear 4 (the follower of this gear train) and rotates at a speed N_4 in the clockwise (CW) sense. (∵ 3 and 4 are in external mesh)

$$\therefore \frac{N_2}{N_1} = \frac{-T_1}{T_2} \Rightarrow N_2 = -N_1\left(\frac{T_1}{T_2}\right)$$

The minus (−) sign indicates that N_1 and N_2 rotate in opposite sense.

$$N_3 = N_2 = -N_1\left(\frac{T_1}{T_2}\right)$$

$$\frac{N_4}{N_3} = \frac{-T_3}{T_4} \Rightarrow N_4 = -N_3\frac{T_3}{T_4}$$

$$\Rightarrow N_4 = -\left(-N_1\frac{T_1}{T_2}\right)\frac{T_3}{T_4}$$

$= +N_1\frac{T_1T_3}{T_2T_4}$ (+ sign indicate N_1 and N_4 are rotating in same sense)

$$\therefore \frac{N_4}{N_1} = \frac{T_1T_3}{T_2T_4}$$

$$= \frac{\text{Speed of last driven (or follower)}}{\text{Speed of first driver}}$$

$$= \text{Train value}$$

$$\therefore \text{Train Value} = \frac{N_4}{N_1}$$

$$= \frac{\text{Product of number of teeth on driving gears}}{\text{Product of number of teeth on follower gears}}$$

$$\text{Speed ratio} = \frac{1}{\text{Train value}} = \frac{N_1}{N_4}$$

$$= \frac{\text{Speed of first driver}}{\text{Speed of last follower}}$$

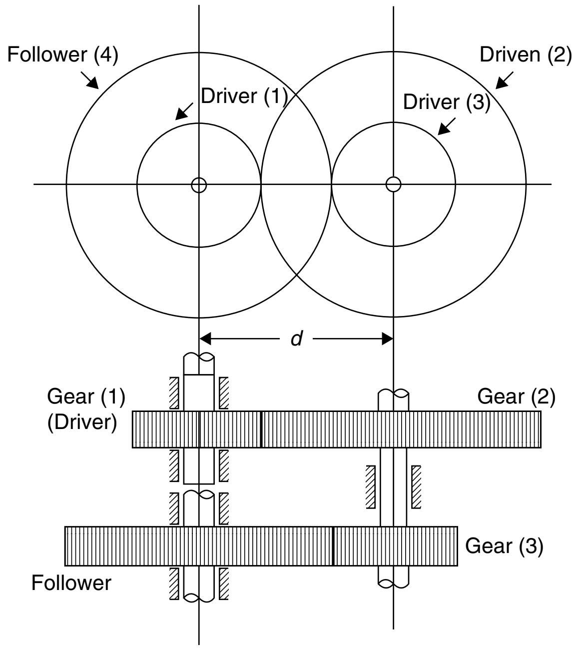
$$= \frac{\text{Product of number of teeth on follower gears}}{\text{Product of number of teeth on driving gears}}$$

Compound gear trains are preferred when large speed ratios are required.

The degree of freedom of a compound gear train is one.

Reverted Gear Trains

In a compound gear train, when the driving shaft and the driven shaft (or follower shaft) are co-axial, it is called a **Reverted gear train**. Such arrangements are used in **lathes and clocks**.



In the reverted gear train shown, the driver (gear 1) and the follower (gear 4) are co-axial. The intermediate shaft mounts gears 2 and 3 on it.

d = centre distance between gears 1 and 2

= centre distance between gears 3 and 4

$$\therefore\ d = R_1 + R_2 = R_3 + R_4 \qquad (1)$$

where R_1, R_2, R_3 and R_4 are pitch circle radii of gears 1, 2, 3 and 4 respectively.

If m_1 is the module for gears 1 and 2 (must be of same module as they are meshing) and m_2 is the module for gears 3 and 4 (must be of same module as they are meshing but need not be equal to m_1), then $R_1 = \frac{m_1 T_1}{2}$, $R_2 = \frac{m_1 T_2}{2}$, $R_3 = \frac{m_3 T_3}{2}$, $R_4 = \frac{m_3 T_4}{2}$ where T_1, T_2, T_3 and T_4 are the number of teeth on gears 1, 2, 3 and 4 respectively.

$$\therefore\ (1) \Rightarrow \frac{m_1(T_1+T_2)}{2} = \frac{m_3(T_3+T_4)}{2}$$

$$\Rightarrow \frac{m_1}{m_3} = \frac{(T_3+T_4)}{(T_1+T_2)} \qquad (2)$$

If $m_1 = m_3$, then $T_1 + T_2 = T_3 + T_4$

$$\text{Speed ratio} = \frac{N_1}{N_4}$$

$$= \frac{\text{Product of number of teeth on driven gears}}{\text{Product of number of teeth on driving gears}}$$

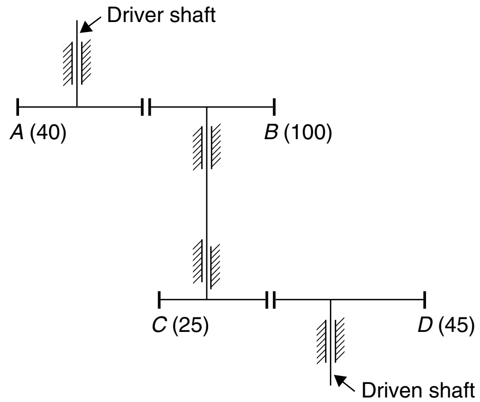
$$= \frac{T_2 \times T_4}{T_1 \times T_3}$$

$$\text{Train value} = \frac{1}{\text{Speed ratio}} = \frac{N_4}{N_1}$$

$$= \frac{\text{Product of number of teeth on driving gears}}{\text{Product of number of teeth on driven gears}}$$

$$= \frac{T_1 \times T_3}{T_2 \times T_4}$$

Example 15:



In the compound gear train shown, gears B and C are mounted on the same shaft, gear A is mounted on the driving shaft which rotates at 1170 rpm in the clockwise sense. The table below gives the number of teeth on each gear.

Gear	A	B	C	D
No. of teeth	40	100	25	45

The speed of driven shaft D (in rpm) and its sense of rotation are (CW = clockwise, CCW = counter clockwise)

(A) 292.5 rpm, CW (B) 260 rpm, CW
(C) 260 rpm, CCW (D) 292.5 rpm, CCW

Solution:

$$\text{Speed ratio} = \frac{N_A}{N_D}$$

$$= \frac{\text{Product of number of teeth on driven gears}}{\text{Product of number of teeth on driving gears}}$$

$$= \frac{T_B \times T_D}{T_A \times T_C} = \frac{100 \times 45}{40 \times 25} = 4.5$$

$\therefore\ N_D$ = speed of shaft of driven gear D

$$= \frac{N_A}{\text{Speed ratio}} = \frac{1170}{4.5} = 260 \text{ rpm}$$

A rotates CW → B rotates CCW

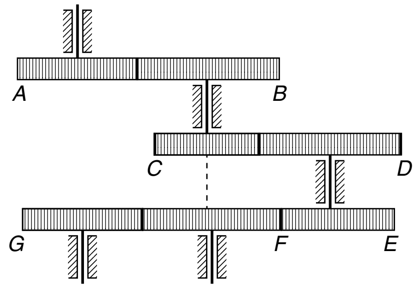
($\because$ A and B have external mesh)

$\therefore$ C rotates CCW

($\because$ B and C on same shaft)

→ D rotates CW
(∵ C and D have external mesh)
∴ Shaft of gear D rotates at 260 rpm in the clockwise sense.

Direction for questions 16 and 17:



A reverted gear train consisting of gears A, B, C, D, E, F and G is shown in figure. Gear A is mounted on the driving shaft which is rotating counter clockwise (CCW) at 1200 rpm. Gear G is mounted on the driven shaft (follower) which is rotating at 64 rpm. Gears B and C and gears D and E are mounted on the same shaft and gear F is mounted on a shaft co-axial with shaft of B and C. Gear F transmits motion from Gear E to gear G which is coaxial with gear A. The number of known teeth on gears are tabulated below.

Gear:	A	B	C	D	E	F
No. of teeth:	18	48	24	54	16	72

Example 16: The number of teeth on gear G and its sense of rotation are
(A) 50, CCW (B) 64, CCW
(C) 90, CW (D) 50, CW

Solution:

Gear F is only an idler and it does not affect the speed ratio.
N_A = speed of gear A
= 1200 rpm, CCW
N_G = speed of gear G = 64 rpm
A rotates CCW → B rotates CW
(∵ A and B have external mesh)
B and C rotate in the same sense (∵ mounted on same shaft)
D rotates CCW (∵ C and D have external mesh)
D and E rotate in the same sense
(∵ mounted in same shaft)
∴ E rotate CCW
F rotates CW (∵ E and F have external mesh)
∴ G rotates CCW (∵ G and F have external mesh)
∴ G rotates at 64 rpm, CCW

We have speed ratio = $\frac{N_A}{N_G}$

$$= \frac{\text{Product of no. of teeth on driven}}{\text{Product of no. of teeth on driver}}$$

$$\frac{N_A}{N_G} = \frac{1200}{64} = \frac{T_B \times T_D \times T_G}{T_A \times T_C \times T_E}$$

(∵ F is an idler, T_F is neglected)

$$\Rightarrow T_G = \frac{1200 \times T_A T_C T_E}{64 \times T_B T_D}$$

$$= \frac{1200 \times 18 \times 24 \times 16}{64 \times 48 \times 54} = 50$$

∴ $T_G = 50$ and G rotates CCW.

Example 17: If m_1, m_2 and m_3 are the modules of gear A, gear C and gear G respectively, $m_1 : m_2 : m_3$ is equal to
(A) 1.8485 : 1.4325 : 1 (B) 1.8485 : 1.1282 : 1
(C) 1.4325 : 1.1282 : 1 (D) 1 : 1.1282 : 1.8485

Solution:

Let d_1 = distance between axis of gears A and B
= distance between axis of gears G and F (data)
Gear A and B have the same module (m_1), Gears C and D have same module (m_2) and gears E, F and G have same module (m_3)

$$\Rightarrow d_1 = R_A + R_B = R_G + R_F$$

$$\Rightarrow m_1\left(\frac{T_A}{2} + \frac{T_B}{2}\right) = m_3\left(\frac{T_G}{2} + \frac{T_F}{2}\right)$$

$$\left(\because R = \frac{mT}{2}\right)$$

$$\Rightarrow \frac{m_1}{m_3} = \frac{(T_G + T_F)}{(T_A + T_B)}$$

$$= \frac{(50+72)}{(18+48)} = \frac{122}{66}$$

$$= 1.8485$$

Also, let d_2 = distance between axis of gears C and D
= distance between axis of gears F and E

$$\Rightarrow d_2 = R_C + R_D = R_F + R_E$$

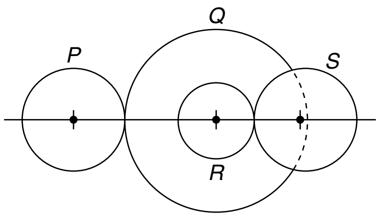
$$\Rightarrow m_2\left(\frac{T_C + T_D}{2}\right) = m_3\left(\frac{T_F + T_E}{2}\right)$$

$$\Rightarrow \frac{m_2}{m_3} = \frac{T_F + T_E}{T_C + T_D} = \frac{(72+16)}{(24+54)}$$

$$= \frac{88}{78} = 1.1282$$

$$\therefore m_1 : m_2 : m_3 = 1.8485 : 1.1282 : 1$$

Example 18:



A compound gear train with gears *P*, *Q*, *R* and *S* has number of teeth 20, 40, 15 and 20, respectively. Gears *Q* and *R* are mounted on the same shaft as shown in the figure below. The diameter of the gear *Q* is twice that of gear *R*. If the module of gear *R* is 2 mm, the centre distance (in mm) between gears *P* and *S* is

(A) 40 (B) 80 (C) 120 (D) 160

Solution:

Data

$T_P = 20$, $T_Q = 40$, $T_R = 15$ and $T_S = 20$

Module of gear $R = m_2 = 2$ mm

(= same as module of gear *S*, because *R* and *S* are in mesh)

Diameter of *Q*, $D_Q = 2D_R$

where

D_R = diameter of *R*

Let d_2 = distance between centres of gears *R* and *S*

$$= R_R + R_S$$

$$= \frac{m_2 T_R}{2} + \frac{m_2 T_s}{2} = m_2 \frac{(T_R + T_s)}{2}$$

$$= \frac{2[15+20]}{2} = 35\,\text{mm}$$

Diameter of *R*, $D_R = m_2 T_R = 2 \times 15$

$= 30$ mm

$\therefore$ Diameter of *Q*, $D_Q = 2D_R = 2 \times 30$

$= 60$ mm

Module of *Q*, $m_1 = \frac{D_Q}{T_Q} = \frac{60 \text{ mm}}{40}$

$= 15$ mm

Module of P = same as module of *Q*

($\because$ *P* and *Q* are in mesh)

Let d_1 = distance between centres of *P* and *Q*

$$= \frac{m_1(T_P + T_Q)}{2}$$

$$= \frac{1.5(20+40)}{2} = 45 \text{ mm}$$

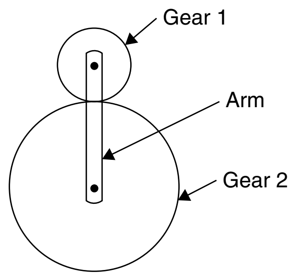
$\therefore$ Centre distance between *P* and *S*

$$= d_1 + d_2 = 45 + 35 = 80 \text{ mm}$$

Epicyclic Gear Trains

Epicyclic gear means one gear revolving upon and around another. Thus, there is relative motion between the axes of the gears in an epicyclic gear train. They are also known as **planetary gear trains**. If at least the axis of one gear moves relative to the frame, then it is an epicyclic gear train. When an annular wheel is added to an epicyclic gear train, it is called as **sun and planet gear train**.

In a **simple planetary gear train** (or **epicyclic gear train)**, there are two gears in the mesh pivoted on a link (called arm)



If the arm is fixed, then gear 1 and gear 2 can only rotate about their fixed axes and in such a case it becomes a simple gear train. But if either gear 1 is fixed (or only gear 2 is fixed), then the arm connecting the gears can also move and the arrangement becomes an epicyclic gear train. The fixed gear becomes 'the Sun' and the moving gear becomes the planet. The whole arrangement can be enclosed is an annual wheel, which can be fixed (instead of fixing the gears).

If there are more than one arm in a planetary gear train, then it is known as a **compound planetary gear train**. Planetary gear trains are used in machines where a high speed ratio between the input and output speeds is required within a small space.

The degree of freedom of an epicyclic gear train is two.

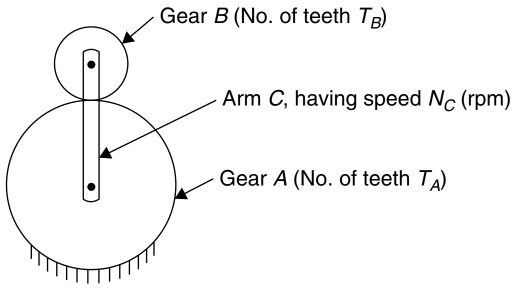
Speed Ratio of Epicyclic (Planetary) Gear Trains

In determining the speed ratio of epicyclic gear trains, the following concepts are applied.

1. The relative motion between a pair of mating gears remains the same, whether the axes of rotation are fixed or moving.
2. Ratio of relative motion is equal to inverse ratio of the number or teeth on the gears.
3. Gears with external meshing rotate in opposite sense while gears with internal meshing rotate in same sense. There are two methods for speed ratio determination of epicyclic gear trains. (i) **Relative velocity method (or Algebraic method** (ii) **Tabular method**. The procedure is the same in both cases. The results are expressed in the form of equation in the algebraic method, while they are expressed in the form of a table in tabular method.

Relative Velocity (Algebraic) Method

Consider an epicyclic gear train as shown in figure. Gear *A* is fixed. Gear *B* can move over Gear *A*. Arm *c* also rotates.



We have to find the speed of gear B which we denote as N_B. (If Gear B was fixed, instead of gear A, then we have to find speed of gear A which will be denoted as N_A)

Relative speed of A with respect to C

$$= N_A - N_C$$

Relative speed of B with respect to C

$$= N_B - N_C$$

Gears A and B are in external mesh, so they will rotate in the opposite sense. As the relative motion between gear A and B is the same whether their axes are fixed or not and this ratio of relative motion is equal to the inverse ratio of the number of teeth on the gears

$$\therefore \frac{\text{Speed of } B \text{ relative to arm } C}{\text{Speed of } A \text{ relative to arm } C} = \frac{-T_A}{T_B}$$

(−sign because A and B rotate in opposite sense)

$$\frac{N_B - N_C}{N_A - N_C} = \frac{-T_A}{T_B} \qquad (1)$$

If gear A is fixed, then $N_A = 0$

$$\Rightarrow \frac{-N_B}{N_C} + 1 = \frac{-T_A}{T_B}$$

$$\Rightarrow \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B} \Rightarrow N_B = N_C\left[1 + \frac{T_A}{T_B}\right]$$

Thus, the speed of gear B can be determined if speed of arm N_C, number of teeth on gears A and B are known. Similarly, if gear B was fixed and we have to determine the speed of gear A, we will get $N_A = N_C\left[1 + \frac{T_B}{T_A}\right]$. If the arm is fixed ($N_C = 0$), then it becomes a simple gear and equation (1)

$$\Rightarrow \frac{N_B}{N_A} = \frac{-T_A}{T_B}.$$

Tabular Method

In this method, we start off by considering the arm c as fixed ($N_C = 0$) and giving +1 rotation to arm A (+1 means one anticlockwise rotation, so − will mean clockwise rotation), so $N_A = +1$. As B is in external mesh with A, the corresponding rotation of B will be $N_B = -\left(\frac{T_A}{T_B}\right)$ (− indicates anticlockwise rotation) Next, keeping the arm fixed, gear A is given $+x$ rotations (i.e. x rotation is the anticlockwise direction). i.e. $N_C = 0$, $N_A = +x$ and so $N_B = -x\left(\frac{T_A}{T_B}\right)$. Now, all elements are given y rotations anticlockwise. i.e. $N_C = +y$, $N_A = +x + y$ and $N_B = -x\left(\frac{T_A}{T_B}\right) + y$.

The operations and resulting rotation of each element (gear A, gear B and arm C) is written in a tabular form as given below.

Sl. No.	Operation	Revolution of elements		
		Arm C (N_C)	Gear A (N_A)	Gear B (N_B)
1.	Arm is fixed. Gear A is given +1 rotation (ie anticlockwise)	0	+1	$-\left(\frac{T_A}{T_B}\right)$
2.	Arm is fixed. Gear A is given $+x$ rotation (ie x rotations in the anti-clockwise direction)	0	$+x$	$-x\left(\frac{T_A}{T_B}\right)$
3.	All elements are given $+y$ rotation (i.e. y rotation in anticlockwise direction for all elements)	$+y$	$+y$	$+y$
4.	Resultant motion (= sum of second and third rows)	$+y$	$x + y$	$-x\left(\frac{T_A}{T_B}\right) + y$

If gear A is fixed, then $x + y = 0$. If N_C is given in problem, then y is known and N_B can be determined. If N_B is given, then we have two equations connecting x and y (usually T_A and T_B are given in problems, so we are not treating T_A and T_B as variables) and hence x and y can be solved. If arm is rotating at 10 rpm in clockwise direction, then $y = -10$ ($\because$ clockwise rotation)

Example 19: The arm C of an epicyclic gear train rotates at 150 rpm in the clockwise direction. The arm carries two gears A and B having 60 and 75 teeth respectively. The gear A is fixed, while the arm rotates about the centre of gear A. The gear B meshes externally with gear A. The speed of gear B is

(A) 225 rpm, CW
(B) 270 rpm, CCW
(C) 225 rpm, CCW
(D) 270 rpm, CW

Solution:

(i) Algebraic Method

Let N_A, N_B and N_C be the speed of gear A, gear B and arm C respectively.

$$T_A = 60 \text{ and } T_B = 75$$

$$= \frac{\text{Relative velocity of } B \text{ WRT } C}{\text{Relative velocity of } A \text{ WRT } C}$$

$$= -\frac{T_A}{T_B} \text{ (– because external mesh between } A \text{ and } B)$$

i.e., $\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$ is the governing equation.

Given gear A is fixed (i.e., $N_A = 0$) and $N_C = -150$ rpm (because clockwise)

$$\Rightarrow \frac{N_B - (-150)}{0 - (-150)} = -\frac{60}{75} = -\frac{4}{5}$$

$$\Rightarrow \frac{N_B + 150}{150} = -\frac{4}{5}$$

$$\Rightarrow N_B = -\frac{4}{5} \times 150 - 150 = -120 - 150 = -270$$

∴ Gear B makes 270 rpm, clockwise.

(ii) Tabular Method

Sl. No.	Operation	Revolution of elements		
		Arm C (N_C)	**Gear A (N_A)**	**Gear B (N_B)**
1.	Arm C is fixed and Gear A is given +1 rotation	0	+1	$-\left(\frac{T_A}{T_B}\right)$
2.	Arm C is fixed and Gear A is given $+x$ rotation	0	$+x$	$-x\left(\frac{T_A}{T_B}\right)$
3.	All elements are given $+y$ rotation	$+y$	$+y$	$+y$
4.	Resultant motion (= sum of rows 2 and 3)	$+y$	$x + y$	$-x\left(\frac{T_A}{T_B}\right)+y$

Given gear A is fixed ∴ $x + y = 0$ (1)

Arm c makes 150 rpm CW

$$\Rightarrow y = -150 \quad (2)$$

$$\therefore x = -y = -(-150) = 150$$

$$\therefore N_B = -x\left(\frac{60}{75}\right) + y$$

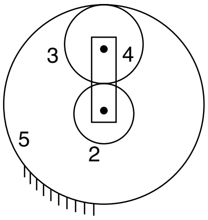
$$= -50 \times \frac{60}{75} - 150$$

$$= -120 - 150$$

$$= -270$$

∴ B makes 270 rpm in the clockwise direction.

Example 20:



An epicyclic gear train is shown schematically in figure. The sun gear 2 on the input shaft is a 20 teeth external gear. The planet gear 3 is a 40 teeth external gear. The ring gear 5 is a 100 teeth internal gear. The gear 5 is fixed and the gear 2 is rotating at 60 rpm CCW

(CCW = counter clockwise, CW = clockwise). The arm 4 attached to the output shaft will rotate at

(A) 10 rpm CCW (B) 10 rpm CW
(C) 12 rpm CCW (D) 12 rpm CW

(i) Algebraic Method

Solution:

The number of teeth are $T_2 = 20$, $T_3 = 40$, $T_s = 100$

Let the speeds of 2, 3, 4 and 5 be N_2, N_3, N_4 and N_5

∴ Speed of arm 4 = N_4

Speed of gear 3 relative to arm 4 = $N_3 - N_4$

Speed of gear 2 relative to arm 4 = $N_2 - N_4$

We have $\frac{(N_2 - N_4)}{(N_3 - N_4)} = -\frac{T_3}{T_2}$

(∵ 2 and 3 have external mesh they rotate in opposite sense)

$$\therefore \frac{N_2 - N_4}{N_3 - N_4} = -\frac{40}{20} = -2 \quad (1)$$

Relative velocity of 5 with respect to arm 4 = $N_5 - N_4$

$\therefore \frac{N_5 - N_4}{N_3 - N_4} = \frac{T_3}{T_5}$ (∵ 3 and 5 have internal mesh, they rotate in same sense)

$$\therefore \frac{N_5 - N_4}{N_3 - N_4} = \frac{40}{100} = 0.4 \quad (2)$$

Given $N_5 = 0$ (∵ 5 is fixed) and $N_2 = +60$ (∵ CCW is taken as positive)

$$\text{(i)} \rightarrow \frac{60 - N_4}{N_3 - N_4} = -2 \quad (3)$$

$$\text{(ii)} \rightarrow \frac{0 - N_4}{N_3 - N_4} = 0.4 \quad (4)$$

There are two unknowns N_3 and N_4 and we can obtain their values by solving equation (3) and equation (4).

$\Rightarrow N_3 = -15$ rpm and

$N_4 = +10$ rpm

i.e. Gear 3 makes 15 rpm (CW) and arm (4) makes 10 rpm (CCW)

(ii) Tabular Method

Sl. No.	Operation	Number of rotations of			
		Arm 4	Gear 2 ($T_2 = 20$)	Gear 3 ($T_3 = 40$)	Gear 5 ($T_5 = 100$)
1.	Arm 4 is fixed and gear 2 is given + 1 rotation (CCW)	0	+1	$-1 \times \frac{T_2}{T_3} = -\frac{1}{2}$	$-1 \times \frac{T_2}{T_3} \times \frac{T_3}{T_5} = \frac{-1}{5}$
2.	Arm 4 is fixed and gear 2 is given + x rotation (CCW)	0	$+x$	$-\frac{x}{2}$	$-\frac{x}{5}$
3.	All elements are given + y rotation (CCW)	$+y$	$+y$	$+y$	$+y$
4.	Resultant motion (sum of rows 2 and 3)	$+y$	$x + y$	$-\frac{x}{2} + y$	$-\frac{x}{5} + y$

Given gear 5 is fixed $\Rightarrow -\frac{x}{5} + y = 0$

i.e. $x = 5y$ (1)

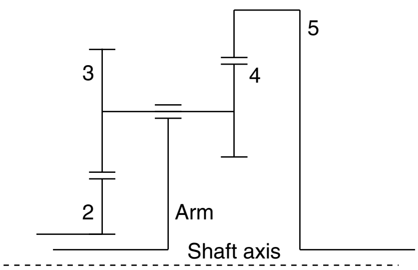
Gear 2 rotates at 60 rpm CCW

$\Rightarrow x + y = 60$ (2)

i.e. $5y + y = 60 \Rightarrow y = \frac{60}{6} = 10$

$\therefore$ The arm 4 rotates at $y = 10$ rpm (CCW)

Example 21:



N_i = Number of teeth for gear i

$N_2 = 20$

$N_3 = 24$

$N_4 = 32$

$N_5 = 80$

For the epicyclic gear arrangement shown in figure, $\omega_2 = 100$ rad/s clockwise (CW) and $\omega_{arm} = 80$ rad/s counter clockwise (CCW). The angular velocity ω_5 (in rad/s) is

(A) 0 (B) 70 CW

(C) 140 CCW (D) 140 CW

Solution:

(i) Algebraic Method (Relative velocity method)

Let ω_2, ω_3, ω_4, ω_5 and ω_a be the angular velocities of gears 2, 3, 4, 5 and arm respectively. $\omega_3 = \omega_4$ ($\because$ they are on same shaft)

Then $\frac{(\omega_2 - \omega_a)}{(\omega_5 - \omega_a)} = \frac{(\omega_2 - \omega_a)}{(\omega_3 - \omega_a)} \times \frac{(\omega_4 - \omega_a)}{(\omega_5 - \omega_a)}$

$= -\left(\frac{N_3}{N_2}\right) \times \left(\frac{N_5}{N_4}\right)$ ($\because$ 2 and 3 mesh externally; 4 and 5 mesh internally)

$= -\frac{24}{20} \times \frac{80}{32} = -3$

$\therefore \frac{\omega_2 - \omega_a}{\omega_5 - \omega_a} = -3$ (1)

Given $\omega_2 = -100$ rad/s ($\because$ CW)

and $\omega_a = +80$ rad/s ($\because$ CCW)

$\therefore (1) \Rightarrow \frac{-100 - 80}{\omega_5 - 80} = -3$

$\Rightarrow -180 = -3\omega_5 + 240$

$\Rightarrow 3\omega_5 = 240 + 180 = 420$

$\Rightarrow \omega_5 = \frac{420}{3} = +140$ ($+ \rightarrow$ CCW)

$\therefore$ Angular velocity ω_5 is 140 rad/s CCW.

(ii) Tabular Method

Sl. No.	Operation	No. of revolutions of				
		Arm	Gear 2 ($N_2 = 20$)	Gear 3 ($N_3 = 24$)	Gear 4 ($N_4 = 32$)	Gear 5 ($N_2 = 80$)
1.	Arm is fixed and gear 2 is given +1 rotation (CCW)	0	+1	$-\frac{20}{24} = -\frac{5}{6}$	$-\frac{20}{24} = \frac{-5}{6}$	$\frac{-20}{24} \times \frac{32}{80} = -\frac{1}{3}$
2.	Arm is fixed and gear 2 is given $+x$ rotation (CCW)	0	$+x$	$-\frac{5x}{6}$	$-\frac{5x}{6}$	$-\frac{x}{3}$

(*Continued*)

Sl. No.	Operation	No. of revolutions of				
		Arm	Gear 2 ($N_2 = 20$)	Gear 3 ($N_3 = 24$)	Gear 4 ($N_4 = 32$)	Gear 5 ($N_2 = 80$)
3.	All elements are given + y (CCW) rotation	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Resultant motion (sum of rows 2 and 3)	$+y$	$x+y$	$-\frac{5}{6}x+y$	$-\frac{5}{6}x+y$	$-\frac{x}{3}+y$

Given $x + y = -100$ ($\because$ CW) (1)

and $y = +80$ (2)

$\therefore x = -100 - y = -100 - 80$

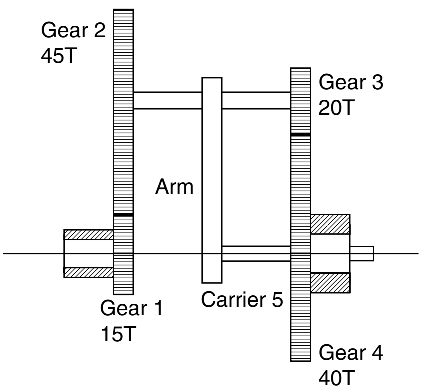
$= -180$

$$\therefore \omega_5 = -\frac{x}{3} + y = \frac{-(-180)}{3} + 80$$

$= 60 + 80 = 140$

$\therefore \omega_5 = 140$ rad/s CCW.

Direction for questions (Examples 4 and 5):



A planetary gear train has four gears and one arm (carrier). Angular velocities of the gears are ω_1, ω_2, ω_3 and ω_4 respectively. The carrier rotates with angular velocity ω_5.

Example 22: The relation between the angular velocities of gear 1 and gear 4 is given by the equation

(A) $\frac{(\omega_1 - \omega_5)}{(\omega_4 - \omega_5)} = 6$ (B) $\frac{(\omega_4 - \omega_5)}{(\omega_1 - \omega_5)} = 6$

(C) $\frac{(\omega_1 - \omega_2)}{(\omega_4 - \omega_5)} = -\left(\frac{2}{3}\right)$ (D) $\frac{\omega_2 - \omega_5}{\omega_4 - \omega_5} = -\left(\frac{8}{9}\right)$

Solution:

$$\frac{(\omega_1 - \omega_5)}{(\omega_4 - \omega_5)} = \frac{(\omega_1 - \omega_5)}{(\omega_2 - \omega_5)} \times \frac{(\omega_3 - \omega_5)}{(\omega_4 - \omega_5)}$$

[$\because$ $\omega_2 = \omega_3$ as they are on same shaft]

$$= \frac{-T_2}{T_1} \times \frac{-T_4}{T_3}$$

[$\because$ 1 and 2 in external mesh, 3 and 4 in external mesh]

$$= -\frac{45}{15} \times -\frac{40}{20} = -3 \times -2 = 6$$

$$\therefore \frac{(\omega_1 - \omega_5)}{(\omega_4 - \omega_5)} = 6.$$

Example 23: For $\omega_1 = 60$ rpm clockwise (CW) when looked from left, what is the angular velocity of the carrier and its direction so that gear 4 rotates in counter clockwise direction (CCW) at twice the angular velocity of gear 1 when looked from left?

(A) 130 rpm, CW (B) 223 rpm, CCW
(C) 256 rpm, CW (D) 156 rpm, CCW

Solution:

We have $\frac{(\omega_1 - \omega_5)}{(\omega_4 - \omega_5)} = 6$ (1)

Given $\omega_1 = -60$ rpm ($\because$ CW),

$\omega_4 = +2\,|\omega_1|$ ($\because$ CCW)

$= 2 \times 60 = 120$ rpm (CCW)

$$\therefore (1) \Rightarrow \frac{-60 - \omega_5}{120 - \omega_5} = 6$$

$\Rightarrow -60 - \omega_5 = 720 - 6\omega_5$

$\Rightarrow 6\omega_5 - \omega_5 = 720 + 60$

i.e. $5\omega_5 = 780$

$$\Rightarrow \omega_5 = \frac{780}{5} = 156 \; (+ \rightarrow \text{CCW})$$

$\therefore \omega_5 = 156$ rpm, CCW.

Example 24: 100 kW power is supplied to a machine through a gear box which uses an epicyclic gear train. The power is supplied at 100 rad/s. The speed of the output shaft of the gear box is 10 rad/s in a sense opposite to the input speed. The magnitude of the holding torque on the fixed gear of the train is

(A) 10 kN m (B) 8 kN m
(C) 9 kN m (D) 11 kN m

Solution:

$P_i = P_o = 100$ kW $= 100 \times 10^3$ W

Input power P_i = output power P_o

($\because$ Efficiency is taken as 100%)

$\omega_1 = 100$ rad/s CW (assumed)

$\therefore$ Input torque $T_i = \dfrac{P_i}{\Omega_i}$

$$= \frac{100 \times 10^3}{100} = 10^3 \text{ N m}$$

T_o = output torque

$$= \frac{P_o}{\omega_o} = \frac{100 \times 10^3}{(-10)} = -10^4 \text{ N m}$$

($\because$ $\omega_o = -10$ rad/s, opposite to ω_i)

If T_h is the holding torque, for rotational equilibrium of gear box, we have

$$T_i + T_o + T_h = 0$$
$$\Rightarrow 10^3 - 10^4 + T_h = 0$$
$$\Rightarrow T_h = 9 \times 10^3 \text{ N m}$$
$$= 9 \text{ kN m}$$

$\therefore$ Holding torque on fixed gear = 9 kN m

Example 25: A pinion and a gear are in mesh with each other. The gear ratio is 2 and the moment of inertias of the pinion and gear about their axes of rotation are 3 kg m^2 and 5 kg m^2 respectively. For the gear to have an angular acceleration of 4 rad/s^2, the torque to be applied to the pinion shaft is

(A) 5 N m (B) 10 N m (C) 14 N m (D) 17 N m

Solution:

100% efficiency is assumed

$\therefore$ Power given to gear = Power given to Pinion

$\Rightarrow$ Torque on gear $\times$ ω_{gear} = Torque on pinion $\times$ ω_{pinion}

$$\Rightarrow \text{Torque on pinion} = \text{Torque on gear} \times \frac{\omega_{gear}}{\omega_{pinion}}$$

$$= \text{Torque on gear} \times \frac{\text{No. of teeth on pinion}}{\text{No. of teeth on gear}}$$

$$= (I_{gear} \times \alpha_{gear}) \times \frac{1}{\text{gear ratio}}$$

$$= 5 \times 4 \times \frac{1}{2}$$

$$= 10 \text{ N m}$$

$$\left(\therefore \frac{\omega_{gear}}{\omega_{pinion}} = \frac{\text{No. of teeth on pinion}}{\text{No. of teeth on gear}} = \frac{1}{\text{Gear ratio}}\right)$$

Example 26: Consider the following statements given below.

(1) The mating spur gears must have the same pressure angle
(2) The mating spur gears must have the same module
(3) The mating spur gears must be of the same material

The true statements are:

(A) 1, 2 and 3 (B) 1 and 2 only
(C) 2 and 3 only (D) 1 and 3 only

Solution:

The mating spur gears must be of same pressure angle and same module but they can be made of different materials.

Example 27: Two parallel shafts whose axes are fixed and separated by a distance of 40 mm are to be connected by a spur gear set so that the output shaft rotates at $\frac{1}{3}$ rd of the speed of the input shaft. Which of the following could be the pitch circle diameters of the gears?

(A) 20 mm and 60 mm (B) 30 mm and 50 mm
(C) 15 mm and 65 mm (D) 40 mm and 120 mm

Solution:

Distance between centres, $d = \dfrac{D_1 + D_2}{2}$

$$\Rightarrow 40 = \frac{D_1 + D_2}{2}$$

$\Rightarrow D_1 + D_2 = 2 \times 40 = 80$ mm (1)

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} = \frac{D_2}{D_1} = 3 \left[\because \frac{\omega_1}{\omega_2} = 3, \text{ given}\right)$$

$\therefore D_2 = 3D_1$ (2)

From (1) and (2) $D_1 + 3D_1 = 80$

$$\Rightarrow D_1 = \frac{80}{4} = 20 \text{ mm}$$

$D_2 = 3D_1 = 20 \times 3 = 60$ mm

$\therefore$ The diameters are 20 mm and 60 mm.

EXCERCISE

Practice Problems I

1. A circular solid disc of uniform thickness 25 mm, radius 250 mm and mass 25 kg is used as a flywheel. Its kinetic energy (in joule) when it rotates at 900 rpm is ________
2. The speed of a flywheel of mass moment of inertia 15 kg m^2 fluctuates by 30 rpm for a fluctuation in energy of 2467 joule. The mean speed of the flywheel (in rpm) is
   (A) 450 (B) 500
   (C) 550 (D) 600
3. The speed of an engine varies from 250 rad/s to 240 rad/s. Change in kinetic energy during the cycle is 420 J. Mass moment of inertia of the flywheel (in kg m^2) is
   (A) 0.1714 (B) 0.1962 (C) 0.2272 (D) 0.2986
4. The mean speed of a vertical double acting steam engine developing 80 kW is 250 rpm. The maximum and minimum speeds are not to vary more than 1% on either side of the mean speed and maximum fluctuation of energy is one third of the indicated work per stroke. If the mean radius of gyration of the flywheel is 0.6 metre, its mass is (in kg) ______.

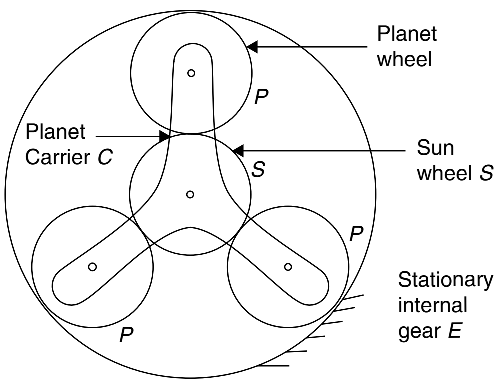
5. A punching press is required to punch 30 mm diameter holes, in a plate of 15 mm thickness, at the rate of 30 holes per minute. It requires 6 J of energy per mm^2 of sheared area. Punching of a hole takes 0.2 s and the speed of the flywheel varies from 160 rpm to 140 rpm. If radius of gyration is 1 metre, mass of the flywheel is.
(A) 192 kg (B) 204 kg (C) 232 kg (D) 248 kg

6. A simple gear train has total of seven gears including the driver and the driven, all having external meshing. If the driver rotates in the clockwise direction (CW), the driven will rotate in the
(A) clockwise sense (CW)
(B) counter clockwise sense (CCW)
(C) either clockwise (CW) or counter clockwise (CCW)
(D) Cannot be predicted

7. A simple gear train consists of gears A and B, having module 2 mm and centre distance of shafts equal to 115 mm. If the pitch circle diameter of the driver (gear A) is 46 mm, the train value of the gear train is
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2.5}$

8. Two parallel shafts are to be connected by spur gears. The shafts are 600 mm apart approximately. Speed of the driver shaft is 360 rpm and the speed of the other is 120 rpm. The circular pitch is 25 mm. Fill up the blanks
(i) No. of teeth on driver gear is ________
(ii) No. of teeth on driven gear is ________
(iii) Pitch circle diameter of gear is _______
(iv) Exact centre distance between shafts is __________

Direction for questions 9 and 10:

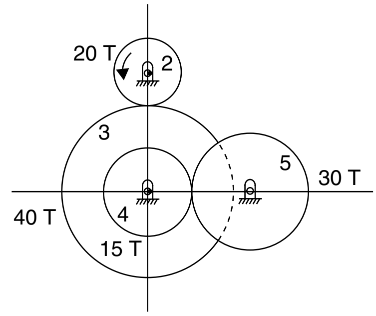


An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star shaped carrier C. The size of the different toothed wheels are such that the planet carrier C rotates at $\frac{1}{4}^{\text{th}}$ of the speed of the sun wheel S. The minimum number of teeth on any wheel is 18. The driving toque on the sun wheel is 80 N m.

9. The number of teeth on Sun gear, Internal gear and planet gears are respectively
(A) 18, 54, 24 (B) 22, 66, 33
(C) 26, 78, 52 (D) 18, 54, 18

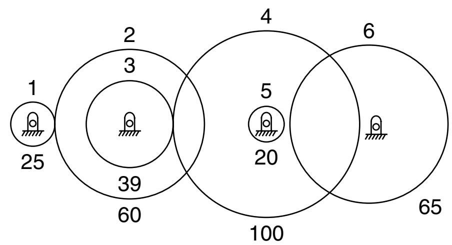
10. The magnitude of the torque necessary to keep the internal gear stationary is
(A) 320 N m (B) 240 N m
(C) 400 N m (D) −80 N m

11. In the figure shown, gear 2 rotates at 1200 rpm in counter clockwise direction and engages with gear 3. Gear 3 and Gear 4 are mounted on the same shaft. Gear 5 engages with gear 4. The numbers of teeth on gears 2, 3, 4 and 5 are 20, 40, 15 and 30 respectively. The speed of gear 5 is



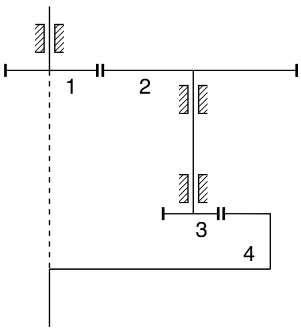
(A) 300 rpm, CCW
(B) 300 rpm, CW
(C) 4800 rpm, CCW
(D) 4800 rpm, CW

12.



In the compound gear train shown, there are 6 gears (1, 2, 3, 4, 5 and 6) with gear 1 being the driver and gear 6 being the driven. The number of teeth on gears 1, 2, 3, 4, 5 and 6 are, respectively, 25, 60, 39, 100, 20 and 65. Gears 2 and 3 are mounted on same shaft while gears 4 and 5 are mounted on same shaft. Gear 1 meshes with 2, gear 3 meshes with 4 and gear 5 meshes with 6, all meshing being external. If gear 1 rotates at 1320 rpm in the clockwise direction, the speed of rotation of gear 6 (in rpm) and its direction are respectively.
(A) 33 rpm, CCW (B) 66 rpm, CW
(C) 66 rpm, CCW (D) 33 rpm, CW

13. For the gear train in Qn. 12 above, if the modules of gears 1, 3 and 6 are 1 mm, 1.5 mm and 2 mm respectively, the distance between the shafts on which gear 1 and gear 6 are mounted is
(A) 199.75 mm (B) 264.35 mm
(C) 292.25 mm (D) 231.75 mm

14. 

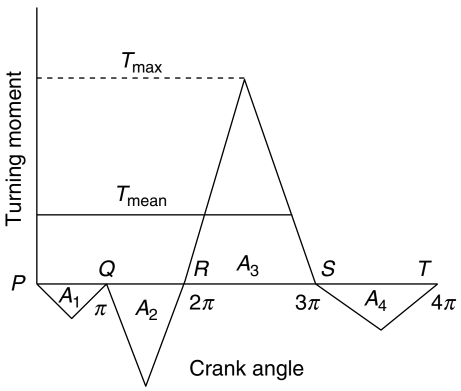
In the reverted gear train shown in figure, all the gears have same module. Gear 2 and gear 3, which are mounted on the same shaft, have 57 teeth and 22 teeth respectively. If the internal gear 4 (which meshes with gear 3) has 98 teeth, the number of teeth on gear 1 is
(A) 17 (B) 18 (C) 19 (D) 21

15. For the gear train in Qn. 14 above, if the driver (gear 1) rotates at 735 rpm clockwise, the speed and direction of rotation of driven (gear 4) is
(A) 147 rpm, CW (B) 55 rpm, CCW
(C) 55 rpm, CW (D) 147 rpm, CCW

Practice Problems 2

1. Mass of a flywheel is 5000 kg and radius of gyration is 1.8 m. From the turning moment diagram maximum fluctuation of energy is found to be 52 kJ. If the mean speed of the engine is 120 rpm, its maximum speed is
(A) 115.26 rpm (B) 121.22 rpm
(C) 128.34 rpm (D) 132.58 rpm

2. In the turning moment diagram of a four stroke gas engine, areas representing various strokes are as follows.
Suction stroke, $A_1 = 0.45 \times 10^{-3}$ m^2
Compression stroke, $A_2 = 1.7 \times 10^{-3}$ m^2
Expansion stroke, $A_3 = 6.8 \times 10^{-3}$ m^2
Exhaust stroke, $A_4 = 0.65 \times 10^{-3}$ m^2
One m^2 of the area represents 3 MNm of energy.(ie MJ)
Maximum fluctuation of energy (in J) is ______



3. A rivetting machine is driven by a constant torque motor of power 3 kW. The moving parts, including the flywheel, have an equivalent mass of 150 kg at 0.5 m radius. One rivetting operation takes 1 s and absorbs 9000 J of energy. Speed of the flywheel is 300 rpm before rivetting. Speed of flywheel immediately after rivetting is
(A) 196.4 rpm (B) 208.3 rpm
(C) 232.8 rpm (D) 246.6 rpm

Direction for questions 4 and 5: A punching machine makes 20 working strokes per minute and is used for punching 30 mm diameter holes in 20 mm thick steel plates of ultimate shear strength 300 MPa. The punching operations takes place during $\frac{1}{10}^{th}$ of a revolution of the crank shaft.

The flywheel revolves at 10 times the speed of the crank shaft. The permissible coefficient of fluctuation of speed is 0.1

4. If the mechanical efficiency is 95%, power required for the driving motor is
(A) 1.984 kW (B) 2.232 kW
(C) 2.948 kW (D) 3.526 kW

5. If radius of gyration of the flywheel is to be 0.7 m, the minimum mass required for the flywheel is
(A) 249.24 kg (B) 236.78 kg
(C) 159.22 kg (D) 136.73 kg

6. Consider a flywheel whose mass m is distributed almost equally between a heavy ring-like ring of radius R and a concentric disk-like feature of radius $\frac{R}{2}$. Other parts of the flywheel such as spokes etc have negligible mass. The best approximation of α, if the moment of inertia of the flywheel about its axis of rotation is expressed as αMR^2, is ______.

7. A certain machine requires a torque of $(200 + 20 \sin \theta)$ kNm to drive it, where θ is the angle of rotation of shaft measured from certain datum. The machine is directly coupled to an engine which produces a torque $(200 + 20 \sin 2\theta)$ kNm in a cycle. The number of times the value of torque of machine and engine will be identical in a cycle is
(A) 8 (B) 4 (C) 2 (D) 1

8. Maximum fluctuation of energy for flywheel is the
(A) sum of maximum and minimum energies.
(B) difference between the maximum and minimum energies.

(C) difference between the maximum and mean energies.
(D) ratio of the maximum energy to the minimum energy.

9. The amount of energy absorbed by a flywheel is determined from the
(A) speed-energy diagram.
(B) speed-space diagram.
(C) Torque-crank angle diagram.
(D) Acceleration-crank angle diagram

10. If the rotating mass of a rim type flywheel is distributed on another rim type flywheel, whose mean radius is half the mean radius of the former, then the energy stored in the latter at the same speed will be
(A) 4 times the first one. (B) same as the first one.
(C) $\frac{1}{4}^{th}$ of the first one. (D) $\frac{1}{2}$ of the first one.

11. The average speed of a flywheel is 1200 rpm and its polar moment of inertia is I(kg m^2). The fluctuation of speed is 4%. The average speed of the flywheel is to be halved and the fluctuation of speed is to be brought down to 2%. Assuming that the fluctuation of energy in the flywheel remains constant, the polar moment of inertia of the flywheel has to be changed to
(A) $2I$ (B) $4I$ (C) $4\sqrt{2}I$ (D) $8I$

12. If V is the maximum peripheral speed of flywheel, ρ is the density of flywheel material and σ is the allowable hoop stress in the flywheel, then
(A) $V = \sqrt{\frac{\sigma}{\rho^3}}$ (B) $V = \sqrt{\rho\sigma}$
(C) $V = \sqrt{\frac{\rho}{\sigma}}$ (D) $V = \sqrt{\frac{\sigma}{\rho}}$

13. Why is the mass of the flywheel concentrated in the rim?
(A) To store maximum energy.
(B) To make it strong.
(C) To store minimum energy.
(D) To let it rotate freely.

14. The density of a flywheel material is 7.82 g/cm^3 and the safe hoop stress in it is 24.8 MN/m^2. The maximum safe peripheral velocity of the flywheel (in m/s) is
(A) 1.89 (B) 56.31 (C) 1780.83 (D) 66.34

15. In which of the following case, the turning moment diagram will have the least variation?
(A) Double arting steam engine.
(B) Four stroke, single cylinder petrol Engine.
(C) 8 cylinder, 4 stroke diesel engine.
(D) Pelton wheel.

16. The radius of gyration of a uniform solid disc type flywheel of diameter D is
(A) D (B) $\frac{D}{\sqrt{8}}$ (C) $\frac{D}{\sqrt{2}}$ (D) $\frac{\sqrt{3}D}{2}$

17. Flywheels are fitted for single cylinder and multi-cylinder engine of the same power rating. Which of the following statement is true?
(A) The size of flywheel for an engine depends on the compression ratio.
(B) The flywheel will be smaller for single cylinder engine when compared to that for multi-cylinder engine.
(C) The flywheel will be smaller a for multi-cylinder engine as compared to that of a single cylinder engine.
(D) The flywheel of the two engines will be identical.

18. The power developed by a 4 stroke engine is 150 kW at 100 rpm. The fluctuation of energy is 0.58 times the energy developed per cycle. The fluctuation of energy (in kJ) per cycle is.
(A) 480 (B) 360 (C) 256.7 (D) 208.8

19. Consider the following statements.
The flywheel in an IC engine
(i) acts as a reservoir of energy.
(ii) minimises cyclic fluctuations in engine speed.
(iii) takes care of load fluctuations in the engine and controls speed variation.
The correct statements are:
(A) (i) and (iii) (B) (i), (ii) and (iii)
(C) (i) and (ii) (D) (ii) and (iii)

20. The turning moment diagram of a 4 stroke IC engine during compression stroke is
(A) positive throughout
(B) negative throughout
(C) positive during major portion of stroke
(D) negative during major portion of stroke.

21. A simple gear train consists of a pair of spur gears with module 5 mm and a centre distance of 450 mm. If the speed ratio is 5:1, the number of teeth on the pinion is ______.

22. In a simple gear train, two mating spur gears have 40 and 120 teeth, respectively. The pinion rotates at 1200 rpm and transmits a torque of 20 N m. The torque transmitted by the gear is
(A) 6.6 N m (B) 20 N m
(C) 40 N m (D) 60 N m

23. In a simple gear train, the pinion rotates at a speed of 1440 rpm and transmits a power of 1000 W. The speed ratio for this unit is 10 : 1 with pinion being the driver. If the torque transmitted by the gear is 56.36 N m, the mechanical efficiency of the transmission is about
(A) 78% (B) 85% (C) 63% (D) 96%

24. The velocity ratio in the case of compound train of wheels is equal to
(A) $\frac{\text{Number of teeth on first driver}}{\text{Number of teeth on last follower}}$

(B) $\dfrac{\text{Number of teeth on last follower}}{\text{Number of teeth on first driver}}$

(C) $\dfrac{\text{Product of teeth on the drivers}}{\text{Product of teeth on the followers}}$

(D) $\dfrac{\text{Product of teeth on the followers}}{\text{Product of teeth on the drivers}}$

25. The gear train usually employed in clocks is a
(A) reverted gear train.
(B) simple gear train.
(C) sun and planet gear.
(D) differential gear.

26. Train value of a gear train is
(A) equal to speed ratio.
(B) half of speed ratio.
(C) equal to speed ratio plus one.
(D) reciprocal of speed ratio.

27. A gear train, in which at least one of the gear axes is in motion relative to the frame, is called
(A) compound gear train. (B) Epicyclic gear train.
(C) reverted gear train. (D) double bevel gear train.

28. A reverted gear train is one in which the output shaft and input shaft
(A) rotate in opposite directions.
(B) are co-axial.
(C) are at right angles to each other.
(D) are at an angle to each other.

29. Consider the following specifications of gears *A*, *B*, *C* and *D*

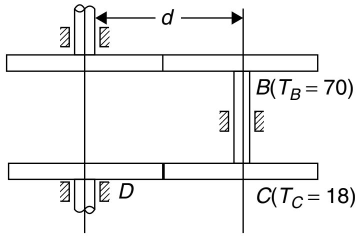
Gears	A	B	C	D
No. of teeth	20	60	20	60
Pressure angle	$14\frac{1}{2}^\circ$	$14\frac{1}{2}^\circ$	20′	$14\frac{1}{2}^\circ$
Module	1	3	3	1
Material	Steel	Brass	Brass	Steel

Which of these gears form a pair of spur gears to achieve a gear ratio of 3?
(A) *A* and *B* (B) *A* and *D*
(C) *B* and *C* (D) *C* and *D*

30. In a reverted gear train, two gears *A* and *B* are in external mesh. *B*-*C* is a compound gear and *C* and *D* are in external mesh. The module of *A* and *C* are 3 mm and 4 mm, respectively. The number of teeth in *A*, *B* and *C* are 20, 44 and 18, respectively. The number of teeth in *D* is
(A) 26 (B) 34 (C) 46 (D) 30

31. In a compound gear train, gear *P* drives gear *Q*. Gear *Q* and gear *R* are mounted on same shaft. Gear *R* drives gear *S*. All gears are in external mesh. The number of teeth on gear *P* and gear *R* are same while the number of teeth on gear *Q* and gear *S* are same. When *P* rotates at 156 rpm, *S* rotates at 100 rpm. The rotational speed of compound gear *Q*-*R* is nearly
(A) 125 rpm (B) 135 rpm
(C) 118 rpm (D) 142 rpm

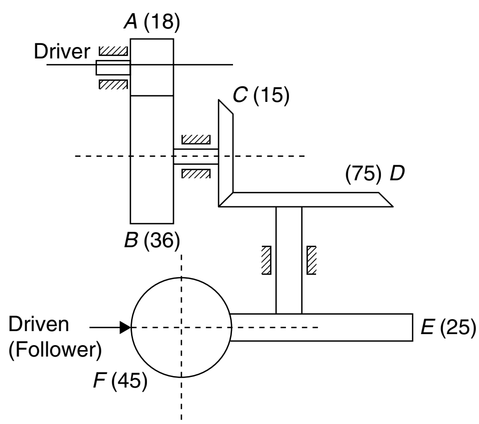
32.



In the reverted gear train shown, gear *A* is the driver and rotates at 2100 rpm. Gear *B* and gear *C* are mounted on the same shaft. The module of Gear *B* is 1.2 mm and it has 70 teeth while Gear *C* has 18 teeth. The distance between centres of the shafts is 54 mm. The module of gear *D* is (in mm)
(A) 1.2 mm (B) 1.5 mm
(C) 1.75 mm (D) 2.0 mm

33. In a simple gear train, the idle wheel
(A) has no influence on speed ratio. It only affects the direction of rotation.
(B) changes the speed ratio and also direction of rotation.
(C) may change the speed ratio but may not affect the direction of rotation.
(D) does not change the centre distance between the driver and driven shafts.

34.



A compound gear train consists of spur, bevel and spiral gears as is shown in the figure with the name of gear in bold capital letters followed by the number of teeth on that gear in parenthesis (bracket). Gear *A* is the driver and gear *F* is the driven (follower). The overall speed ratio of the train is
(A) 9 (B) 18
(C) 16 (D) 24

35. Pinion A and gear *B* for a simple gear train are made of spur gears. The speed ratio is 4 and the module is 2.5 mm. If the gear rotates at 400 rpm and has 84 teeth, the centre distance between shafts (in mm) and the pitch line velocity (in m/s) are respectively

(A) 131.25 mm, 5.6125 m/s
(B) 122.65 mm, 4.3982 m/s
(C) 131.25 mm, 4.3982 m/s
(D) 122.65 mm, 5.6125 m/s

36. A fixed gear *A* having 117 teeth is in external mesh with another gear *B* having 39 teeth. The number of revolutions made by the smaller gear for one revolution of the arm(connecting the two gear) about the entire of the bigger gear is
(A) 3 (B) 4
(C) 5 (D) 2

Direction for questions 37 and 38: In an epicyclic gear train, an annular wheel *C* having 150 teeth is in internal mesh with a planet wheel *B* having 60 teeth. The sun wheel *A* is in external mesh with planet *B*. The centres of wheel *A* and *B* are connected by an arm *D*.

37. The number of teeth on sun wheel *A* is
(A) 20 (B) 30 (C) 40 (D) 45

38. If the sun wheel *A* is fixed and the arm *D* is given 5 rotations clockwise, the number of rotations made by the annular wheel *C* is
(A) 4 rotations, CCW (B) 6 rotation, CW
(C) 3 rotations, CCW (D) 4 rotations, CW

39. The first and last gear in a simple gear train, with all gears having external mesh, are having 20 teeth and 70 teeth, respectively. If the first gear is the driver, then the train value and speed ratio of this train are, respectively,
(A) cannot be determined from given data.
(B) $\frac{7}{2}$ and $\frac{2}{7}$
(C) $\frac{2}{7}$ and $\frac{7}{2}$
(D) $\frac{2}{5}$ and $\frac{5}{2}$

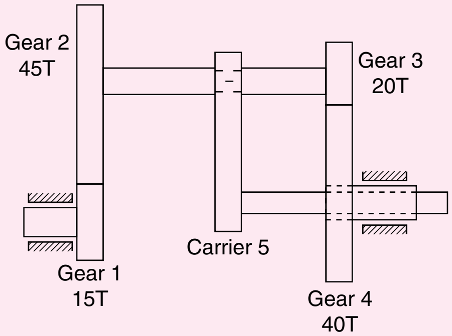
40. A spur gear having 40 teeth and having circular pitch of 25 mm is rotating at a speed of 150 rpm. Its diametral pitch (in per mm) and pitch line velocity (in m/s) are, respectively,
(A) 0.1257/mm, 2.5 m/s (B) 0.1467/mm, 2.3 m/s
(C) 0.1257/mm, 2.8 m/s (D) 0.1467/mm, 2.5 m/s

PREVIOUS YEARS' QUESTIONS

1. Two mating spur gears have 40 and 120 teeth, respectively. The pinion rotates at 1200 rpm and transmits a torque of 20 Nm. The torque transmitted by the gear is **[2004]**
(A) 6.6 Nm (B) 20 Nm
(C) 40 Nm (D) 60 Nm

2. If C_f, is the coefficient of speed fluctuation of a flywheel then the ratio of $\omega_{max} / \omega_{min}$ will be: **[2006]**
(A) $\frac{1-2C_f}{1+2C_f}$ (B) $\frac{2-C_f}{2+C_f}$
(C) $\frac{1+2C_f}{1-2C_f}$ (D) $\frac{2+C_f}{2-C_f}$

Direction for questions 3 and 4: A planetary gear train has four gears and one carrier. The angular velocities of the gears are ω_1, ω_2, ω_3 and ω_4, respectively. The carrier rotates with angular velocity ω_5.



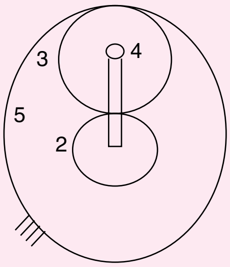
3. What is the relation between the angular velocities of gear 1 and gear 4? **[2006]**
(A) $\frac{\omega_1-\omega_5}{\omega_4-\omega_5}=6$ (B) $\frac{\omega_4-\omega_5}{\omega_1-\omega_2}=6$
(C) $\frac{\omega_1-\omega_2}{\omega_4-\omega_3}=\left(\frac{2}{3}\right)$ (D) $\frac{\omega_2-\omega_5}{\omega_4-\omega_5}=\frac{8}{9}$

4. For ω_1 = 60 rpm clockwise (CW) when looked from the left, what is the angular velocity of the carrier and its direction so that gear 4 rotates in counterclockwise (CCW) direction at twice the angular velocity of gear 1 when looked from the left **[2006]**
(A) 130 rpm, CW (B) 223 rpm, CCW
(C) 256 rpm, CW (D) 156 rpm, CCW

5. The speed of an engine varies from 210 rad/s to 190 rad/s. During a cycle the change in kinetic energy is found to be 400 Nm. The inertia of the flywheel in kgm^2 is **[2007]**
(A) 0.10 (B) 0.20
(C) 0.30 (D) 0.40

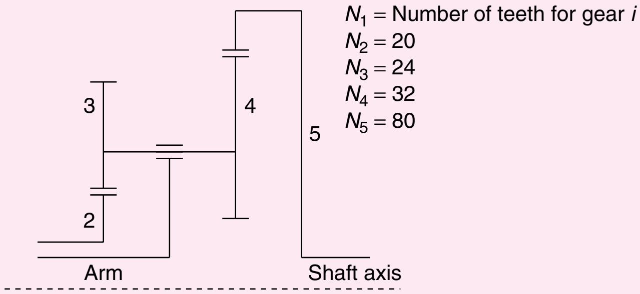
6. An epicyclic gear train is shown schematically in the adjacent figure. The sun gear 2 on the input shaft is a 20 teeth external gear. The planet gear 3 is a 40 teeth external gear. The ring gear 5 is a 100 teeth internal gear. The ring gear 5 is fixed and the gear 2 is rotating at 60 rpm CCW (CCW = counter clockwise and CW = clockwise)

The arm 4 attached to the output shaft will rotate at **[2009]**



(A) 10 rpm CCW (B) 10 rpm CW
(C) 12 rpm CW (D) 12 rpm CCW

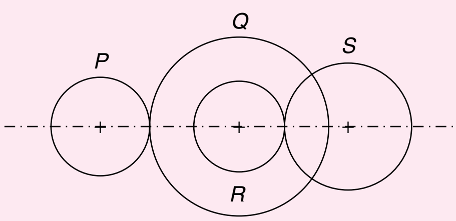
7. For the epicyclic gear arrangement shown in the figure, ω_2 = 100 rad/s clockwise (CW) and ω_{arm} = 80 rad/s counter clockwise (CCW). The angular velocity ω_5(in rad/s) is **[2010]**



(A) 0 (B) 70 CW
(C) 140 CCW (D) 140 CW

8. A circular solid disc of uniform thickness 20 mm, radius 200 mm and mass 20 kg, is used as a flywheel. If it rotates at 600 rpm, the kinetic energy of the flywheel, in Joules is **[2012]**
(A) 395 (B) 790
(C) 1580 (D) 3160

9. A compound gear train with gears *P*, *Q*, *R* and *S* has number of teeth 20, 40, 15 and 20, respectively. Gears *Q* and *R* are mounted on the same shaft as shown in the figure below. The diameter of the gear *Q* is twice that of the gear *R*. If the module of the gear *R* is 2 mm, the center distance in mm between gears *P* and *S* is **[2013]**



(A) 40 (B) 80
(C) 120 (D) 160

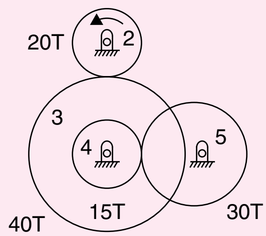
10. A flywheel connected to a punching machine has to supply energy of 400 Nm while running at a mean angular speed of 20 rad/s. If the total fluctuation of speed is not to exceed ±2%, the mass moment of inertia of the flywheel in kg-m^2 is **[2013]**
(A) 25 (B) 50
(C) 100 (D) 125

11. A pair of spur gears with module 5 mm and a center distance of 450 mm is used for a speed reduction of 5 : 1. The number of teeth on pinion is _____ **[2004]**

12. Consider a flywheel whose mass m is distributed almost equally between a heavy, ring–like rim of radius *R* and a concentric disk–like feature of radius *R*/2. Other parts of the flywheel, such as spokes, etc, have negligible mass. The best approximation for α, if the moment of inertia of the flywheel about its axis of rotation is expressed as αMR^2, is _____ **[2014]**

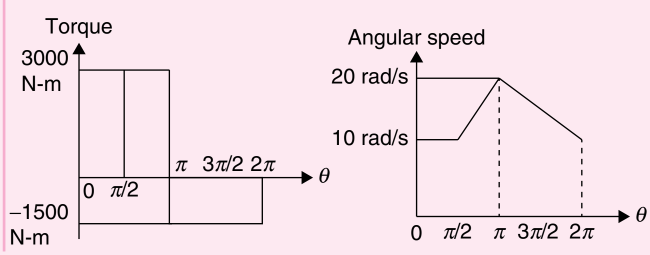
13. Maximum fluctuation of kinetic energy in an engine has been calculated to be 2600 J. Assuming that the engine runs at an average speed of 200 rpm, the polar mass moment of intertia (in kg.m^2) of a flywheel to keep the speed fluctuation within ± 0.5% of the average speed is _____ **[2014]**

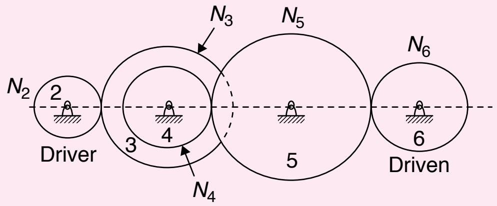
14. Gear 2 rotates at 1200 rpm in counter clockwise direction and engages with Gear 3. Gear 3 and Gear 4 are mounted on the same shaft. Gear 5 engages with Gear 4. The numbers of teeth on Gears 2, 3, 4 and 5 are 20, 40, 15 and 30, respectively. The angular speed of Gear 5 is **[2014]**



(A) 300 rpm, CCW (B) 300 rpm, CW
(C) 4800 rpm, CCW (D) 4800 rpm, CW

15. Torque and angular speed data over one cycle for a shaft carrying a flywheel are shown in the figures. The moment of inertia (in kg.m^2) of the flywheel is ____ **[2014]**



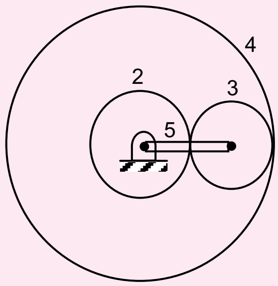
16. 

A gear train is made up of five spur gears as shown in the figure. Gear 2 is driver and gear 6 is driven member. N_2, N_3, N_4, N_5 and N_6 represent number of teeth on gears 2, 3, 4, 5 and 6 respectively. The gear(s) which act(s) as idler(s) is/are: **[2015]**

(A) Only 3 (B) Only 4
(C) Only 5 (D) Both 3 and 5

17. The torque (in N-m) exerted on the crank shaft of a two stroke engine can be described as $T = 10000 + 1000 \sin 2\theta - 1200 \cos 2\theta$, where θ is the crank angle as measured from inner dead center position. Assuming the resisting torque to be constant, the power (in kW) developed by the engine at 100 rpm is ____. **[2015]**

18. In the gear train shown, gear 3 is carried on arm 5, Gear 3 meshes with gear 2 and gear 4. The number of teeth on gear 2, 3 and 4 are 60, 20 and 100, respectively. If gear 2 is fixed and gear 4 rotates with an angular velocity of 100 rpm in the counterclockwise direction, the angular speed of arm 5 (in rpm) is: **[2016]**



(A) 166.7 counterclockwise
(B) 166.7 clockwise
(C) 62.5 counterclockwise
(D) 62.5 clockwise

Answer Keys

Exercises

Practice Problems I

1. 3469.78 J **2.** B **3.** A **4.** 648.46 kg **5.** C **6.** A **7.** C
8. (i) 38 (ii) 114 (iii) 302.39 mm (iv) 604.79 mm **9.** D **10.** B **11.** A **12.** C **13.** D
14. C **15.** B

Practice Problems 2

1. B **2.** 17510 J **3.** D **4.** A **5.** B **6.** 0.5625 **7.** B **8.** B **9.** C **10.** C
11. D **12.** D **13.** A **14.** B **15.** D **16.** B **17.** C **18.** D **19.** C **20.** B
21. 30 **22.** D **23.** B **24.** D **25.** A **26.** C **27.** B **28.** B **29.** B **30.** D
31. A **32.** D **33.** A **34.** B **35.** C **36.** B **37.** B **38.** D **39.** C **40.** A

Previous Years' Questions

1. D **2.** D **3.** A **4.** D **5.** A **6.** A **7.** C **8.** B **9.** B **10.** A
11. 29 to 31 **12.** 0.55 to 0.57 **13.** 590 to 595 **14.** A **15.** 30 to 32 **16.** C
17. 104 to 105 **18.** C