CLASS: XI CHAPTER 3 - TRIGONOMETRIC FUNCTIONS

EXERCISE 3.4

Find the principal and Gen Solution of the following equations:

QNo 1: $\tan x = \sqrt{3}$

Sol $\tan x = \sqrt{3}$ and $\tan \frac{\pi}{3} = \sqrt{3}$

$\Rightarrow \tan x = \tan \frac{\pi}{3}$.

$\Rightarrow x = n\pi + \frac{\pi}{3}$; $n \in Z$ (General Sol.)

[$\because \tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ General sol]

$\Rightarrow x = \frac{\pi}{3}, \frac{4\pi}{3}$ (Principal Sol.)

QNo 2: $\text{Sec}\, x = 2$

Sol $\text{Sec}\, x = 2 \Rightarrow \frac{1}{\text{Sec}\, x} = \frac{1}{2}$ i.e $\cos x = \frac{1}{2}$ and $\cos \pi/3 = \frac{1}{2}$

$\therefore \text{Cos}\, x = \cos \pi/3$

$\Rightarrow x = 2n\pi \pm \pi/3$ (General sol.)

[$\because \text{Cos}\,\theta = \text{Cos}\,\alpha \Rightarrow \theta = 2n\pi \pm \alpha$.]

$= \frac{\pi}{3}, \frac{5\pi}{3}$ (Principal Sol.)

[By putting $n = 0, \pm 1, \pm 2 \cdots$ only those values which lie between $[0, 2\pi]$.]

QNo 3. $\text{Cot}\, x = -\sqrt{3}$

Sol $\Rightarrow \tan x = -\frac{1}{\sqrt{3}}$.

$\Rightarrow \tan x = -\tan\left(\frac{\pi}{6}\right) = \tan\left(\pi - \frac{\pi}{6}\right) = \tan \frac{5\pi}{6}$

$\Rightarrow x = n\pi + \frac{5\pi}{6}$ (General Soln)

$\therefore x = \frac{5\pi}{6}, \frac{11\pi}{6}$ (Principal Sol.)

QNo 4: $\text{Cosec}\, x = -2$

Sol $\text{Cosec}\, x = -2 \Rightarrow \sin x = -\frac{1}{2} \Rightarrow \sin x = -\sin(\pi/6)$

$\Rightarrow \sin x = \sin(-\pi/6) = \sin\left(\pi - (-\pi/6)\right) = \sin \frac{7\pi}{6}$

$\therefore x = n\pi + (-1)^n \frac{7\pi}{6}$ (General sol.

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ (Principal Sol.)

find the general solution for each of following:

QNo 5: $\cos 4x = \cos 2x$

Sol. $\cos 4x = \cos 2x$

$\Rightarrow \cos 4x - \cos 2x = 0$

$\Rightarrow -2 \sin \frac{4x+2x}{2} \sin \frac{4x-2x}{2} = 0$ [Applying C-D formula]

$\Rightarrow -2 \sin 3x \cdot \sin x = 0$

$\Rightarrow$ Either $\sin 3x = 0$ or $\sin x = 0$ [$\because \sin\theta = 0 \Rightarrow \theta = n\pi, n \in Z$]

$\Rightarrow 3x = n\pi; n \in Z$ or $x = n\pi; n \in Z$

$\Rightarrow x = \frac{n\pi}{3}$ or $x = n\pi; n \in Z.$

QNo 6: $\cos 3x + \cos x - \cos 2x = 0$

Sol: $(\cos 3x + \cos x) - \cos 2x = 0$

$\Rightarrow 2 \cos \frac{3x+x}{2} \cos \frac{3x-x}{2} - \cos 2x = 0$ [Applying C-D formula]

$\Rightarrow 2 \cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x [2\cos x - 1] = 0.$

$\Rightarrow$ Either $\cos 2x = 0$ or $2\cos x - 1 = 0$. ie $\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow 2x = (2n+1)\pi/2$ or $x = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = (2n+1)\frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}.$

QNo 7 $\sin 2x + \cos x = 0$

$\Rightarrow 2 \sin x \cos x + \cos x = 0$ [$\because \sin 2\theta = 2\sin\theta\cos\theta$]

$\Rightarrow \cos x [2\sin x + 1] = 0.$

$\Rightarrow \cos x = 0$ or $2\sin x + 1 = 0.$

$\Rightarrow$ Either $\cos x = 0$ or $\sin x = -1/2 = \sin\frac{7\pi}{6}.$

$\Rightarrow x = (2n+1)\frac{\pi}{2}$ or $x = n\pi + (-1)^n \frac{7\pi}{6}; n \in Z$

QNo 8: $\sec^2 2x = 1 - \tan 2x$

Sol

$$\sec^2 2x = 1 - \tan 2x \Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$$

$$\Rightarrow \tan^2 2x + \tan 2x = 0.$$

$$\Rightarrow \tan 2x (\tan 2x + 1) = 0$$

$$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0.$$

$$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x = -1 = \tan \frac{3\pi}{4}.$$

$$\Rightarrow 2x = n\pi \quad \text{or} \quad 2x = 2n\pi + \frac{3\pi}{4}$$

$$\Rightarrow x = \frac{n\pi}{2} \quad \text{or} \quad x = n\pi + \frac{3\pi}{8}$$

QNo 9 $\sin x + \sin 3x + \sin 5x = 0$

Sol.

$$(\sin x + \sin 5x) + \sin 3x = 0.$$

$$\Rightarrow 2 \sin \frac{x+5x}{2} \cos \frac{5x-x}{2} + \sin 3x = 0. \quad \text{[Applying C-D formula]}$$

$$\Rightarrow 2 \sin 3x \cos 2x + \sin 3x = 0$$

$$\Rightarrow \sin 3x [2 \cos 2x + 1] = 0.$$

$$\Rightarrow \text{Either } \sin 3x = 0 \quad \text{or} \quad 2 \cos 2x + 1 = 0$$

$$\Rightarrow 3x = n\pi \quad \text{or} \quad \cos 2x = -\frac{1}{2}$$

$$\Rightarrow x = \frac{n\pi}{3}; \ n \in Z \quad \text{or} \quad \cos 2x = -\cos \frac{\pi}{3} = \cos\left(\pi - \pi/3\right)$$

$$\Rightarrow x = \frac{n\pi}{3}; \ n \in Z \quad \text{or} \quad \cos 2x = \cos \frac{2\pi}{3}$$

$$\Rightarrow x = \frac{n\pi}{3}; \ n \in Z \quad \text{or} \quad 2x = 2n\pi \pm \frac{2\pi}{3}$$

$$\Rightarrow x = \frac{n\pi}{3}; \ n \in Z \quad \text{or} \quad x = n\pi \pm \frac{\pi}{3}.$$

$$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad n\pi \pm \frac{\pi}{3}; \ n \in Z$$

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