classmate
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** Multiplication Theorem :

$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$

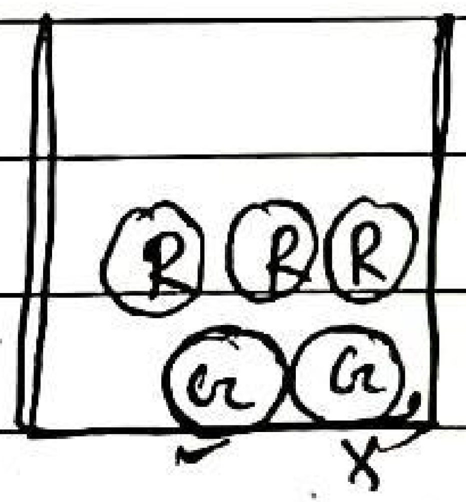
$$P(B/A) = \frac{P(A\cap B)}{P(A)}$$

both are conditional probability.

- Multiplication theorem (Dependent) –

$$\boxed{\begin{array}{l} P(A\cap B) = P(B)\ P(A/B) \\ P(A\cap B) = P(A)\ P(B/A) \end{array}}$$ → for 2 event.

Ex-1 The Box containing 2 Green and 3 red balls. 2 balls are randomly drawn from the box, without replacement. What is the probability that ~~the balls are~~ ~~of~~ the first ball are green and second ball are red.

→ 

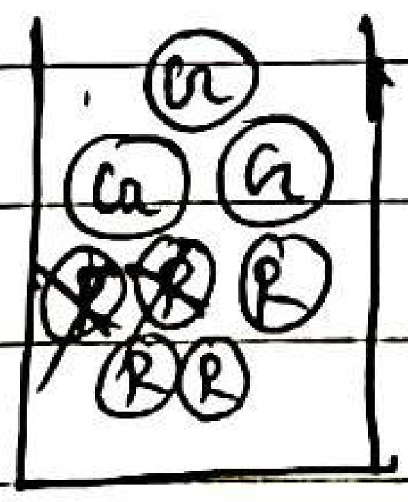
$$P(G_1\cap R_2) = P(G_1)\ P(R_2/G_1)$$

$$= \frac{2}{5} \times \frac{3}{4}$$

$$= \frac{3}{10}$$

(*) Chain rule → $$\boxed{P(E_1\cap E_2\cap E_3) = P(E_1)\ P(E_2/E_1)\ P(E_3/E_1\cap E_2)}$$ → for 3 events

Ex-2 The Box containing total 8 balls. Where 5 Red balls and 3 Green balls. 3 balls are drawn one by one from the box without replacement. What is the propability that 3 balls are red.

→ 

$$P(R_1\cap R_2\cap R_3) = P(R_1)\ P(R_2/R_1)\ P(R_3/R_1\cap R_2)$$

$$= \left(5/8 \times 4/7 \times 3/6\right)$$

$$= 5/28.$$

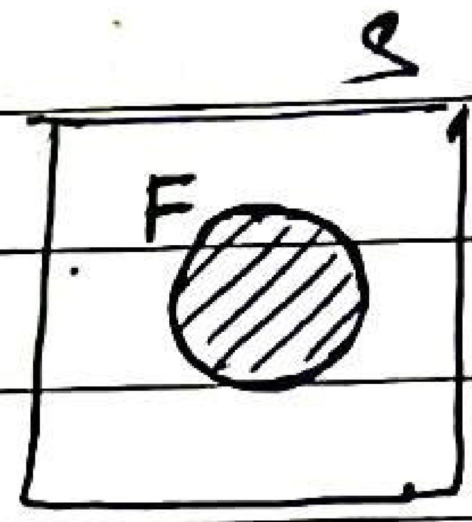
• PROPERTIES OF CONDITIONAL PROBABILITY:

(1) $P(S/F) = 1$

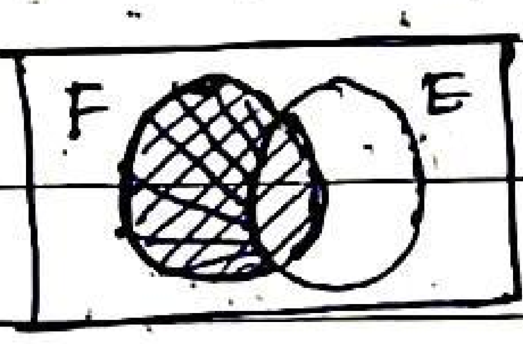
(2) $P(\bar{E}/F) = 1 - P(E/F)$.

(3) $P(A \cup B/F) = P(A/F) + P(B/F) - P\left(\frac{A \cap B}{F}\right)$.

(1) $P(S/F) \Rightarrow \frac{P(S \cap F)}{P(F)} \Rightarrow \frac{P(F)}{P(F)} \Rightarrow 1$



(2) $P(\bar{E}/F) = 1 - P(E/F)$

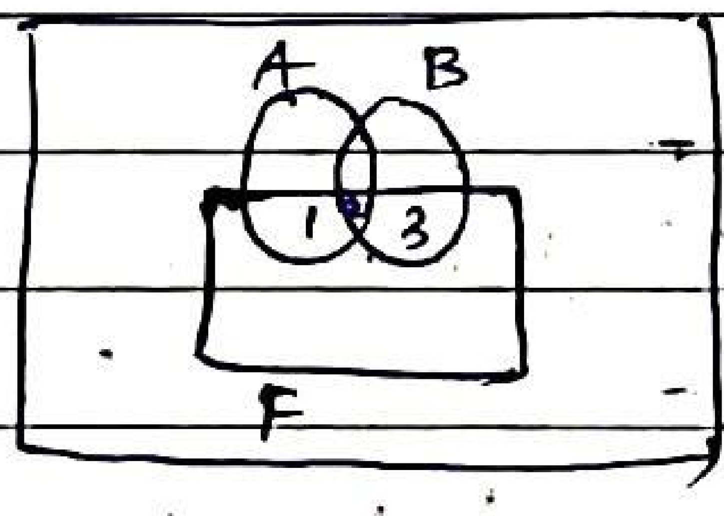


$P(F) = 1$

$\Rightarrow P(F) = P(\bar{E}/F) + P(E/F)$

$\Rightarrow 1 = P(\bar{E}/F) + P(E/F)$

(3) $P(A \cup B/F) = \underbrace{P(A/F)}_{1,2} + \underbrace{P(B/F)}_{2,3} - \underbrace{\frac{P(A \cap B)}{F}}_{2}$



Ex-3

Ravi can either take biology or calculus. If he takes biology, then ~~to~~ he will get an 'A' grade with probability $1/2$. If he ~~to~~ takes calculus, then he will get an 'A' grade with probability $1/3$. He desides to base his decision on the flip of a fair coin. What is the probability that,

(a) He will get an 'A' in calculas.
(b) He will get an 'A' in biology.

→ Probability to take Biology(B) and calculas by flip of a coin,

$$P(B) = 1/2$$
$$P(C) = 1/2$$

given,

$P(A/B) = 1/2$ (Prob to get 'A' grade when he take Bio.)

$P(A/C) = 1/3$

(a) $P(C \cap A) = P(C)\ P(A/C)$

$$= 1/2 \times 1/2 \Rightarrow 1/4$$

(b) $P(B \cap A) = P(B)\ P(A/B)$

$$= 1/2 \times 1/3 \Rightarrow 1/6$$

Ex-4 suppose that each of 3 men at a party throws his hat into center of the room. The hats are first mixed up and each man randomly selects a hat. What is the probability that –

(a) All of the 3 men selects his own hat.

(b) Non of the 3 men selects his own hat.

→ $\cancel{E}_1, \cancel{E}_2, \cancel{E}_3$

(a) Three events that 3 man selects his own hat.

E_1, E_2, E_3

$P(E_1) = 1/3$ (Pro to get first person his own heat)

$P(E_2/E_1) = 1/2$ (" " secon " " ")

$P(E_3/E_1 \cap E_2) = 1$ (" " 3rd " " ")

$$P(E_1 \cap E_2 \cap E_3) = P(E_1)\, P(E_2/E_1)\, P(E_3/E_1 \cap E_2)$$
$$= 1/3 \times 1/2 \times 1$$
$$= 1/6$$

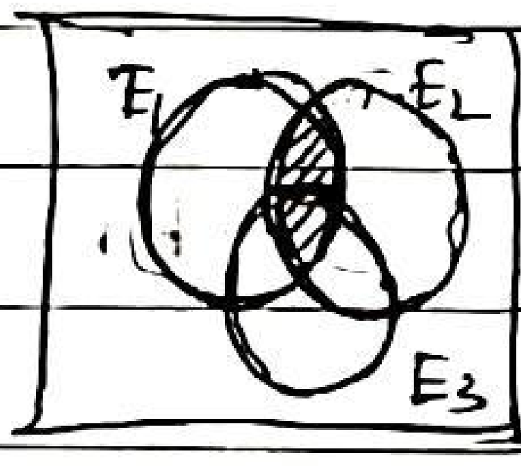
(b) Non of the 3 men selects his own hat -

$\bar{E}_1, \bar{E}_2, \bar{E}_3$

$$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = P(\overline{E_1 \cup E_2 \cup E_3})$$

** $\boxed{P(\bar{E}) = 1 - P(E)}$

$$= 1 - P(E_1 \cup E_2 \cup E_3)$$



$$= 1 - \frac{2}{3}$$

$$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = \frac{1}{3}$$

$P(E_1 \cup E_2 \cup E_3)$

$= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$

$= 1/3 + 1/3 + 1/3 - 1/6 - 1/6 - 1/6 + 1/6$

$= (1 - 1/3) = \frac{2}{3}$

$$P(E_1 \cap E_2) = P(E_1)\, P\left(E_1/E_2\right)$$

$$= 1/3 \times 1/2 = 1/6$$

$$P(E_2 \cap E_3) = 1/3 \times 1/2 = 1/6$$

$$P(E_1 \cap E_3) = P(E_1)\, P\left(P_1/E_3\right)$$

$$= 1/3 \times 1/2 = 1/6$$