Chapter : 20. SUMMATIVE ASSESSMENT I

Exercise : SAMPLE PAPER I

Question: 1

Euclid's Division

Solution:

Euclid's division lemma :

Given positive integers a and b, there exist unique integers q and r satisfying $a = bq + r, 0 \le r < b$

Question: 2

In the given figu

Solution:

The zeroes of polynomial means that value of polynomial becomes zero.

In the above graph, the curve depicts the polynomial and it gets zero at two points, therefore p(x) has two zeroes.

Question: 3

In ΔABC, it is gi

Solution:

In ΔADE and ΔABC

∠ADE = ∠ABC [Corresponding angles as DE || BC]

∠AED = ∠ACB [Corresponding angles as DE || BC]

ΔADE ~ ΔABC [By Angle-Angle Similarity criterion]

$\Rightarrow \frac{AB}{AD} = \frac{BC}{DE}$ [Corresponding sides of similar triangles are in the same ratio]

Now,

Given, AD = 3 cm

DB = 2 cm

DE = 6 cm

⇒ AB = AD + DB = 3 + 2 = 5 cm

Using this in above equation,

$\Rightarrow \frac{5}{3} = \frac{BC}{6}$

⇒ BC = 10 cm

Question: 4

If sin 3θ = cos (

Solution:

Given, we know that

sin θ = cos(90° - θ)

Replacing θ by 3θ

⇒ sin(3θ) = cos(90° - 3θ)

$\Rightarrow \cos(\theta - 2°) = \cos(90° - 3\theta)$

[Given, $\sin 3\theta = \cos(\theta - 2°)$]

$\Rightarrow \theta - 2° = 90° - 3\theta$

$\Rightarrow 4\theta = 92°$

$\Rightarrow \theta = 23°$

Question: 5

If $\tan \theta = \sqrt{3}$, th

Solution:

Given,

$\tan \theta = \sqrt{3}$

$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \sec^2\theta - 1 = 3$ [As $\tan^2\theta + 1 = \sec^2\theta$]

$\Rightarrow \sec^2\theta = 4$ ...[1]

Also,

$$\cot\theta = \frac{1}{\tan\theta}$$

$\Rightarrow \cot\theta = \frac{1}{\sqrt{3}}$ as $\tan\theta = \sqrt{3}$

Squaring both sides,

$$\Rightarrow \cot^2\theta = \frac{1}{3}$$

$\Rightarrow \operatorname{cosec}^2\theta - 1 = \frac{1}{3}$ [As $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$]

$\Rightarrow \operatorname{cosec}^2\theta = \frac{4}{3}$ ...[2]

Putting the values from [1] and [2] into given eqn

$$\frac{\sec^2\theta - \operatorname{cosec}^2\theta}{\sec^2\theta + \operatorname{cosec}^2\theta} = \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}}$$

$$\Rightarrow \frac{\frac{12-4}{3}}{\frac{12+4}{3}} = \frac{8}{16} = \frac{1}{2}$$

Question: 6

The decimal expan

Solution:

$$\frac{49}{40} = \frac{49}{2 \times 2 \times 2 \times 5} = \frac{49}{2^3 5}$$

We know that if $\frac{p}{q}$ is a rational number, such that p and q are co-prime and q has factors in the form of $2^m.5^n$, then, decimal expansion of $\frac{p}{q}$ will terminate after the highest power of 2 or 5 (whichever is greater).

Therefore, $\frac{49}{40}$ will terminate after 3 places of decimal.

Question: 7

The pair of linea

Solution:

Comparing the equation with the set of equations

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

we have,

$a_1 = 6, a_2 = 2$

$b_1 = -3, b_2 = -1$

$c_1 = 10, c_2 = 9$

and we have,

$\frac{a_1}{a_2} = \frac{6}{2} = 3$ and $\frac{b_1}{b_2} = -\frac{3}{-1} = 3$ and $\frac{c_1}{c_2} = \frac{10}{9}$

So, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

and in this case, we know that equations have no solution.

Question: 8

For a given data

Solution:

As we know that, the x-coordinate of the point of intersection of the

more than ogive and less than ogive give us a median of the data.

So, the median of the data is 18.5

Question: 9

Is (7 x 5 x 3 X 2

Solution:

(7 x 5 x 3 x 2 + 3) = (210 + 3) = 213

And 213 = 71 x 3

As, this number is expressible as product of two no's other, the given number is composite.

[Composition no's are those no's which has factors other than 1 and itself]

Question: 10

When a polynomial

Solution:

No, because degree of remainder cannot be equal to the degree of divisor

And in this case degree of divisor, i.e. 2x + 1 = 1

And degree of remainder, i.e. x -1 = 1 is equal.

Question: 11 A

If 3 $\cos^2$

Solution:

Given,

$3\cos^2\theta + 7\sin^2\theta = 4$

$\Rightarrow 3\cos^2\theta + 3\sin^2\theta + 4\sin^2\theta = 4$

$\Rightarrow 3(\cos^2\theta + \sin^2\theta) + 4\sin^2\theta = 4$

$\Rightarrow 3 + 4\sin^2\theta = 4$

[as $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow 4\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30°$

[as $\sin\theta = \frac{1}{2}$]

$\Rightarrow \cot\theta = \sqrt{3}$

[as $\cot 30° = \sqrt{3}$]

Question: 11 B

If tan θ = 8/15 e

Solution:

$\tan\theta = \frac{8}{15}$

Now, To find :

$\frac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$

$\Rightarrow \frac{2(1 + \sin\theta)(1 - \sin\theta)}{2(1 + \cos\theta)(1 - \cos\theta)}$

$\Rightarrow \frac{1 - \sin^2\theta}{1 - \cos^2\theta}$

[As, $(a + b)(a - b) = a^2 - b^2$]

$\Rightarrow \frac{\cos^2\theta}{\sin^2\theta}$

[As $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow \cot^2\theta = \frac{1}{\tan^2\theta}$

$\left[\text{as } \frac{\cos\theta}{\sin\theta} = \cot\theta = \frac{1}{\tan\theta}\right]$

$\Rightarrow \frac{1}{\left(\frac{8}{15}\right)^2} = \frac{1}{\frac{64}{225}} = \frac{225}{64}$

Question: 12

In the given figu

Solution:

DE || AC [Given]

And we know, By Basic Proportional Theorem

If a line is drawn parallel to the one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in same ratio

$\Rightarrow \frac{BE}{EC} = \frac{BD}{AD}$ ...[1]

And DF || AE

By Basic Proportional Theorem,

$\Rightarrow \frac{BF}{FE} = \frac{BD}{AD}$

$\Rightarrow \frac{BF}{FE} = \frac{BE}{EC}$ [From [1]]

$\Rightarrow \frac{EC}{BE} = \frac{FE}{BF}$

Hence, Proved

Question: 13

In the given figu

Solution:

We have,

BC = BD + CD

$BC = \frac{1}{3}CD + CD = \frac{4}{3}CD \left[AS\ BD = \frac{1}{3}CD\right]$

$\Rightarrow CD = \frac{3}{4}BC$ [1]

As, AD ⊥ BC

⇒ ΔADC is a right-angled triangle

By Pythagoras theorem,[i.e. hypotenuse^2 = perpendicular^2 + base^2]

$AD^2 + CD^2 = CA^2$

$\Rightarrow AD^2 = CA^2 - CD^2$ ....[2]

Also, ΔABD is a right-angled triangle

By Pythagoras theorem,

$AD^2 + BD^2 = AB^2$

From [2]

$CA^2 - CD^2 + BD^2 = AB^2$

$\Rightarrow CA^2 - CD^2 + \left(\frac{1}{3}CD\right)^2 = AB^2 \left[AS\ BD = \frac{1}{3}CD\right]$

$\Rightarrow CA^2 - CD^2 + \frac{1}{9}CD^2 = AB^2$

$\Rightarrow CA^2 - \frac{8}{9}CD^2 = AB^2$

$\Rightarrow CA^2 - \frac{8}{9}\left(\frac{3}{4}BC\right)^2 = AB^2$ [From [1]]

$\Rightarrow CA^2 - \frac{8}{9} \times \frac{9}{16} \times BC^2 = AB^2$

$\Rightarrow CA^2 - \frac{1}{2}BC^2 = AB^2$

$\Rightarrow 2CA^2 - BC^2 = 2AB^2$

$\Rightarrow 2CA^2 = 2AB^2 + BC^2$

Hence, Proved.

Question: 14

Find the mode of

Solution:

In the given data,

The maximum class frequency is 32. So, the modal class is 30-40.

Lower limit(l) of modal class = 30

Class size(h) = 40 - 30 = 10

Frequency(f_1) of modal class = 32

Frequency(f_0) of class preceding the modal class = 12

Frequency(f_2) of class succeeding the modal class = 20

And we know,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Substituting values, we get

$$\text{Mode} = 30 + \left(\frac{32 - 12}{2(32) - 12 - 20}\right)(10) = 30 + \frac{200}{32}$$

$$\Rightarrow \text{Mode} = \frac{960 + 200}{32} = \frac{1160}{32} = 36.25$$

Question: 15

Show that any pos

Solution:

Let a be an positive odd integer, and let b = 4

By, using Euclid's division lemma,

a = 4q + r, where r is an integer such that, $0 \leq r < 4$

So, only four cases are possible

a = 4q or

a = 4q + 1 or

a = 4q + 2 or

a = 4q + 3

But 4q and 4q + 2 are divisible by 2, therefore these cases are not possible, as a is an odd integer.

Therefore,

a = 4q + 1 or a = 4q + 3.

Question: 16 A

Prove that (5 - √

Solution:

Let 5 - √3 be rational,

Then, 5 - √3 can be expressed as $\frac{p}{q}$ where, p and q are co-prime integers and

q ≠ 0,

we have,

$$5-\sqrt{3} = \frac{p}{q}$$

$$\Rightarrow 5-\frac{p}{q} = \sqrt{3}$$

$$\Rightarrow \frac{5q-p}{q} = \sqrt{3}$$

As p and q are integers, 5q - p is also an integer

⇒ $\frac{5q-p}{q}$ is a rational number.

But √3 is an irrational number, so the equality is not possible.

This contradicts our assumption, that 5 - √3 is a rational number.

Therefore, 5 - √3 is an irrational number.

Question: 16 B

Prove that

Solution:

Let $\frac{3\sqrt{3}}{5}$ be rational,

Then, $\frac{3\sqrt{3}}{5}$ can be expressed as $\frac{p}{q}$ where p and q are co-prime integers and

q ≠ 0,

we have,

$$\frac{3\sqrt{3}}{5} = \frac{p}{q}$$

$$\Rightarrow \frac{5p}{3q} = \sqrt{3}$$

As p and q are integers, 5p and 3q are also integers

⇒ $\frac{5p}{3q}$ is a rational number.

But √3 is an irrational number, so the equality is not possible.

This contradicts our assumption, that $\frac{3\sqrt{3}}{5}$ is a rational number.

Therefore, $\frac{3\sqrt{3}}{5}$ is an irrational number.

Question: 17 A

A man can row a b

Solution:

Speed of boat in still water = 4 km/h

Let the speed of stream be 'x'

Therefore,

Speed of the boat upstream = Speed of boat in still water - Speed of stream = 4 - x

Speed of the boat downstream = Speed of boat in still water + Speed of stream = 4 + x

Time taken to go upstream = $\frac{\text{distance}}{\text{speed}} = \frac{30}{4-x}$ hours

Time taken to go downstream = $\frac{\text{distance}}{\text{speed}} = \frac{30}{4+x}$ hours

Given, time taken in upstream is thrice as in downstream

$\Rightarrow \frac{30}{4-x} = 3\left(\frac{30}{4+x}\right)$

$\Rightarrow \frac{30}{4-x} = \frac{90}{4+x}$

$\Rightarrow \frac{1}{4-x} = \frac{3}{4+x}$

⇒ 4 + x = 12 - 3x

⇒ 4x = 8

⇒ x = 2

i.e. the speed of stream = x is 2 km/hour.

Question: 17 B

In a competitive

Solution:

Let the number of correct answers = x

Let the number of wrong answers = y

Total no of questions attempted = x + y = 120

⇒ y = 120 - x ....[1]

Marks for each correct answer = 5

Marks for x correct answers = 5x

As 2 marks are deducted for each wrong question,

Marks deducted for y wrong answers = 2y

Total marks obtained by student will be 5x - 2y,

⇒ 5x - 2y = 348

⇒ 5x - 2(120 - x) = 348

⇒ 5x - 240 + 2x = 348

⇒ 7x = 588

⇒ x = 84

Hence, no of correct answers = x = 84

Question: 18

If α and β are th

Solution:

We know that, for a quadratic equation ax^2 + box + c

Sum of zeroes = $-\frac{b}{a}$

Product of zeroes = $\frac{c}{a}$

Given equation = $2x^2 + x - 6$ and zeroes are α and β

Therefore,

$\alpha + \beta = -\frac{1}{2}$ **....**[1] and

$\alpha\beta = -\frac{6}{2} = -3$ ....[2]

Now, any quadratic equation having α and β as zeroes will have the form

$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

⇒ equation having α and β as zeroes will have the form

$p(x) = x^2 - (2\alpha + 2\beta)x + (2\alpha)(2\beta)$

$\Rightarrow p(x) = x^2 - 2(\alpha + \beta)x + 4\alpha\beta$

From [1] and [2]

$\Rightarrow p(x) = x^2 - 2\left(-\frac{1}{2}\right)x + 4(-3) = x^2 + x - 12$

Hence required equation is $x^2 + x - 12$.

Question: 19

Prove that: (cose

Solution:

Taking L.H.S

= (cosecθ - sinθ)(secθ - cosθ)

$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)$

$= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right)\left(\frac{1 - \cos^2\theta}{\cos\theta}\right)$

We know, $\sin^2\theta + \cos^2\theta = 1$

Therefore,

$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} = \cos\theta\ \sin\theta$

Taking R.H.S

$= \frac{1}{\tan\theta + \cot\theta}$

$= \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$

$= \frac{1}{\left(\frac{(\sin^2\theta + \cos^2\theta)}{\sin\theta\cos\theta}\right)}$

$= \sin\theta\cos\theta$ [as $\sin^2\theta + \cos^2\theta = 1$]

LHS = RHS

Hence, Proved.

Question: 20

If cosθ + sinθ =

Solution:

Given,

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ ...[1]

Squaring both side,

$(\cos\theta + \sin\theta)^2 = 2\cos^2\theta$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow 2\cos\theta\sin\theta = 2\cos^2\theta - \cos^2\theta - \sin^2\theta$

$\Rightarrow 2\cos\theta\sin\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow 2\cos\theta\sin\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

$\Rightarrow 2\cos\theta\sin\theta = (\cos\theta - \sin\theta)(\sqrt{2}\cos\theta)$ [From [1]]

$$\Rightarrow \cos\theta - \sin\theta = \frac{2\cos\theta\sin\theta}{\sqrt{2}\cos\theta} = \sqrt{2}\sin\theta$$

Hence, Proved.

Question: 21

ΔABC and ΔDBC are

Solution:

Given: ΔABC and ΔDBC with common base BC.

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction: Draw AM ⊥ BC and DN ⊥ BC

Proof:

In ΔAMO and ΔDNO

∠AOM = ∠DON [Vertically opposite angle]

∠AMO = ∠DNO [Both 90°]

ΔAMO ~ ΔDNO [By Angle-Angle sum criterion]

$\Rightarrow \frac{AM}{DN} = \frac{AO}{DO}$ [Corresponding sides of similar triangles are in the same ratio] [1]

Now, we know that

Area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

Therefore,

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN} = \frac{BC}{DN}$$

$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$ [From [1]]

Hence, Proved

Question: 22

In Δ ABC, the AD

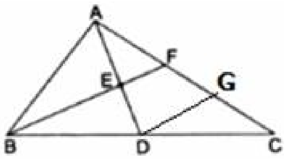
Solution:

Proof:

Given: In ΔABC, the AD is a median and E is mid-point of the AD and BE is produced to meet AC in F.

To Prove: $AF = \frac{1}{3}AC$

Construction: Draw DG || BF as shown in figure



Proof:

Now, In ΔBFC

DG || BF [By construction]

As AD is a median on BC, D is a mid-point of BC

Therefore,

G is a mid-point of CF [By mid-point theorem]

⇒ CG = FG ...[1]

Now, In ΔADG

EF || DG [By Construction]

As E is a mid-point of AD [Given]

Therefore,

F is a mid-point of AG [By mid-point theorem]

⇒ FG = AF ...[2]

From [1] and [2]

AF = CG = FG ...[3]

And

AC = AF + FG + CG

⇒ AC = AF + AF + AF [From 3]

⇒ AC = 3AF

$$\Rightarrow AF = \frac{1}{3}AC$$

Hence Proved

Question: 23 A

Find the mean of

Solution:

Let us first calculate the mid-values(x_i) for each class-interval, By using the formula

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Class-interval	Frequency f_i	Mid-Values x_i	$u_i = \frac{x_i - a}{h}$ $u_i = \frac{x_i - 75}{50}$	f_iu_i
0-50	17	25	-1	-17
50-100	35	75	0	0
100-150	43	125	1	43
150-200	40	175	2	80
200-250	21	225	3	63
250-300	24	275	4	96
Total	$\sum f_i = 180$			$\sum f_iu_i = 265$

Let us assume the assumed mean(a) = 75

and from that, we get the data as shown in above table.

And we know, By step-deviation method

$$\text{mean}(\bar{x}) = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$$

Where, a = assumed mean

h = class size

$$\Rightarrow \bar{x} = 75 + \left(\frac{265}{180}\right)(50)$$

$$\Rightarrow \bar{x} = 75 + 73.61 = 148.61$$

Question: 23 B

The mean of the f

Solution:

Let us first calculate the mid-values(x_i) for each class-interval, By using the formula

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

By which, we get the following data

Class-interval	Frequency (f_i)	Mid-values(x_i)	f_ix_i
0-10	15	5	75
10-20	20	15	300
20-30	35	25	875
30-40	p	35	35p
40-50	10	45	450
	$\sum f_i = 80 + p$		$\sum f_ix_i = 1700 + 35p$

We know, that

$$\text{mean}(\bar{x}) = \frac{\sum f_ix_i}{\sum f_i}$$

Given, mean = 24

$$\Rightarrow 24 = \frac{1700 + 35p}{80 + p}$$

⇒ 1920 + 24p = 1700 + 35p

$\Rightarrow 11p = 220$

$\Rightarrow p = 20$

Question: 24

Find the median o

Solution:

First, let us make a cumulative frequency distribution of less than type.

Class Interval	Frequency(f)	Cumulative Frequency(cf)
0-10	5	5
10-20	3	8
20-30	4	12
30-40	3	15
40-50	3	18
50-60	4	22
60-70	7	29
70-80	9	38
80-90	7	45
90-100	8	53
	Total : 53	

In this case,

Sum of all frequencies, n = 53

$$\Rightarrow \frac{n}{2} = \frac{53}{2} = 26.5$$

Now, we know the median class is whose cumulative frequency is greater than and nearest to $\frac{n}{2}$.

As, a Cumulative frequency greater than and nearest to 26.5 is 29, the median class is 60 - 70.

Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

where l = lower limit of median class,

n = number of observations,

cf = cumulative frequency of class preceding the median class,

f = frequency of median class,

h = class size

In this case,

l = 60

n = 53

cf = 22

f = 7

h = 10

Putting values, we get,

$\text{Median} = 60 + \left(\frac{26.5-22}{7}\right)(10)$

$= 60 + \frac{45}{7} = 66.4$

Question: 25

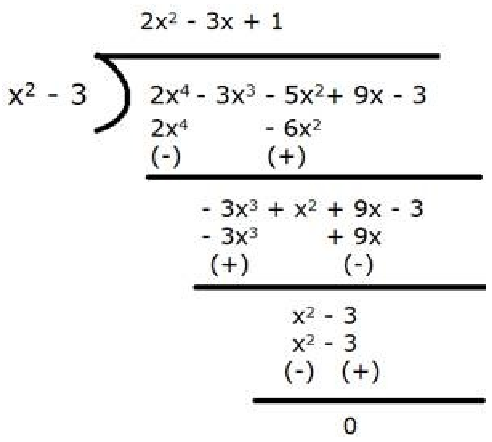
Let p(x) = 2x

Solution:

Two zeroes are √3 and -√3,

Therefore (x -√3)(x - (-√3) = (x - √3)(x + √3) is a factor of p(x).

Let us divide p(x) by (x - √3)(x + √3) = (x^2 - 3)



⇒ ($2x^4$ - $3x^3$ - $5x^2$ + 9x - 3) = (x^2 - 3)($2x^2$ - 3x + 1)

= (x - √3)(x + √3)($2x^2$ - 2x - x + 1)

= (x - √3)(x + √3)(2x(x - 1) - 1(x - 1))

= (x - √3)(x + √3)(2x - 1)(x - 1)

Hence,

2x - 1 = 0 or x - 1 = 0

⇒ $x = \frac{1}{2}$ or x = 1

Hence, other two zeroes are $\frac{1}{2}$ or 1.

Question: 26 A

Prove that the ra

Solution:

Let ΔPQR and ΔABC be two similar triangles,

⇒ $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC}$ [Corresponding sides of similar triangles are in the same ratio] [1]

And as corresponding angles of similar triangles are equal

∠A = ∠P

∠B = ∠Q

∠C = ∠R

Construction: Draw PM ⊥ QR and AN ⊥ BC

In ΔPQR and ΔABC

∠PMR = ∠ANC [Both 90°]

$\angle R = \angle C$ [Shown above]

$\Delta PQR \sim \Delta ABC$ [By Angle-Angle Similarity]

$\Rightarrow \frac{PM}{AN} = \frac{PR}{AC}$ [Corresponding sides of similar triangles are in the same ratio] [2]

Now, we know that

Area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

Therefore,

$$\frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{\frac{1}{2} \times PQ \times PM}{\frac{1}{2} \times AB \times AN} = \frac{PQ \times PM}{AB \times AN}$$

$\Rightarrow \frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{PQ}{AB} \times \frac{PR}{AC}$ [From 2]

$\Rightarrow \frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{PQ}{AB} \times \frac{PQ}{AB} = \left(\frac{PQ}{AB}\right)^2$ [From 1]

$\Rightarrow \frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \left(\frac{PQ}{AB}\right)^2 = \left(\frac{PR}{AC}\right)^2 = \left(\frac{QR}{BC}\right)^2$ [From 1]

Hence, Proved.

Question: 26 B

In a triangle, if

Solution:

Let us consider a triangle ABC, in which

$AC^2 = BC^2 + AB^2$ ...[1]

To Prove: Angle opposite to the first side i.e. AC is right angle or

$\angle ABC = 90°$

Construction:

Let us draw another right-angled triangle PQR right-angled at Q, with

AB = PQ

BC = QR

Now, By Pythagoras theorem, In ΔPQR

$PR^2 = QR^2 + PQ^2$

But QR = BC and PQ = AB

$\Rightarrow PR^2 = BC^2 + AB^2$

But From [1] we have,

$AC^2 = PR^2$

$\Rightarrow AC = PR$

In ΔABC and ΔPQR

AB = PQ [Assumed]

BC = QR [Assumed]

AC = PR [Proved above]

$\Rightarrow \Delta ABC \cong \Delta PQR$ [By Side-Side-Side Criterion]

$\Rightarrow \angle ABC = \angle PQR$ [Corresponding parts of congruent triangles are equal]

But, ∠PQR = 90°

⇒ ∠ABC = 90°

Hence, Proved !

Question: 27 A

Prove that

Solution:

Taking LHS

$$= \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

Dividing by cosθ in numerator and denominator

$$= \frac{\left(\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)}{\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}\right)}$$

Using $\frac{\sin\theta}{\cos\theta} = \tan\theta$ and $\frac{1}{\cos\theta} = \sec\theta$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

Putting 1 = $\sec^2\theta - \tan^2\theta$ in numerator

$$= \frac{\tan\theta - (\sec^2\theta - \tan^2\theta) + \sec\theta}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta + (\tan^2\theta - \sec^2\theta)}{\tan\theta - \sec\theta + 1}$$

Using $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{(\tan\theta + \sec\theta) + (\tan\theta + \sec\theta)(\tan\theta - \sec\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)(1 + \tan\theta - \sec\theta)}{1 + \tan\theta - \sec\theta}$$

= tan θ + sec θ

Now, taking RHS

$$= \frac{1}{\sec\theta - \tan\theta}$$

Multiplying and dividing by secθ + tanθ = 1

$$= \frac{1}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

= tanθ + secθ [As $\sec^2\theta - \tan^2\theta = 1$]

LHS = RHS

Hence Proved.

Question: 27 B

Evaluate:

Solution:

Using cosec(90° - θ) = secθ

and cot(90° - θ) = tanθ

we have,

$$\frac{\sec\theta\,\text{cosec}(90-\theta) - \tan\theta\cot(90-\theta) + \sin^2 65° + \sin^2 25°}{\tan 10° \tan 20° \tan 60° \tan 70° \tan 80°}$$

$$= \frac{\sec\theta\sec\theta - \tan\theta\tan\theta + \sin^2(90-25) + \sin^2 25°}{\tan 10° \tan 20° \tan 60° \tan(90-20)\tan(90-10)}$$

Now, sin(90 - θ) = cos θ and

tan(90 - θ) = cot θ we have

$$= \frac{\sec^2\theta - \tan^2\theta + \cos^2 25° + \sin^2 25°}{\tan 10° \tan 20° \tan 60° \cot 20° \cot 10°}$$

$$= \frac{1 + 1}{\tan 10° \tan 20° \tan 60° \left(\frac{1}{\tan 20°}\right)\left(\frac{1}{\tan 10°}\right)} = \frac{2}{\tan 60°} = \frac{2}{\sqrt{3}}$$

[Since,

$\tan^2\theta - \sec^2\theta = 1$

$\sin^2\theta + \cos^2\theta = 1$

tan 60° = √3]

Question: 28

If secθ + tanθ =

Solution:

Taking RHS

$$\frac{x^2 - 1}{x^2 + 1} = \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$$

Now, $\sec^2\theta - \tan^2\theta = 1$ and $(a + b)^2 = a^2 + b^2 + 2ab$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - (\sec^2\theta - \tan^2\theta)}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + (\sec^2\theta - \tan^2\theta)}$$

$$= \frac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\sec\theta + \tan\theta)} = \frac{\tan\theta}{\sec\theta}$$

Now, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\frac{1}{\sec\theta} = \cos\theta$, using these we have

$$= \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta$$

= LHS

Hence, Proved !

Question: 29

Solve the followi

Solution:

Equation 1:

2x - y = 1

X	0	1
Y	-1	1

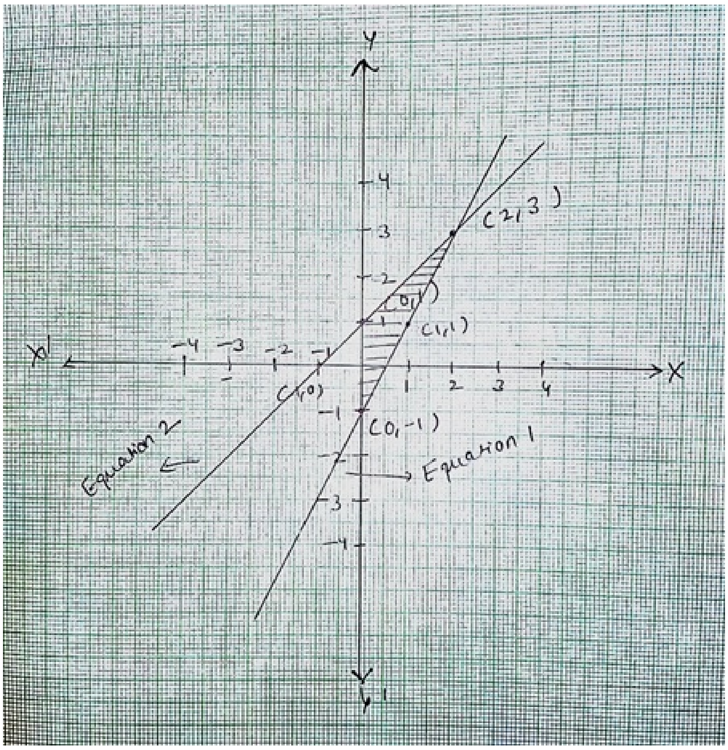
Plot the line with equation 1 on graph.

Equation 2:

x - y = -1

X	0	- 1
Y	1	0

Plot the line with equation 2 on graph.



From the graph We observe point of intersection of two lines is (2, 3)

Region bound by these lines and y-axis is shaded in the graph.

Question: 30

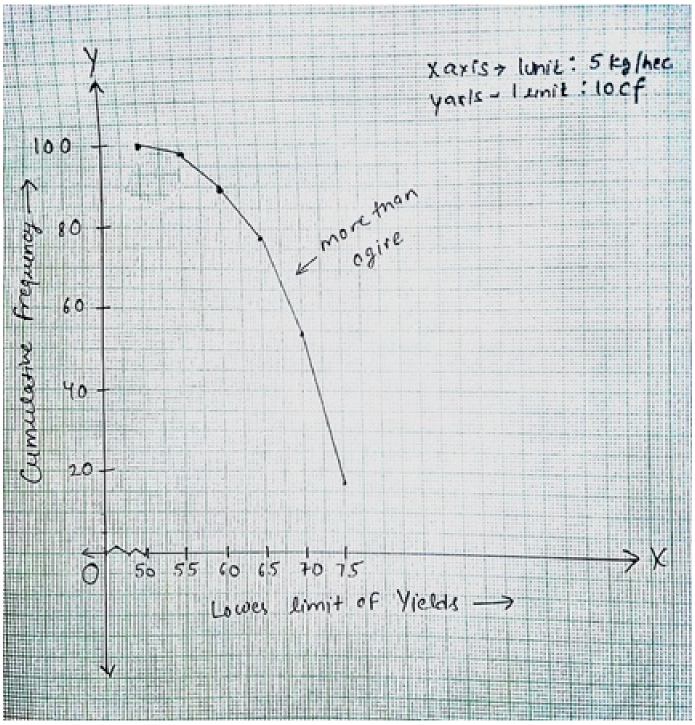
The following tab

Solution:

Let us draw cumulative frequency with table for the above data

Yield (in kg/hectare)	Number of farms Or frequency(f)	Yield [More than or equal to]	Cumulative frequency (cf)
50-55	2	50	100
55-60	8	55	98
60-65	12	60	90
65-70	24	65	78
70-75	38	70	54
75-80	16	75	16

Taking Yield as x-axis and Cumulative frequencies as y-axis, we draw its more than 'ogive'



Question: 31

Solve for x and y

Solution:

Eqn1 : ax + by - a + b = 0

⇒ ax + by = a - b

Multiplying both side by b

⇒ $abx + b^2y = ab - b^2$ ...[1]

Eqn2 : bx - ay - a - b = 0

⇒ bx - ay = a + b

Multiplying both side by a

⇒ $abx - a^2y = a^2 + ab$ ...[2]

Subtracting [2] from [1]

$abx - a^2y - (abx + b^2y) = a^2 + ab - (ab - b^2)$

⇒ $abx - a^2y - abx - b^2y = a^2 + ab - ab + b^2$

⇒ $-y(a^2 + b^2) = a^2 + b^2$

⇒ -y = 1

$\Rightarrow$ y = 1

Putting value of y in eqn1, we get

ax + b(-1) - a + b = 0

$\Rightarrow$ ax - b - a + b = 0

$\Rightarrow$ ax = a

$\Rightarrow$ x = 1

So, x = 1 and y = -1

Question: 32

Prove that:

Solution:

Taking LHS

$$\frac{1-\cos\theta}{1+\cos\theta}$$

Multiplying and dividing by (1 - cos θ)

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

As $\sin^2\theta + \cos^2\theta = 1$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

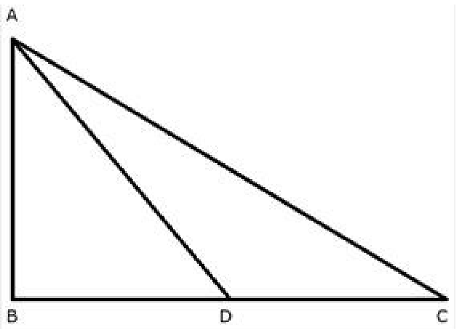
$$= (\operatorname{cosec}\theta - \cot\theta)^2$$

Hence Proved.

Question: 33

Δ ABC is right an

Solution:



Given: A ΔABC right-angled at B, and D is the mid-point of BC, i.e. BD = CD

To Prove: $AC^2 = (4AD^2 - 3AB^2)$

Proof:

In ΔABD,

By Pythagoras theorem, [i.e. Hypotenuse^2 = Base^2+ Perpendicular^2]

$AD^2 = AB^2 + BD^2$

[as D is mid-point of BC, therefore, $BC = \frac{1}{2}BD$]

$\Rightarrow AD^2 = AB^2 + \left(\frac{1}{2}BC\right)^2 = AB^2 + \frac{BD^2}{4}$

$\Rightarrow 4AD^2 = 4AB^2 + BC^2$

$\Rightarrow BC^2 = 4AD^2 - 4AB^2$ [1]

Now, In ΔABC, again By Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = AB^2 + 4AD^2 - 4AB^2$ [From 1]

$AC^2 = 4AD^2 - 3AB^2$

Hence Proved !

Question: 34

Find the mean, mo

Solution:

Let us make the table for above data and containing cumulative frequency and mid-values for each data

Class	Frequency(f_i)	Mid-values (x_i)	f_ix_i	Cumulative Frequency (cf)
0-10	5	5	25	5
10-20	10	15	150	15
20-30	18	25	450	33
30-40	30	35	1050	63
40-50	20	45	900	83
50-60	12	55	660	95
60-70	5	65	325	100
	$\sum f_i = 100$		$\sum f_ix_i = 3560$	

MEAN

We know, that

$$\text{mean}(\bar{x}) = \frac{\sum f_ix_i}{\sum f_i}$$

$$\Rightarrow \bar{x} = \frac{3560}{100}$$

$$\Rightarrow \bar{x} = 35.6$$

MODE

In the given data,

The maximum class frequency is 30. So, the modal class is 30-40.

Lower limit(l) of modal class = 30

Class size(h) = 40 - 30 = 10

Frequency(f_1) of modal class = 30

Frequency(f_0) of class preceding the modal class = 18

Frequency(f_2) of class succeeding the modal class = 20

And we know,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Substituting values, we get

$$\text{Mode} = 30 + \left(\frac{30 - 18}{2(30) - 18 - 20}\right)(10) = 30 + \frac{120}{22}$$

$$\Rightarrow \text{Mode} = 30 + \frac{60}{11} = 30 + 5.45 = 35.45$$

MEDIAN

In this case,

Sum of all frequencies, n = 100

$$\Rightarrow \frac{n}{2} = \frac{100}{2} = 50$$

Now, we know the median class is whose cumulative frequency is greater than and nearest to $\frac{n}{2}$.

As, Cumulative frequency greater than and nearest to 50 is 63, the median class is 30 - 40.

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

where l = lower limit of median class,

n = number of observations,

cf = cumulative frequency of class preceding the median class,

f = frequency of median class,

h = class size

In this case,

l = 30

n = 100

cf = 33

f = 30

h = 10

Putting values, we get,

$$\text{Median} = 30 + \left(\frac{50-33}{30}\right)(10)$$

$$= 30 + \frac{17}{3} = 30 + 5.67 = 35.67$$

Exercise : SAMPLE PAPER II

Question: 1

What is the large

Solution:

We know that Dividend = Divisor × Quotient + Remainder

According to the problem :

Dividend 1 = 245

Dividend 2 = 1029

Dividend - Remainder = Divisor × Quotient

So Dividend 1- Remainder = 240 = Divisor × Quotient 1

Prime Factor of 240 = $2^4 \times 3 \times 5$

Dividend 2 – Remainder = 1024 = Divisor × Quotient 2

Prime Factor of 1024 = $2^4 \times 2^6$

Since, the Divisor is common for both the numbers we need to find the Highest Common Factor between both the numbers. From the Prime factors, we find the

<u>Highest Common Factor between the two numbers is 2^4 = 16</u>

Question: 2

If the product of

Solution:

Given Equation :$ax^2 - 6x - 6 = 0$

which is of the form $ax^2 + bx + c = 0$ (General Form)

The product of the roots of the general form of equation = $\frac{c}{a}$

So according to the given Equation Product of the roots = $-\frac{6}{a}$

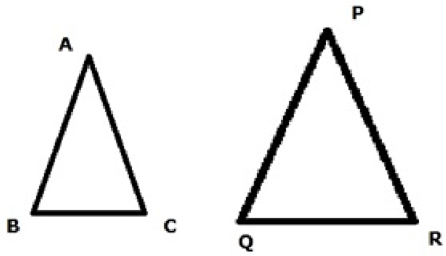
$$\Rightarrow -\frac{6}{a} = 4$$

The Value Of a for which the equation has product of root 4 = $a = -\frac{3}{2}$

Question: 3

The areas of two

Solution:



Given :

Area of ΔABC = 25 cm^2

Area of ΔPQR = 49 cm^2

Length of QR = 9.8 cm.

Since both the triangles are similar so according to the Area –Length relations of similar triangle we can write

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{BC^2}{QR^2}$$

$$\frac{25}{49} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \frac{BC}{QR} = \sqrt{\frac{25}{49}}$$

$$\Rightarrow BC = \frac{5 \times 9.8}{7}$$

The length Of The side BC is 7 cm.

Question: 4

If sin (θ + 34°)

Solution:

Given sin $(\theta + 34^0) = \cos\theta$ ...Equation 1

Since sin θ & cos θ are complementary to each other

so $\sin\theta = \cos(90^0 - \theta)$

Using the above relations in Equation 1 we get

$\cos(90^0 - \theta - 34^0) = \cos\theta$

Since both L.H.S. and R.H.S. are functions of cosine and $\theta + 34^0$ is acute so we can write

$90^0 - \theta - 34^0 = \theta$

$\Rightarrow 2\theta = 56^0$

$\Rightarrow \theta = 28^0$

Question: 5

lf cos θ = 0.6, t

Solution:

Given cos θ = 0.6

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

$\Rightarrow \sin\theta = 0.8$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \tan\theta = \frac{4}{3}$$

According to the question, the required problem needs us to find

5 sin θ- 3 tan θ

$$\Rightarrow 5 \times 0.8 - 3 \times \frac{4}{3}$$

The value of the expression is 0

Question: 6

The simplest form

Solution:

Prime factorization of 1095 = 5×3×73

Prime factorization of 1168 = $2^4 \times 73$

So $\frac{1095}{1168} = \frac{5 \times 3 \times 73}{2^4 \times 73}$

Since 73 is a common factor for both numerator and denominator so it cancels out

The Simplest form is $\frac{15}{16}$

Question: 7

The pair of linea

Solution:

Equation 1: 4x - 5y = 20

Equation 2: 3x + 5y = 15

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

According to the problem:

$a_1 = 4$

$a_2 = 3$

$b_1 = -5$

$b_2 = 5$

$c_1 = 20$

$c_2 = 15$

We compare the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2} \& \frac{c_1}{c_2}$

$$\frac{a_1}{a_2} = \frac{4}{3}$$

$$\frac{b_1}{b_2} = \frac{-1}{1}$$

$$\frac{c_1}{c_2} = \frac{4}{3}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, So

It has a Unique solution

Question: 8

If mode = x(media

Solution:

Given: mode = x(median) - y(mean)

According to an empirical relation, the relation between Mean, Median & Mode is given by

Mode = 3 Median – 2 Mean ...Eq(1)

This empirical relation is very much close to the actual value of mode which is calculated. So this relation is valid.

Comparing the Relation given with equation 1 we find

x = 3 & y = 2

Question: 9

Check whether 6"

Solution:

When a number ends with 0 it has to be divisible by the factors of 10 which are 5 and 2

Now $6^n = (3 \times 2)^n$ ...Equation 1

From Equation 1 We can see the factors of 6 are only 3 & 2.

There are no factors as powers of 5 in the factorization of 6

Hence 6^n cannot end with 0

Question: 10

Find the zeros of

Solution:

Given Equation : $9x^2 - 5 = 0$

which is of the form $ax^2 + bx + c = 0$ (General Form)

For finding the zeroes of the polynomial we use the method of Factorization

$9x^2 - 5 = 0$

$\Rightarrow 9x^2 = 5$

$\Rightarrow x^2 = \frac{5}{9}$

$\Rightarrow x = \pm\frac{\sqrt{5}}{3}$

The zeroes of the polynomial expression are $\frac{\sqrt{5}}{3}$ & $-\frac{\sqrt{5}}{3}$

Question: 11 A

If $2 \sin 2\theta = \sqrt{3}$

Solution:

Given $2 \sin 2\theta = \sqrt{3}$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin 2\theta = \sin 60^0$

$\Rightarrow 2\theta = 60^0$

$\Rightarrow \theta = 30^0$

Question: 11 B

If $7 \sin^2$

Solution:

Given: $7 \sin^2 \theta + 3 \cos^2 \theta = 4$

Since $\sin^2 \theta + \cos^2 \theta = 1$ ...Equation 1

So the equation becomes

$4 \sin^2 \theta = 1$

$\Rightarrow \sin^2 \theta = \frac{1}{4}$

From Equation 1 we get

$\cos^2 \theta = \frac{3}{4}$

Since $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

Hence Proved

Question: 12

In ΔABC, D and E

Solution:

Given :

AD = 5 cm

DB = 8 cm

AC = 6.5 cm

DE ||BC

In ΔABC & ΔADE

∠ADE = ∠ABC (Corresponding Angles)

∠AED = ∠ACB (Corresponding Angles)

So ΔABC & ΔADE are similar by the A.A. (Angle-Angle) axiom of Similarity

AB = AD + BD = 13 cm.

Since the two triangles are similar so their lengths of sides must be in proportion.

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow AE = \frac{6.5\times5}{13}$

AE = 2.5cm.

Question: 13

D is a point on t

Solution:

Given:

∠ADC = ∠BAC

D is a point on the side BC

∠ACB = ∠ACD (Common Angle)

So ΔABC & ΔADC are similar by the A.A. (Angle-Angle) axiom of Similarity

Since the two triangles are similar so their lengths of sides must be in proportion

$\frac{CB}{CA} = \frac{CA}{DC}$

Cross Multiplying We Get

$CA^2 = DC \times CB$

Which is the required expression

Hence Proved

Question: 14

Calculate the mod

Solution:

Class corresponding to maximum frequency = (4-8)

f_1 (Frequency of the modal class) = 8

f_0 (Frequency of the class preceding the modal class) = 4

f_2 (Frequency of the succeeding modal class) = 5

l(lower limit) = 4

h(width of class) = 4

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\Rightarrow$ Mode = $4 + \left(\frac{8-4}{2\times 8-4-5}\right) \times 4$

Mode = 6.29

Question: 15

Show that any pos

Solution:

According to Euclid's algorithm p = 6q + r

where r is any whole number 0< = r<6 and p is a positive integer

Since 6q is divisible by 2 so the value of r will decide whether it is odd or even.

Also since r<6 so only 6 cases are possible

For r = 1 , 3, 5 we get three odd numbers and for r = 0 , 2 , 4 we get three even numbers

So (6q + 1) , (6q + 3) & (6q + 5) represents positive odd integers .

Hence Proved

Question: 16 A

Prove that (3 -√1

Solution:

Let us assume (3 -√15) is rational

$3 - \sqrt{15} = \frac{a}{b}$ (Assume)

where a & b are integers (b≠0)

$\Rightarrow 3 - \frac{a}{b} = \sqrt{15}$

$\Rightarrow \frac{3b-a}{b} = \sqrt{15}$

Now let's solve the R.H.S. Of the above equation

Let $\sqrt{15} = \frac{p}{q}$

Squaring we get

$15 = \frac{p^2}{q^2}$

$15q^2 = p^2$

In The above equation since 15 divides p^2 so it must also divide p so p is a multiple of 15

let p = 15k where k is an integer

Putting in Equation 1 the value of p we get

$15q^2 = 225k^2$

$\Rightarrow q^2 = 15k^2$

Since 15 divides q^2 so it must also divide q

so q is a multiple of 15

But this contradicts our previously assumed data since we had considered p & q has been resolved in their simplest form and they shouldn't have any common factors.

So √15 is irrational and hence

(3 -√15) is also irrational

<u>Hence Proved</u>

Question: 16 B

Prove that

Solution:

Let us consider $\frac{2\sqrt{2}}{3}$ to be rational

$\frac{2\sqrt{2}}{3} = \frac{a}{b}$ where a & b are integers (b≠0)

Rearranging we get

$$\sqrt{2} = \frac{3a}{2b}$$

The R.H.S of the above expression is a rational number since it can be expressed as a numerator by a denominator

Let L.H.S = $\frac{p}{q}$ where p and q are integers (q≠0)

$\Rightarrow \sqrt{2} = \frac{p}{q}$

$\Rightarrow q\sqrt{2} = p$

Squaring both sides we get

$2q^2 = p^2$...Equation 1

Since 2 divides p^2 so it must also divide p

so p is a multiple of 2

let p = 2k where k is an integer

Putting in Equation 1 the value of p we get

$2q^2 = 4k^2$

$\Rightarrow q^2 = 2k^2$

Since 2 divides q^2 so it must also divide q

so q is a multiple of 2

But this contradicts our previously assumed data since we had considered p & q has been resolved in their simplest form and they shouldn't have any common factors.

So √2 is irrational and hence

$\frac{2\sqrt{2}}{3}$ is also irrational

Hence Proved

Question: 17 A

What number must

Solution:

Let the number added to each of the numbers to make them in proportion be x

When any four numbers (a, b, c, d)are in proportion then

$$\frac{a}{b} = \frac{c}{d}$$

Applying the above equation for our problem we get

$$\frac{5 + x}{9 + x} = \frac{17 + x}{27 + x}$$

$\Rightarrow (5 + x)(27 + x) = (17 + x)(9 + x)$

$\Rightarrow 135 + 32x + x^2 = 153 + 26x + x^2$

$\Rightarrow 6x = 18$

The number added should be 3

Question: 17 B

The sum of two nu

Solution:

Let the two numbers be x & Y

x + y = 18 (Given) …Equation 1

$\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$ (Given) …Equation 2

Solving Equation 2 We get

$$\Rightarrow \frac{x + y}{xy} = \frac{1}{4}$$

Putting the value from Equation 1 we get

$\Rightarrow xy = 72$

$\Rightarrow y = \frac{72}{x}$ …Equation 3

Putting the value of Equation 3 in Equation 1 We get

$$\Rightarrow x + \frac{72}{x} = 18$$

$\Rightarrow x^2 + 72 = 18x$

$\Rightarrow x^2 - 18x + 72 = 0$

$\Rightarrow (x-6)^2 = 0$

$\Rightarrow x = 6$

Putting the value of x in Equation 1 we get y = 12

The two numbers are 6 & 12

Question: 18

If α, β are the z

Solution:

Given Equation : $x^2 - x - 12 = 0$

which is of the form $ax^2 + bx + c = 0$ (General Form)

The product of the roots of the general form of equation = $\frac{c}{a}$

Sum of Roots of the general equation = $-\frac{b}{a}$

So $\alpha + \beta = -\frac{b}{a}$

⇒ $\alpha + \beta = 1$

⇒ $2(\alpha + \beta) = 2$ ....Equation 1

Similarly

$\alpha\times\beta = -12$

⇒ $2\alpha\times2\beta = -48$ ...Equation 2

The new equation will be formed by combining the results of Equation 1 & 2

The New Polynomial Formed from the new roots is $x^2 - 2x - 48$

Question: 19

Prove that (sin θ

Solution:

Given L.H.S. = $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

We know

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

⇒ $\sin^2\theta + \operatorname{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$

Also From the Trigonometrical identities

$\sin^2\theta + \cos^2\theta = 1$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$

⇒ $1 + 1 + \cot^2\theta + 2 + 1 + \tan^2\theta + 2$

⇒ $7 + \cot^2\theta + \tan^2\theta$

So, L.H.S = R.H.S

Hence Proved

Question: 20

If sec θ + tan θ

Solution:

Given $\sec\theta + \tan\theta = m$

$$\sec\theta = \frac{1}{\cos\theta} \;\&\; \frac{\sin\theta}{\cos\theta} = \tan\theta$$

So, we can write

$$\frac{1 + \sin\theta}{\cos\theta} = m$$

Squaring both sides we get

$$\frac{(1 + \sin\theta)^2}{\cos^2\theta} = m^2$$

Since $\cos^2\theta = 1 - \sin^2\theta$

$$\Rightarrow \frac{(1 + \sin\theta)^2}{1 - \sin^2\theta} = m^2$$

$$\Rightarrow \frac{1 + \sin^2\theta + 2\sin\theta}{1 - \sin^2\theta} = m^2$$

Applying Componendo & Dividendo i.e.

$$\frac{a}{b} = \frac{c}{d}$$

is equivalent to $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

we get

$$\Rightarrow \frac{\sin^2\theta + \sin\theta}{1 + \sin\theta} = \frac{m^2 - 1}{m^2 + 1}$$

$$\Rightarrow \frac{\sin\theta\,(1 + \sin\theta)}{1 + \sin\theta} = \frac{m^2 - 1}{m^2 + 1}$$

$$\Rightarrow \sin\theta = \frac{m^2 - 1}{m^2 + 1}$$

Hence Proved

Question: 21

In a trapezium AB

Solution:

Given :

AB|| CD

AB = 2 x CD

$$\Rightarrow \frac{AB}{CD} = 2$$

∠AOB = ∠COD (Vertically Opposite angles)

∠DCO = ∠OAB (Alternate Angles)

So ΔAOB & ΔDOC are similar by the A.A. (Angle Angle) axiom of Similarity

Since both the triangles are similar so according to the Area –Length relations of similar triangle we can write

$$\frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta DOC} = \frac{AB^2}{CD^2}$$

$$\Rightarrow \frac{84}{\text{Area of } \Delta DOC} = 4$$

Area of ΔDOC = 21cm^2

Question: 22

In the given figu

Solution:

Given:

AB ⊥ BC

GF ⊥ BC

DE ⊥ AC

Since AB ⊥ BC so ∠DAE & ∠GCF are complementary angles i.e.

∠DAE + ∠GCF = 90^0 ....Equation 1

Similarly since GF ⊥ BC so ∠CFG & ∠GCF are complementary angles i.e.

∠CGF + ∠GCF = 90^0 ....Equation 2

Combining Equation 1 & 2 We can say that

∠CGF = ∠DAE

Also ∠CFG = ∠DEA (Perpendicular Angles)

So ΔCGF is similar to ΔADE By A.A. (Angle Angle)axiom of similarity

Hence Proved

Question: 23 A

Find the mean of

Solution:

Class	Frequency(f_i)	Class Mark(x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0-10	7	5	-2	-14
10-20	12	15	-1	-12
20-30	13	25	0	0
30-40	10	35	1	10
40-50	8	45	2	16
	$\Sigma f_i = 50$			$\Sigma f_i u_i = 0$

h (Represents the class width) = 10

a (Assumed mean) = 25

So Mean according to Step Deviation method:

$$\text{Mean} = a + h \times \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\Rightarrow 25 + \frac{10 \times 0}{50}$$

Mean = 25

Question: 23 B

The mean of the f

Solution:

Class Interval	Frequency(f_i)	Class Mark(x_i)	$f_i x_i$
50-60	8	55	440
60-70	6	65	390
70-80	12	75	900
80-90	11	85	935
90-100	p	95	95p
	$\Sigma f_i = 37 + p$		$\Sigma f_i x_i = 2665 + 95p$

Mean = 78 (Given)

According to the direct method

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow 78 = \frac{2665 + 95p}{37 + p}$

$\Rightarrow 2886 + 78p = 2665 + 95p$

$\Rightarrow 17p = 221$

Value of p is 13

Question: 24

Find the median o

Solution:

Weight (in kg)	Number of students	Weight Less than(Kg)	Cumulative Frequency
40-45	2	45	2
45-50	3	50	5
50-55	8	55	13
55-60	6	60	19
60-65	6	65	25
65-70	3	70	28
70-75	2	75	30

Total frequency(n) = 30

$\frac{n}{2} = 15$

15 lies in the interval 55-60

so l (lower limit) = 55

c_f(Cumulative frequency of the preceding class of median class) = 13

f (frequency of median class) = 6

h (class size) = 5

Median = $l + \left(\frac{\frac{n}{2} - c_f}{f}\right) \times h$

Median = $55 + \frac{15-13}{6} \times 5$

Median = 56.67Kg

Question: 25

If two zeroes of

Solution:

Given: $p(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$

Since $x = \sqrt{2}$ & $-\sqrt{2}$ is a solution so

$x - \sqrt{2}$ & $x + \sqrt{2}$ are two factors of p(x)

Multiplying the two factors we get $x^2 - 2$ ...Equation 1

which is also a factor of p(x)

To get the other two factors we need to perform long division

On performing long division we will get

$2x^2 + 7x - 15$ ...Equation 2

Equation 2 is also a factor of p(x)

To find the other two zeroes of the polynomial we need to solve Equation 2

We use the method of factorization for solving Equation 2

$2x^2 + 7x - 15 = 0$

$\Rightarrow 2x^2 + 10x - 3x - 15 = 0$

$\Rightarrow 2x(x + 5) - 3(x + 5) = 0$

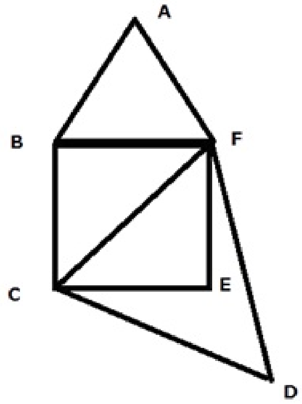
$\Rightarrow (2x-3)(x + 5) = 0$

The two roots are $\frac{3}{2}$ and -5

Question: 26 A

Prove that the ar

Solution:



Let us assume BFEC is a square , ΔABF is an equilateral triangle described on the side of the square & Δ CFD is an equilateral triangle describes on diagonal of the square

Now since ΔABF & Δ CFD are equilateral so they are similar

Let side CE = a,

So EF = a

$CF^2 = a^2 + a^2$

$CF^2 = 2a^2$

Since both the triangles are similar so according to the Area –Length relations of similar triangle we can write

$$\frac{\text{Area of }\Delta AFB}{\text{Area of }\Delta DFC} = \frac{BF^2}{CF^2}$$

$\Rightarrow \frac{\text{Area of } \Delta AFB}{\text{Area of } \Delta DFC} = \frac{1}{2}$

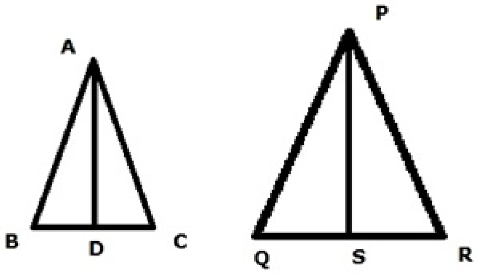
So Area Of Δ CFD = 2 ΔABF

Hence Proved

Question: 26 B

Prove that the ra

Solution:



Let us assume ΔABC & ΔPQR are similar

Area of ΔABC = 0.5 ×AD ×BC

Area of ΔPQR = 0.5 ×PS ×QR

Now since the two triangles are similar so the length of sides and perpendiculars will also be in proportion

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RS} = \frac{AD}{PS}$ ...Equation 1

$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{0.5 \times AD \times BC}{0.5 \times PS \times QR}$ ...Equation 2

From Equation 1 We get

$$\frac{AD}{PS} = \frac{BC}{QR}$$

Putting in Equation 2 we get

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{0.5 \times BC \times BC}{0.5 \times QR \times QR}$$

$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{BC^2}{QR^2}$

So we can see ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides

Hence Proved

Question: 27 A

Prove that:

Solution:

Given: L.H.S. $= \frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1}$

Since we know $\sec\theta = \frac{1}{\cos\theta}$ & $\tan\theta = \frac{\sin\theta}{\cos\theta}$

So L.H.S. $= \frac{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta} + 1} = \frac{1 + \sin\theta - \cos\theta}{\sin\theta - 1 + \cos\theta}$

Multiplying Numerator & Denominator with sin θ - (1 - cos θ) we get

L.H.S. $=\frac{\sin^2\theta-(1-\cos\theta)^2}{(\sin\theta-1+\cos\theta)^2}$

$= \frac{\sin^2\theta-1+2\cos\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta+1-2\sin\theta-2\cos\theta+2\sin\theta\cos\theta}$

Since $\sin^2\theta + \cos^2\theta = 1$

$= \frac{-\cos^2\theta+2\cos\theta-\cos^2\theta}{1+1-2\sin\theta-2\cos\theta+2\sin\theta\cos\theta}$ Taking 2 common out of numerator and denominator

$= \frac{\cos\theta-\cos^2\theta}{1-\sin\theta-\cos\theta+\sin\theta\cos\theta}$

$= \frac{\cos\theta(1-\cos\theta)}{1(1-\sin\theta)-\cos\theta(1-\sin\theta)}$

$= \frac{\cos\theta(1-\cos\theta)}{(1-\sin\theta)(1-\cos\theta)}$

$= \frac{\cos\theta}{(1-\sin\theta)}$

L.H.S. = R.H.S.

Hence Proved

Question: 27 B

Evaluate:

Solution:

Given: $\frac{\sec\theta\ \mathrm{cosec}(90°-\theta)-\tan\theta\ \cot(90°-\theta)+\sin^2 55°+\sin^2 35°}{\tan\ 10°\ \tan\ 20°\ \tan\ 60°\ \tan\ 70°\ \tan\ 80°}$ ...Equation 1

We know

$\sec\theta = \mathrm{cosec}\ (90^0 - \theta)$

$\tan\theta = \cot\ (90^0 - \theta)$

$\sin\theta = \cos\ (90^0 - \theta)$

Using the above three relations in Equation 1 we get

$\frac{\sec^2\theta\ -\ \tan^2\theta\ +\ \sin^2 55^0\ +\ \cos^2 55^0}{\tan 10^0 \tan 20^0 \tan 60^0 \cot 20^0 \cot 10^0}$

We also know

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2 55 + \cos^2 55 = 1$

And, $\tan\theta\ =\ \frac{1}{\cot\theta}$

$\therefore \tan 10°\ =\ \frac{1}{\cot 20°}$

$\Rightarrow \frac{1+1}{\tan 10^0 \times \tan 20° \times \tan 60^0 \times \frac{1}{\tan(10^0)} \times \frac{1}{\tan(20^0)}}$

$\Rightarrow \frac{2}{\tan 60°}$

$\Rightarrow \frac{2}{\sqrt{3}}$

$\frac{2}{\sqrt{3}}$

Question: 28

If sec θ + tan θ

Solution:

Given sec θ + tan θ = m

$\sec\theta = \frac{1}{\cos\theta}$ & $\frac{\sin\theta}{\cos\theta} = \tan\theta$ So we can write

$$\frac{1 + \sin\theta}{\cos\theta} = m$$

Squaring both sides we get

$$\frac{(1 + \sin\theta)^2}{\cos^2\theta} = m^2$$

Since $\cos^2\theta = 1 - \sin^2\theta$

$$\Rightarrow \frac{(1+\sin\theta)^2}{1-\sin^2\theta} = m^2$$

$$\Rightarrow \frac{1+\sin^2\theta+2\sin\theta}{1-\sin^2\theta} = m^2$$

Applying Componendo & Dividendo i.e.

$$\frac{a}{b} = \frac{c}{d}$$

is equivalent to $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

we get

$$\Rightarrow \frac{\sin^2\theta + \sin\theta}{1 + \sin\theta} = \frac{m^2 - 1}{m^2 + 1}$$

$$\Rightarrow \frac{\sin\theta\,(1 + \sin\theta)}{1 + \sin\theta} = \frac{m^2 - 1}{m^2 + 1}$$

$$\Rightarrow \sin\theta = \frac{m^2 - 1}{m^2 + 1}$$

Hence Proved

Question: 29

Draw the graph of

Solution:

Given: The equations 3x + y- 11 = 0 and x-y- 1 = 0.**To find:** the region bounded by these lines and the y-axis.**Solution:**For 3x + y - 11 = 0y = 11 - 3xNow for x = 0 y = 11 - 3(0)y = 11For x = 3y = 11 - 3(3)y = 11 - 9y = 2Table for equation 3x + y -11 = 0 is

x	0	3
y	11	2

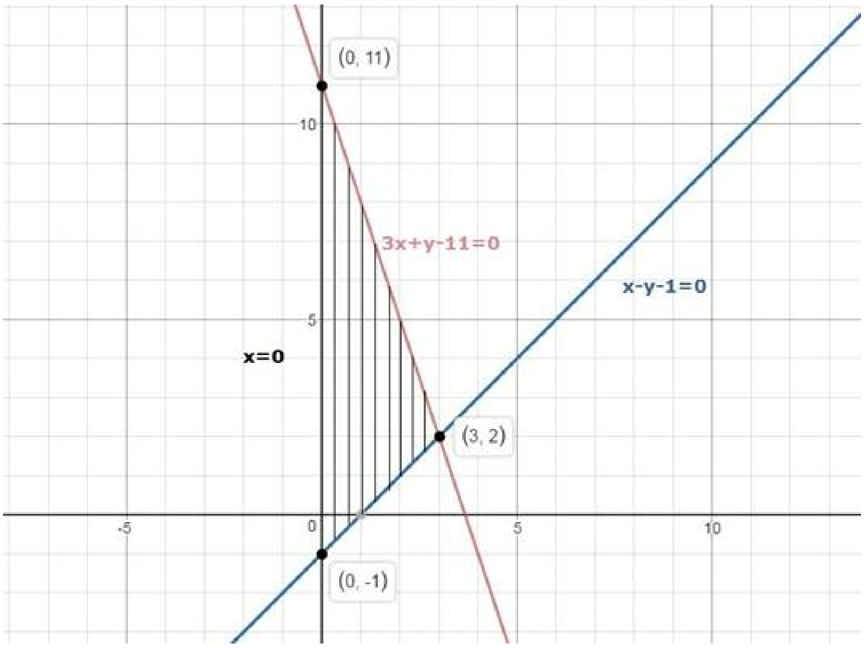
Plot the points (0,11),(3,2)

For x-y- 1 = 0y = x - 1

Now for x = 0 y = 0 - 1y = -1For x = 3y = 3 - 1y = 2Table for equation x-y- 1 = 0 is

x	0	3
y	-1	2

Plot the points (0,-1),(3,2)The graph is shown below:

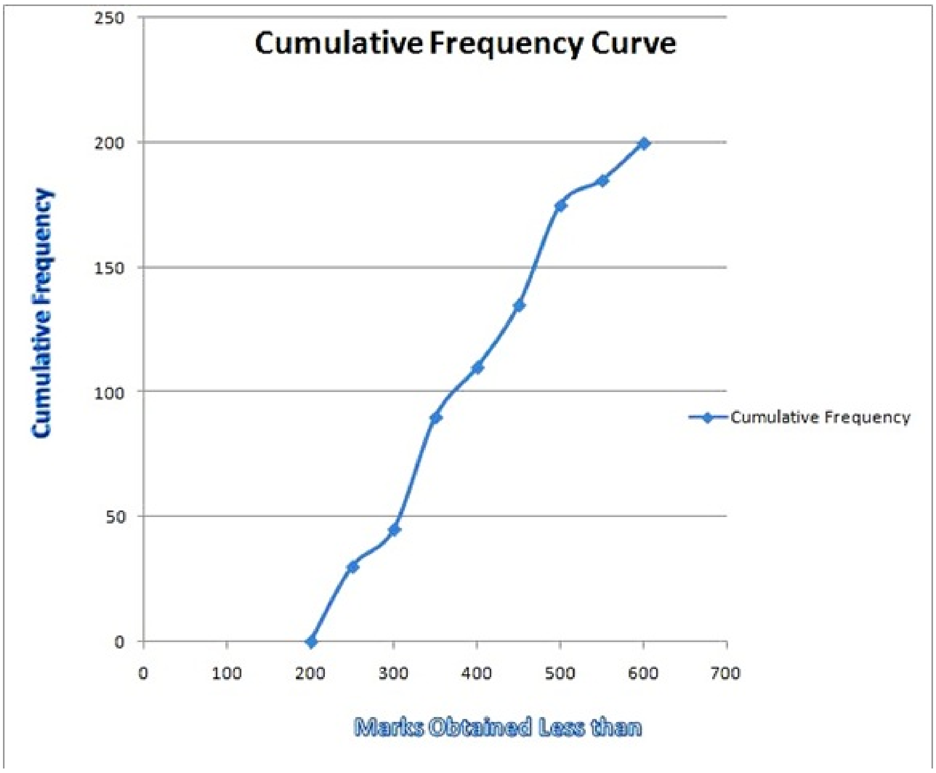


Question: 30

The table given b

Solution:

Scores	No. Of Candidates	Score Less Than	Cumulative Frequency
200-250	30	250	30
250-300	15	300	45
300-350	45	350	90
350-400	20	400	110
400-450	25	450	135
450-500	40	500	175
500-550	10	550	185
550-600	15	600	200



Question: 31

For what value of

Solution:

Given:

Equation 1: 2x - 3y = 7

Equation 2: (k + 1)x + (1 - 2k)y = (5k - 4)

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

For the system of linear equations to have infinitely many solutions, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 2$

$a_2 = k + 1$

$b_1 = -3$

$b_2 = 1-2k$

$c_1 = 7$

$c_2 = 5k-4$

Putting the above values in equation (i) we get:

$$\mathbf{\frac{2}{k + 1} = \frac{-3}{1-2k}}$$

⇒ 2(1-2k) = -3(k + 1)
⇒ 2-4k = -3k-3
⇒ k = 5
The value of k for which the system of equations has infinitely many solutions is k = 5

Question: 32

Prove that: (sin

Solution:

To Prove: $(\sin\theta - \operatorname{cosec}\theta)(\cos\theta - \sec\theta) = \frac{1}{(\tan\theta + \cot\theta)}$

L.H.S. = (sin θ –cosec θ)(cos θ- sec θ)

$\Rightarrow (\sin\theta - \frac{1}{\sin\theta}) \times (\cos\theta - \frac{1}{\cos\theta})$

$\Rightarrow \frac{(\sin^2\theta - 1)}{\sin\theta} \times \frac{(\cos^2\theta - 1)}{\cos\theta}$

Since $\sin^2\theta + \cos^2\theta = 1$, So

$\Rightarrow \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta}$

After Cancellation we get

L.H.S. = sin θ cos θ

Dividing the numerator and denominator with cos θ we get

$\Rightarrow \frac{\sin\theta}{\cos\theta} \times \cos^2\theta$

We know $\frac{\sin\theta}{\cos\theta} = \tan\theta$ & $\cos^2\theta = \frac{1}{\sec^2\theta}$

$\Rightarrow \frac{\tan\theta}{\sec^2\theta}$

Since $\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \frac{\tan\theta}{1 + \tan^2\theta}$

Dividing The Numerator and denominator by tan θ we get

$\Rightarrow \frac{1}{\frac{1}{\tan\theta} + \tan\theta}$

Since $\frac{1}{\tan\theta} = \cot\theta$

$\Rightarrow \frac{1}{\cot\theta + \tan\theta}$ = R.H.S

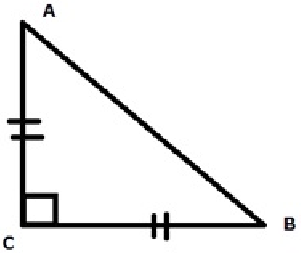
Since L.H.S. = R.H.S

Hence Proved

Question: 33

ΔABC is an isosce

Solution:



Given:

AC = BC

$AB^2 = 2AC^2$ ...(Equation 1)

Equation 1 can be rewritten as

$AB^2 = AC^2 + AC^2$

Since AC = BC we can write

$AB^2 = AC^2 + BC^2$ ...Equation 2

Equation 2 represents the Pythagoras theorem which states that

$Hypotenuse^2 = Base^2 + Perpendicular^2$

Since Pythagoras theorem is valid only for right-angled triangle so

So $\triangle$ ABC is a right angled triangle right angled at C

Hence Proved

Question: 34

The table given b

Solution:

Daily Expenditure (Rs.)	Number of households (f_i)	Class Mark (x_i)	$f_i x_i$	Daily expenditure Less than(Rs.)	Cumulative frequency
100-150	6	125	750	150	6
150-200	7	175	1225	200	13
200-250	12	225	2700	250	25
250-300	3	275	825	300	28
300-350	2	325	650	350	30
	$\Sigma f_i = 30$		$\Sigma f_i x_i = 6150$		

According to the direct method

Mean = $\frac{\sum f_i x_i}{\sum f_i}$

⇒ Mean = $\frac{6150}{30}$

⇒ Mean = 205

Total frequency(n) = 30

$\frac{n}{2} = 15$

15 lies in the interval 200-250

so l (lower limit) = 200

c_f(Cumulative frequency of the preceding class 200-250) = 13

f (frequency of median class) = 12

h (class size) = 50

Median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

Median = $200 + \frac{15-13}{12} \times 50$

Median = 208.33