

4. ELLIPSE

1. **Standard Equation :** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$ & $b^2 = a^2(1 - e^2)$.

Eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}}$, ($0 < e < 1$), **Directrices :** $x = \pm \frac{a}{e}$.

Foci : $S \equiv (\pm ae, 0)$. Length of, major axes = $2a$ and minor axes = $2b$

Vertices : $A' \equiv (-a, 0)$ & $A \equiv (a, 0)$.

Latus Rectum : $= \frac{2b^2}{a} = 2a(1 - e^2)$

2. **Auxiliary Circle :** $x^2 + y^2 = a^2$
 3. **Parametric Representation :** $x = a \cos \theta$ & $y = b \sin \theta$
 4. **Position of a Point w.r.t. an Ellipse:**

The point $P(x_1, y_1)$ lies outside, inside or on the ellipse according as ; $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 > < \text{ or } = 0$.

5. **Line and an Ellipse:** The line $y = mx + c$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in two points real, coincident or imaginary according as c^2 is $< = \text{ or } > a^2m^2 + b^2$.

6. **Tangents:** Slope form: $y = mx \pm \sqrt{a^2m^2 + b^2}$, Point form : $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$,

Parametric form: $\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1$

7. **Normals:** $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$, $ax \sec \theta - by \operatorname{cosec} \theta = (a^2 - b^2)$, $y = mx - \frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}$.

8. **Director Circle:** $x^2 + y^2 = a^2 + b^2$