CHAPTER 2

Algebraic Expression & Inequalities

VARIABLE

An unknown quantity used in any equation may be constant known as variable. Variables are generally denoted by the last English alphabet x, y, z etc.
An equation is a statement of equality of two algebraic expressions, which involve one or more variables.

LINEAR EQUATION

An equation in which the highest power of variables is one, is called a linear equation. These equations are called linear because the graph of such equations on the x–y cartesian plane is a straight line.

Linear Equation in one variable

A linear equation which contains only one variable is called **linear equation in one variable**.
The general form of such equations is $ax + b = c$, where a, b and c are constants and $a \neq 0$.
All the values of x which satisfy this equation are called its solution(s).

NOTE : *An equation satisfied by all values of the variable is called an identity. For example : $2x + x = 3x$.*

EXAMPLE 1. ***Solve*** **$2x - 5 = 1$**

Sol. $2x - 5 = 1$

$\Rightarrow \quad 2x = 1 + 5$

$\Rightarrow \quad 2x = 6 \Rightarrow x = \dfrac{6}{2} = 3.$

EXAMPLE 2. ***Solve*** **$7x - 5 = 4x + 11$**

Sol. $7x - 5 = 4x + 11$

$\Rightarrow \quad 7x - 4x = 11 + 5$ (Bringing like terms together)

$\Rightarrow \quad 3x = 16 \Rightarrow \quad x = \dfrac{16}{3} = 5\dfrac{1}{3}.$

EXAMPLE 3. ***Solve*** **$\dfrac{4}{x} - \dfrac{3}{2x} = 5$**

Sol. $\dfrac{4}{x} - \dfrac{3}{2x} = 5 \Rightarrow \dfrac{8-3}{2x} = 5$

$\Rightarrow \quad \dfrac{5}{2x} = 5 \Rightarrow 10x = 5$

$\Rightarrow \quad x = \dfrac{5}{10} = \dfrac{1}{2}$

Application of linear equations with one variables

EXAMPLE 4. The sum of the digits of a two digit number is 16. If the number formed by reversing the digits is less than the original number by 18. Find the original number.

Sol. Let unit digit be x.

Then tens digit $= 16 - x$

$\therefore \quad$ Original number $= 10 \times (16 - x) + x$
$= 160 - 9x.$

On reversing the digits, we have x at the tens place and $(16 - x)$ at the unit place.

$\therefore \quad$ New number $= 10x + (16 - x) = 9x + 16$

Original number – New number = 18

$(160 - 9x) - (9x + 16) = 18$

$160 - 18x - 16 = 18$

$-18x + 144 = 18$

$-18x = 18 - 144 \quad \Rightarrow 18x = 126$

$\Rightarrow x = 7$

$\therefore$ In the original number, we have unit digit = 7

Ten's digit $= (16 - 7) = 9$

Thus, original number = 97

EXAMPLE 5. The denominator of a rational number is greater than its numerator by 4. If 4 is subtracted from the numerator and 2 is added to its denominator, the new number becomes $\dfrac{1}{6}$. Find the original number.

Sol. Let the numerator be x.
Then, denominator $= x + 4$

$\therefore \quad \dfrac{x - 4}{x + 4 + 2} = \dfrac{1}{6}$

$\Rightarrow \quad \dfrac{x - 4}{x + 6} = \dfrac{1}{6}$

$\Rightarrow \quad 6(x - 4) = x + 6$

$\Rightarrow \quad 6x - 24 = x + 6 \Rightarrow \quad 5x = 30$

$\therefore \quad x = 6$

Thus, Numerator = 6, Denominator = 6 + 4 = 10.

Hence the original number $= \dfrac{6}{10}$.

EXAMPLE 6. A man covers a distance of 33 km in $3\frac{1}{2}$ hours; partly on foot at the rate of 4 km/hr and partly on bicycle at the rate of 10 km/hr. Find the distance covered on foot.

Sol. Let the distance covered on foot be x km.

$\therefore$ Distance covered on bicycle = (33 – x) km

$\therefore$ Time taken on foot $= \frac{\text{Distance}}{\text{Speed}} = \frac{x}{4}$ hr.

$\therefore$ Time taken on bicycle $= \frac{33-x}{10}$ hr.

The total time taken $= \frac{7}{2}$ hr.

$$\frac{x}{4} + \frac{33-x}{10} = \frac{7}{2}$$

$$\frac{5x+66-2x}{20} = \frac{7}{2}$$

$6x + 132 = 140$
$6x = 140 - 132$
$6x = 8$

$x = \frac{8}{6} = 1.33$ km.

$\therefore$ The distance covered on foot is 1.33 km.

Linear equation in two variables

General equation of a linear equation in two variables is $ax + by + c = 0$, where a, $b \neq 0$ and c is a constant, and x and y are the two variables.

The sets of values of x and y satisfying any equation are called its solution(s).

Consider the equation $2x + y = 4$. Now, if we substitute $x = -2$ in the equation, we obtain $2.(-2) + y = 4$ or $-4 + y = 4$ or $y = 8$. Hence (–2, 8) is a solution. If we substitute x = 3 in the equation, we obtain $2.3 + y = 4$ or $6 + y = 4$ or $y = -2$

Hence (3, –2) is a solution. The following table lists six possible values for x and the corresponding values for y, i.e. six solutions of the equation.

x	–2	–1	0	1	2	3
y	8	6	4	2	0	–2

Systems of Linear equation

Consistent System : A system (of 2 or 3 or more equations taken together) of linear equations is said to be consistent, if it has at least one solution.

Inconsistent System: A system of simultaneous linear equations is said to be inconsistent, if it has no solutions at all.

e.g. $X + Y = 9$; $3X + 3Y = 8$

Clearly there are no values of X & Y which simultaneously satisfy the given equations. So the system is inconsistent.

REMEMBER

★ The system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ has :

- a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
- Infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
- No solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

★ The homogeneous system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ has the only solution $x = y = 0$ when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

★ The homogeneous system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ has a non-zero solution only when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, and in this case, the system has an infinite number of solutions.

EXAMPLE 7. Find k for which the system $3x - y = 4$, $kx + y = 3$ has a infinitely many solution.

Sol. The given system will have inifinite solution,

if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ i.e. $\frac{3}{k} = \frac{-1}{1}$ or $k = -3$.

EXAMPLE 8. Find k for which the system $6x - 2y = 3$, $kx - y = 2$ has a unique solution.

Sol. The given system will have a unique solution,

if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. $\frac{6}{k} \neq \frac{-2}{-1}$ or $k \neq 3$.

EXAMPLE 9. What is the value of k for which the system $x + 2y = 3$, $5x + ky = -7$ is inconsistent?

Sol. The given system will be inconsistent if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e. if $\frac{1}{5} = \frac{2}{k} \neq \frac{3}{-7}$ or $k = 10$.

EXAMPLE 10. Find k such that the system $3x + 5y = 0$, $kx + 10y = 0$ has a non-zero solution.

Sol. The given system has a non zero solution,

if $\frac{3}{k} = \frac{5}{10}$ or $k = 6$

QUADRATIC EQUATION

An equation of the degree two of one variable is called quadratic equation.

General form : $ax^2 + bx + c = 0$...........(1) where a, b and c are all real number and $a \neq 0$.

For Example :

$2x^2 - 5x + 3 = 0$; $2x^2 - 5 = 0$; $x^2 + 3x = 0$

If $b^2 - 4ac \geq 0$, then the quadratic equation gives two and only two values (either same or different) of the unknown variable and both these values are called the roots of the equation.

The roots of the quadratic equation (1) can be evaluated using the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad ...(2)$$

The above formula provides both the roots of the quadratic equation, which are generally denoted by α and β,

say $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

The expression inside the square root $b^2 - 4ac$ is called the DISCRIMINANT of the quadratic equation and denoted by D. Thus, Discriminant (D) = $b^2 - 4ac$.

☞ Shortcut Approach

Shortcut Approach to solve Quadratic equation $ax^2 + bx + c = 0$, if $b^2 - 4ac \geq 0$,

$\boxed{a}x^2 + bx + \boxed{c}$

ac ← [Product of coefficient a of x^2 and constant term c]

$p \quad q$ ← [Divide ac in two parts p and q such that their sum is equal to coefficient b of x and product equal to ac]

$\frac{p}{a} \quad \frac{q}{a}$ ← [Divide p and q by a]

$-\frac{p}{a} \quad -\frac{q}{a}$ ← [Change the sign]

Here $\frac{-p}{a}$ and $\frac{-q}{a}$ are two roots or solutions of quadratic equation $ax^2 + bx + c = 0$ i.e. $x = -\frac{p}{a}$ or $-\frac{q}{a}$.

EXAMPLE 11. Which of the following is a quadratic equation?

(a) $x^{\frac{1}{2}} + 2x + 3 = 0$

(b) $(x-1)(x+4) = x^2 + 1$

(c) $x^4 - 3x + 5 = 0$

(d) $(2x+1)(3x-4) = 2x^2 + 3$

Sol. (d) Equations in options (a) and (c) are not quadratic equations as in (a) max. power of x is fractional and in (c), it is not 2 in any of the terms.

For option (b), $(x-1)(x+4) = x^2 + 1$

or $x^2 + 4x - x - 4 = x^2 + 1$

or $3x - 5 = 0$

which is not a quadratic equations but a linear.

For option (d), $(2x+1)(3x-4) = 2x^2 + 3$

or $6x^2 - 8x + 3x - 4 = 2x^2 + 3$

or $4x^2 - 5x - 7 = 0$

which is clearly a quadratic equation.

EXAMPLE 12. *Solve* $2x^2 + 6 = 7x$

Sol. $2x^2 + 6 = 7x$

$\Rightarrow 2x^2 - 7x + 6 = 0$

$\boxed{2}x^2 + 7x + \boxed{6} = 0$

12 ← $[2 \times 6 = 12]$

$-4 \quad -3$ ← $\begin{bmatrix}(-4) + (-3) = -7 \\ (-4) \times (-3) = 12\end{bmatrix}$

$-4 \quad -\frac{3}{2}$ ← [Divide by coefficient 2 or x^2]

$2 \quad \frac{3}{2}$ ← [Change the sign]

$\therefore$ Solutions or roots of given quadratic equation $2x^2 + 6 = 7x$ are $\frac{3}{2}$ and 2.

EXAMPLE 13. *Solve* $x - \frac{1}{x} = 1\frac{1}{2}$

Sol. $x - \frac{1}{x} = 1\frac{1}{2} \Rightarrow \frac{x^2 - 1}{x} = \frac{3}{2}$

$\Rightarrow 2(x^2 - 1) = 3x$

$\Rightarrow 2x^2 - 2 = 3x$

$\Rightarrow 2x^2 - 3x - 2 = 0$

$\boxed{2}x^2 + (-3)x + \boxed{(-2)} = 0$

-4 ← $[2 \times (-2) = -4]$

$-4 \quad 1$ ← $\begin{bmatrix}-4 + 1 = -3 \\ -4 \times 1 = -4\end{bmatrix}$

$-2 \quad \frac{1}{2}$ ← [Divide by 2]

$2 \quad -\frac{1}{2}$ ← [Change the sign]

Either $2x + 1 = 0$ or $x - 2 = 0$

$\Rightarrow 2x = -1$ or $x = 2$

$\Rightarrow x = \frac{-1}{2}$ or $x = 2$

$\therefore x = \frac{-1}{2}$, 2 are solutions.

Nature of Roots

The nature of roots of the equation depends upon the nature of its discriminant D.

1. If $D < 0$, then the roots are non-real complex, Such roots are always conjugate to one another. That is, if one root is $p + iq$ then other is $p - iq$, $q \neq 0$.

2. If $D = 0$, then the roots are real and equal. Each root of the equation becomes $-\frac{b}{2a}$. Equal roots are referred as repeated roots or double roots also.
3. If $D > 0$ then the roots are real and unequal.

Sign of Roots:

Let α, β are real roots of the quadratic equation $ax^2 + bx + c = 0$ that is $D = b^2 - 4ac \geq 0$. Then

1. Both the roots are positive if a and c have the same sign and the sign of b is opposite.
2. Both the roots are negative if a, b and c all have the same sign.
3. The Roots have opposite sign if sign of a and c are opposite.
4. The Roots are equal in magnitude and opposite in sign if $b = 0$ [that is its roots α and $-\alpha$]
5. The roots are reciprocal if $a = c$.
   [that is the roots are α and $\frac{1}{\alpha}$]

EXAMPLE 14. The solutions of the equation $\sqrt{25 - x^2} = x - 1$ are :

(a) $x = 3$ and $x = 4$ (b) $x = 5$ and $x = 1$
(c) $x = -3$ and $x = 4$ (d) $x = 4$ and $x = -3$

Sol. (d) $\sqrt{25 - x^2} = x - 1$

or $25 - x^2 = (x-1)^2$ or $25 - x^2 = x^2 + 1 - 2x$

or $2x^2 - 2x - 24 = 0$ or $x^2 - x - 12 = 0$

or $(x-4)(x+3) = 0$ or $x = 4, x = -3$

EXAMPLE 15. Which of the following equations has real roots ?

(a) $3x^2 + 4x + 5 = 0$ (b) $x^2 + x + 4 = 0$
(c) $(x-1)(2x-5) = 0$ (d) $2x^2 - 3x + 4 = 0$

Sol. (c) Roots of a quadratic equation

$ax^2 + bx + c = 0$ are real if $b^2 - 4ac \geq 0$
Let us work with options as follows.

Option (a) : $3x^2 + 4x + 5 = 0$

$b^2 - 4ac = (4)^2 - 4(3)(5) = -44 < 0$.
Thus, roots are not real.

(b) : $x^2 + x + 4 = 0$

$b^2 - 4ac = (1)^2 - 4(1)(4) = 1 - 16 = -15 < 0$
Thus, roots are not real.
(c) : $(x-1)(2x-5) = 0 \Rightarrow 2x^2 - 7x + 5 = 0$
$b^2 - 4ac = (-7)^2 - 4 \times 2 \times 5 = 49 - 40 = 9 > 0$
Thus roots are real.

or $x = 1$ and $x = \frac{5}{2} > 0$; Thus, equation has real roots.

(d) : $2x^2 - 3x + 4 = 0$

$b^2 - 4ac = (-3)^2 - 4(2)(4) = 9 - 32 = -23 < 0$
Thus, roots are not real.
Hence, option (c) is correct.

EXAMPLE 16. If $2x^2 - 7xy + 3y^2 = 0$, then the value of x : y is :

(a) 3 : 2 (b) 2 : 3
(c) 3 : 1 or 1 : 2 (d) 5 : 6

Sol. (c) $2x^2 - 7xy + 3y^2 = 0$

$$2\left(\frac{x}{y}\right)^2 - 7\left(\frac{x}{y}\right) + 3 = 0$$

$$\frac{x}{y} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{49 - 24}}{2 \times 2} = \frac{7 \pm 5}{4} = 3, \frac{1}{2}$$

$$\Rightarrow \frac{x}{y} = \frac{3}{1} \text{ or } \frac{x}{y} = \frac{1}{2}$$

EXAMPLE 17. If $a + b + c = 0$ and a,b,c, are rational numbers then the roots of the equation
$(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are

(a) rational (b) irrational
(c) non real (d) none of these.

Sol. (a) The sum of coefficients

$= (b + c - a) + (c + a - b) + (a + b - c) = a + b + c = 0$ (given)

$\therefore$ $x = 1$ is a root of the equation

$\therefore$ The other root is $\frac{a+b-c}{b+c-a}$, which is rational as a, b, c, are rational
Hence, both the roots are rational.

<u>OTHER METHOD</u> :

$$D = (c + a - b)^2 - 4(b + c - a)(a + b - c)$$

$$= (-2b)^2 - 4(-2a)(-2c) = 4b^2 - 16ac$$

$$= 4(a+c)^2 - 16ac = 4[(a+c)^2 - 4ac] = [2(a-c)]^2$$

D is a perfect square. Hence, the roots of the equation are rational.

EXAMPLE 18. Both the roots of the equation
$(x-b)(x-c) + (x-c)(x-a) + (x-a)(x-b) = 0$ are

(a) dependent on a, b, c (b) always non real
(c) always real (d) rational

Sol. (c) The equation is

$$3x^2 - 2(a + b + c)x + (bc + ca + ab) = 0$$

The discriminant

$$D = 4(a + b + c)^2 - 4.3.(bc + ca + ab)$$

$$= 4[a^2 + b^2 + c^2 - ab - bc - ca]$$

$$= 2[(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)]$$

$$= 2[(a-b)^2 + (b-c)^2 + (c-a)^2] \geq 0$$

$\therefore$ Roots are always real.

Symmetric Functions of Roots :

An expression in α, β is called a symmetric function of α, β if the function is not affected by interchanging α and β. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ then,

Sum of roots : $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

and Product of roots : $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Formation of quadratic Equation with Given Roots :

- An equation whose roots are α and β can be written as $(x - \alpha)(x - \beta) = 0$ or $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ or $x^2 -$ (sum of the roots) $x +$ product of the roots $= 0$.
- Further if α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then
  $ax^2 + bx + c = a(x - \alpha)(x - \beta)$ is an identity.

EXAMPLE 19. Of the following quadratic equations, which is the one whose roots are 2 and – 15 ?

(a) $x^2 - 2x + 15 = 0$ **(b)** $x^2 + 15x - 2 = 0$

(c) $x^2 + 13x - 30 = 0$ **(d)** $x^2 - 30 = 0$

Sol. (c) Sum of roots = 2 – 15 = – 13

Product of roots = 2 × (– 15) = – 30

Required equation

$= x^2 - x$ (sum of roots) + product of roots = 0

$\Rightarrow \quad x^2 + 13x - 30 = 0$

EXAMPLE 20. If a and b are the roots of the equation $x^2 - 6x + 6 = 0$, then the value of $a^2 + b^2$ is :

(a) 36 (b) 24 (c) 17 (d) 6

Sol. (b) The sum of roots = a + b = 6

Product of roots = ab = 6

Now, $a^2 + b^2 = (a + b)^2 - 2ab = 36 - 12 = 24$

EXAMPLE 21. If a, b are the two roots of a quadratic equation such that a + b = 24 and a – b = 8, then the quadratic equation having a and b as its roots is :

(a) $x^2 + 2x + 8 = 0$ **(b)** $x^2 - 4x + 8 = 0$

(c) $x^2 - 24x + 128 = 0$ **(d)** $2x^2 + 8x + 9 = 0$

Sol. (c) a + b = 24 and a – b = 8

$\Rightarrow$ a = 16 and b = 8 $\Rightarrow$ ab = 16 × 8 = 128

A quadratic equation with roots a and b is

$x^2 - (a + b)x + ab = 0$ or $x^2 - 24x + 128 = 0$

INEQUATIONS :

A statement or equation which states that one thing is not equal to another, is called an inequation.

Symbols :

'<' means "is less than"

'>' means "is greater than"

'$\leq$' means "is less than or equal to"

'$\geq$' means "is greater than or equal to"

For example :

(a) $x < 3$ means x is less than 3.

(b) $y \geq 9$ means y is greater than or equal to 9.

Properties

1. Adding the same number to each side of an inequation does not effect the sign of inequality, i.e. if $x > y$ then, $x + a > y + a$.
2. Subtracting the same number to each side of an inequation does not effect the sign of inequaltiy, i.e., if $x < y$ then, $x - a < y - a$.
3. Multiplying each side of an inequality with same positive number does not effect the sign of inequality, i.e., if $x \leq y$ then $ax \leq ay$ (where, $a > 0$).
4. Multiplying each side of an inequality with a negative number reverse the sign of inequality i.e., if $x < y$ then $ax > ay$ (where $a < 0$).
5. Dividing each side of an inequation by a positive number does not effect the sign of inequality, i.e., if $x \leq y$ then $\frac{x}{a} \leq \frac{y}{a}$ (where $a > 0$).
6. Dividing each side of an inequation by a negative number reverses the sign of inequality, i.e., if $x > y$ then $\frac{x}{a} < \frac{y}{a}$ (where $a < 0$).

REMEMBER

- ★ If $a > b$ and a, b, n are positive, then $a^n > b^n$ but $a^{-n} < b^{-n}$. For example $5 > 4$; then $5^3 > 4^3$ or $125 > 64$, but $5^{-3} < 4^{-3}$ or $\frac{1}{125} < \frac{1}{64}$.
- ★ If $a > b$ and $c > d$, then $(a + c) > (b + d)$.
- ★ If $a > b > 0$ and $c > d > 0$, then $ac > bd$.
- ★ If the signs of all the terms of an inequality are changed, then the sign of the inequality will also be reversed.

MODULUS :

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

1. If a is positive real number, x and y be the fixed real numbers, then

(i) $|x - y| < a \Leftrightarrow y - a < x < y + a$

(ii) $|x - y| \leq a \Leftrightarrow y - a \leq x \leq y + a$

(iii) $|x - y| > a \Leftrightarrow x > y + a$ or $x < y - a$

(iv) $|x - y| \geq a \Leftrightarrow x \geq y + a$ or $x \leq y - a$

2. Triangle inequality :

(i) $|x+y| \le |x| + |y|, \ \forall\, x, y \in R$

(ii) $|x-y| \ge |x| - |y|, \ \forall\, x, y \in R$

EXAMPLE 22. If a – 8 = b, then determine the value of $|a-b|-|b-a|$.

(a) 16 (b) 0 (c) 4 (d) 2

Sol. (b) $|a-b| = |8| = 8$

$\Rightarrow |b-a| = |-8| = 8$

$\Rightarrow |a-b| - |b-a| = 8 - 8 = 0$

EXAMPLE 23. *Solve :* $3x+4 \le 19, \ x \in N$

Sol. $3x+4 \le 19$

$3x+4-4 \le 19-4$ [Subtracting 4 from both the sides]

$3x \le 15$

$\frac{3x}{3} \le \frac{15}{3}$ [Dividing both the sides by 3]

$x \le 5; \ x \in N$

$\therefore x = \{1, 2, 3, 4, 5\}$.

EXAMPLE 24. *Solve* $5 \le 2x-1 \le 11$

Sol. $5 \le 2x-1 \le 11$

$5+1 \le 2x-1+1 \le 11+1$

[Adding 1 to each sides]

$6 \le 2x \le 12$

$\frac{6}{2} \le \frac{2x}{2} \le \frac{12}{2}$ [Dividing each side by 2]

$3 \le x \le 6$

$\Rightarrow x = \{3, 4, 5, 6\}$.

Applications Formulation of Equations/ Expressions :

A formula is an equation, which represents the relations between two or more quantities.

For example :

Area of parallelogram (A) is equal to the product of its base (b) and height (h), which is given by

$A = b \times h$

or $A = bh$.

Perimeter of triangle (P),

$P = a + b + c$, where a, b and c are length of three sides.

EXAMPLE 25. Form the expression for each of the following:

(a) 5 less than a number is 7.

(b) Monika's salary is 1500 less than thrice the salary of Surbhi.

Sol. (a) Expression is given by

$x - 5 = 7$, where x is any number

(b) Let the salary of Surbhi be Rs. x and salary of Monika be Rs. y.

Now, according to the question

$y = 3x - 1500$

More Applications of Equations :

Problems on Ages can be solved by linear equations in one variable, linear equations in two variables, and quadratic equations.

EXAMPLE 26. Kareem is three times as old as his son. After ten years, the sum of their ages will be 76 years. Find their present ages.

Sol. Let the present age of Kareem's son be x years.

Then, Kareem's age = 3x years

After 10 years, Kareem's age = 3x + 10 years

and Kareem's son's age = x + 10 years

$\therefore (3x+10)+(x+10) = 76$

$\Rightarrow 4x = 56 \Rightarrow x = 14$

$\therefore$ Kareem's present age = $3x = 3 \times 14 = 42$ years

Kareem's son's age = x = 14 years.

EXAMPLE 27. The present ages of Vikas and Vishal are in the ratio 15 : 8. After ten years, their ages will be in the ratio 5 : 3. Find their present ages.

Sol. Let the present ages of Vikas and Vishal be 15x years and 8x years.

After 10 years,

Vikas's age = 15x + 10 and

Vishal's age = 8x + 10

$\therefore \frac{15x+10}{8x+10} = \frac{5}{3}$

$\Rightarrow 3(15x+10) = 5(8x+10)$

$\Rightarrow 45x+30 = 40x+50$

$\Rightarrow 5x = 20 \Rightarrow x = \frac{20}{5} = 4$

$\therefore$ Present age of Vikas = $15x = 15 \times 4 = 60$ years

Present age of Vishal = $8x = 8 \times 4 = 32$ years.

SHORTCUT METHOD

(Ratio of present age of vikas and vishal) (After 10 years, ratio of age of vikash and vishal)

$\frac{15}{8} \quad \frac{5}{3} \xrightarrow{\text{Difference}} 2 \xrightarrow{\text{Difference after 10 years}} 10 \times 2 = 20$

Difference = $15 \times 3 - 8 \times 5 = 5$

Common part $= \frac{20}{5} = 4$

Vikas Present age = $15 \times 4 = 60$ years

Vishal Present age = $8 \times 4 = 32$ years

EXAMPLE 28. Father's age is 4 less than five times the age of his son and the product of their ages is 288. Find the father's age.

Sol. Let the son's age be x years.

So, father's age = 5x – 4 years.

$\therefore x(5x-4) = 288$

$\Rightarrow 5x^2 - 4x - 288 = 0 \Rightarrow 5x^2 - 40x + 36x - 288 = 0$

$\Rightarrow 5x(x-8) + 36(x-8) = 0$

$\Rightarrow (5x+36)(x-8) = 0$

Either $x - 8 = 0$ or $5x + 36 = 0 \Rightarrow x = 8$ or $x = \frac{-36}{5}$

x cannot be negative; therefore, x = 8 is the solution.

$\therefore$ Son's age = 8 years and Father's age = $5x - 4 = 36$ years.

> **☞ Shortcut Approach**
>
> If present age of the father is F times the age of his son. T years hence, the father's age become Z times the age of son then present age of his son is given by $\frac{(Z-1)T}{(F-Z)}$

EXAMPLE 29. Present age of the father is 9 times the age of his son. One year later, father's age become 7 times the age of his son. What are the present ages of the father and his son.

Sol. By the formula

$$\text{Son's age} = \frac{(7-1)}{(9-7)} \times 1 = \frac{6}{2} \times 1 = 3 \text{ years.}$$

So, father's age = 9 × son's age = 9 × 3 = 27 years.

SHORTCUT METHOD

(Ratio of present age of father and son) (After one year ratio of father and son)

$$\frac{9}{1} \quad \frac{7}{1} \xrightarrow{\text{Difference}} 6 \xrightarrow{\text{Difference after one year}} 1 \times 6 = 6$$

Difference = 9 × 1 – 1 × 7 = 2

Common part = $\frac{6}{2} = 3$

Present age of father = 9 × 3 = 27 years

Present age of son = 1 × 3 = 3 years

> **☞ Shortcut Approach**
>
> If T_1 years earlier the age of the father was n times the age of his son, T_2 years hence, the age of the father becomes m times the age of his son then his son's age is given by
>
> $$\text{Son's age} = \frac{T_2(n-1) + T_1(m-1)}{n-m}$$

EXAMPLE 30. 10 years ago, Shakti's mother was 4 times older than her. After 10 years, the mother will be twice older than the daughter. What is the present age of Shakti?

Sol. By using formula,

$$\text{Shakti's age} = \frac{10(4-1) + 10(2-1)}{4-2} = 20 \text{ years.}$$

SHORTCUT METHOD

Comparisons of ages is given 10 years before the present time and 10 years after the present time.

Therefore time gap = 10 + 10 = 20 years.

(10 years before ratio of age of mother and daughter) (10 years after, ratio of age of mother and daughter)

$$\frac{4}{1} \quad \frac{2}{1} \xrightarrow{\text{Difference}} 1 \xrightarrow{\text{Difference after 20 years}} 20 \times 1 = 20$$

Difference = 4 × 1 – 1 × 2 = 2

Common part = $\frac{20}{2} = 10$

Present age of Shakti's mother = 4 × 10 + 10 = 50 years

Present age fo Shakti = 10 +10 = 20 years

> **☞ Shortcut Approach**
>
> Present age of Father : Son = a : b
>
> After / Before T years = m : n
>
> Then son's age = $b \times \frac{T(m-n)}{an-bm}$
>
> and Father's age = $a \times \frac{T(m-n)}{an-bm}$

EXAMPLE 31. The ratio of the ages of the father and the son at present is 3 : 1. Four years earliar, the ratio was 4 : 1. What are the present ages of the son and the father?

Sol. Ratio of present age of Father and Son = 3 : 1

4 years before = 4 : 1

$$\text{Son's age} = 1 \times \frac{4(4-1)}{4 \times 1 - 3 \times 1} = 12 \text{ years.}$$

$$\text{Father's age} = 3 \times \frac{4(4-1)}{4 \times 1 - 3 \times 1} = 36 \text{ years.}$$

SHORTCUT METHOD

(Ratio of present age of father and son) (4 years before, ratio of age of father and son)

$$\frac{3}{1} \quad \frac{4}{1} \xrightarrow{\text{Difference}} 3 \xrightarrow{\text{Difference after 4 years}} 4 \times 3 = 12$$

Difference = 4 – 3 = 1

Common part = $\frac{12}{1} = 12$

Present age of father = 3 × 12 = 36 years

Present age of son = 1 × 12 = 12 years

EXERCISE

Directions (Qs. 1-5): In each question one/two equations are provided. On the basis of these you have to find out the relation between p and q.

Give answer (a) if $p = q$
Give answer (b) if $p > q$
Give answer (c) if $q > p$
Give answer (d) if $p \geq q$, and
Give answer (e) if $q \geq p$

1. **I.** $pq + 30 = 6p + 5q$
2. **I.** $2p^2 + 12p + 16 = 0$
   II. $2q^2 + 14q + 24 = 0$
3. **I.** $2p^2 + 48 = 20p$
   II. $2q^2 + 18 = 12q$
4. **I.** $q^2 + q = 2$
   II. $p^2 + 7p + 10 = 0$
5. **I.** $p^2 + 36 = 12p$
   II. $4q^2 + 144 = 48q$

Directions (Qs.6-10): For the two given equations I and II give answer

(a) if p is greater than q
(b) if p is smaller than q
(c) if p is equal to q.
(d) if p is either equal to or greater than q
(e) if p is either equal to or smaller than q.

6. **I.** $6p^2 + 5p + 1 = 0$
   II. $20q^2 + 9q = -1$
7. **I.** $3p^2 + 2p - 1 = 0$
   II. $2q^2 + 7q + 6 = 0$
8. **I.** $3p^2 + 15p = -18$
   II. $q^2 + 7q + 12 = 0$
9. **I.** $p = \dfrac{\sqrt{4}}{\sqrt{9}}$
   II. $9q^2 - 12q + 4 = 0$
10. **I.** $p^2 + 13p + 42 = 0$
    II. $q^2 = 36$

Directions (Qs. 11 - 14): In each of the following questions, one or two equation(s) is/are given. On their basis you have to determine the relation between x and y and then give answer

(a) if $x < y$ (b) if $x > y$
(c) if $x \leq y$ (d) if $x \geq y$
(e) if $x = y$

11. **I** $x^2 + 3x + 2 = 0$ **II.** $2y^2 = 5y$
12. **I.** $2x^2 + 5x + 2 = 0$ **II.** $4y^2 = 1$
13. **I.** $y^2 + 2y - 3 = 0$ **II.** $2x^2 - 7x + 6 = 0$
14. **I.** $x^2 - 5x + 6 = 0$ **II.** $y^2 + y - 6 = 0$

Directions (Qs. 15-18): In each of the following questions two equations are given. You have to solve them and Give answer

(a) if $p < q$ (b) if $p > q$
(c) if $p \leq q$ (d) if $p \geq q$
(e) if p = q

15. **I.** $p^2 - 7p = -12$
    II. $q^2 - 3q + 2 = 0$
16. **I.** $12p^2 - 7p = -1$
    II. $6q^2 - 7q + 2 = 0$
17. **I.** $p^2 - 8p + 15 = 0$
    II. $q^2 - 5q = -6$
18. **I.** $2p^2 + 20p + 50 = 0$
    II. $q^2 - 25$

Directions (Qs.19-23): In each of these questions two equations are given. You have to solve these equations and Give answer

(a) if $x < y$ (b) if $x > y$
(c) if $x = y$ (d) if $x \geq y$
(e) if $x \leq y$

19. **I.** $x^2 - 6x = 7$
    II. $2y^2 + 13y + 15 = 0$
20. **I.** $3x^2 - 7x + 2\ 0$
    II. $2y^2 - 11y + 15 = 0$
21. **I.** $10x^2 - 7x + 1 = 0$
    II. $35y^2 - 12y + 1 = 0$
22. **I.** $4x^2 = 25$
    II. $2y^2 - 13y + 21 = 0$
23. **I.** $3x^2 + 7x = 6$
    II. $6(2y^2 + 1) = 17y$

Directions (Qs. 24-26): In each question below one or more equation(s) is /are given. On the basis of these, you have to find out the relationship between p and q.

Give answer (a) if $p = q$
Give answer (b) if $p > q$
Give answer (c) if $p < q$
Give answer (d) if $p \leq q$
Give answer (e) if $p \geq q$

24. I. $2p^2 = 23p - 63$
    II. $2q\ (q^{-8}) = q^{-36}$
25. I. $p\ (p^{-1}) = (p^{-1})$
    II. $q^2 = 4q^{-1}$
26. I. $2p\ (p - 4) = 8\ (p + 5)$
    II. $q^2 + 12 + 7q$

Directions (Qs. 27-30): In each of the following questions two equations I and II are given. You have to solve both the equations and give answer

(a) if $a < b$ (b) if $a \leq b$
(c) if $a \geq b$ (d) if $a = b$
(e) if $a > b$

27. I. $a^2 - 5a + 6 = 0$
    II. $b^2 - 3b + 2 = 0$
28. I. $2a + 3b = 31$
    II. $3a = 2b + 1$
29. I. $2a^2 + 5a + 3 = 0$
    II. $2b^2 - 5b + 3 = 0$

30. I. $4a^2 = 1$
II. $4b^2 - 12b + 5 = 0$

Directions (Qs. 31-35) : In each of the following questions there are two equations. Solve them and choose the correct option

(a) If $P < Q$ (b) If $P > Q$
(c) If $P \le Q$ (d) If $P \ge Q$
(e) If $P = Q$

31. **I.** $4P^2 - 8P + 3 = 0$ **II.** $2Q^2 - 13Q + 15 = 0$
32. **I.** $P^2 + 3P - 4 = 0$ **II.** $3Q^2 - 10Q + 8 = 0$
33. **I.** $3P^2 - 10P + 7 = 0$ **II.** $15Q^2 - 22Q + 8 = 0$
34. **I.** $20P^2 - 17P + 3 = 0$ **II.** $20Q^2 - 9Q + 1 = 0$
35. **I.** $20P^2 + 31P + 12 = 0$ **II.** $21Q^2 + 23Q + 6 = 0$

Diretions (Qs. 36-40) : For the two given equations I and II, give answer

(a) if a is greater than b
(b) if a is smaller than b
(c) if a is equal to b
(d) if a is either equal to or greater than b
(e) if a is either equal to or smaller than b

36. **I.** $\sqrt{2304} = a$
II. $b^2 = 2304$
37. **I.** $12a^2 - 7a + 1 = 0$
II. $15b^2 - 16b + 4 = 0$
38. **I.** $a^2 + 9a + 20 = 0$
II. $2b^2 - 10b + 12 = 0$
39. **I.** $3a + 2b = 14$
II. $a + 4b - 13 = 0$
40. **I.** $a^2 - 7a + 12 = 0$
II. $b^2 - 9b + 20 = 0$

Directions (Qs. 41-45) : In each question one or more equation(s) is (are) provided. On the basis of these you have
Give answer (a) if $p = q$
Give answer (b) if $p > q$
Give answer (c) if $q > p$
Give answer (d) if $p \ge q$ and
Give answer (e) if $q \ge p$

41. (i) $\frac{5}{28} \times \frac{9}{8} p = \frac{15}{14} \times \frac{13}{16} q$
42. (i) $p - 7 = 0$ (ii) $3q^2 - 10q + 7 = 0$
43. (i) $4p^2 = 16$ (ii) $q^2 - 10q + 25 = 0$
44. (i) $4p^2 - 5p + 1 = 0$ (ii) $q^2 - 2q + 1 = 0$
45. (i) $q^2 - 11q + 30 = 0$ (ii) $2p^2 - 7p + 6 = 0$

Directions (Qs. 46-48) : In each question below one or more equation(s) is/are provided. On the basis of these, you have to find out relation between p and q.
Give answer (a) if $p = q$, Give answer
(b) if $p > q$, Give answer
(c) if $q > p$, Give answer
(d) if $p \ge q$ and Give answers
(e) if $q \ge p$.

46. I. $4q^2 + 8q = 4q + 8$
II. $p^2 + 9p = 2p - 12$
47. I. $2p^2 + 40 = 18p$
II. $q^2 = 13q - 42$
48. I. $6q^2 + \frac{1}{2} = \frac{7}{2} q$
II. $12p^2 + 2 = 10p$

Directions (Qs. 49-53): In each of the following questions two equations are given. You have to solve them and give answer

(a) if $x > y$; (b) if $x < y$;
(c) if $x = y$; (d) if $x \ge y$;
(e) if $x \le y$;

49. **I.** $y^2 - 6y + 9 = 0$ **II.** $x^2 + 2x - 3 = 0$
50. **I.** $x^2 - 5x + 6 = 0$ **II.** $2y^2 + 3y - 5 = 0$
51. **I.** $x = \sqrt{256}$ **II.** $y = (-4)^2$
52. **I.** $x^2 - 6x + 5 = 0$ **II.** $y^2 - 13y + 42 = 0$
53. **I.** $x^2 + 3x + 2 = 0$ **II.** $y^2 - 4y + 1 = 0$

54. If $3x - 5y = 5$ and $\frac{x}{x+y} = \frac{5}{7}$, then what is the value of $x - y$?
(a) 9 (b) 6
(c) 4 (d) 3
(e) None of these

55. $\frac{5}{7}$ of $\frac{4}{15}$ of a number is 8 more than $\frac{2}{5}$ of $\frac{4}{9}$ of the same number. What is half of that number?
(a) 630 (b) 315
(c) 210 (d) 105
(e) None of these

56. The difference between a two-digit number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number?
(a) 4 (b) 9
(c) 3 (d) Cannot be determined
(e) None of theses

57. By the how much is two-fifth of 200 greater than three - fifths of 125?
(a) 15 (b) 3
(c) 5 (d) 30
(e) None of these

58. If $\frac{x^2 - 1}{x + 1} = 2$, then, $x = ?$
(a) 1 (b) 0
(c) 2 (d) Can't be determined
(e) None of these

59. The difference between a number and its one-third is double of its one-third. What is the number?
(a) 60 (b) 18
(c) 30 (d) Cannot be determined
(e) None of these

60. Two pens and three pencils cost ₹ 86. Four pens and a pencil cost ₹ 112. What is the difference between the cost of a pen and that of a pencil?
(a) ₹ 25 (b) ₹ 13
(c) ₹ 19 (d) Cannot be determined
(e) None of these

61. The difference between a two-digit number and the number after interchanging the position of the two digits is 36. What is the difference between the two digits of the number?
(a) 4 (b) 6
(c) 3 (d) Cannot be determined
(e) None of these

62. If the digit in the unit's place of a two-digit number is halved and the digit in the ten's place is doubled, the number thus, obtained is equal to the number obtained by interchanging the digits. Which of the following is **definitely true**?
(a) Digits in the unit's place and the ten's place are equal.
(b) Sum of the digits is a two-digit number.
(c) Digit in the unit's place is half of the digit in the ten's place.
(d) Digit in the unit's place is twice the digit in the ten's place.
(e) None of these

63. If A and B are positive integers such that $9A^2 = 12A + 96$ and $B^2 = 2B + 3$, then which of the following is the value of $5A + 7B$?
(a) 31 (b) 41
(c) 36 (d) 43
(e) 27

64. On Children's Day, sweets were to be equally distributed among 175 children in a school. Actually on the Children's Day 35 children were absent and therefore, each child got 4 sweets extra. How many sweets were available in all for distribution?
(a) 2480 (b) 2680
(c) 2750 (d) 2400
(e) None of these

65. A two-digit number is seven times the sum of its digits. If each digit is increased by 2, the number thus obtained is 4 more than six times the sum of its digits. Find the number.
(a) 42 (b) 24
(c) 48 (d) Data inadequate
(e) None of these

66. One-third of Ramani's savings in National Savings Certificate is equal to one-half of his savings in Public Provident Fund. If he has ₹ 150000 as total savings, how much he saved in Public Provident Fund?
(a) ₹60000 (b) ₹50000
(c) ₹90000 (d) ₹30000
(e) None of these

67. $\frac{1}{5}$ of a number is equal to $\frac{5}{8}$ of the second number. If 35 is added to the first number then it becomes 4 times of second number. What is the value of the second number?
(a) 125 (b) 70
(c) 40 (d) 25
(e) None of these

68. In a two-digit number, the digit at unit place is 1 more than twice of the digit at tens place. If the digit at unit and tens place be interchanged, then the difference between the new number and original number is less than 1 to that of original number. What is the original number?
(a) 52 (b) 73
(c) 25 (d) 49
(e) 37

69. Free notebooks were distributed equally among children of a class. The number of notebooks each child got was one-eighth of the number of children. Had the number of children been half, each child would have got 16 notebooks. How many notebooks were distributed in all?
(a) 432 (b) 640
(c) 256 (d) 512
(e) None of these

70. Twenty times a positive integer is less than its square by 96. What is the integer?
(a) 24 (b) 20
(c) 30 (d) Cannot be determined
(e) None of these

71. The digit in the units place of a number is equal to the digit in the tens place of half of that number and the digit in the tens place of that number is less than the digit in units place of half of the number by 1. If the sum of the digits of the number is seven, then what is the number?
(a) 52 (b) 16
(c) 34 (d) Data inadequate
(e) None of these

72. The difference between a two-digit number and the number obtained by interchanging the digits is 9. What is the difference between the two digits of the number?
(a) 8 (b) 2
(c) 7 (d) Cannot be determined
(e) None of these

73. The difference between a number and its three-fifths is 50. What is the number?
(a) 75 (b) 100
(c) 125 (d) Cannot be determined
(e) None of these

74. If the numerator of a fraction is increased by 2 and the denominator is increased by 1, the fraction becomes $\frac{5}{8}$ and if the numerator of the same fraction is increased by 3 and the denominator is increased by I the fraction becomes $\frac{3}{4}$. What is the original fraction?
(a) Data inadequate (b) $\frac{2}{7}$
(c) $\frac{4}{7}$ (d) $\frac{3}{7}$
(e) None of these

75. If $2x + 3y = 26$; $2y + z = 19$ and $x + 2z = 29$, what is the value of $x + y + z$?
(a) 18 (b) 32
(c) 26 (d) 22
(e) None of these

76. If the sum of a number and its square is 182, what is the number?
(a) 15 (b) 26
(c) 28 (d) 91
(e) None of these

77. A certain number of tennis balls were purchased for ₹ 450. Five more balls could have been purchased for the same amount if each ball was cheaper by ₹ 15. Find the number of balls purchased.
(a) 15 (b) 20
(c) 10 (d) 25
(e) None of these

78. What will be the value of $n^4 - 10n^3 + 36n^2 - 49n + 24$, if $n = 1$?
(a) 21 (b) 2
(c) 1 (d) 22
(e) None of these

79. Out of total number of students in a college 12% are interested in sports. $\frac{3}{4}$th of the total number of students are interested in dancing. 10% of the total number of students are interested in singing and the remaining 15 students are not interested in any of the activities. What is the total number of students in the college?
(a) 450 (b) 500
(c) 600 (d) Cannot be determined
(e) None of these

80. The sum of four numbers is 64. If you add 3 to the first number, 3 is subtracted from the second number, the third is multiplied by 3 and the fourth is divided by 3, then all the results are equal. What is the difference between the largest and the smallest of the original numbers?
(a) 32 (b) 27
(c) 21 (d) Cannot be determined
(e) None of these

81. A classroom has equal number of boys and girls. Eight girls left to play Kho-kho, leaving twice as many boy as girls in the classroom. What was the total number of girls and boys present initially?
(a) Cannot be determined (b) 16
(c) 24 (d) 32
(e) None of these

82. The difference between the digits of a two-digit number is one-ninth of the difference between the original number and the number obtained by interchanging positions of the digits. What definitely is the sum of digits of that number?
(a) 5 (b) 14
(c) 12 (d) Data inadequate
(e) None of these

83. The denominator of a fraction is 2 more than thrice its numerator. If the numerator as well as denominator is increased by one, the fraction becomes 1/3. What was the original fraction?
(a) $\frac{4}{13}$ (b) $\frac{3}{11}$
(c) $\frac{5}{13}$ (d) $\frac{5}{11}$
(e) None of these

84. If $2x + y = 15$, $2y + z = 25$ and $2z + x = 26$, what is the value of z?
(a) 4 (b) 7
(c) 9 (d) 12
(e) None of these

85. Which of the following values of P satisfy the inequality $P(P-3) < 4P - 12$?
(a) $P > 4$ or $P \leq 3$ (b) $24 \leq P < 71$
(c) $P > 13; P < 51$ (d) $3 < P < 4$
(e) $P = 4, P = +3$

86. If the ages of P and R are added to twice the age of Q, the total becomes 59. If the ages of Q and R are added to thrice the age of P, the total becomes 68. And if the age of P is added to thrice the age of Q and thrice the age of R, the total becomes 108. What is the age of P?
(a) 15 years (b) 19 years
(c) 17 years (d) 12 years
(e) None of these

87. The product of the ages of Harish and Seema is 240. If twice the age of Seema is more than Harish's age by 4 years, what is Seema's age in years?
(a) 12 years (b) 20 years
(c) 10 years (d) 14 years
(e) Data inadequate

88. What would be the maximum value of Q in the following equation? $5P9 + 3R7 + 2Q8 = 1114$
(a) 8 (b) 7
(c) 5 (d) 4
(e) None of the above

89. Two-fifths of one-fourth of three-sevenths of a number is 15. What is half of that number?
(a) 96 (b) 196
(c) 94 (d) 188
(e) None of these

90. The sum of the digits of a two-digit number is 1 /11 of the sum of the number and the number obtained by interchanging the position of the'digits. What is the difference between the digits of that number?
(a) 3 (b) 2
(c) 6 (d) Data inadequate
(e) None of these

91. If a fraction's numerator is increased by 1 and the denominator is increased by 2 then the fraction becomes $\frac{2}{3}$. But when the numerator is increased by 5 and the denominator is increased by 1 then the fraction becomes $\frac{5}{4}$. What is the value of the original fraction?
(a) $\frac{3}{7}$ (b) $\frac{5}{8}$
(c) $\frac{5}{7}$ (d) $\frac{6}{7}$
(e) None of these

92. In a two-digit number the digit in the unit's place is more than the digit in the ten's place by 2. If the difference between the number and the number obtained by interchanging the digits is 18 what is the original number?
(a) 46 (b) 68
(c) 24 (d) Data inadequate
(e) None of these

93. If $2x + y = 17y$, $2z = 15$ and $x + z = 9$ then what is the value of $4x + 3y + z$?
(a) 41 (b) 43
(c) 55 (d) 45
(e) None of these

94. If the numerator of a fraction is increased by 2 and denominator is increased by 3, the fraction becomes 7/9; and if numerator as well as denominator are decreased by 1 the fraction becomes 4/5. What is the original fraction?
(a) $\frac{13}{16}$ (b) $\frac{9}{11}$
(c) $\frac{5}{6}$ (d) $\frac{17}{21}$
(e) None of these

95. The inequality $3n^2 - 18n + 24 > 0$ gets satisfied for which of the following values of n?
(a) $n < 2$ & $n > 4$ (b) $2 < n < 4$
(c) $n > 2$ (d) $n > 4$
(e) None of these

96. A sum is divided among Rakesh, Suresh and Mohan. If the difference between the shares of Rakesh and Mohan is ₹7000 and between those of Suresh and Mohan is ₹ 3000, what was the sum?
(a) ₹30,000 (b) ₹ 13,000
(c) ₹ 10,000 (d) Cannot be determined
(e) None of these

97. Three-fifths of a number is 30 more than 50 per cent of that number. What is 80 per cent of that number?
(a) 300 (b) 60
(c) 240 (d) Cannot be determined
(e) None of these

98. The difference between a two-digit number and the number obtained by interchanging the position of the digits of the number is 27. What is the difference between the digits of that number?
(a) 2 (b) 3
(c) 4 (d) Cannot be determined
(e) None of these

99. The sum of the ages of a father and his son is 4 times the age of the son. If the average age of the father and the son is 28 years, what is the son's age?
(a) 14 years (b) 16 years
(c) 12 years (d) Data inadequate
(e) None of these

100. Two-fifths of one-fourth of five-eighths of a number is 6. What is 50 per cent of that number?
(a) 96 (b) 32
(c) 24 (d) 48
(e) None of these

101. The sum of the digits of a two-digit number is $\frac{1}{5}$ of the difference between the number and the number obtained by interchanging the positions of the digits. What definitely is the difference between the digits of that number?
(a) 5 (b) 9
(c) 7 (d) Data inadequate
(e) None of these

102. Ashok gave 40 per cent of the amount he had to Jayant. Jayant in turn gave one-fourth of what he received from Ashok to Prakash. After paying ₹ 200 to the taxi-driver out of the amount he got from Jayant, Prakash now has ₹ 600 left with him. How much amount did Ashok have?
(a) ₹ 1,200 (b) ₹ 4,000
(c) ₹ 8,000 (d) Data inadequate
(e) None of these

103. What should be the maximum value of Q in the following equation?
$5P9 - 7Q2 + 9R6 = 823$
(a) 7 (b) 5
(c) 9 (d) 6
(e) None of these

104. The difference between a two-digit number and the number obtained by interchanging the position of the digits of that number is 54. What is the sum of the digits of that number?
(a) 6 (b) 9
(c) 15 (d) Data inadequate
(e) None of these

105. The product of two numbers is 192 and the sum of these two numbers is 28. What is the smaller of these two numbers?
(a) 16 (b) 14
(c) 12 (d) 18
(e) None of these

106. The age of Mr. Ramesh is four times the age of his son. After ten years the age of Mr. Ramesh will be only twice the age of his son. Find the present age of Mr. Ramesh's, son.
(a) 10 years (b) 11 years
(c) 12 years (d) Cannot be determined
(e) None of these

107. In an exercise room some discs of denominations 2 kg and 5 kg are kept for weightlifting. If the total number of discs is 21 and the weight of all the discs of 5 kg is equal to the weight of all the discs of 2 kg, find the weight of all the discs together.
(a) 80 kg (b) 90 kg
(c) 56 kg (d) Cannot be determined
(e) None of these

108. If the number of barrels of oil consumed doubles in a 10-year period and if B barrels were consumed in the year 1940, what multiple of B will be consumed in the year 2000?
(a) 64 (b) 60
(c) 12 (d) 32
(e) None of these

109. The sum of three consecutive even numbers is 14 less than one-fourth of 176. What is the middle number?
(a) 8 (b) 10
(c) 6 (d) Data inadequate
(e) None of these

110. The price of four tables and seven chairs is ₹ 12,090. **Approximately,** what will be the price of twelve tables and twenty-one chairs?
(a) ₹32,000 (b) ₹46,000
(c) ₹38,000 (d) ₹36,000
(e) ₹39,000

111. If the price of 253 pencils is ₹ 4263.05, what will be the **approximate** value of 39 such pencils'?
(a) ₹650 (b) ₹550
(c) ₹450 (d) ₹700
(e) ₹750

112. Sundari, Kusu and Jyoti took two tests each. Sundari secured $\frac{24}{60}$ marks in the first test and $\frac{32}{40}$ marks in the second test. Kusu secured $\frac{35}{70}$ marks in the first test and $\frac{54}{60}$ marks in the second test. Jyoti secured $\frac{27}{90}$ marks in the first test and $\frac{45}{50}$ marks in the second test. Who among them did register maximum progress?
(a) Only Sundari (b) Only Kusu
(c) Only Jyoti (d) Both Sundari and Kusu
(e) Both Kusu and Jyoti

ANSWER KEY

1	(c)	13	(b)	25	(c)	37	(b)	49	(b)	61	(a)	73	(c)	85	(d)	97	(c)	109	(b)
2	(b)	14	(d)	26	(b)	38	(b)	50	(a)	62	(d)	74	(d)	86	(d)	98	(b)	110	(d)
3	(b)	15	(b)	27	(c)	39	(a)	51	(c)	63	(b)	75	(d)	87	(a)	99	(a)	111	(a)
4	(e)	16	(a)	28	(a)	40	(e)	52	(b)	64	(e)	76	(e)	88	(e)	100	(d)	112	(c)
5	(a)	17	(d)	29	(a)	41	(b)	53	(b)	65	(a)	77	(c)	89	(e)	101	(a)		
6	(b)	18	(c)	30	(b)	42	(b)	54	(d)	66	(a)	78	(b)	90	(d)	102	(c)		
7	(a)	19	(b)	31	(c)	43	(c)	55	(d)	68	(c)	79	(b)	91	(c)	103	(a)		
8	(d)	20	(a)	32	(a)	44	(e)	56	(a)	68	(e)	80	(a)	92	(d)	104	(d)		
9	(c)	21	(d)	33	(b)	45	(c)	57	(c)	69	(d)	81	(d)	93	(d)	105	(c)		
10	(e)	22	(a)	34	(d)	46	(c)	58	(e)	70	(a)	82	(d)	94	(c)	106	(e)		
11	(a)	23	(e)	35	(a)	47	(c)	59	(d)	71	(a)	83	(b)	95	(a)	107	(e)		
12	(c)	24	(b)	36	(c)	48	(d)	60	(b)	72	(e)	84	(e)	96	(d)	108	(a)		

Hints & Explanations

1. (c) I. $pq + 30 = 6p + 5q$
or, $(6p - 30) + (5q - pq) = 0$
or, $6(p - 5) - q(p - 5) = 0$
or, $(p - 5)(6 - q) = 0$
$\therefore$ $p = 5$
or, $q = 6$
Hence, $q > p$

2. (b) I. $2p^2 - 12p + 16 = 0$
II. $2q^2 + 14q + 24 = 0$
or, $p^2 - 6p + 8 = 0$
or, $(p - 4)(p - 2) = 0$
or, $q^2 + 7q + 12 = 0$
or, $(q + 4)(q + 3) = 0$
$p = +4$ or, $+2$
$\therefore$ $q = -3$ or, -4
When $p = +2$, $q = -3$, then, $p > q$
When $p = +4$, $q = -4$, then, $p > q$
When $p = +4$, $q = -3$, then, $p > q$
Hence $p > q$

3. (b) I. $2p^2 - 20p + 48 = 0$
$p^2 - 10p + 24 = 0$
$(p - 4)(p - 6) = 0$
or $p = 4$; $p = 6$
II. $2q^2 - 12q + 18 = 0$
$q^2 - 6q + 9 = 0$
$(q - 3)(q - 3) = 0$
or $q = 3$; $q = 3$
hence $p > q$

4. (e) I. $q^2 + q = 2$
or, $q^2 + q - 2 = 0$
II. $p^2 + 7p + 10 = 0$
or, $p^2 + 5p + 2p + 10 = 0$
or, $(q + 2)(q - 1) = 0$
or, $(q + 5)(p + 2) = 0$
$\therefore$ $q = -2$ or 1
$\therefore$ $p = -5$ or -2
Hence, $q \geq p$

5. (a) I. $p^2 + 36 = 12p$
or, $p^2 - 12p + 36 = 0$
II. $4q^2 + 144 = 48q$
or, $(p - 6)^2 = 0$
or, $q^2 - 12q + 36 = 0$
$\therefore$ $p = 6$
or, $(q - 6)^2 = 0$
$\therefore$ $q = 6$
Hence, $p = q$

6. (b) **I.** $6p^2 + 5p + 1 = 0$
or, $6p^2 + 3p + 2p + 1 = 0$
or, $3p(2p + 1) + 1(2p + 1) = 0$
or, $(3p + 1)(2p + 1) = 0$
Hence, $p = \frac{-1}{3}, \frac{-1}{2}$
II. $20q^2 + 9q + 1 = 0$
or, $20q^2 + 5q + 4q + 1 = 0$
or, $5q(4q + 1) + 1(4q + 1) = 0$
or, $(5q + 1)(4q + 1) = 0$
Hence, $q = \frac{-1}{5}, \frac{-1}{4}$ Thus, $p < q$.

7. (a) **I.** $3p^2 + 2p - 1 = 0$
or, $3p^2 + 3p - p - 1 = 0$
or, $3p(p + 1) - 1(p + 1) = 0$
or, $(3p - 1)(p + 1) = 0$
Therefore, $p = \frac{1}{3}, -1$
II. $2q^2 + 7q + 6 = 0$
or, $2q^2 + 4q + 3q + 6 = 0$
or, $2q(q + 2) + 3(q + 2) = 0$
or, $(2q + 3)(q + 2) = 0$
Therefore, $q = \frac{-3}{2}, -2$ Thus $p > q$

8. (d) **I.** $3p^2 + 15p + 18 = 0$
or, $3p^2 + 9p + 6p + 18 = 0$
or, $3p(p + 3) + 6(p + 3) = 0$
or, $(3p + 6)(p + 3) = 0$
or, $p = \frac{-6}{3}, -3 = -2, -3$
II. $q^2 + 7q + 12 = 0$
or, $q^2 + 4q + 3q + 12 = 0$
or, $q(q + 4) + 3(q + 4) = 0$
or, $(q + 3)(q + 4) = 0$
or, $q = -3, -4$
Therefore, $p \geq q$

9. (c) **I.** $p = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$
II. $9q^2 - 12q + 4 = 0$
or, $9q^2 - 6q - 6q + 4 = 0$
or, $3q(3q - 2) - 2(3q - 2) = 0$
or, $(3q - 2)(3q - 2) = 0$

or, $q = \frac{2}{3}$

Therefore, $p = q$

10. (e) I. $p^2 + 13p + 42 = 0$
or, $p^2 + 7p + 6p + 42 = 0$
or, $p(p+7) + 6(p+7) = 0$
or, $(p+6)(p+7) = 0$
or, $p = -6, -7$
II. $q^2 = 36$
$q = \sqrt{36}$
$\therefore$ $q = +6, -6$
Therefore, $p \leq q$.

11. (a) **I.** $x^2 + 3x + 2 = 0$
or, $x^2 + 2x + x + 2 = 0$
or, $(x+2)(x+1) = 0$
or, $x = -2, -1$
II. $2y^2 = 5y$
or, $2y^2 - 5y = 0$
or, $y(2y - 5) = 0$
or, $y = 0, \frac{5}{2}$
Hence, $y > x$

12. (c) **I.** $2x^2 + 5x + 2 = 0$
or, $2x^2 + 4x + x + 2 = 0$
or, $(x+2)(2x+1) = 0$
or, $x = -2, -\frac{1}{2}$
II. $4y^2 = 1$
or $y^2 = \frac{1}{4}$
or, $y = \pm\frac{1}{2}$
Hence, $y \geq x$

13. (b) **I.** $y^2 + 2y - 3 = 0 \Rightarrow (y-1)(y+3) = 0$
or, $y = 1, -3$
II. $2x^2 - 7x + 6 = 0$
or, $2x^2 - 4x - 3x + 6 = 0$
or, $(2x-3)(x-2) = 0$
or, $x = 2, \frac{3}{2}$
Hence, $x > y$

14. (d) **I.** $x^2 - 5x + 6 = 0$
or, $x^2 - 3x - 2x + 6 = 0$
or, $x(x-3) - 2(x-3) = 0$
or, $(x-3)(x-2) = 0$
or, $x = 2, 3$
II. $y^2 + y - 6 = 0$
or, $y^2 + 3y - 2y - 6 = 0$
or, $y(y+3) - 2(y+3) = 0$
or, $(y+3)(y-2) = 0$
or, $y = 2, -3$
Hence, $x \geq y$

15. (b) **I.** $p^2 - 7p = -12$
or, $p^2 - 7p + 12 = 0$
or, $(p-3)(p-4) = 0$
or, $p = 3$ or 4
II. $q^2 - 3q + 2 = 0$
or, $(q-2)(q-1) = 0$
or, $q = 1$ or 2
Hence, $p > q$

16. (a) **I.** $12p^2 - 7p = -1$
or, $12p^2 - 7p + 1 = 0$
or, $(3p-1)(4p-1) = 0$
or, $p = \frac{1}{4}$ or $\frac{1}{3}$
II. $6q^2 - 7q + 2 = 0$
or, $(3q-2)(2q-1) = 0$
or, $q = \frac{1}{2}$ or $\frac{2}{3}$
Hence, $q > p$

17. (d) I. $p^2 - 8p + 15 = 0$
or, $(p-3)(p-5) = 0$
or, $p = 3$ or 5
II. $q^2 - 5q + 6 = 0$
or, $(q-2)(q-3) = 0$
or, $q = 2$ or 3
Hence, $p \geq q$

18. (c) **I.** $2p^2 + 20p + 50 = 0$
or, $p^2 + 10p + 25 = 0$
or, $(p+5)^2 = 0$
or, $p = -5$
II. $q^2 = 25$
or, $q = \pm 5$
Hence, $p \leq q$

19. (b) **I.** $x^2 - 6x = 7$
or, $x^2 - 6x - 7 = 0$
or, $(x-7)(x+1) = 0$
or, $x = 7, -1$
II. $2y^2 + 13y + 15 = 0$
or, $2y^2 + 3y + 10y + 15 = 0$
or, $(2y+3)(y+5) = 0$ or, $y = \frac{-3}{2}, -5$
Hence, $x > y$

20. (a) **I** $3x^2 - 7x + 2 = 0$
or, $3x^2 - 6x - x + 2 = 0$
or, $(x-2)(3x-1) = 0$
or, $x = 2, 1/3$
II. $2y^2 - 11y + 15 = 0$

or, $2y^2 - 6y - 5y + 15 = 0$
or, $(2y - 5)(y - 3) = 0$
or, $y = 5/2, 3$
Hence, $y > x$

21. (d) **I.** $10x^2 - 7x + 1 = 0$
or, $10x^2 - 5x - 2x + 1 = 0$
or, $(2x - 1)(5x - 1) = 0$
or, $x = 1/2, 1/5$
II. $35y^2 - 12y + 1 = 0$
or, $35y^2 - 7y - 5y + 1 = 0$
or, $(5y - 1)(7y - 1) = 0$
or, $y = \frac{1}{5}, \frac{1}{7}$
Hence, $x \geq y$

22. (a) **I.** $4x^2 = 25$
or $x = \pm\frac{5}{2}$
II. $2y^2 - 13y + 21 = 0$
or, $2y^2 - 6y - 7y + 21 = 0$
or, $(y - 3)(2y - 7) = 0$
or, $y = 3, \frac{7}{2}$
Hence, $y > x$

23. (e) **I.** $3x^2 + 7x - 6 = 0$
or, $3x^2 + 9x - 2x - 6 = 0$
or, $(x + 3)(3x - 2) = 0$
or, $x = -3, \frac{2}{3}$
II. $6(2y^2 + 1) = 17y$
or, $12y^2 + 6 - 17y = 0$
or, $12y^2 - 9y - 8y + 6 = 0$
or, $(4y - 3)(3y - 2) = 0$
or, $y = \frac{3}{4}, \frac{2}{3}$
Hence, $y \geq x$

24. (b) I. $2p^2 = 23p - 63$
or, $2p^2 - 23p + 63 = 0$
II. $2q\,(q^{-8}) = q^{-36}$
or, $(2p - 9)(p - 7) = 0$
or, $q^{-7} \times q^{36} = \frac{1}{2}$
$\therefore\ p = \frac{9}{2}\,or\,7$
or, $q^{29} = \frac{1}{2}$
$\therefore\ q = \left(\frac{1}{2}\right)^{1/29}$
Hence, $p > q$

25. (c) I. $p(p^{-1}) = p^{-1}$
II. $q^2 = 4(q^{-1})$
or, $p \times \frac{1}{p} = \frac{1}{p}$
$\therefore\ p = 1$
or, $q^3 = 4$
$\therefore\ q = (4)^{1/3} > 1$
Hence, $q > p$

26. (b) I. $2p\,(p - 4) = 8\,(p + 5)$
or $p^2 - 8p - 20 = 0$
or $(p + 2)(p - 10) = 0$
$\Rightarrow p = -2, \text{ or } +10$
II. $q^2 + 7q + 12 = 0$
$(q + 3)(q + 4) = 0$
$q = -3 \text{ or } -4$
$p > q$

27. (c) For eqn 1, the roots (a) will be 2, 3. As $-2 \times -3 = 6\,(ac)$ and $(-2) + (-3) = -5$
(b). Similarly, for eqn II, the roots (b) will be 2, 1.

28. (a) $2a + 3b = 31$ ... (i)
$3a - 2b = 1$ ..(ii)
Multiply (i) by 2 and (ii) by 3 and then adding
(i) and (ii), we get $a = \frac{65}{13} = 5$. Putting the value of 'a'
in any equation, we get $b = 7$.
Hence, $b > a$ or $a < b$.

29. (a) $a = -3/2\ \&\ -1;\ b = \frac{3}{2}\ \&\ 1$

30. (b) $a = \pm 1/2;\ b = 1/2, 5/2$

31. (c) **I.** $4P^2 - 8P + 3 = 0$
$4P^2 - 2P - 6P + 3 = 0$
$2P\,(2P - 1) - 3\,(2P - 1) = 0$
$(2P - 3)(2P - 1) = 0$
$\Rightarrow\ P = 1/2, 3/2$
II. $2Q^2 - 13Q + 15 = 0$
$2Q^2 - 10Q - 3Q + 15 = 0$
$2Q(Q - 5) - 3(Q - 5) = 0$
$\Rightarrow\ (2Q - 3)(Q - 5) = 0$
$\Rightarrow\ Q = 3/2, 5$
$\therefore\ Q \geq P$

32. (a) **I.** $P^2 + 3P - 4 = 0$
$P^2 + 4P - P - 4 = 0$
$\Rightarrow P(P + 4) - 1(P + 4) = 0$
$\Rightarrow\ P = 1, -4$
II. $3Q^2 - 10Q + 8 = 0$
$3Q^2 - 6Q - 4Q + 8 = 0$
$3Q\,(Q - 2) - 4\,(Q - 2) = 0$
$(3Q - 4)(Q - 2) = 0$
$\Rightarrow\ Q = 4/3, 2$
$\therefore\ Q > P$

33. (b) **I.** $3P^2 - 10P + 7 = 0$

$3P^2 - 3P - 7P + 7 = 0$

$3P(P-1) - 7(P-1) = 0$

$\Rightarrow (3P-7)(P-1) = 0$

$\Rightarrow P = 7/3, 1$

II. $15Q^2 - 22Q + 8 = 0$

$15Q^2 - 10Q - 12Q + 8 = 0$

$5Q(3Q-2) - 4(3Q-2) = 0$

$(5Q-4)(3Q-2) = 0$

$\Rightarrow Q = \frac{4}{5}, \frac{2}{3}$

$\therefore P > Q$

34. (d) **I.** $20P^2 - 17P + 3 = 0$

$20P^2 - 12P - 5P + 3 = 0$

$5P(4P-1) - 3(4P-1) = 0$

$\Rightarrow P = 3/5, 1/4$

II. $20Q^2 - 9Q + 1 = 0$

$20Q^2 - 4Q - 5Q + 1 = 0$

$4Q(5Q-1) - 1(5Q-1) = 0$

$(4Q-1)(5Q-1) = 0$

$\Rightarrow Q = 1/4, 1/5$

$\therefore P \geq Q$

35. (a) **I.** $20P^2 + 31P + 12 = 0$

$20P^2 + 16P + 15P + 12 = 0$

$5P(4P+3) + 4(4P+3) = 0$

$\therefore P = -4/5, \frac{-3}{4}$

II. $21Q^2 + 23Q + 6 = 0$

$21Q^2 + 14Q + 9Q + 6 = 0$

$7Q(3Q+2) + 3(3Q+2) = 0$

$(7Q+3)(3Q+2) = 0$

$\Rightarrow Q = -3/7, -2/3$

$\therefore Q > P$

36. (c) **From I :**

If $\sqrt{2304} = a$

then $a = \pm 48$

(Do not consider – 48 as value of a)

Again,

From II :

If $b^2 = 2304$ then $b = \pm 48$

Hence $a = b$.

37. (b) **I.** $12a^2 - 7a + 1 = 0$

II. $15b^2 - 16b + 4 = 0$

Sum of the two values of a, i.e., $(a_1, + a_2)$

$= \frac{-(-7)}{12} = \frac{7}{12}$

Similarly,

Sum of the two values of b,

i.e., $(b_1 + b_2) = \frac{-(-16)}{15} = \frac{16}{15}$

Since $\frac{7}{12} < \frac{16}{15}$

Therefore, $a < b$,

Now check the equality of root

$(12\times 4 - 15\times 1)^2 = \{12\times(-16) - 15\times(-7)\}$

$\{(-7)\times 4 - (-16)\times 1\}$

$\Rightarrow 33^2 = \{-87\}\{-12\}$

$\Rightarrow$ 1089 = 1044, which is not true.

Therefore, our answer should be $a < b$.

38. (b) **I.** $a^2 + 9a + 20 = 0$

Break 9 as F_1 and F_2, so that $F_1 \times F_2 = 20$ and $F_1 + F_2 = 9$.

Therefore, $F_1 = 5, F_2 = 4$

Now one value of $a = \frac{-5}{1} = -5$

other value of $a = \frac{-20}{5} = -4$

II. $2b^2 + 10b + 12 = 0$

The two parts of 10, ie $F_1 = 6$ and $F_2 = 4$

$\therefore$ Value of $b = \frac{-6}{2} = -3$ and $\frac{-12}{6} = -2$

Obviously $b > a$.

If general form of quadratic equation is $ax^2 + bx + c = 0$,

then split b into two parts so that $b_1 + b_2 = b$ and $b_1 \times b_{2e} = a \times c$

Now say b_1 as F_1 and b_2 as F_2. Then the values of 'x' will be $\frac{-F_1}{a}$ and $\frac{-C}{F_1}$ or

$\frac{-F_2}{a}$ and $\frac{-C}{F_2}$

39. (a) **I.** $3a + 2b = 14$

II. $a + 4b = 13$

Substract equation I from equation II after multiplying II by 3.

We get $3a + 12b - 3a - 2b = 39 - 14$

$\Rightarrow 10b = 25$

$\Rightarrow b = 2.5$

Put value of b in equation II. We set $a + 4 \times 2.5 = 13$.

Therefore, $a = 3$. Thus, $a > b$

40. (e) **I.** $a^2 - 7a + 12 = 0$

Here, $F_1 = -4$ and $F_2 = -3$

Now, values of $a = \frac{-(-4)}{1} = 4$

and $\frac{-12}{-4} = 3$

II. $b^2 - 9b + 20 = 0$

Here $F_1 = -5$ and $F_2 = -4$

Now, values of $a = \frac{-(-5)}{1} = 5$

and $\frac{-20}{-5} = 4$

Thus $b \geq a$.

41. (b) $\frac{5}{28} \times \frac{9}{8} p = \frac{15}{14} \times \frac{13}{16} a$

or, $\frac{45p}{224} = \frac{195q}{224}$

or, $3p = 13q$

$\therefore p > q$

42. (b) (i) $p - 7 = 0$ or, $p = 7$

(ii) $3q^2 - 10q + 7 = 0$

or, $3q^2 - 3q - 7q + 7 = 0$

or, $3q(q-1) - 7(q-1) = 0$

or, $(3q-7)(q-1)$ or, $q = 1$ or, $\frac{7}{3}$

$\therefore p > q$

43. (c) (i) $4p^2 2 = 16; p = \sqrt{4} = 2$

(ii) $q^2 - 10q + 25 = 0 \Rightarrow (q-5)(q-5) = 0$

or, $q = 5 \therefore q > p$

44. (e) (i) $4p^2 - 5p + 1 = 0$ or, $4p^2 - 4p - p + i = 0$

or, $4p(p-1) - 1(p-1) = 0$

or, $(4p-1)(p-1) = 0$

or, $p = 1$ and $p = \frac{1}{4}$

(ii) $q^2 - 2q + 1 = 0$

$\Rightarrow (q-1)(q-1) = 0$

or, $q = 1$

$\therefore q \geq p$

45. (c) $q = 5, 6$ & $p = \frac{3}{2}, 2$

46. (c) I. $4q^2 + 8q = 4q + 8$ II. $p^2 + 9p = 2p - 12$

or, $q^2 + q - 2 = 0$ or, $p^2 + 7p + 12 = 0$

or, $(q-1)(q+2) = 0$ or, $(p+p)(p+(c) = 0$

$\therefore q = 1$ or -2 $\therefore p = -3$ or -4

Hence, q > p

47. (c) I. $2p^2 + 40 = 18p$ II. $q^2 = 13q - 42$

or, $p^2 - 9p + 20 = 0$ or, $q^2 - 13q + 42 = 0$

or, $(p-4)(p-5) = 0$ or, $(q-7)(q-6) = 0$

$\therefore p = 4$ or 5 $\therefore q = 6$ or 7

Hence, $q > p$

48. (d) I. $6q^2 + \frac{1}{2} = \frac{7}{2} q$ II. $12p^2 + 2 = 10p$

or, $12q^2 - 7q + 1 = 0$ or, $6p^2 - 5p + 1 = 0$

or $\left(q - \frac{1}{4}\right)\left(q - \frac{1}{3}\right) = 0$ or, $\left(p - \frac{1}{3}\right)\left(p - \frac{1}{2}\right) = 0$

$\therefore q = \frac{1}{4}$ or $\frac{1}{3}$ $\therefore p = \frac{1}{3}$ or $\frac{1}{2}$

Hence, $p \geq q$

49. (b) **I.** $y^2 - 6y + 9 = 0$

or, $(y-3)^2 = 0$ or, $y = 3$

II. $x^2 + 2x - 3 = 0$ or, $x = 1, -3$

Hence, $y > x$

50. (a) **I.** $x^2 - 5x + 6 = 0$

or, $(x-3)(x-2) = 0$ or, $x = 2, 3$

II. $2y^2 - 3y - 5 = 0$

or, $y = 1, -\frac{5}{2}$

Hence, $x > y$

51. (c) **I.** $x = \sqrt{256} = 16$

II. $y = (-4)^2 = 16$

Hence, $x = y$

52. (b) **I.** $x^2 - 6x + 5 = 0$ or, $x = 1, 5$

II. $y^2 - 13y + 42 = 0$

or, $(y-7)(y-6) = 0$ or, $y = 6, 7$

Hence, $y > x$

53. (b) **I.** $x^2 + 3x + 2 = 0$

or, $(x+2)(x+1) = 0$

or, $x = -2$ or, -1

II. $y^2 - 4y + 1 = 0$ or, $y = 2 \pm \sqrt{3}$

Hence, $y > x$

54. (d) $3x - 5y = 5$ ...(i)

And $\frac{x}{x+y} = \frac{5}{7} \Rightarrow 7x = 5x + 5y$

$\Rightarrow 2x = 5y$ ...(ii)

From (i) and (ii), $x = 5$ and $y = 2$

$\therefore x - y = 3$

55. (d) Let the number be x.

$\therefore \frac{5}{7} \times \frac{4}{15} \times x - \frac{2}{5} \times \frac{4}{9} \times x = 8$

or, $x = \frac{8 \times 315}{12} = 210$

$\therefore$ Half of the number = 105

56. (a) Let the two-digit number be $10x + y$.

Then, $(10x + y) - (10y + x) = 36$

or. $x - y = 4$

57. (c) Reqd no. $= \frac{2}{5} \times 200 - \frac{3}{5} \times 125$

$= 80 - 75 = 5$

58. (e) $(x-1) = 2 \Rightarrow x = 3$

59. (d) Let the no. be x.

Then, $x-\frac{x}{3}=\frac{2}{3}x$

or, $\frac{2}{3}x=\frac{2}{3}x$

So, can't be determined is the correct choice.

60. (b) Let the cost of a pen and a pencil be ₹ 'x' and ₹ 'y' respectively. We have to find $(x-y)$.
From the question,
$2x+3y=86$ ..... (i)
$4x+y=112$ ......(ii)
Subtracting (i) from (ii), we get
$2x-2y=26$ or, $x-y=13$

61. (a) Let the two-digit no. be $10x+y$.
Then, $(10x+y)-(10y+x)=36$
or, $9(x-y)=36$
or, $x-y=4$

62. (d) Suppose the two-digit number is
$10x+y$
Then, $10y+x=20x+y/2$
or $20y+2x=40x+y$ or, $y=2x$

63. (b) $9A^2=12A+96 \Rightarrow 3A^2-4A-32=0$

$\therefore A=\frac{4\pm\sqrt{16+384}}{6}=4,-\frac{8}{3}$

$B^2=2B+3 \Rightarrow B^2-2B-3=0$

$\therefore B=\frac{2\pm\sqrt{4+12}}{2}=3,-1$

$\therefore 5A+7B=5\times4+7\times3=20+21=41$

64. (e) Let the original number of sweets be x.
According to the question,

$\frac{x}{140}-\frac{x}{175}=4$

or, $175x-140x=4\times140\times175$

or, $x=\frac{4\times140\times175}{35}=2800$

65. (a) Let the two-digit number be $10x+y$.
$10x+y=7(x+y) \Rightarrow x=2y$...(i)
$10(x+2)+y+2=6(x+y+4)+4$
or $10x+y+22=6x+6y+28 \Rightarrow 4x-5y=6$ ... (ii)
Solving equations (i) and (ii), we get $x=4$ and $y=2$

66. (a) Ratio of Ramani's savings in NSC and PPF = 3 : 2

His savings in PPF = $\frac{2}{5}\times150000$ $=60000$

68. (c) Let x be the first number and y be the second number.

$\frac{1}{5}x=\frac{5}{8}y$ $\quad\therefore \frac{x}{y}=\frac{25}{8}$ .... (i)

$x+35=4y$ or, $\frac{25}{8}y+35=4y$

$\therefore y=40$

68. (e) Let the original number be $10x+y$
$y=2x+1$....(i)
and $(10y+x)-(10x+y)=10x+y-1$
or, $9y-9x=10x+y-1$
or, $19x-8y=1$ ...(ii)
Putting the value of (i) in equation (ii) we get,
$19x-8(2x+1)=1$
or, $19x-16x-8=1$
or, $3x=9$ or, $x=3$
So, $y=2\times3+1=7$
$\therefore$ original number $=10\times3+7=37$

69. (d) **In case I:** Let the no. of children = x.

Hence, total no. of notebooks distributed

$\frac{1}{8}x.x$ or $\frac{x^2}{8}$ .....(i)

In case II: no. of children = $\frac{x}{2}$

Now, the total no. of notebooks

$=16\times\frac{x}{2}$ ..... (ii)

Comparing (i) & (ii), we get

$\frac{x^2}{8}=8x$

or, $x=64$

Hence, total no. of notebooks

$=\frac{64\times64}{8}=512$

70. (a) Let the positive integer be x.
Now, $x^2-20x=96$
or, $x^2-20x-96=0$
or, $x^2-24x+4x-96=0$
or, $x(x-24)+4(x-24)=0$
or, $(x-24)(x+4)=0$
or, $x=24,-4$
$x\neq-4$ because x is a positive integer

71. (a) Let ½ of the no. = $10x+y$
and the no. = 10V + W From the given conditions, W = x and V = $y-1$
Thus the no. = $10(y-1)+x$ …..(A)
$\therefore$ $2(10x+y)=10(y-1)+x \Rightarrow 8y-19x=10$…(i)
Again, from the question,
$V+W=7 \Rightarrow y-1+x=7$
$\therefore$ $x+y=8$ …(ii)
Solving equations (i) and (ii), we get $x=2$ and $y=6$.
$\therefore$ From equation ((A), Number $=10(y-1)+x=52$

72. (e) Suppose the two-digit number be $10x + y$.
Then we have been given
$10x + y - (10y + x) = 9$
$\Rightarrow 9x - 9y = 9$
$\Rightarrow x - y = 1$
Hence, the required difference = 1
Note that if the difference between a two-digit number and the number obtained by interchanging the digits is D, then the difference between the two digits of the number = $\frac{D}{9}$

73. (c) Suppose the number is N.
Then $N - \frac{3}{5}N = 50$
$\Rightarrow \frac{2N}{5} = 50, \quad \therefore N = \frac{50 \times 5}{2} = 125$

74. (d) Let the original fraction be $\frac{x}{y}$.
Then $\frac{x+2}{y+1} = \frac{5}{8}$ or, $8x - 5y = -11$ .... (i)
Again, $\frac{x+3}{y+1} = \frac{3}{4}$ or, $4x - 3y = -9$ .... (ii)
Solving, (i) and (ii) we get $x = 3$ and $y = 7$
$\therefore$ fraction = $\frac{3}{7}$

75. (d) On solving equation we get
$x = 7, y = 4, z = 11$

76. (e) Let the number = x
Then, $x^2 + x = 182$
or, $x^2 + x - 182 = 0$
or,x $+14x - 13x - 182 = 0$
or, $x(x + 14) - 13(x + 14) = 0$
or,$(x - 13)(x + 14) = 0$
or, $x = 13$ (negative value is neglected)

77. (c) Let the no. of balls = b
Rate = $\frac{450}{b}$
$(b+5)\left(\frac{450}{b} - 15\right) = 450$
or, $450 - 15b + \frac{2250}{b} - 75 = 450$
or, $b^2 + 5b - 150 = 0$
or, $(b + 15)(b - 10) = 0$
or, $b = 10$ (Neglecting negative value)

78. (b) $n^4 - 10n^3 + 36n^2 - 49n + 24$
$1 - 10 + 36 - 49 + 24 = 2$

79. (b) Let 'x' be the total number of students in college
$$x - \left[\frac{12x}{100} + \frac{3x}{4} + \frac{10x}{100}\right] = 15$$
$$x - \left[\frac{48x + 300x + 40x}{400}\right] = 15 \quad \therefore x = 500$$

80. (a) Let the first, second, third and fourth numbers be a, b, c and d respectively.
According to the question,
$a + b + c + d = 64$ ....(i)
and $a + 3 = b - 3 = 3c = \frac{d}{3}$
i.e., $a + 3 = b - 3 \Rightarrow b = a + 6$ ....(ii)
Also, $c = \frac{a+3}{3}$ and $d = 3(a + 3)$
Solving the above eqns, we get
$a = 9, b = 15, c = 4$ and $d = 36$
$\therefore$ Difference between the largest and the smallest numbers = $36 - 4 = 32$

81. (d) Let the no. of boys and girls in the classroom is x each.
From the question,
$2(x - 8) = x$ $\therefore x = 16$
$\therefore$ Number of boys and girls = $16 + 16 = 32$

82. (d) $x - y = \frac{1}{9}\{(10x + y) - (10y + x)\} = \frac{1}{9}$
$(9x - 9y) = x - y$

83. (b) By trial and error method.

84. (e) $2x + y = 15 \Rightarrow y = 15 - 2x$ ..... (i)
$2y + z = 25 \Rightarrow 2(15 - 2x) + z = 25$ [from (i)]
$\Rightarrow 4x - z = 5$ ..... (ii)
and $2z + x = 26$ ..... (iii)
Combining equation (ii) and (iii), we get $z = 11$

85. (d) $P(P - 3) < 4(P - 3)$;
$P(P - 3) - 4(P - 3) < 0$
$(P - 3)(P - 4) < 0$
This means that when
$(P - 3) > 0$ then $(P - 4) < 0$ ....(i)
or, when $(P - 3) < 0$ then $(P - 4) > 0$ ....(ii)
From (i),
$P > 3$ and $P < 4$
$\therefore \quad 3 < P < 4$
From (ii)
$P < 3$ and $P > 4$

86. (d) $P + R + 2Q = 59$;
$Q + R + 3P = 68$
and $P + 3(Q + R) = 108$
Solving the above two equations, we get $P = 12$years.

87. (a) Let the ages of Harish and Seema be x and y respectively.
According to the question,
$x.y = 240$ .... (i)
$2y - x = 4$ ....(ii)
Solving equations (i) and (ii), we get
$y = 12$ years

88. (e) $5P9+3R7+2Q8=1114$

For the maximum value of Q, the values of P and R should be the minimum, i.e. zero each.

Now, $509+307+2Q8=1114$

or, $816+2Q8=1114$

or, $2Q8=(1114-816=)\,298$

So, the reqd value of Q is 9.

89. (e) $\frac{2}{5}\times\frac{1}{4}\times\frac{3}{7}\times x=15$

$\therefore\ \frac{x}{2}=\frac{5\times7\times2\times5}{2}=25\times7=175$

90. (d) Let, the two-digit no. be xy, i.e. $10x+y$ then,

$x+y=\frac{1}{11}[(10x+y)+(10y+x)]=x+y$

Thus we see that the difference of x and y can't be determined.

Hence, the answer is data inadequate.

91. (c) Let the fraction be $\frac{x}{y}$ then

$\frac{x+1}{y+2}=\frac{2}{3}$ or, $3x+3=2y+4$ or, $3x=2y+1$ I

Also, we have

$\frac{x+5}{y+1}=\frac{5}{4}$

or, $4x+20=5y+5$

or $4x=5y-15$ II

From I and II, we get

$\frac{2y+1}{3}=\frac{5y-15}{4}$

or, $8y+4=15y-45$

$\therefore\ y=7$ and $x=\frac{2y+1}{3}=\frac{2\times7+1}{3}=\frac{15}{3}=5$

$\therefore$ Reqd original fraction $=\frac{x}{y}=\frac{5}{7}$

92. (d) Let the no. be $10x+y$

then $y=x+2$ or $y-x=2$ ...... (i)

$(10y+x)-(10x+y)=18$

or, $9y-9\boldsymbol{x}=18$

$y-x=2$ ...... (ii)

From eq. (i) and (ii) we can't get any conclusion.

93. (d) $2x+y=17\Rightarrow y=17-2x$ ...... (i)

$y+2z=15\Rightarrow 17-2x+2z=15$

$\Rightarrow 2x-2z=2\Rightarrow x-z=1$ ...... (ii)

and $x+z=9$ ...... (iii)

Solving equations (i) and (ii), we get

$x=5, z=4$

$\therefore\ y=17-2x=17-10=7$

$4x+3y+z=4\times5+3\times7+4$

$=20+21+4=45$

94. (c) Let the numerator and denominator be x and y respectively. Then $\frac{x+2}{y+3}=\frac{7}{9}$

or, $9(x+2)=7(y+3)$

or $9x-7y=3$ ...... (i)

$\frac{x-1}{y-1}=\frac{4}{5}$

$\Rightarrow 5x-4y=1$ ...... (ii)

Solving (i) and (ii), we get

$x=5, y=6$

Reqd fraction = 5/6

95. (a) $3n^2-18n+24=0$

or, $n^2-6n+8=0$ or, $(n-4)(n-2)=0$

$\therefore\ n=4, 2$

$\therefore\ n>4$

96. (d) R – M = 7000 and S – M = 3000

Here, S + M + R can be found only when one more equation in terms of S and R is given. Therefore, Can't be determined is the correct answer.

97. (c) Let the no. be N.

Now, $\frac{3N}{5}-\frac{N}{2}=30$ or, $\frac{N}{10}=30$

or, $N=300$

80% of $N=240$

98. (b) Let the two-digit no. be $10x+y$.

Then, $(10x+y)-(10y+x)=27$

or, $x-y=3$

99. (a) $F+S=4S$

or, $F=3S\Rightarrow F:S=3:1$

The ages of father and son = 56 years

$\therefore$ Son's age $=\frac{1}{4}\times56=14$ years

100. (d) Let the number be x.

$\therefore\ \frac{2}{5}\times\frac{1}{4}\times\frac{5}{8}\times x=6$

$\therefore\ x=\frac{6\times5\times4\times8}{2\times1\times5}\times\frac{1}{2}=48$

101. (a) Let the two-digit number be $10x+y$.

Then $x+y=\frac{1}{5}(10x+y-10y-x)$

or, $x+y=\frac{9}{5}(x-y)$

or, $4x-14y=0\Rightarrow\frac{x}{y}=\frac{7}{2}$

Using componendo & dividendo, we have,

$\frac{x+y}{x-y}=\frac{7+2}{7-2}=\frac{9}{5}$ i.e., $x-y=5k$

Here k has the only possible value, $k=1$. Because the difference of two single-digit numbers will always be of a single digit.

102. (c) $J=\frac{2}{5}A,\ P=\frac{1}{4}\times\frac{2}{5}A=\frac{1}{10}A$

$\frac{1}{10}A-200=600 \therefore \frac{1}{10}A=800$

A = ₹ 8,000

103. (a) For Q to be maximum. P and R will also be maximum, i.e., $P=R=9$.
So, by putting the value of P and R in
$5P9-7Q2+9R6=823$, we get $Q=7$

104. (d) Let the two-digit no. be $10x+y$.
According to question,
$(10x+y)-(10y+x)=54$
$9x-9y=54$ $\quad x-y=6$

105. (c) Let the two numbers be x and y.
$\therefore$ $xy=192$, $x+y=28$ ....(i)
$\therefore$ $(x-y)^2=(x+y)^2-4xy=784-768=16$
$\therefore$ $x-y=4$ ....(ii)
Combining (i) and (ii), $x=16$, and $y=12$.

106. (e) Let the present ages of Mr. Ramesh and his son be x and y respectively.
$\therefore$ $x=4y$ and $(x+10)=2(y+10)$ Solving the above two equations, we get $x=20$ years and $y=5$ years

107. (e) Let the total number of discs of 2 kg and 5 kg be 'a' and 'b' respectively.
Then, $a+b=21$ and $5b=2a$
Solving the above two equations, we get $a=15$, $b=6$
$\therefore$ Weight of all discs together
$=15\times2+6\times5=60$ kg

108. (a) No. of 10-year periods = 6
$2\times2\times2\times2\times2\times2\times B=64B$

109. (b) Let the middle no. $=x$

$(x-2)+x+(x+2)=\frac{176}{4}-14$ or,

$3x=\frac{120}{4}$ or, $x=10$

110. (d) number of tables and chairs and tripled, so price is $12{,}090\times3=36{,}000$

111. (a) Price of 39 pencils $=\frac{4263.05}{253}\times39\approx$ ₹650

112. (c) Percentage of marks obtained by Sundari in first and second papers is 40% and 80% respectively. Percentage of marks obtained by Kusu in first and second papers is 50% and 90%, respectively. Percentage of marks obtained by Jyoti in first and second papers is 30% and 90% respectively.
From the above, we see that Jyoti's progress is maximum.