Chapter 2

Time Response Analysis

CHAPTER HIGHLIGHTS

- *Time Response*
- *Time Response of a First Order System*
- *Unit Ramp Response*
- *Unit Impulse Response*
- *Time Response of a Second Order System*
- *Transient Response Specifications of Second Order System*
- *Delay Time, t_d*
- *Rise Time, t_r*
- *Peak Time, t_p*
- *Peak Overshoot M_p*
- *Settling Time (t_s)*
- *Steady State Error Analysis*
- *Unit Step Input*
- *Steady State Errors for Different Types of System*
- *Parabolic Input*

TIME RESPONSE

The response given by the system which is function of time for the applied excitation is called time response.

Time response can be obtained by solving the differential equations governing the system or from the transfer function of the system and input given to the system.

$$C(t) = C_t(t) + C_{ss}(t)$$

The time response of a control system consists of the following two parts:

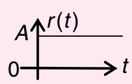
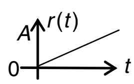
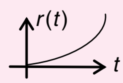
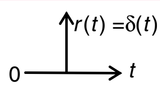
1. Transient response $C_t(t)$
2. Steady-state response $C_{ss}(t)$

TRANSIENT RESPONSE

The output variation during the time it takes to achieve its final value is called transient response.

The time required to achieve the final values is called transient period.

Table 1: Standard Test Signals

Name of the signal	Wave form	Physical interpretation	Mathematical representation	Laplace transform
Step		A sudden change or constant position	$r(t) = A \quad t \geq 0$ $= 0 \quad t < 0$	$\frac{A}{s}$
Ramp		A constant velocity	$r(t) = At \quad t \geq 0$ $= 0 \quad t < 0$	$\frac{A}{s^2}$
Parabolic		A constant acceleration	$r(t) = \frac{At^2}{2} \quad t \geq 0$ $= 0 \quad t < 0$	$\frac{A}{s^3}$
Impulse		A sudden shock	$r(t) = \infty; \quad t = 0$ $= 0 \quad t \neq 0$	1

For a stable operating system, transient response $C_t(t)$ can be written as follows:

$$\underset{t \to \infty}{\ell t}\ C_t(t) = 0$$

Steady-state Response

The response that remains after complete transient response vanishes (transient period). Steady-state response indicates the accuracy of a system.

Standard Test Signals

The information of input signals is required to pre-estimate the response of a system. The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity, and a constant acceleration. Therefore, the equivalent test signals are used as input signals to predict the performance of the system.

Type and Order of the System

Type of the System

Type of a system is defined as the number of open loop poles at origin.

Steady-state behaviour of the system depends on the type of the system.

For example,

$$G(s)H(s) = \frac{K(1+T_{z_1}s)(1+T_{z_2}s)(1+T_{z_3}s)...}{s^n(1+T_{p_1}s)(1+T_{p_2}s)(1+T_{p_3}s)...}$$

If $n = 0 \Rightarrow$ type '0' system
If $n = 1 \Rightarrow$ type '1' system
Therefore, n = type of the system

Order of the System

The highest power of s in the denominator of the closed-loop transfer function is defined as the order of the system.

For example,
Transfer function =

$$\frac{C(S)}{R(S)} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + ...a_n}{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + ... + b_m}$$

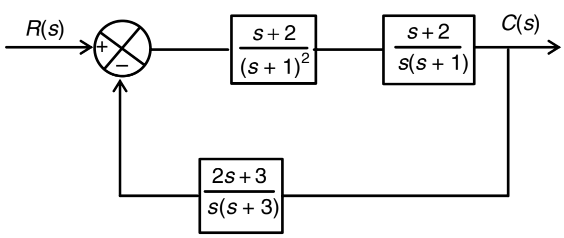
In the above example, the behaviour of the system depends on the order of the system.

Transient behaviour of the system depends on the order of the system.

Solved Examples

Example 1

The feedback control system shown in the figure is



(A) Type '0' system (B) Type '1' system
(C) Type '2' system (D) Type '3' system

Solution

Number of integrators in the open-loop transfer function are '2'. So the type of the system is '2'.

Example 2

A system has the following transfer function

$$G(s) = \frac{10(s+5)(s+50)}{s^4(s+1)(s^2+3s+10)}$$

The type and order of the system are, respectively,
(A) 4 and 9 (B) 4 and 7
(C) 5 and 7 (D) 7 and 5

Solution

Number of integrators = 4 = type of system

Highest power of 's' in denominator = order of the system = 7

Example 3

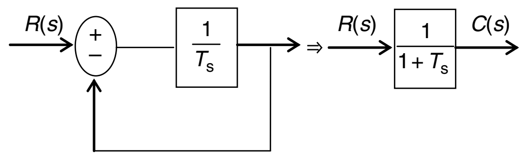
The type of a system depends on the
(A) No. of poles
(B) Difference between number of poles and zeros
(C) Number of real poles
(D) Number of poles it has at the origin

Solution: (D)

Time Response of a First-order System

A system with highest power of s in the denominator of its transfer function equal to 1 is known as first-order system.

Consider a first-order unity feedback control system shown in the figure.



Transfer function of the system = $\frac{C(s)}{R(s)} = \frac{1}{1+T_s}$

Unit-step Response

Unit-step input $r(t) = u(t)$

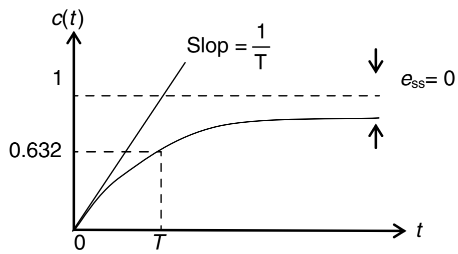
$$R(t) = \frac{1}{s}$$

Unit-step response in 's' domine $[C(s)] = \frac{1}{1+T_s} \times \frac{1}{s}$

Unit-step response in time domine $[c(t)] = 1 – e^{-t/T}$

Steady-state value of the response = $\underset{t\to\infty}{lt}\ c(t) = 1$

Transient response of the system = $e^{-t/\tau}$



Time response for unit-step input

Time constant: = Time takes for the step response to raise to 63% of its final value.

Rise time (T_r) Time taken by the step response to go from 10% to 90% of its final value.

$$T_r = 2.31T - 0.11T = 2.22T$$

Setting time (T_s)
Time taken by the step response to reach and stay with in 2% or 5% of its final value.

$$T_s = 4T \text{ for } \pm 2\% \text{ tolerance}$$
$$= 3T \text{ for } \pm 5\% \text{ tolerance}$$

Unit-ramp Response

Unit-ramp input $r(t) = t\,u(t)$

$$R(s) = \frac{1}{s^2}$$

Unit-ramp response in 's' domain $= \frac{1}{1+T_s} \times \frac{1}{s^2}$

Unit-ramp response in time domain $c(t) = L^{-1}\left\{\frac{1}{s^2(1+T_s)}\right\}$

$$e(t) = (t - \tau + \tau e^{-t/\tau})$$

Error for unit-ramp input $e(t) = r(t) - c(t) = (\tau - \tau e^{-t/\tau})$

The steady-state error for unit-ramp input $= \underset{t\to\infty}{lt}\ e(t)$

$$= \underset{t\to\infty}{lt}\ (\tau - \tau e^{-t/\tau})$$

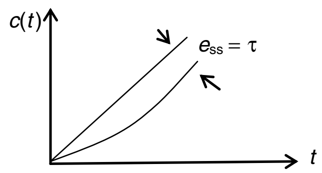


Figure 1 Unit-ramp response of first-order system.

Unit-impulse Response

Unit-impulse input $r(t) = \delta(t)$

$$R(s) = 1$$

Unit-impulse response in 's' domine $c(s) = \frac{1}{1+T_s}$

Unit-impulse response in time domain $c(t) = \frac{1}{T}e^{-t/T}$ for $t \geq 0$

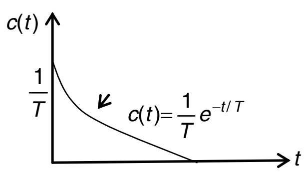


Figure 2 Unit-impulse response of first-order system.

Example 4

Transfer function of a system is given by

$$G(s) = \frac{20}{(s+20)}$$

Rise time of the system is

(A) 0.33 s
(B) 44 s
(C) 0.11 s
(D) 0.2 s

Solution

Transfer function $= \frac{20}{s+20} = \frac{1}{(0.05s+1)}$

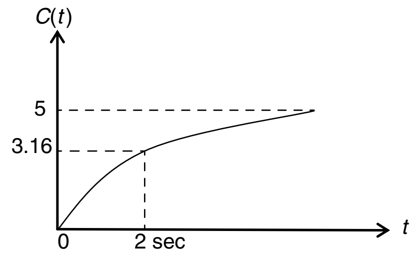
Time constant = 0.05 s

Rise time = 2.2 × time constant

= 0.11 s

Example 5

Time response of the system $G(s) = \frac{K}{s+a}$ is given by



Then the values of 'K' and 'a' are

(A) $K = 5, a = 2$
(B) $K = 5, a = 0.5$
(C) $K = 2.5, a = 0.5$
(D) $K = 2.5, a = 2$

Solution

Time constant = 2 s

Steady-state value = 5

Transfer function $= \frac{5}{2s+1} = \frac{2.5}{(s+0.5)}$

$$\therefore K = 2.5$$
$$a = 0.5$$

Example 6

Transfer function of the system is given by

$$G(s) = \frac{2}{(s+2)}$$

Time required for the system unit-step response to reach 95% of its final value is

(A) 0.5 s (B) 1 s (C) 1.5 s (D) 2 s

Solution

Transfer function $= \frac{2}{s+2} = \frac{1}{1+0.5s}$

Time constant = 0.5

Time required for unit-step response to reach 95% of its final value = 3 × Time constant = 1.5 s

Time Response of a Second-Order System

A system with highest power of s in the denominator of its transfer function equal to 2 is known as second-order system.

Closed-loop transfer function for a standard second-order system is given by the following:

Transfer function =

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{N(s)}{D(s)}$$

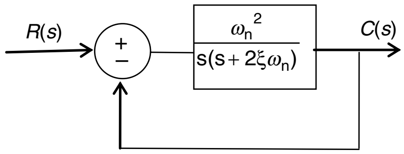


Figure 3 Block diagram of standard second-order system.

Where ω_n = Undamped natural frequency
ζ = Damping ratio constant

The tendency to oppose the oscillatory behaviour of the system is called damping, and is denoted by 'ζ'. As 'ζ' is increased, the response becomes progressively less oscillatory till it becomes critically damped ($\zeta = 1$) and becomes over damped for $\zeta > 1$.

The denominator polynomial of the transfer function $D(s)$ is called characteristic equation. The characteristic equation of the second-order system is

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

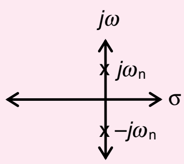
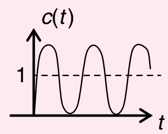
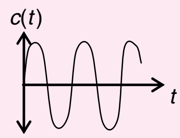
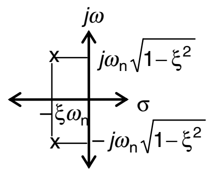
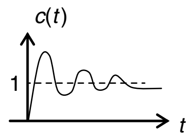
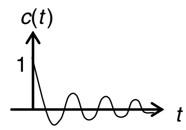
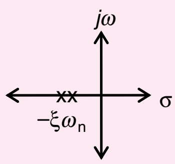
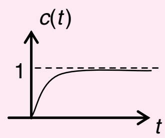
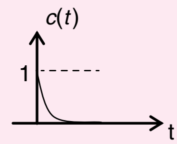
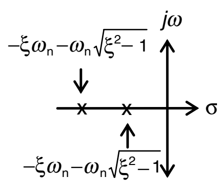
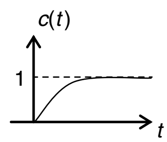
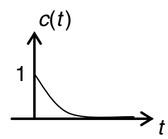
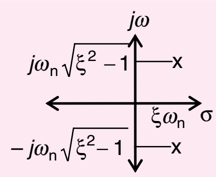
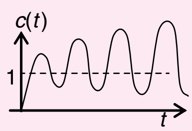
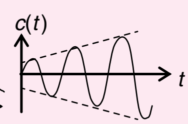
The roots of the characteristic equation are given as follows:

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

Where $\omega_d = \omega_n\sqrt{1-\xi^2}$ $\Rightarrow$ Damping natural frequency.

The time response of any system is characterized by the roots of the denominator polynomial (poles of the system), which depends on the damping ratio (ζ) of the system. Detailed summary of the system response characterization with variation is ζ/poles is given in Table.

Table 2 *Summary of Second-order System Response*

System characterization	Range of ζ	Poles of second-order system	Poles on the s-plane	Nature of the response	Unit–step response	Impulse response
Undamped	$\zeta = 0$	$\pm j\omega_n$		Constant frequency and magnitude oscillations		
Under damped	$0 < \zeta < 1$	$-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$		Damped oscillations		
Critically damped	$\zeta = 1$	$-\xi\omega_n$		Pure exponential		
Over damped	$\zeta > 1$	$-\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$		Slow exponential		
Negative damped	$\zeta < 0$	$\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$		Increasing oscillations		

Transient Response Specifications of Second-order System

The common model for physical problems is under damped second-order system due to its speed in reaching a steady state. The specifications of the under damped second-order system are shown in the following figure.

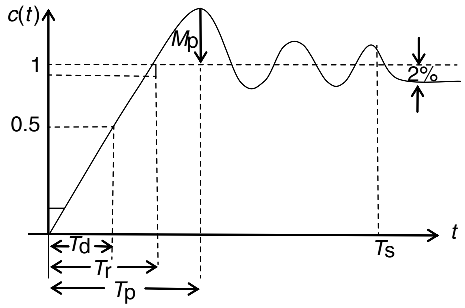


Figure 4 Unit-step response of the under damped second-order system.

Delay Time, t_d

The delay time is the time required for the response to reach half the final value for the very first time.

$$T_d = \frac{1+0.7\xi}{\omega_n}\ \text{s}$$

Rise Time, t_r

It is the time required for the response to rise from 0% to 100% of its final value for an undamped case. For under damped case and over damped case, it is 10% to 90% of its final value.

$$t_r = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{-\sigma}\right) = \frac{\pi-\beta}{\omega_d}$$

Where $\beta = \cos^{-1}\xi$

Peak Time, t_p

The peak time is the time required for the response to reach the first peak of the overshoot.

$$t_p = \frac{\pi}{\omega_d}$$

The peak time t_p corresponds to one half cycle of the frequency of damped oscillation.

Peak Overshoot M_p

The maximum overshoot is the maximum peak value of the response curve measured from unity.

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\%\ \text{peak overshoot} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100$$

Settling Time (t_s)

This is the time required for the response to reach and remain within a specified tolerance band of its final value.

The settling time corresponding to a ±2% or ±5% tolerance band may be measured in terms of the time constant $T = \frac{1}{\xi\omega_n}$

$$t_s = 4T = \frac{4}{\xi\omega_n} \quad \text{(2\% band)}$$

$$t_s = 3T = \frac{3}{\xi\omega_n} \quad \text{(5\% band)}$$

NOTE

Time period of oscillations $T_{osci} = \frac{2\pi}{\omega_d}$

Number of oscillations $(N) = \frac{t_s}{T_{osci}} = \frac{t_s\,\omega_d}{2\pi}$

Example 7

A unity feedback system has open loop transfer function as $G(s) = \frac{25}{s(s+25)}$

The natural frequency of the system is

(A) 10 (B) 5 (C) 25 (D) 2.5

Solution

Given open-loop transfer function = $\frac{25}{s(s+25)}$

Standard open-loop transfer function = $\frac{\omega_n^2}{s(s+2\xi\omega_n)}$

Therefore, $\omega_n^2 = 25$

Natural frequency $\omega_n = 5$ rad/s

Example 8

Given unity feedback with

$$G(s) = \frac{K}{s(s+5)}$$

The value of K for damping ratio of 0.5 is

(A) 1 (B) 25 (C) 5 (D) 2.5

Solution

Standard second-order OLTF = $\frac{\omega_n^2}{s(s+2\xi\omega_n)}$

Comparing with given OLTE $2\zeta\omega_n = 5$

$$2 \times 0.5 \times \omega_n = 5 \Rightarrow \omega_n = 5$$

$$\therefore K = \omega_n^2 = 25$$

Example 9

The unit-impulse response of a second-order system is $\frac{1}{6}e^{-0.8t}\sin(0.6t)$. Then, the natural frequency and damping ratio of the system are, respectively,

(A) 2 and 0.3 (B) 1 and 0.6
(C) 1 and 0.8 (D) 2 and 0.8

Solution

Transfer function = $L\{\text{Impulse response}\}$

$$= \frac{1}{6} \cdot \frac{0.6}{(s+0.8)^2 + (0.6)^2}$$

$$\text{Transfer function} = \frac{0.1}{s^2 + 1.6s + 1}$$

After comparing with standard second-order transfer function

$$\omega_n = 1, 2\zeta\omega_n = 1.6$$

$$\zeta = 0.8$$

Example 10

A second-order system has

$$M(j\omega) = \frac{100}{100 - \omega^2 + j20\omega}$$

If M_P (peak magnitude) is

(A) 100 (B) 1 (C) 0.5 (D) 2

Solution

Comparing with standard second-order system

$$\zeta = 1, \omega_n = 10$$

System is critically damped ⇒ Peak magnitude = 1

Example 11

A unity feedback control system has a forward path transfer function equal to $G(s) = \frac{30.25}{s(s+5.5)}$

The unit-step response of the system starting form rest will have its maximum value at a time equal to

(A) 0.5 s (B) 0.66 s (C) 0 s (D) Infinity

Solution

$$\text{Standard form of second-order OLTF} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

Therefore, $\omega_n = 5.5, \zeta = 0.5$

$$\text{Peak time } (t_p) = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.66 \text{ s}$$

Example 12

A plant has the following transfer function:

$$G(s) = \frac{1}{s^2 + 0.4s + 1}$$

For a step input, it is required that the response value settles to within 2% of its final value. The plant setting time is

(A) 10 s (B) 20 s (C) 30 s (D) Infinity

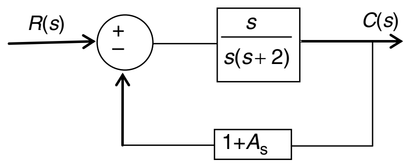
Solution

For the given system transfer function $\omega_n = 1, \zeta = 0.2$

$$\text{Setting time of 2\% tolerance } (t_s) = \frac{4}{\xi\omega_n} = \frac{4}{0.2} = 20 \text{ s}$$

Example 13

The block diagram of a closed-loop control system is given in the figure. The values of K and A such that the system has a damping ratio of 0.8 and undamped natural frequency ω_n of 5 rad/s are, respectively, equal to



(A) 4 and 0.8 (B) 5 and 0.24
(C) 25 and 0.24 (D) 20 and 0.8

Solution

$$\text{Transfer function of the given system} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{K}{s^2 + (2+AK)s + (K)} \quad (1)$$

$$\text{Required transfer function} = \frac{25}{s^2 + 8s + 25} \quad (2)$$

Compare equations (1) and (2)

$K = 25$

$2 + AK = 8$

$25A = 6 \Rightarrow A = \frac{6}{25} = 0.24$

Steady-state Error Analysis

Steady-state error is the difference between the input and the output for a prescribed test input as $t \to \infty$.

A generalized expression for steady-state error is obtained by using feedback system shown in the figure.

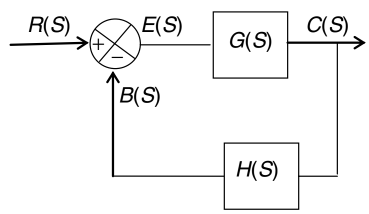


Figure 5 Feedback control system.

From the above block diagram,

$$E(s) = R(s) - B(s) = R(s) - C(s)\,H(s)$$

$$E(s) = R(s) - E(s)\,G(s)\,H(s)$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

The steady-state error using final value theorem can be written as follows:

$$e_{ss} = \underset{t\to\infty}{lt}\ e_{ss}(t) = \underset{s\to 0}{lt}\ sE(S) = \underset{s\to 0}{lt}\ s.\frac{R(s)}{1+G(s)H(s)}$$

From the above expression, it is evident that steady-state error depends on input $R(s)$ and the open-loop transfer function $G(s)\,H(s)$.

The expressions for steady-state errors for various types of standard test signals are given below.

Unit-step Input

Steady-state error =

$$e_{ss} = \underset{s\to 0}{lt}\frac{sR(s)}{1+G(s)H(s)} = \underset{s\to 0}{lt}\frac{s.1/s}{1+G(s)H(s)}$$

$$e_{ss} = \frac{1}{1+\underset{s\to 0}{lt}\,G(s)H(s)} = \frac{1}{1+K_P}$$

$$K_P = \underset{s\to 0}{lt}\,G(s)H(s) = \text{position error constant}$$

Unit-ramp Input

Steady-state error =

$$e_{ss} = \underset{s\to 0}{lt}\frac{s\,.\,1/s^2}{1+G(s)H(s)} = \underset{s\to 0}{lt}\frac{1}{s+sG(s)H(s)}$$

$$= \frac{1}{\underset{s\to 0}{lt}\,G(s)H(s)} = \frac{1}{K_V}$$

$$K_V = \underset{s\to 0}{lt}\,sG(s)H(s) = \text{velocity error constant}$$

Unit-parabolic Input

Steady-state error $e_{ss} = \underset{s\to 0}{lt}\dfrac{s\,.\,1/s^3}{1+G(s)H(s)}$

$$= \frac{1}{\underset{s\to 0}{\mathrm{lt}}\,s^2G(s)H(s)} = \frac{1}{K_a}$$

$$K_a = \underset{s\to 0}{lt}\,s^2G(s)H(s) = \text{Acceleration error constant}$$

Steady-state Errors for Different Types of Systems

Type 0 System

Open-loop transfer function $G(s)\,H(s)$

$$= \frac{K(T_{z1}S+1)(T_{z2}S+1)..........}{(T_{p1}S+1)(T_{p2}S+1)......}$$

For step input $\quad e_{ss} = \dfrac{1}{1+K_P}$

Position error constant $(K_P) = \underset{s\to 0}{lt}\,G(s)H(s) = K$

Steady-state error $(e_{ss}) = \dfrac{1}{1+K}$

For ramp input
Velocity error constant $(K_V) = \underset{s\to 0}{lt}\,sG(s)H(s) = 0$
Steady-state error$(e_{ss}) = \infty$

For parabolic input
Acceleration error constant $(K_a) = \underset{s\to 0}{lt}\,s^2G(s)H(s) = 0$
Steady-state error $(e_{ss}) = \infty$

Type 1 System

Open loop transfer function

$$G(s)H(s) = \frac{K(T_{z1}s+1)(T_{z2}s+1)(T_{z3}s+1)......}{S(T_{p1}s+1)(T_{p2}s+1)(T_{p3}s+1)......}$$

Step input
Position error constant $(K_P) = \underset{s\to 0}{lt}\,G(s)H(s) = \infty$

Steady state error $(e_{ss}) = \dfrac{1}{1+K_p} = 0$

Parabolic input
Acceleration constant $(K_a) = \underset{s\to 0}{lt}\,S^2G(s)H(s) = 0$

Steady-state error $(e_{ss}) = \dfrac{1}{K_a} = \infty$

Type 2 System

Open-loop transfer function

$$G(s)\,H(s) = \frac{K(T_{z1}s+1)(T_{z2}s+1)......}{s^2(T_{p1}s+1)(T_{p2}s+1).....}$$

Step input
Position error constant $(K_p) = \underset{s\to 0}{lt}\,G(s)H(s) = \infty$

Steady-state error $(e_{ss}) = \dfrac{1}{1+K_p} = 0$

Ramp input
Velocity error constant $(K_v) = \underset{s\to 0}{lt}\,sG(s)H(s) = \infty$

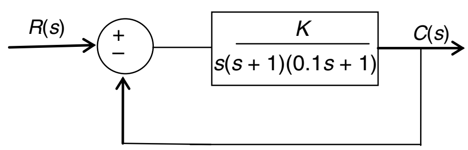
Steady-state error $(e_{ss}) = \dfrac{1}{K_v} = 0$

Parabolic input
Acceleration error constant $(K_a) = \underset{s\to 0}{lt}\,sG(s)\,H(s) = K$

Steady-state error $(e_{ss}) = \dfrac{1}{K_a} = \dfrac{1}{K}$

Example 14

The system shown in the figure has a unit-step input. In order that the steady-state error is 0.1, the value of 'K' required is



(A) 0.1 (B) 9 (C) 0.9 (D) 1.0

Solution

Steady-state error $= \underset{s\to 0}{lt}\,s.\dfrac{R(s)}{1+G(s)(\ \ )H(s)}$

Input is unit step

$$e_{ss} = \frac{1}{1+\underset{s\to 0}{lt}\,G(s)H(s)} = \frac{1}{1+K_p}$$

$$K_p = \underset{s\to 0}{lt}\, G(s)H(s) = \underset{s\to 0}{lt} \frac{K}{(s+1)(0.1s+1)} = K$$

From the given problem

$$e_{ss} = \frac{1}{1+K} = 0.1$$

$$K = 9$$

Example 15

If the time response of the system is given by the following expression:

$$y(t) = 5 + 4\sin(\omega t + \delta_1) + e^{-8t}\sin(\omega t + \delta_2) + e^{-4t}$$

Then, the steady-state part of the above response is given by

(A) $5 + e^{-4t}$
(B) $5 + 4(\sin(\omega t + \delta_1)$
(C) $5 + 4\sin(\omega t + \delta_1) + e^{-8t}(\omega t + \delta_2)$
(D) 5

Solution

Steady-state part = $\underset{t\to\infty}{lt}\; y(t)$

$$= 5 + 4\sin(\omega t + \delta_1)$$

Example 16

Which of the following equations give the steady-state error for a unity feedback system excited by

$$u_s(t) + tu_s(t) + t^2\frac{u_s(t)}{2}\,?$$

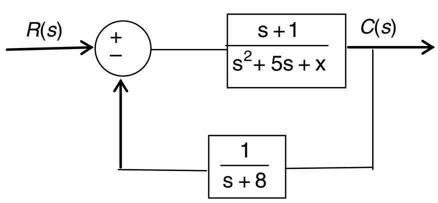
(A) $\frac{1}{1+K_p} + \frac{1}{K_v} + \frac{1}{K_a}$ (B) $\frac{1}{1+K_p} + \frac{1}{K_v} + \frac{2}{K_a}$

(C) $\frac{1}{K_p} + \frac{1}{K_v} + \frac{1}{K_a}$ (D) $\frac{1}{K_p} + \frac{1}{K_v} + \frac{2}{K_a}$

Solution: (A)

Example 17

For what values of x, does the system shown in figure have a zero steady-state error (timed) for a step input?



(A) $x = 5$
(B) $x > 1$
(C) $x = 0$
(D) For no value of x

Solution

When step input is applied, the steady-state error is zero when type of the system is greater than zero (1).

For $X = 0 \Rightarrow$ type of the system = 1

Example 18

The system $G(s) = \frac{0.4}{s^2 + s - 2}$ is excited by a unit-step input. The steady-state output is

(A) 0 (B) 0.83
(C) 1.25 (D) Unbounded

Solution

The given system is unstable. Therefore, the error is unbounded.

Example 19

The transfer function of a system is given by $\frac{(s+2)}{s(2s^3+3s^2+s)}$

The steady-state error of the system with input $r(t) = (8t + 4t^2)\, u(t)$ is

(A) 4 (B) 12 (C) 8 (D) 2

Solution

Given system is type '2' system. Steady-state error for $8t(u(t)$ is zero.

Steady-state error for $4t^2$

$$= \frac{A}{K_a}\left[\because \frac{At^2}{2} = \frac{8}{2}t^2\right]$$

$$= \frac{8}{\underset{s\to 0}{lt}\, s^2 G(s)H(s)} = \frac{8}{2} = 4$$

Example 20

Which of the following is the steady-state error for a step input applied to a unity feedback system with open-loop transfer function $G(s) = \frac{5}{s^2 + 14s + 25}$?

(A) $e_{ss} = 0.83$ (B) $e_{ss} = 0$
(C) $e_{ss} = 1$ (D) $e_{ss} = \infty$

Solution

Steady-state error for step input $= \frac{A}{1+K_p}$

$$= \frac{1}{1 + \underset{s\to 0}{lt}\, G(s)H(s)}.$$

$$= \frac{1}{1+0.2}\; e4 = 0.833$$

Exercises

Practice Problems I

Direction for questions 1 to 25: Select the correct alternative from the given choices.

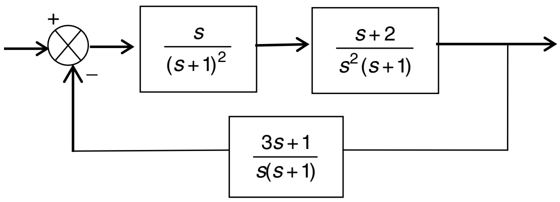
1. The closed-loop transfer function of a second-order system is

$$\frac{C(s)}{R(s)} = \frac{100}{s^2 + 16s + 100}$$

The type of damping in the system is
(A) underdamped. (B) over-damped.
(C) critically damped. (D) undamped.

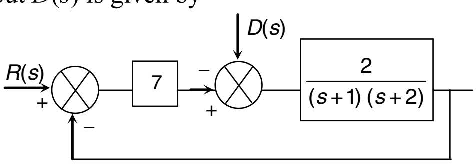
2. The unit-impulse response of a second-order system is $\frac{1}{6}e^{-0.8t}\sin(0.6t)$. The natural frequency and damped frequency oscillations are, respectively,
(A) 1 and 0.8. (B) 1 and 0.6.
(C) 0.8 and 1. (D) 1.6 and 1.

3. The feedback control system shown in the figure represents a



(A) Type 1 system (B) Type 2 system
(C) Type 3 system (D) None

4. The steady-state error due to unit-ramp disturbance input $D(s)$ is given by



(A) 0 (B) 0.012 (C) 0.021 (D) 0.025

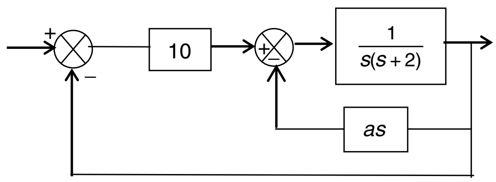
5. A unity feedback control system is represented by the open-loop transfer function $G(s) = \frac{K}{s(s+2)}$. The range values of K, so that the system remain under damped.
(A) $K < 1$ (B) $K > 1$
(C) $K = 1$ (D) None of these

6. For a proportional control action, which among the following are true?
(i) As gain K increases, ζ decreases.
(ii) The system tends to have large overshoots as gain increases.
(iii) As gain increases, the system will be over damped.
(iv) As gain increases, settling time will be more.
(A) All true (B) i, ii, and iv are true
(C) i and ii true (D) ii, iii, and iv true

7. The unit-impulse response of a system is given by $C(t) = 4e^{-3t}$. Find the step response of the system for $t > 0$
(A) $1 - e^{-3t}$ (B) $3\,e^{-3t}$
(C) $\frac{4}{3}(1 - e^{-3t})$ (D) e^{-3t}

8. If the system's transfer function is given by $T(s) = \frac{6}{s(s+2)(s+3)}$, then its time constant is
(A) $\frac{1}{6}$ (B) 6 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Direction for questions 9 and 10:
A feedback system employing output-rate damping is shown in the figure.



9. In the absence of derivative feedback, the damping factor and natural frequency of the system are
(A) 0.1 and 10 (B) 1 and 10
(C) $\frac{1}{\sqrt{10}}$ and $\sqrt{10}$ (D) $\frac{1}{\sqrt{5}}$ and $\sqrt{5}$

10. In the absence of derivative feedback, the steady-state error resulting from unit-ramp input is
(A) 5 (B) 0.2 (C) 2 (D) 0.02

11. A unity feedback system is characterized by an open-loop transfer function $G(s) = \frac{K}{s(s+20)}$. The value of K so that the system will have a damping ratio of 0.5 and the peak time is
(A) 200 and 0.157 s
(B) 400 and 0.1813 s
(C) 100 and 0.157 s
(D) $\sqrt{200}$ and 0.181 s

12. A unity feedback has a loop transfer function of $G(s) = \frac{20(s+5)}{s(s+1)(s+2)}$
The steady-state error for unit-ramp input is
(A) 50. (B) zero.
(C) infinite. (D) 0.02.

13. A second-order control system has

$$M(j\omega) = \frac{64}{64 - \omega^2 + 8\sqrt{2}\,j\omega}$$

Its M_p (Peak magnitude) is
(A) 0.5 (B) 1.5. (C) $\sqrt{2}$. (D) 1.

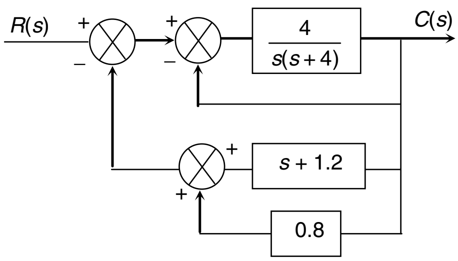
14. A servo system is given by the equation

$$C(t) = 1 - 2e^{-5t} + e^{-10t}$$

Find the natural frequency of the system, if step input is applied to the system.

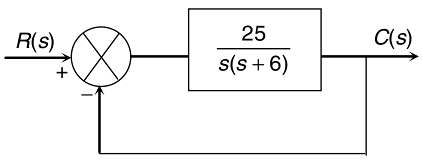
(A) 17.52 (B) 50.01 (C) 7.07 (D) 2.04

15. Obtain the damping factor of the following system:



(A) 0.8 (B) 4 (C) 4.3 (D) 3.46

16. The transfer function of second-order closed-loop control system is shown below. The value of its response is maximum when $t = t_{max}$. Then, $t_{max} = ?$



(A) π/8 (B) π/4 (C) π/2 (D) π

Direction for questions 17 and 18:

A unity feedback system is characterized by the open loop transfer function $G(s) = \dfrac{100}{s(s+10)}$

17. The static error constants (K_p, K_v, K_a) for the system are

(A) 0, 10, and ∞ (B) ∞, 10, and 0
(C) ∞, 0.1, and 0 (D) 0, 10, and 0

18. The steady-state error of the system when subjected to an input given by

$$r(t) = 3 + 4t + \frac{7}{2}t^2$$

(A) 0.4 (B) 0 (C) ∞ (D) 0.35

Direction for questions 19 and 20:

Open loop transfer function of a system is given by $G(s)H(s) = \dfrac{15}{5s^3 + 2s^2 + 3s}$

19. The steady-state error of the system when it is subjected to an input $r(t) = 5t$

(A) 1 (B) 3 (C) 5 (D)15

20. The steady-state error of the system when it is subjected to an input $r(t) = 5 + 8t + 3t^2$

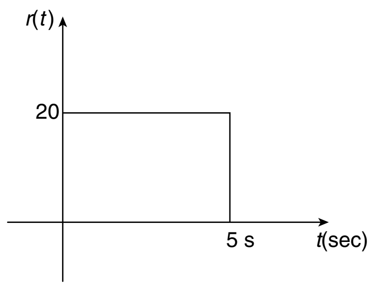
(A) 0 (B) 1 (C) 3 (D) ∞

21. Match the following:

Transfer function	Type of damping
1. $\dfrac{5}{s^2+10s+5}$	P. Undamped
2. $\dfrac{25}{s^2+10s+25}$	Q. Under damped
3. $\dfrac{25}{s^2+s+25}$	R. Critically damped
4. $\dfrac{5}{s^2+5}$	S. Over damped

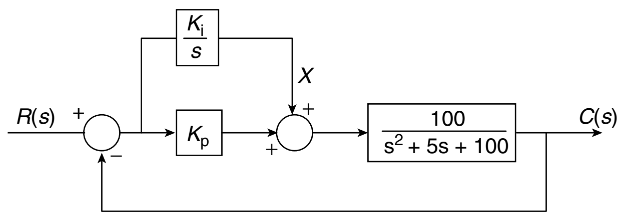
Codes:	P	Q	R	S
(A)	4	3	2	1
(B)	3	2	4	1
(C)	4	2	3	1
(D)	3	4	1	2

22. The steady-state error of a unity feedback linear system for unit-step input is 0.08. The steady-state error of the same system for a pulse input $r(t)$ having a magnitude of 20 and duration of 5 s as shown in the figure is



(A) 0 (B) 08 (C) 1.6 (D) 0.16

23. Consider the feedback system shown below which is subjected to a unit input. The system is stable and has the controller parameters as $K_p = 5$, $K_i = 10$. The steady-state value of X is



(A) 100 (B) 0 (C) 1 (D) 0.1

24. The roots of the characteristic equation are symmetric about origin and are on the real axis of the s-plane; then which one of the following is true?

(A) One row of zeros is present in the RH table.
(B) System is unstable.
(C) One sign change is present in the first column of the RH table.
(D) All of the above

25. If the imaginary part of second-order under damped closed-loop control system is increased and the real part remains same,

(A) ζ increases and ω_n decreases.
(B) ζ decreases and ω_n increases.
(C) ζ and ω_n decreases.
(D) ζ and ω_n increases.

Practice Problems 2

Direction for questions 1 to 15: Select the correct alternative from the given choices.

1. Consider the unity feedback control system with open loop transfer function

$$G(s) = \frac{k}{s(s+a)}$$

The steady-state error of the system due to a unit-step input is

(A) always zero.
(B) depends on the value of k.
(C) depends on a.
(D) depends on both k and a.

2. Given that $e(t) =$

$$C_0 r(t) + C_1\dot{r}(t) + \frac{C_2}{2!}\ddot{r}(t) + \frac{C_3}{3!}\dddot{r}(t) + ... + \frac{Cn}{n!}r^{n\bullet}(t)$$

where $e(t)$ and $r(t)$ represents error and input signal, respectively, and K_p, K_v and K_a represent static error constants then which of the following statements are true?

(i) Generalized error series gives error signal as a function of time.
(ii) Generalized error constants $C_0, C_1, C_{2,}....C_n$ are functions of time.
(iii) Using generalized error constants, steady-state error can be determined for any type of input.
(iv) Using static error constants the steady-state error can be determined for any type of input.

(A) i, ii, and iv (B) i, ii, and iii
(C) i, iii, and iv (D) i and iii

3. If $y(t) = A + X\sin(\omega t + \theta_1) + e^{-bt}\sin(\omega t + \theta_1)$ represents the equation of a system, then the steady-state part of the above response is

(A) A (B) $A + X\sin(\omega t + \theta_1)$
(C) $A + e^{-bt}(\sin \omega t + \theta_1)$ (D) $X\sin(\omega t + \theta_1)$

4. The steady-state error of first-order system with ramp input is

(A) 0 (B) T (C) 1 (D) $1 - T$

5. Which among the following are true?

(i) An over-damped system gives sluggish response.
(ii) An integral controller improves the steady-state response of the system.
(iii) The pole of an over damped system is in the real axis.
(iv) When $\zeta = 0$ the system is un damped one

(A) i and iii are true (B) i, ii, and iv are true
(C) i and ii are true (D) All are true

6. The position and velocity error constants for the following system are

$$G(s)\,H(s) = \frac{100}{(s+2)(s+5)}$$

(A) 10, 0 (B) 10, infinity
(C) .1, 0 (D) 0, 10

7. $\frac{K}{s(\tau s+2)}$ represents the open-loop transfer function of a unity feed back system. Keeping gain constant, what should be the change in time constant in order to reduce the damping factor from .8 to .4?

(A) $T_2 = 4T_1$ (B) $T_2 = 2T_1$
(C) $T_2 = \frac{T_1}{4}$ (D) $T_2 = \frac{T_1}{2}$

Direction for questions 8 and 9:
The open-loop transfer function of a unity feedback system is given by $G(s) = \frac{K}{s(1+sT)}$ where T and K are constants having positive values. By what factor should the amplifier gain be reduced so that

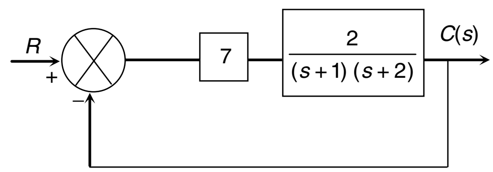
8. the peak overshoot of unit response of the systems is reduced from 75% to 50%

(A) 5.6 (B) 10 (C) 2.5 (D) 20

9. the damping ratio increases from 0.1 to 0.5

(A) 25 (B) 5 (C) $\sqrt{5}$ (D) $\frac{1}{\sqrt{5}}$

10. The maximum undershoot of the system given in the figure below will be



(A) 0.14 (B) 0.28
(C) 0.079 (D) 0.092

11. The open-loop transfer function of a unity feed back system is given as $\frac{10}{s(s+5)}$. The system is subjected to an input $r(t) = 1 + 5t$. Find the steady-state error.

(A) 2 (B) α (C) 2.5 (D) 1

12. A unity feedback system is characterized by the open-loop transfer function

$$G(s) = \frac{100}{s(5s+10)(2s+10)}$$

The steady-state errors for unit–step, unit–ramp, and unit–acceleration inputs are
(A) 1, 1, ∞ (B) 0, 1, 1 (C) 0, 1, ∞ (D) 1, 1, ∞

13. A servo system is given by the equation

$$C(t) = 1 - 2e^{-5t} + e^{-10t}$$

Find the natural frequency of the system, if step input is applied to the system.

(A) 17.52 (B) 50.01
(C) 7.07 (D) 2.04

14. A unity feedback control system has its open-loop transfer function $G(s) = \frac{8s+1}{16s^2}$. Determine an expression for time response when it is subjected to a unit-impulse input.

(A) $\frac{e^{-0.5t}}{20}[1+t]$ (B) $\frac{e^{-0.25t}}{20}[1+t]$

(C) $\frac{e^{-0.25t} - 4te^{-0.25t}}{20}$ (D) $\frac{te^{-.25t}+1}{20}$

15. The steady-state error of the system with loop transfer function $\frac{K}{(s+1)\,(s+2)}$ is given as 0.1. Find the value of gain K, if system is subjected to step input.

(A) 10 (B) 16
(C) 18 (D) 20

Previous Years' Questions

1. The open-loop transfer function of a unity feed back system is

$G(s) = \frac{K}{s(s^2+s+2)(s+3)}$. The range of K for which the system is stable is **[2004]**

(A) $\frac{21}{4} > K > 0$ (B) $13 > K > 0$

(C) $\frac{21}{4} < K < \infty$ (D) $-6 < K < \infty$

2. A ramp input applied to a unity feedback system results in 5% steady-state error. The type number and zero frequency gain of the system are, respectively, **[2005]**

(A) 1 and 20 (B) 0 and 20

(C) 0 and $\frac{1}{20}$ (D) 1 and $\frac{1}{20}$

3. Consider two transfer functions:

$$G_1(s) = \frac{1}{s^2+as+b} \text{ and } G_2(s) = \frac{s}{s^2+as+b}$$

The 3-dB band widths of their frequency response are, respectively, **[2006]**

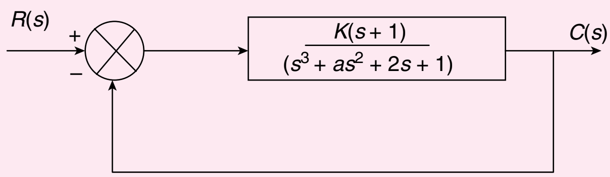
(A) $\sqrt{a^2-4b}, \sqrt{a^2+4b}$

(B) $\sqrt{a^2+4b}, \sqrt{a^2-4b}$

(C) $\sqrt{a^2-4b}, \sqrt{a^2-4b}$

(D) $\sqrt{a^2+4b}, \sqrt{a^2+4b}$

4. The positive values of K and a so that the system shown in the following figure oscillates at a frequency of 2 rad/s, respectively, are **[2006]**



(A) 1, 0.75 (B) 2, 0.75
(C) 1, 1 (D) 2, 2

5. If the closed–loop transfer function of a control system is given as $T(s) = \frac{s-5}{(s+2)(s+3)}$, then it is **[2007]**

(A) an unstable system
(B) an uncontrollable system
(C) a minimum phase system
(D) ss non-minimum phase system

6. The transfer function of a plant is $T(s) = \frac{5}{(s+5)(s^2+s+1)}$. The second-order approximation of $T(s)$ using dominant pole concept is **[2007]**

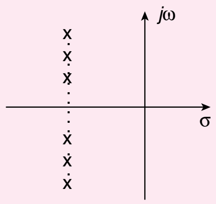
(A) $\frac{1}{(s+5)(s+1)}$

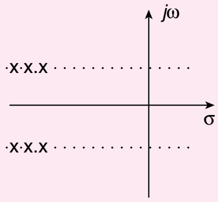
(B) $\frac{5}{(s+5)(s+1)}$

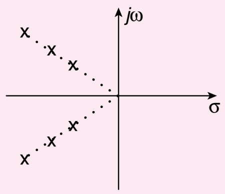
(C) $\frac{5}{s^2+s+1}$

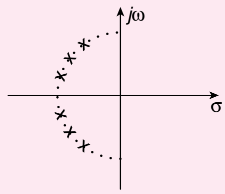
(D) $\frac{1}{s^2+s+1}$

7. Step responses of a set of three second-order underdamped systems all have the same percentage overshoot. Which of the following diagrams represents the poles of the three systems? **[2008]**

(A)


(B)


(C)


(D)


8. Group I lists a set of four transfer functions. Group II gives a list of possible step responses $y(t)$. Match the step responses with the corresponding transfer functions.

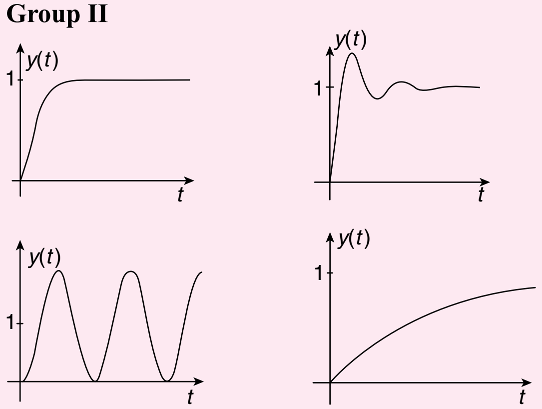
Group I

$P = \dfrac{25}{s^2 + 25}$ $\quad Q = \dfrac{36}{s^2 + 20s + 36}$

$R = \dfrac{36}{s^2 + 12s + 36}$ $\quad S = \dfrac{49}{s^2 + 7s + 49}$

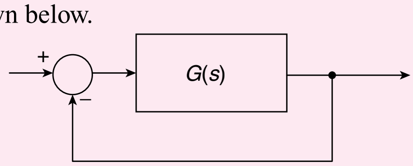
[2008]

Group II



(A) P – 3, Q – 1, R – 4, S – 2
(B) P – 3, Q – 2, R – 4, S – 1
(C) P – 2, Q – 1, R – 4, S – 3
(D) P – 3, Q – 4, R – 1, S – 2

9. A certain system has transfer function $G(s) = \dfrac{s+8}{s^2 + \alpha s - 4}$, α is a parameter. Consider the standard negative unity feedback configuration as shown below.



Which of the following statements is true? **[2008]**

(A) The closed-loop system in never stable for any value of α.
(B) For some positive values of α, the closed-loop system is stable, but not for all positive values.
(C) For all positive values of α, the closed-loop system is stable.
(D) The closed-loop system is stable for all values of α, both positive and negative.

10. The magnitude of frequency response of an under-damped second-order system is 5 at 0 rad/s and peaks to $\dfrac{10}{\sqrt{3}}$ at $5\sqrt{2}$ rad/sec. The transfer function of the system is **[2008]**

(A) $\dfrac{500}{s^2 + 10s + 100}$ (B) $\dfrac{375}{s^2 + 5s + 175}$

(C) $\dfrac{720}{s^2 + 12s + 144}$ (D) $\dfrac{1125}{s^2 + 25s + 225}$

11. The unit-step response of an under-damped second-order system has steady state value of –2. Which one of the following transfer functions has these properties? **[2009]**

(A) $\dfrac{-2.24}{s^2 + 2.59s + 1.12}$ (B) $\dfrac{-3.82}{s^2 + 1.91s + 1.91}$

(C) $\dfrac{-2.24}{s^2 - 2.59s + 1.12}$ (D) $\dfrac{-3.82}{s^2 - 1.91s + 1.91}$

12. Given $f(t) = L^{-1}\left[\dfrac{3s+1}{s^3 + 4s^2 + (K-3)s}\right]$. If $\lim_{t\to\infty} f(t) = 1$, then the value of K is **[2010]**

(A) 1 (B) 2 (C) 3 (D) 4

13. A unity negative feedback closed-loop system has a plant with the transfer function $G(s) = \dfrac{1}{s^2 + 2s + 2}$ and a controller $G_c(s)$ in the feed forward path. For a unit-step input, the transfer function of the controller that gives minimum steady-state error is **[2010]**

(A) $G_c(s) = \dfrac{s+1}{s+2}$

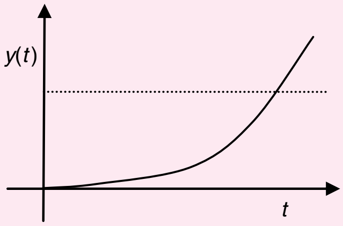
(B) $G_c(s) = \dfrac{s+2}{s+1}$

(C) $G_c(s) = \dfrac{(s+1)(s+4)}{(s+2)(s+3)}$

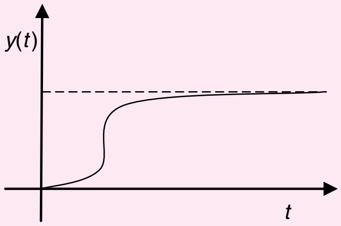
(D) $G_c(s) = 1 + \dfrac{2}{s} + 3s$

14. The differential equation $100\dfrac{d^2y}{dt^2} - 20\dfrac{dy}{dt} + y = x(t)$ describes a system with an input $x(t)$ and an output $y(t)$. The system, which is initially relaxed, is excited by a unit-step input. The output $y(t)$ can be represented by the waveform. **[2011]**

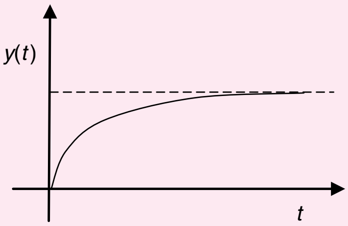
(A)



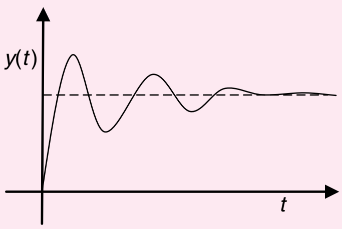
(B)



(C)



(D)



15. A system with transfer function

$$G(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

is excited by $\sin(\omega t)$. the steady-state output of the system is zero at **[2012]**

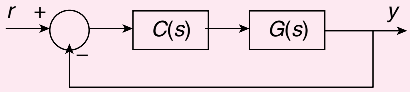
(A) $\omega = 1$ rad/s (B) $\omega = 2$ rad/s
(C) $\omega = 3$ rad/s (D) $\omega = 4$ rad/s

16. The forward path transfer function of a unity negative feedback system is given by

$$G(s) = \frac{K}{(s+2)(s-1)}$$

The value of K which will place both the poles of the closed-loop system at the same location, is ___________. **[2014]**

17. For the following feedback system $G(s) = \frac{1}{(s+1)(s+2)}$. The 2%-settling time of the step response is required to be less than 2 s. **[2014]**



Which one of the following compensators $C(s)$ achieves this? **[2014]**

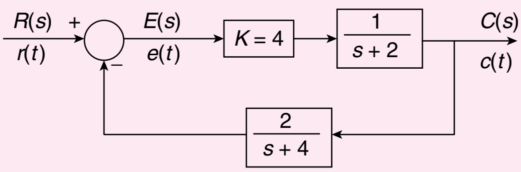
(A) $3\left(\frac{1}{s+5}\right)$ (B) $5\left(\frac{0.03}{s}+1\right)$

(C) $2(s+4)$ (D) $4\left(\frac{s+8}{s+3}\right)$

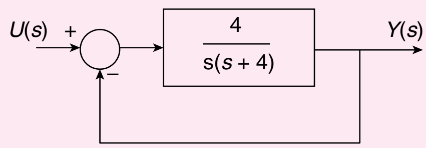
18. The natural frequency of an undamped second-order system is 40 rad/s. If the system is damped with a damping ratio 0.3, the damped natural frequency in rad/s is _____________. **[2014]**

19. The input $-3e^{2t}u(t)$, where $u(t)$ is the unit-step function, is applied to a system with transfer function $\frac{s-2}{s+3}$. If the initial value of the output is –2, then the value of the output at steady state is _____. **[2014]**

20. The steady-state error of the system shown in the figure for a unit-step input is _____________ **[2014]**



21. For the second-order closed-loop system shown in the figure, the natural frequency (in rad/s) is **[2014]**



(A) 16 (B) 4 (C) 2 (D) 1

22. A unity negative feedback system has an open-loop transfer function $G(s) = \frac{K}{s(s+10)}$. The gain K for the system to have a damping ratio of 0.25 is_______.

[2015]

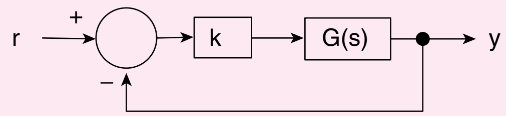
23. The open-loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(s+2)}.$$

For the peak overshoot of the closed loop system to a unit step input to be 10% the value of K is ________ . **[2016]**

24. In the feedback system shown below $G(s) = \frac{1}{(s^2 + 2s)}$.

The step response of the closed loop system should have minimum settling time and have no overshoot.

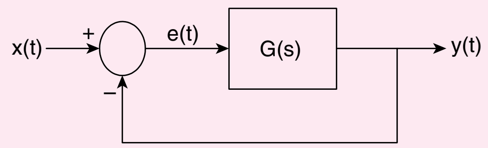


The required value of gain k to achieve this is ________. **[2016]**

25. For the unity feedback control system shown in the figure, the open loop transfer function $G(s)$ is given as:

$$G(s) = \frac{2}{s(s+1)}.$$

The steady state error e_{ss} due to a unit step input is **[2016]**



(A) 0 (B) 0.5
(C) 1.0 (D) ∞

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** B	**3.** B	**4.** A	**5.** B	**6.** C	**7.** C	**8.** C	**9.** C	**10.** B
11. B	**12.** D	**13.** D	**14.** C	**15.** B	**16.** A	**17.** B	**18.** C	**19.** A	**20.** D
21. A	**22.** A	**23.** B	**24.** D	**25.** B					

Practice Problems 2

1. A	**2.** B	**3.** B	**4.** B	**5.** D	**6.** A	**7.** A	**8.** A	**9.** A	**10.** C
11. C	**12.** C	**13.** C	**14.** B	**15.** C					

Previous Years' Questions

1. A	**2.** A	**3.** C	**4.** B	**5.** D	**6.** D	**7.** C	**8.** D	**9.** C	**10.** A
11. B	**12.** D	**13.** D	**14.** A	**15.** C	**16.** 2.24 to 2.26		**17.** C	**18.** 38.13 to 38.19	
19. 0	**20.** 0.49 to 0.51		**21.** C	**22.** 400	**23.** 2.77	**24.** 1	**25.** A		