CHAPTER

9.1

LINEAR ALGEBRA

1. If $\mathbf{A} = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -2 & \lambda \end{bmatrix}$ is a singular matrix, then λ is

(A) 0 (B) –2

(C) 2 (D) –1

2. If **A** and **B** are square matrices of order 4×4 such that $\mathbf{A} = 5\mathbf{B}$ and $|\mathbf{A}| = \alpha \cdot |\mathbf{B}|$, then α is

(A) 5 (B) 25

(C) 625 (D) None of these

3. If **A** and **B** are square matrices of the same order such that $\mathbf{AB} = \mathbf{A}$ and $\mathbf{BA} = \mathbf{A}$, then **A** and **B** are both

(A) Singular (B) Idempotent

(C) Involutory (D) None of these

4. The matrix, $\mathbf{A} = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ is

(A) Idempotent (B) Involutory

(C) Singular (D) None of these

5. Every diagonal element of a skew–symmetric matrix is

(A) 1 (B) 0

(C) Purely real (D) None of these

6. The matrix, $\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ is

(A) Orthogonal (B) Idempotent

(C) Unitary (D) None of these

7. Every diagonal elements of a Hermitian matrix is

(A) Purely real (B) 0

(C) Purely imaginary (D) 1

8. Every diagonal element of a Skew–Hermitian matrix is

(A) Purely real (B) 0

(C) Purely imaginary (D) 1

9. If **A** is Hermitian, then $i\mathbf{A}$ is

(A) Symmetric (B) Skew–symmetric

(C) Hermitian (D) Skew–Hermitian

10. If **A** is Skew–Hermitian, then $i\mathbf{A}$ is

(A) Symmetric (B) Skew–symmetric

(C) Hermitian (D) Skew–Hermitian.

11. If $\mathbf{A} = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then adj. **A** is equal to

(A) **A** (B) $\mathbf{c}^t$

(C) $3\mathbf{A}^t$ (D) 3**A**

12. The inverse of the matrix $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$ is

(A) $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix}$ (D) None of these

13. Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 3 & 1 & 2 \end{bmatrix}$, then $\mathbf{A}^{-1}$ is equal to

(A) $\frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ -10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix}$ (B) $\frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ -10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix}$ (D) None of these

14. If the rank of the matrix, $\mathbf{A} = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 7 & \lambda \\ 1 & 4 & 5 \end{bmatrix}$ is 2, then the value of λ is

(A) −13 (B) 13

(C) 3 (D) None of these

15. Let **A** and **B** be non–singular square matrices of the same order. Consider the following statements.

(I) $(\mathbf{AB})^T = \mathbf{A}^T\mathbf{B}^T$ (II) $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

(III) $\text{adj}(\mathbf{AB}) = (\text{adj.}\mathbf{A})(\text{adj.}\mathbf{B})$ (IV) $\rho(\mathbf{AB}) = \rho(\mathbf{A})\rho(\mathbf{B})$

(V) $|\mathbf{AB}| = |\mathbf{A}| \cdot |\mathbf{B}|$

Which of the above statements are false ?

(A) I, III & IV (B) IV & V

(C) I & II (D) All the above

16. The rank of the matrix $\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$ is

(A) 3 (B) 2

(C) 1 (D) None of these

17. The system of equations $3x - y + z = 0$, $15x - 6y + 5z = 0$, $\lambda x - 2y + 2z = 0$ has a non–zero solution, if λ is

(A) 6 (B) -6

(C) 2 (D) -2

18. The system of equation $x - 2y + z = 0$, $2x - y + 3z = 0$, $\lambda x + y - z = 0$ has the trivial solution as the only solution, if λ is

(A) $\lambda \neq -\frac{4}{5}$ (B) $\lambda = \frac{4}{3}$

(C) $\lambda \neq 2$ (D) None of these

19. The system equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = 12$ is inconsistent, if λ is

(A) 3 (B) −3

(C) 0 (D) None of these.

20. The system of equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ has

(A) a unique solution

(B) no solution

(C) an infinite number of solutions

(D) none of these

21. If **A** is an n–row square matrix of rank $(n-1)$, then

(A) $\text{adj}\,\mathbf{A} = 0$ (B) $\text{adj}\,\mathbf{A} \neq 0$

(C) $\text{adj}\,\mathbf{A} = I_n$ (D) None of these

22. The system of equations $x - 4y + 7z = 14$, $3x + 8y - 2z = 13$, $7x - 8y + 26z = 5$ has

(A) a unique solution

(B) no solution

(C) an infinite number of solution

(D) none of these

23. The eigen values of $\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 9 & -5 \end{bmatrix}$ are

(A) ± 1 (B) 1, 1

(C) −1, −1 (D) None of these

24. The eigen values of $\mathbf{A} = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are

(A) 0, 3, −15 (B) 0, −3, −15

(C) 0, 3, 15 (D) 0, −3, 15

25. If the eigen values of a square matrix be 1, −2 and 3, then the eigen values of the matrix 2**A** are

(A) $\frac{1}{2}, -1, \frac{3}{2}$ (B) 2, −4, 6

(C) 1, −2, 3 (D) None of these.

26. If **A** is a non–singular matrix and the eigen values of **A** are 2, 3, −3 then the eigen values of $\mathbf{A}^{-1}$ are

(A) 2, 3, −3 (B) $\frac{1}{2}, \frac{1}{3}, \frac{-1}{3}$

(C) $2|\mathbf{A}|, 3|\mathbf{A}|, -3|\mathbf{A}|$ (D) None of these

27. If $-1, 2, 3$ are the eigen values of a square matrix $\mathbf{A}$ then the eigen values of $\mathbf{A}^2$ are

(A) $-1, 2, 3$ (B) 1, 4, 9

(C) 1, 2, 3 (D) None of these

28. If $2, -4$ are the eigen values of a non–singular matrix $\mathbf{A}$ and $|\mathbf{A}| = 4$, then the eigen values of $\text{adj}\,\mathbf{A}$ are

(A) $\frac{1}{2}, -1$ (B) $2, -1$

(C) $2, -4$ (D) $8, -16$

29. If 2 and 4 are the eigen values of $\mathbf{A}$ then the eigenvalues of $\mathbf{A}^T$ are

(A) $\frac{1}{2}, \frac{1}{4}$ (B) 2, 4

(C) 4, 16 (D) None of these

30. If 1 and 3 are the eigenvalues of a square matrix $\mathbf{A}$ then $\mathbf{A}^3$ is equal to

(A) $13(\mathbf{A} - \mathbf{I}_2)$ (B) $13\mathbf{A} - 12\mathbf{I}_2$

(C) $12(\mathbf{A} - \mathbf{I}_2)$ (D) None of these

31. If $\mathbf{A}$ is a square matrix of order 3 and $|\mathbf{A}| = 2$ then $\mathbf{A}\,(\text{adj}\,\mathbf{A})$ is equal to

(A) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) None of these

32. The sum of the eigenvalues of $\mathbf{A} = \begin{bmatrix} 8 & 2 & 3 \\ 4 & 5 & 9 \\ 2 & 0 & 5 \end{bmatrix}$ is equal to

(A) 18 (B) 15

(C) 10 (D) None of these

33. If 1, 2 and 5 are the eigen values of the matrix $\mathbf{A}$ then $|\mathbf{A}|$ is equal to

(A) 8 (B) 10

(C) 9 (D) None of these

34. If the product of matrices

$$\mathbf{A} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \text{ and}$$

$$\mathbf{B} = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

is a null matrix, then θ and ϕ differ by

(A) an odd multiple of π

(B) an even multiple of π

(C) an odd multiple of $\frac{\pi}{2}$

(D) an even multiple $\frac{\pi}{2}$

35. If $\mathbf{A}$ and $\mathbf{B}$ are two matrices such that $\mathbf{A} + \mathbf{B}$ and $\mathbf{AB}$ are both defined, then $\mathbf{A}$ and $\mathbf{B}$ are

(A) both null matrices

(B) both identity matrices

(C) both square matrices of the same order

(D) None of these

36. If $\mathbf{A} = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$

then $(\mathbf{I} - \mathbf{A}) \cdot \begin{bmatrix} \cos\alpha & -\sin\frac{\alpha}{2} \\ \sin\alpha & \cos\alpha \end{bmatrix}$ is equal to

(A) $\mathbf{I} + \mathbf{A}$ (B) $\mathbf{I} - \mathbf{A}$

(C) $\mathbf{I} + 2\mathbf{A}$ (D) $\mathbf{I} - 2\,\mathbf{A}$

37. If $\mathbf{A} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then for every positive integer n, $\mathbf{A}^n$ is equal to

(A) $\begin{bmatrix} 1+2n & 4n \\ n & 1+2n \end{bmatrix}$ (B) $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

(C) $\begin{bmatrix} 1-2n & 4n \\ n & 1+2n \end{bmatrix}$ (D) None of these

38. If $\mathbf{A}_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then consider the following statements :

I. $\mathbf{A}_\alpha \cdot \mathbf{A}_\beta = \mathbf{A}_{\alpha\beta}$ II. $\mathbf{A}_\alpha \cdot \mathbf{A}_\beta = \mathbf{A}_{(\alpha+\beta)}$

III. $(\mathbf{A}_\alpha)^n = \begin{bmatrix} \cos^n\alpha & \sin^n\alpha \\ -\sin^n\alpha & \cos^n\alpha \end{bmatrix}$

IV. $(\mathbf{A}_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

Which of the above statements are true ?

(A) I and II (B) I and IV

(C) II and III (D) II and IV

39. If **A** is a 3-rowed square matrix such that $|\mathbf{A}|=3$, then adj(adj **A**) is equal to :

(A) 3A (B) 9A

(C) 27A (D) none of these

40. If **A** is a 3-rowed square matrix, then $|\text{adj}(\text{adj}\,\mathbf{A})|$ is equal to

(A) $|\mathbf{A}|^6$ (B) $|\mathbf{A}|^3$

(C) $|\mathbf{A}|^4$ (D) $|\mathbf{A}|^2$

41. If **A** is a 3-rowed square matrix such that $|\mathbf{A}|=2$, then $|\text{adj}(\text{adj}\,\mathbf{A}^2)|$ is equal to

(A) 2^4 (B) 2^8

(C) 2^{16} (D) None of these

42. If $\mathbf{A}=\begin{bmatrix}2x & 0\\ x & x\end{bmatrix}$ and $\mathbf{A}^{-1}=\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$, then the value of x is

(A) 1 (B) 2

(C) $\frac{1}{2}$ (D) None of these

43. If $\mathbf{A}=\begin{bmatrix}1 & 2\\ 2 & 1\\ 1 & 1\end{bmatrix}$ then $\mathbf{A}^{-1}$ is

(A) $\begin{bmatrix}1 & 4\\ 3 & 2\\ 2 & 5\end{bmatrix}$ (B) $\begin{bmatrix}1 & -2\\ -2 & 1\\ 1 & 2\end{bmatrix}$

(C) $\begin{bmatrix}2 & 3\\ 3 & 1\\ 2 & 7\end{bmatrix}$ (D) Undefined

44. If $\mathbf{A}=\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$ then **AB** is

(A) $\begin{bmatrix}-1 & -8 & -10\\ -1 & -2 & 5\\ 9 & 22 & 15\end{bmatrix}$ (B) $\begin{bmatrix}0 & 0 & -10\\ -1 & -2 & -5\\ 0 & 21 & -15\end{bmatrix}$

(C) $\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$ (D) $\begin{bmatrix}0 & -8 & -10\\ 1 & -2 & -5\\ 9 & 21 & 15\end{bmatrix}$

45. If $\mathbf{A}=\begin{bmatrix}1 & 2 & 0\\ 3 & -1 & 4\end{bmatrix}$, then $\mathbf{AA}^T$ is

(A) $\begin{bmatrix}1 & 3\\ -1 & 4\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0 & 1\\ -1 & 2 & 3\end{bmatrix}$

(C) $\begin{bmatrix}2 & 1\\ 1 & 26\end{bmatrix}$ (D) Undefined

46. The matrix, that has an inverse is

(A) $\begin{bmatrix}3 & 1\\ 6 & 2\end{bmatrix}$ (B) $\begin{bmatrix}5 & 2\\ 2 & 1\end{bmatrix}$

(C) $\begin{bmatrix}6 & 2\\ 9 & 3\end{bmatrix}$ (D) $\begin{bmatrix}8 & 2\\ 4 & 1\end{bmatrix}$

47. The skew symmetric matrix is

(A) $\begin{bmatrix}0 & -2 & 5\\ 2 & 0 & 6\\ -5 & -6 & 0\end{bmatrix}$ (B) $\begin{bmatrix}1 & 5 & 2\\ 6 & 3 & 1\\ 2 & 4 & 0\end{bmatrix}$

(C) $\begin{bmatrix}0 & 1 & 3\\ 1 & 0 & 5\\ 3 & 5 & 0\end{bmatrix}$ (D) $\begin{bmatrix}0 & 3 & 3\\ 2 & 0 & 2\\ 1 & 1 & 0\end{bmatrix}$

48. If $\mathbf{A}=\begin{bmatrix}1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$, the product of **A** and **B** is

(A) $\begin{bmatrix}1\\ 0\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

(C) $\begin{bmatrix}1\\ 2\end{bmatrix}$ (D) $\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$

49. Matrix **D** is an orthogonal matrix $\mathbf{D}=\begin{bmatrix}A & B\\ C & 0\end{bmatrix}$. The value of $|\mathbf{B}|$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) 1 (D) 0

50. If $\mathbf{A}_{n\times n}$ is a triangular matrix then det **A** is

(A) $\prod_{i=1}^{n}(-1)a_{ii}$ (B) $\prod_{i=1}^{n}a_{ii}$

(C) $\sum_{i=1}^{n}(-1)a_{ii}$ (D) $\sum_{i=1}^{n}a_{ii}$

51. If $\mathbf{A}=\begin{bmatrix} t^2 & \cos t \\ e^t & \sin t \end{bmatrix}$, then $\frac{d\mathbf{A}}{dt}$ will be

(A) $\begin{bmatrix} t^2 & \sin t \\ e^t & \sin t \end{bmatrix}$ (B) $\begin{bmatrix} 2t & \cos t \\ e^t & \sin t \end{bmatrix}$

(C) $\begin{bmatrix} 2t & -\sin t \\ e^t & \cos t \end{bmatrix}$ (D) Undefined

52. If $\mathbf{A}\in\mathbf{R}_{n\times n}$, $\det\mathbf{A}\neq 0$, then

(A) **A** is non singular and the rows and columns of **A** are linearly independent.

(B) **A** is non singular and the rows **A** are linearly dependent.

(C) **A** is non singular and the **A** has one zero rows.

(D) **A** is singular.

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SOLUTIONS

1. (B) **A** is singular if $|\mathbf{A}|=0$

$$\Rightarrow \begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -2 & \lambda \end{vmatrix}=0$$

$$\Rightarrow \quad -(-1)\begin{vmatrix} 1 & -2 \\ -2 & \lambda \end{vmatrix}+2\begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix}+0\begin{vmatrix} 0 & 3 \\ -2 & \lambda \end{vmatrix}=0$$

$$\Rightarrow (\lambda-4)+2(3)=0 \quad\Rightarrow \lambda-4+6=0 \;\Rightarrow\; \lambda=-2$$

2. (C) If k is a constant and **A** is a square matrix of order $n\times n$ then $|k\mathbf{A}|=k^n|\mathbf{A}|$.

$\mathbf{A}=5\mathbf{B} \Rightarrow |\mathbf{A}|=|5\mathbf{B}|=5^4|\mathbf{B}|=625|\mathbf{B}|$

$\Rightarrow \alpha=625$

3. (B) **A** is singular, if $|\mathbf{A}|=0$,

A is Idempotent, if $\mathbf{A}^2=\mathbf{A}$

A is Involutory, if $\mathbf{A}^2=I$

Now, $\mathbf{A}^2=\mathbf{AA}=(\mathbf{A}B)\,\mathbf{A}=\mathbf{A}(B\mathbf{A})=\mathbf{A}B=\mathbf{A}$

and $\mathbf{B}^2=\mathbf{BB}=(\mathbf{BA})\mathbf{B}=\mathbf{B}(\mathbf{AB})=\mathbf{BA}=\mathbf{B}$

$\Rightarrow \quad \mathbf{A}^2=\mathbf{A}$ and $\mathbf{B}^2=\mathbf{B}$,

Thus **A** & **B** both are Idempotent.

4. (B) Since, $\mathbf{A}^2=\begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=\mathbf{I}, \quad \mathbf{A}^2=\mathbf{I} \quad\Rightarrow\quad \mathbf{A}$ is involutory.

5. (B) Let $\mathbf{A}=[a_{ij}]$ be a skew–symmetric matrix, then

$\mathbf{A}^T=-\mathbf{A}$, $\quad\Rightarrow\quad a_{ij}=-a_{ij}$,

if $i=j$ then $a_{ii}=-a_{ii} \quad\Rightarrow\quad 2a_{ii}=0 \quad\Rightarrow\quad a_{ii}=0$

Thus diagonal elements are zero.

6. (C) **A** is orthogonal if $\mathbf{AA}^T=\mathbf{I}$

A is unitary if $\mathbf{AA}^Q=\mathbf{I}$, where $\mathbf{A}^Q$ is the conjugate transpose of **A** i.e., $\mathbf{A}^Q=(\overline{\mathbf{A}})^T$.

Here,

$$\mathbf{AA}^Q=\begin{bmatrix} \frac{1}{\sqrt2} & \frac{i}{\sqrt2} \\ -\frac{i}{\sqrt2} & -\frac{1}{\sqrt2} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt2} & \frac{i}{\sqrt2} \\ -\frac{i}{\sqrt2} & -\frac{1}{\sqrt2} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\mathbf{I}_2$$

Thus **A** is unitary.

7. (A) A square matrix $\mathbf{A}$ is said to be Hermitian if $\mathbf{A}^Q = \mathbf{A}$. So $a_{ij} = \bar{a}_{ji}$. If $i = j$ then $a_{ii} = \bar{a}_{ii}$ i.e. conjugate of an element is the element itself and a_{ii} is purely real.

8. (C) A square matrix $\mathbf{A}$ is said to be Skew-Hermitian if $\mathbf{A}^Q = -\mathbf{A}$. If $\mathbf{A}$ is Skew–Hermitian then $\mathbf{A}^Q = -\mathbf{A}$

$\Rightarrow \quad \bar{a}_{ji} = -a_{ij}$,

if $i = j$ then $\bar{a}_{ii} = -a_{ii} \Rightarrow \quad a_{ii} + \bar{a}_{ii} = 0$

it is only possible when a_{ii} is purely imaginary.

9. (D) $\mathbf{A}$ is Hermitian then $\mathbf{A}^Q = \mathbf{A}$

Now, $(i\mathbf{A})^Q = \bar{i}\,\mathbf{A}^Q = -i\mathbf{A}^Q = -i\mathbf{A}, \Rightarrow \quad (i\mathbf{A})^Q = -(i\mathbf{A})$

Thus $i\mathbf{A}$ is Skew–Hermitian.

10. (C) $\mathbf{A}$ is Skew–Hermitian then $\mathbf{A}^Q = -\mathbf{A}$

Now, $(i\mathbf{A})^Q = \bar{i}\mathbf{A}^Q = -(-\mathbf{A}) = i\mathbf{A}$ then $i\mathbf{A}$ is Hermitian.

11. (C) If $\mathbf{A} = [a_{ij}]_{n\times n}$ then $\det \mathbf{A} = [c_{ij}]^T_{n\times n}$

Where c_{ij} is the cofactor of a_{ij}

Also $c_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is the minor of a_{ij}, obtained by leaving the row and the column corresponding to a_{ij} and then take the determinant of the remaining matrix.

Now, M_{11} = minor of a_{11} i.e. $-1 = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = -3$

Similarly

$$M_{12} = \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = 6 \; ; \; M_{13} = \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -6$$

$$M_{21} = \begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -6 \; ; \; M_{22} = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = 3 \; ;$$

$$M_{23} = \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = 6 \; ; \; M_{31} = \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = 6 \; ;$$

$$M_{32} = \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = 6 \; ; \; M_{33} = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = 3$$

$C_{11} = (-1)^{1+1} M_{11} = -3$; $C_{12} = (-1)^{1+2} M_{12} = -6$;

$C_{13} = (-1)^{1+3} M_{13} = -6$; $C_{21} = (-1)^{2+1} M_{21} = 6$;

$C_{22} = (-1)^{2+2} M_{22} = 3$; $C_{23} = (-1)^{2+3} M_{23} = -6$;

$C_{31} = (-1)^{3+1} M_{31} = 6$; $C_{32} = (-1)^{3+2} M_{32} = -6$;

$C_{33} = (-1)^{3+3} M_{33} = 3$

$$\det \mathbf{A} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = 3\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}^T = 3\mathbf{A}^T$$

12. (A) Since $\mathbf{A}^{-1} = \dfrac{1}{|\mathbf{A}|}\,\text{adj}\,\mathbf{A}$

Now, Here $|\mathbf{A}| = \begin{vmatrix} -1 & 2 \\ 3 & -5 \end{vmatrix} = -1$

Also, $\text{adj}\,\mathbf{A} = \begin{bmatrix} -5 & -3 \\ -2 & -1 \end{bmatrix}^T \Rightarrow \quad \text{adj}\,\mathbf{A} = \begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix}$

$$\mathbf{A}^{-1} = \frac{1}{-1}\begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$$

13. (A) Since, $\mathbf{A}^{-1} = \dfrac{1}{|\mathbf{A}|}\text{adj}\,\mathbf{A}$

$$|\mathbf{A}| = \begin{vmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 3 & 1 & 2 \end{vmatrix} = 4 \neq 0,$$

$$\text{adj}\,\mathbf{A} == \begin{bmatrix} 4 & 10 & -10 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & 0 & 0 \\ 10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ 10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

14. (B) A matrix $\mathbf{A}_{(m\times n)}$ is said to be of rank r if

(i) it has at least one non–zero minor of order r, and

(ii) all other minors of order greater than r, if any; are zero. The rank of $\mathbf{A}$ is denoted by $\rho(\mathbf{A})$. Now, given that $\rho(\mathbf{A}) = 2 \to$ minor of order greater than 2 i.e., 3 is zero.

$$\text{Thus } |\mathbf{A}| = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 7 & \lambda \\ 1 & 4 & 5 \end{vmatrix} = 0$$

$\Rightarrow \quad 2(35 - 4\lambda) + 1(20 - \lambda) + 3(16 - 7) = 0,$

$\Rightarrow \quad 70 - 8\lambda + 20 - \lambda + 27 = 0,$

$\Rightarrow \quad 9\lambda = 117 \quad \Rightarrow \quad \lambda = 13$

15. (A) The correct statements are

$(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$, $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$,

$\text{adj}\,(\mathbf{AB}) = \text{adj}\,(\mathbf{B})\,\text{adj}\,(\mathbf{A})$

$\rho(\mathbf{AB}) \neq \rho(\mathbf{A})\,\rho(\mathbf{B})$, $|\mathbf{A}B| = |\mathbf{A}| \cdot |\mathbf{B}|$

Thus statements I, II, and IV are wrong.

16. (B) Since

$|\mathbf{A}| = 2(-9 + 8) + 2(-2 + 3) = -2 + 2 = 0$

$\Rightarrow \quad \rho(\mathbf{A}) < 3$

Again, one minor of order 2 is $\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 6 \neq 0$

$\Rightarrow \quad \rho(\mathbf{A}) = 2$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 5 \\ -4 & -5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(-5-\lambda)+16=0 \Rightarrow -15+\lambda^2+2\lambda+16=0$$

$$\Rightarrow \lambda^2+2\lambda+1=0 \Rightarrow (\lambda+1)^2=0 \Rightarrow \lambda=-1,-1$$

Thus eigen values are $-1, -1$

24. (C) Characteristic equation is $|\mathbf{A}-\lambda\mathbf{I}|=0$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2-18\lambda^2+45\lambda=0$$

$$\Rightarrow \lambda(\lambda-3)(\lambda-15)=0 \Rightarrow \lambda=0, 3, 15$$

25. (B) If eigen values of **A** are $\lambda_1, \lambda_2, \lambda_3$ then the eigen values of k**A** are $k\lambda_1, k\lambda_2, k\lambda_3$. So the eigen values of 2**A** are $2, -4$ and 6

26. (B) If $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ are the eigen values of a non–singular matrix **A**, then $\mathbf{A}^{-1}$ has the eigen values $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots\ldots, \frac{1}{\lambda_n}$. Thus eigen values of $\mathbf{A}^{-1}$ are $\frac{1}{2}, \frac{1}{3}, \frac{-1}{3}$.

27. (B) If $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ are the eigen values of a matrix **A**, then $\mathbf{A}^2$ has the eigen values $\lambda_1^2, \lambda_2^2, \ldots\ldots, \lambda_n^2$. So, eigen values of $\mathbf{A}^2$ are 1, 4, 9.

28. (B) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of **A** then the eigen values adj **A** are $\frac{|\mathbf{A}|}{\lambda_1}, \frac{|\mathbf{A}|}{\lambda_2}, \ldots\ldots, \frac{|\mathbf{A}|}{\lambda_n}; |\mathbf{A}| \neq 0$. Thus eigenvalues of adj **A** are $\frac{4}{2}, \frac{-4}{4}$ i.e. 2 and–1.

29. (B) Since, the eigenvalues of **A** and $\mathbf{A}^T$ are square so the eigenvalues of $\mathbf{A}^T$ are 2 and 4.

30. (B) Since 1 and 3 are the eigenvalues of **A** so the characteristic equation of **A** is

$$(\lambda-1)(\lambda-3)=0 \Rightarrow \lambda^2-4\lambda+3=0$$

Also, by Cayley–Hamilton theorem, every square matrix satisfies its own characteristic equation so

$$\mathbf{A}^2-4\mathbf{A}+3\mathbf{I}_2=0$$

$$\Rightarrow \mathbf{A}^2=4\mathbf{A}-3\mathbf{I}_2$$

$$\Rightarrow \mathbf{A}^3=4\mathbf{A}^2-3\mathbf{A}=4(4\mathbf{A}-3\mathbf{I})-3\mathbf{A}$$

$$\Rightarrow \mathbf{A}^3=13\mathbf{A}-12\mathbf{I}_2$$

31. (A) Since $\mathbf{A}(\text{adj } \mathbf{A}) = |\mathbf{A}|\mathbf{I}_3$

$$\Rightarrow \mathbf{A}(\text{adj } \mathbf{A}) = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

32. (A) Since the sum of the eigenvalues of an n–square matrix is equal to the trace of the matrix (i.e. sum of the diagonal elements)

so, required sum $= 8+5+5=18$

33. (B) Since the product of the eigenvalues is equal to the determinant of the matrix so $|\mathbf{A}| = 1 \times 2 \times 5 = 10$

34. (C)

$$\mathbf{AB} = \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix} = \mathbf{A}$$

null matrix when $\cos(\theta-\phi)=0$

This happens when $(\theta-\phi)$ is an odd multiple of $\frac{\pi}{2}$.

35. (C) Since $\mathbf{A}+\mathbf{B}$ is defined, **A** and **B** are matrices of the same type, say $m \times n$. Also, **AB** is defined. So, the number of columns in **A** must be equal to the number of rows in **B** i.e. $n = m$. Hence, **A** and **B** are square matrices of the same order.

36. (A) Let $\tan\frac{\alpha}{2} = t$, then, $\cos\alpha = \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \dfrac{1-t^2}{t+t^2}$

and $\sin\alpha = \dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \dfrac{2t}{1+t^2}$

$$(\mathbf{I}-\mathbf{A})\cdot\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix} \times \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{(1+t^2)} \\ \frac{2t}{(1+t^2)} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = (\mathbf{I}+\mathbf{A})$$

37. (B) $\mathbf{A}^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where $n = 2$.

38. (D) $\mathbf{A}_\alpha \cdot \mathbf{A}_\beta = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = \mathbf{A}_{\alpha+\beta}$

Also, it is easy to prove by induction that

$(\mathbf{A}_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

39. (A) We know that $\operatorname{adj}(\operatorname{adj}\mathbf{A}) = |\mathbf{A}|^{n-2} \cdot \mathbf{A}$.

Here $n = 3$ and $|\mathbf{A}| = 3$.

So, $\operatorname{adj}(\operatorname{adj}\mathbf{A}) = 3^{(3-2)} \cdot \mathbf{A} = 3\mathbf{A}$.

40. (C) We have $|\operatorname{adj}(\operatorname{adj}\mathbf{A})| = |\mathbf{A}|^{(n-1)^2}$

Putting $n = 3$, we get $|\operatorname{adj}(\operatorname{adj}\mathbf{A})| = |\mathbf{A}|^4$.

41. (C) Let $\mathbf{B} = \operatorname{adj}(\operatorname{adj}\mathbf{A}^2)$.

Then, $\mathbf{B}$ is also a 3×3 matrix.

$|\operatorname{adj}\{\operatorname{adj}(\operatorname{adj}\mathbf{A}^2)\}| = |\operatorname{adj}\mathbf{B}| = |\mathbf{B}^3|^{3-1} = |\mathbf{B}|^2$

$= |\operatorname{adj}(\operatorname{adj}\mathbf{A}^2)|^2 = \left[|\mathbf{A}^2|^{(3-1)^2}\right]^2 = |\mathbf{A}|^{16} = 2^{16}$

$[\because\ |\mathbf{A}^2| = |\mathbf{A}|^2]$

42. (C) $\begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, So, $2x = 1 \Rightarrow x = \dfrac{1}{2}$.

43. (D) Inverse matrix is defined for square matrix only.

44. (C) $\mathbf{AB} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$

$= \begin{bmatrix} (2)(1)+(-1)(3) & (2)(-2)+(-1)(4) & (2)(-5)+(-1)(0) \\ (1)(1)+(0)(3) & (1)(-2)+(0)(4) & (1)(-5)+(0)(0) \\ (-3)(1)+(4)(3) & (-3)(-2)+(4)(4) & (-3)(-5)+(4)(0) \end{bmatrix}$

$= \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & -22 & 15 \end{bmatrix}$

45. (C) $\mathbf{AA}^T = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} (1)(1)+(2)(2)+(0)(0) & (1)(3)+(2)(-1)+(0)(4) \\ (3)(1)+(-1)(2)+(4)(0) & (3)(3)+(-1)(-1)+(4)(4) \end{bmatrix}$

$= \begin{bmatrix} 5 & 1 \\ 1 & 26 \end{bmatrix}$

46. (B) if $|\mathbf{A}|$ is zero, $\mathbf{A}^{-1}$ does not exist and the matrix $\mathbf{A}$ is said to be singular. Only (B) satisfy this condition.

$|\mathbf{A}| = \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix} = (5)(1) - (2)(2) = 1$

47. (A) A skew symmetric matrix $\mathbf{A}_{n \times n}$ is a matrix with $\mathbf{A}^T = -\mathbf{A}$. The matrix of (A) satisfy this condition.

48. (C) $\mathbf{AB} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} (1)(1)+(1)(0)+(0)(1) \\ (1)(1)+(0)(0)+(1)(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

49. (C) For orthogonal matrix

$\det \mathbf{M} = 1$ And $\mathbf{M}^{-1} = \mathbf{M}^T$, therefore Hence $\mathbf{D}^{-1} = \mathbf{D}^T$

$\mathbf{D}^T = \begin{bmatrix} A & C \\ B & 0 \end{bmatrix} = \mathbf{D}^{-1} = \dfrac{1}{-BC}\begin{bmatrix} 0 & -B \\ -C & A \end{bmatrix}$

This implies $B = \dfrac{-C}{-BC} \Rightarrow B = \dfrac{1}{B} \Rightarrow B = \pm 1$

Hence $B = 1$

50. (B) From linear algebra for $\mathbf{A}_{n \times n}$ triangular matrix

$\det \mathbf{A} = \prod_{i=1}^{n} a_{ii}$, The product of the diagonal entries of $\mathbf{A}$

51. (C) $\dfrac{d\mathbf{A}}{dt} = \begin{bmatrix} \dfrac{d(t^2)}{dt} & \dfrac{d(\cos t)}{dt} \\ \dfrac{d(e^t)}{dt} & \dfrac{d(\sin t)}{dt} \end{bmatrix} = \begin{bmatrix} 2t & -\sin t \\ e^t & \cos t \end{bmatrix}$

52. (A) If $\det \mathbf{A} \neq 0$, then $\mathbf{A}_{n \times n}$ is non-singular, but if $\mathbf{A}_{n \times n}$ is non-singular, then no row can be expressed as a linear combination of any other. Otherwise $\det \mathbf{A} = 0$

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