

CBSE Class 11 Mathematics
Sample Papers 09 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

- i. All the questions are compulsory.
 - ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
 - iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
 - iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
 - v. Use of calculators is not permitted.
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Section A

1. The relation $R = \{(1, 1), (2, 2), (3, 3)\}$ on the set $\{1, 2, 3\}$ is
 - a. an equivalence relation
 - b. reflexive only
 - c. symmetric only
 - d. transitive only
2. The number of ways in which 6 “+” and 4 “-” signs can be arranged in a line such that no two “-” signs occur together is
 - a. 5040

-
- b. 35
- c. 120
- d. 210
3. The largest term in the expansion of $(1 + x)^{19}$ when $x = \frac{1}{2}$ is
- a. 6th
- b. 8th
- c. 5th
- d. 7th
4. The number of ways in which 4 persons can be seated at a round table so that all shall not have same neighbours in any two arrangements is:
- a. 3
- b. none of these
- c. 24
- d. 6
5. The number of bijective functions from the set A to itself when A contains 106 elements is
- a. $(106)^2$
- b. 106
- c. 2^{106}
- d. 106
6. The greatest positive integer, which divides $(n + 1)(n + 2)(n + 3) \dots (n + r) \forall n \in W$, is
- a. $n+r$
- b. r

c. $(r + 1) !$

d. $r !$

7. The chance that an event E 'occurs' or does not occur's

a. 1

b. 0

c. none of these

d. 2

8. The centre of the sphere , which passes through $(a , 0 , 0) , (0 , b , 0) (0 , 0 , c)$ and $(0 , 0 , 0)$ is ? where $abc \neq 0$

a. $(a/2 , 0 , 0)$

b. $(a/2 , b/2 , c/2)$

c. $(0 , 0 , c/2)$

d. $(0 , b/2 , 0)$

9. Both A and B throw a dice. The chance that B throws a number not less than that thrown by A is

a. $1/2$

b. $21/36$

c. $19/36$

d. $15/36$

10. $(\sqrt{3} + 1)^{2n+1} + (\sqrt{3} - 1)^{2n+1}$ is

a. an even positive integer

b. an irrational number

c. an odd positive integer

d. a rational number

11. Fill in the blanks:

If A and B are non-empty sets and either A and B is an infinite set, then $A \times B$ will also be an _____ set.

12. Fill in the blanks:

The coefficient of x^n in the binomial expansion of $(1 + x)^{2n-1} =$ _____.

13. Fill in the blanks:

The value of $4! - 3!$ is _____.

14. Fill in the blanks:

The coordinates of a point are the perpendicular distance from the _____ on the respective axes.

OR

Fill in the blanks:

The length of the longest piece of a string that can be stretched straight in a rectangular room whose dimension are 10, 13 and 8 units are _____.

15. Fill in the blanks:

The derivative of $\sec x$ is _____.

OR

Fill in the blanks:

The value of limit $\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2}$ is _____.

16. If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10,$

15, 20}, find: $C - B$

17. Find the number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same lines.
18. Solve $9x^2 + 16 = 0$.

OR

Express $(5 + 4i) + (5 - 4i)$ in the form of $a + ib$.

19. Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A to B.
20. If ${}^nC_{10} = {}^nC_{12}$, then find the value of ${}^{23}C_n$.
21. Assume that $P(A) = P(B)$ then show that $A = B$.

OR

State whether $A = \{x : x \text{ is a letter in the word LOYAL}\}$ and $B = \{x : x \text{ is a letter of the word ALLOY}\}$ are equal? Justify your answer.

22. An experiment consists of recording boy-girl composition of families with 2 children.
- (i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?
- (ii) What is the sample space if we are interested, in the number of girls in the family?
23. If the term free from x in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then find the value of k .
24. A line passes through (x_1, y_1) and (h, k) . If slope of the line is m , show that $k - y_1 = m(h - x_1)$.

OR

Find the equation of line passing through the points $(-1, 1)$ and $(2, -4)$.

25. Consider the statement, S: 80 is a multiple of 5 and 4. Check its validity.

26. Prove that $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

27. Let A and B be two sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, prove that $A = B$.

28. Let $A = \{-2, -1, 0, 1, 2\}$ and $f: A \rightarrow \mathbb{Z}$ be a function defined by $f(x) = x^2 - 2x - 3$. Find range of f i.e. $f(A)$.

OR

If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$, then verify that

i. $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

ii. $A \times C$ is a subset of $B \times D$.

29. Evaluate $\lim_{x \rightarrow 3} \frac{e^x - e^3}{x - 3}$

30. Find the modulus and argument of complex number $\frac{1}{1+i}$.

31. Solve the inequality $2x + y > 3$ graphically.

OR

Solve the inequation $\frac{x+3}{x-2} \geq 4$.

32. Prove by the principle of mathematical induction that for all $n \in \mathbb{N}$:

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

33. Prove that: $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$.

OR

Find the general solution of the equation, $\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$.

34. The Sum of two no. is 6 times their geometric mean, show that no. are in the ratio $(3 + 3\sqrt{2}) : (3 - 2\sqrt{2})$

35. Find the equation of a circle passing through the points (2, - 6), (6, 4) and (- 3, 1).

OR

Find the equation of a circle concentric with the circle $x^2 + y^2 + 4x + 6y + 11 = 0$ and passing through the point (5, 4).

36. Calculate the coefficient of variation from the following data:

| | | | | | | |
|-------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| Income (in Rs.): | 1000-1700 | 1700-2400 | 2400-3100 | 3100-3800 | 3800-4500 | 4500-5200 |
| No. of families: | 12 | 18 | 20 | 25 | 35 | 10 |

CBSE Class 11 Mathematics
Sample Papers 09

Solution
Section A

1. (a) an equivalence relation
2. (b) 35

Explanation: Since all the plus signs are identical, we have number of ways in which 6 plus signs can be arranged = 1.

Now we will have 7 empty slots between these 6 identical + signs

Hence number of possible places of - sign = 7

Therefore number of ways in which the 4 minus sign can take any of the possible 7 places = ${}^7C_4 = \frac{7!}{4!(7-4)!} = \frac{5 \cdot 6 \cdot 7}{1 \cdot 2 \cdot 3} = 35$

3. (d) 7th **Explanation:** We have the general term in the expansion of $(1+x)^{19}$ is given

by $T_{r+1} = {}^{19}C_r (x)^r$

$$\begin{aligned}\text{Now } \frac{T_{r+1}}{T_r} &= \frac{{}^{19}C_r (x)^r}{{}^{19}C_{r-1} (x)^{r-1}} \\ &= \frac{19!(x)^r}{(19-r)! \cdot r!} \times \frac{(20-r)!(r-1)!}{19!(x)^{r-1}} \\ &= \frac{(20-r)x}{r} \\ &= \frac{(20-r)}{2r} \text{ since given } x = \frac{1}{2}\end{aligned}$$

$$\text{Now } \frac{T_{r+1}}{T_r} \geq 1$$

$$\Rightarrow \frac{(20-r)}{2r} \geq 1$$

$$\Rightarrow 20 - r \geq 2r$$

$$\Rightarrow r \leq \frac{20}{3}$$

$$\Rightarrow r \leq 6\frac{2}{3}$$

Hence the maximum value of r is 6 and hence the greatest term is $T_{r+1} = T_7$

4. (a) 3 **Explanation:**

The number of ways of seating 4 persons at a round table = $(4 - 1)! = 3!$

But in clockwise and anticlockwise arrangements the same persons are neighbours and hence these two arrangements will be same.

Therefore the required number of arrangements = $\frac{3!}{2} = \frac{6}{2} = 3$

5. (d) 106 **Explanation:**

If x and y are any two infinite sets having the same number of elements say n, then the number of injective function from x to y is $n!$.

6. (d) $r!$ **Explanation:**

If $n = 0$ the given expression becomes $1.2.3.4.....r = r!$ Also when $n = 1$ one more extra term will be there in the product $2.3.4.....(r + 1)$ which is also divisible by $r!$.

7. (a) 1 **Explanation:**

Let E denotes the given event. Then, \overline{E} denotes E does not occur

We have, $E \cup \overline{E} = S$

$$\Rightarrow P(E \cup \overline{E}) = P(S) = 1 [\because P(S) = 1]$$

\therefore Required probability is, $P(E \cup \overline{E}) = 1$

8. (b) $(a/2, b/2, c/2)$

Explanation:

General equation of the sphere is $x^2 + y^2 + z^2 + 2gx + 2fy + 2hz + d = 0$ -----
-----1)

Since 1) passes through the point (0,0,0) using this in 1) we get $d=0$

Similarly 1) passes through $(a, 0, 0), (0, b, 0), (0, 0, c)$ using these values in 1)

$$a^2 + 2ag = 0 \Rightarrow a(a + 2g) = 0$$

$$b^2 + 2bf = 0 \Rightarrow b(b + 2f) = 0$$

$$c^2 + 2ch = 0 \Rightarrow c(c + 2h) = 0$$

But as $abc \neq 0$ So, $a \neq 0, b \neq 0, c \neq 0$

So from above equations, we have $a = -2g, b = -2f, c = -2h$

centre is $(-f, -g, -h) = (a/2, b/2, c/2)$

9. (b) $21/36$

Explanation:

Given B getting number not less than A means B can get number on dice greater than or equal to A.

For throwing A and B one dice, the number of elements in the sample space is $6 \times 6 = 36$ i.e. $n(S) = 36$

Let E be the event of "B getting number not less than A", So it can happen in 21 ways out of 36 ways.

The ways are $E = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)\}$ i.e. $n(E) = 21$

Therefore, $P(E) = n(E)/n(S) = 21/36$

10. (b) an irrational number

Explanation: We have $(a + b)^n + (a - b)^n$

$$= [{}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + {}^nC_3 a^{n-3}b^3 + \dots + {}^nC_n b^n] + [{}^nC_0 a^n - {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 - {}^nC_3 a^{n-3}b^3 + \dots + (-1)^n {}^nC_n b^n]$$

$$= 2[{}^nC_0 a^n + {}^nC_2 a^{n-2}b^2 + \dots]$$

Let $a = \sqrt{3}$ and $b = 1$ and $n = 2n + 1$

$$\text{Now we get } (\sqrt{3} + 1)^{2n+1} + (\sqrt{3} - 1)^{2n+1} = 2[{}^{2n+1}C_0 (\sqrt{3})^{2n+1} + {}^{2n+1}C_2 (\sqrt{3})^{2n-1} + {}^{2n+1}C_4 (\sqrt{3})^{2n-3} + \dots]$$

Since there are odd powers of $\sqrt{3}$ we have $(\sqrt{3} + 1)^{2n+1} + (\sqrt{3} - 1)^{2n+1}$ is always an irrational number

11. infinite

12. ${}^{2n-1}C_n$

13. 18

14. given point

OR

$$\sqrt{333}$$

15. $\sec x \tan x$

OR

$$-\frac{1}{4}$$

16. Here $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10, 15, 20\}$

$$\begin{aligned} C - B &= \{2, 4, 6, 8, 10, 12, 14, 16\} - \{4, 8, 12, 16, 20\} \\ &= \{2, 6, 10, 14\} \end{aligned}$$

17. We know that a triangle will be formed by taking three points at a time.

$$\begin{aligned} \therefore \text{Required number of triangles} &= {}^{12}C_3 - {}^7C_3 \\ &= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} - \frac{7 \times 6 \times 5}{3 \times 2} \\ &= 220 - 35 = 185 \end{aligned}$$

Hence, the total number of such triangle is 185.

18. We have, $9x^2 + 16 = 0$

$$\Rightarrow 9x^2 - 16i^2 = 0 \quad [\because i^2 = -1]$$

$$\Rightarrow (3x)^2 - (4i)^2 = 0$$

$$\Rightarrow (3x - 4i)(3x + 4i) = 0$$

$$\Rightarrow 3x - 4i = 0 \text{ or } 3x + 4i = 0$$

$$\therefore x = \frac{4}{3}i \text{ or } x = -\frac{4}{3}i$$

Hence, the roots of the given equation are $\frac{4}{3}i$ and $-\frac{4}{3}i$.

OR

$$\begin{aligned}\text{Given, } (5 + 4i) + (5 - 4i) \\ = (5 + 5) + i(4 - 4) = 10 + 0i\end{aligned}$$

19. Here $A = \{x, y, z\}$ and $B = \{1, 2\}$

Number of elements in set $A = 3$

Number of elements in set $B = 2$

Number of subsets of $A \times B = 3 \times 2 = 6$

Number of relations from A to $B = 2^6$

20. We know that, ${}^nC_x = {}^nC_y \Leftrightarrow x + y = n$ or $x = y$

Here $x \neq y$, so $x + y = n$.

$$\Rightarrow n = 10 + 12 = 22$$

$$\text{Now, } {}^{23}C_n = {}^{23}C_{22} = 23$$

21. Let $X \in A \Rightarrow \{x\} \in P(A)$

$$\Rightarrow \{x\} \in P(B) [\because P(A) = P(B)]$$

$$\Rightarrow x \in B$$

$$\therefore A \subset B \dots (i)$$

$$\text{Let } x \in B \Rightarrow \{x\} \in P(B)$$

$$\Rightarrow \{x\} \in P(A) [\because P(A) = P(B)]$$

$$\Rightarrow x \in A \dots (ii)$$

$$\therefore B \subset A$$

From (i) and (ii) we have $A = B$

OR

We have, $A = \{L, 0, Y, A, L\} = \{L, 0, Y, A\}$

and, $B = \{A, L, L, 0, Y\} = \{L, 0, Y, A\}$

Clearly, $A = B$.

22. There are two children in a family which are to be boys or girls.

(i) Hence $S = \{BB, BG, GB, GG\}$

(ii) When the outcome is BB then number of girl is 0

When the outcome is BG then number of girl is 1

When the outcome is GB then number of girl is 1

When the outcome is GG then number of girl is 2

Hence, when we are interested to know the number of girls in the family then the required sample space $S = \{0, 1, 2\}$.

23. General term of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is $T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r} \left(-\frac{k}{x^2}\right)^r$
 $= {}^{10}C_r x^{\frac{10-r}{2}-2r} (-k)^r = {}^{10}C_r (-k)^r x^{\frac{10-5r}{2}}$

For the term free from x,

Put $\frac{10-5r}{2} = 0 \Rightarrow 10 - 5r = 0 \Rightarrow r = 2$

On putting $r = 2$, we get

$$T_{2+1} = {}^{10}C_2 (-k)^2$$

But term free from x = 405 [given]

$$\therefore {}^{10}C_2 k^2 = 405$$

$$\Rightarrow \frac{10 \times 9}{2} \times k^2 = 405 \Rightarrow k^2 = \frac{405 \times 2}{10 \times 9} = 9$$

$$\therefore k = \pm 3$$

24. Let $A(x_1, y_1)$ and $B(h, k)$ be two points.

$$\therefore \text{Slope of AB} = \frac{k-y_1}{h-x_1}$$

Slope of AB = m (given)

$$\therefore m = \frac{k-y_1}{h-x_1} \Rightarrow k - y_1 = m(h - x_1)$$

OR

Given points are $A(x_1, y_1) = (-1, 1)$ and $B(x_2, y_2) = (2, -4)$, then equation of line AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\Rightarrow y - 1 = \frac{-4-1}{2+1} (x + 1) [\because x_1 = -1, y_1 = 1, x_2 = 2, y_2 = -4]$$

$$\Rightarrow y - 1 = \frac{-5}{3} (x + 1) \Rightarrow 3y - 3 = -5x - 5$$

$$\Rightarrow 5x + 3y + 2 = 0$$

25. The given compound statement is

$S : 80$ is a multiple of 5 and 4.

The component statements of the given compound statement are

p : 80 is a multiple of 5.

q : 80 is a multiple of 4.

Then, $S \equiv p \wedge q$

Now, since we know that 80 is multiple of 5 as well as 4.

Therefore, the statements p and q are true.

Hence, the compound statement $S \equiv p \wedge q$ is also true i.e., $S \equiv p \wedge q$ is a valid statement.

$$\begin{aligned} 26. \text{ LHS} &= \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} \\ &= \frac{2 \sin\left(\frac{7x+5x}{2}\right) \cdot \cos\left(\frac{7x-5x}{2}\right) + 2 \sin\left(\frac{9x+3x}{2}\right) \cdot \cos\left(\frac{9x-3x}{2}\right)}{2 \cos\left(\frac{7x+5x}{2}\right) \cdot \cos\left(\frac{7x-5x}{2}\right) + 2 \cos\left(\frac{9x+3x}{2}\right) \cdot \cos\left(\frac{9x-3x}{2}\right)} \\ &[\because \sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \times \cos\left(\frac{x-y}{2}\right) \text{ and } \cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \times \cos\left(\frac{x-y}{2}\right)] \\ &= \frac{2 \sin 6x \cdot \cos x + 2 \sin 6x \cdot \cos 3x}{2 \cos 6x \cdot \cos x + 2 \cos 6x \cdot \cos 3x} \\ &= \frac{2 \sin 6x (\cos x + \cos 3x)}{2 \cos 6x (\cos x + \cos 3x)} = \frac{\sin 6x}{\cos 6x} = \tan 6x \\ &= \text{RHS} \end{aligned}$$

Hence proved.

27. Here $A \cup X = B \cup X$ for some set X

$$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$$

$$\Rightarrow A = (A \cap B) \cup (A \cap X) [\because A \cap (A \cup X) = A]$$

$$\Rightarrow A = (A \cap B) \cup \phi [\because A \cap X = \phi]$$

$$\Rightarrow A = A \cap B$$

$$\Rightarrow A \subset B \dots (i)$$

$$\text{Also } A \cup X = B \cup X$$

$$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$$

$$\Rightarrow (B \cap A) \cup (B \cap X) = B [\because B \cap (B \cup X) = B]$$

$$\Rightarrow (B \cap A) \cup \phi = B [\because B \cap X = \phi]$$

$$\Rightarrow B \cap A = B$$

$$\Rightarrow B \subset A \dots (ii)$$

From (i) and (ii), we have

$$A = B.$$

28. We have,

$$f(x) = x^2 - 2x - 3$$

Now,

$$f(-2) = (-2)^2 - 2(-2) - 3$$

$$= 4 + 4 - 3$$

$$= 5$$

$$f(-1) = (-1)^2 - 2(-1) - 3$$

$$= 1 + 2 - 3$$

$$= 0$$

$$f(0) = (0)^2 - 2 \times 0 - 3$$

$$= -3$$

$$f(1) = (1)^2 - 2 \times 1 - 3$$

$$= 1 - 2 - 3$$

$$= -4$$

$$f(2) = (2)^2 - 2 \times 2 - 3$$

$$= 4 - 4 - 3$$

$$= -3$$

$$\text{Range}(f) = \{-4, -3, 0, 5\}$$

OR

i. We have, $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$

$$\therefore (B \cap C) = \{1, 2, 3, 4\} \cap \{5, 6\} = \phi$$

$$\text{and } A \times (B \cap C) = \{1, 2\} \times \phi = \phi \dots\dots (i)$$

$$\text{Now, } A \times B = \{1, 2\} \times \{1, 2, 3, 4\}$$

$$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$$

$$\text{and } A \times C = \{1, 2\} \times \{5, 6\}$$

$$= \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

$$\therefore (A \times B) \cap (A \times C) = \phi \dots\dots (ii)$$

From Eqs. (i) and (ii),

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

ii. Now, $A \times C = \{1, 2\} \times \{5, 6\}$

$$= \{(1, 5), (1, 6), (2, 5), (2, 6)\} \dots \dots (iii)$$

$$\text{and } B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\}$$

$$= \{$$

$$(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, \dots \dots (iv)$$

From Eqs. (iii) and (iv), we can say,

$A \times C$ is a subset of $B \times D$.

$$\begin{aligned} 29. \lim_{x \rightarrow 3} \frac{e^x - e^3}{x - 3} &= \lim_{x \rightarrow 3} e^3 \left[\frac{e^{x-3} - 1}{x-3} \right] \\ &= e^3 \lim_{x \rightarrow 3} \left[\frac{e^{x-3} - 1}{x-3} \right] \\ &= e^3 \lim_{y \rightarrow 0} \left[\frac{e^y - 1}{y} \right] \text{ [put } y = x - 3 \text{ so that } y \rightarrow 0 \text{ as } x \rightarrow 3 \text{]} \\ &= e^3 \times 1 \\ &= e^3 \end{aligned}$$

$$\begin{aligned} 30. \text{ We have, } \frac{1}{1+i} &= \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1^2-i^2} = \frac{1-i}{1+1} \\ &= \frac{1-i}{2} = \frac{1}{2} - \frac{i}{2} \end{aligned}$$

$$\text{Let } r \cos \theta = \frac{1}{2} \dots (i)$$

$$\text{and } r \sin \theta = -\frac{1}{2} \dots (ii)$$

On squaring and adding Eqs. (i) and (ii), we get

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2$$

$$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \Rightarrow r = \frac{1}{\sqrt{2}} \text{ [}\because r \text{ is positive]}$$

On putting the value of r in Eqs. (i) and (ii), we get

$$\cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$

Since, $\cos \theta$ is positive and $\sin \theta$ is negative.

So, θ lies in IV quadrant.

$$\therefore \theta = -\frac{\pi}{4}$$

Hence, modulus of $\frac{1}{1+i}$ is $\frac{1}{\sqrt{2}}$ and argument is $\frac{-\pi}{4}$.

$$31. \text{ We have, } 2x + y > 3$$

Step I In equation form, given inequality can be written as

$$2x + y = 3 \dots (i)$$

Step II On putting $x = 0$ in Eq. (i), we get

$$2(0) + y = 3 \Rightarrow y = 3$$

\therefore The line meets Y - axis at $y = 3$ i.e., at point A (0,3).

On putting $y = 0$ in Eq. (i), we get

$$2x + 0 = 3 \Rightarrow x = \frac{3}{2}$$

\therefore The line meets X - axis at $x = \frac{3}{2}$ i.e., at point B $\left(\frac{3}{2}, 0\right)$

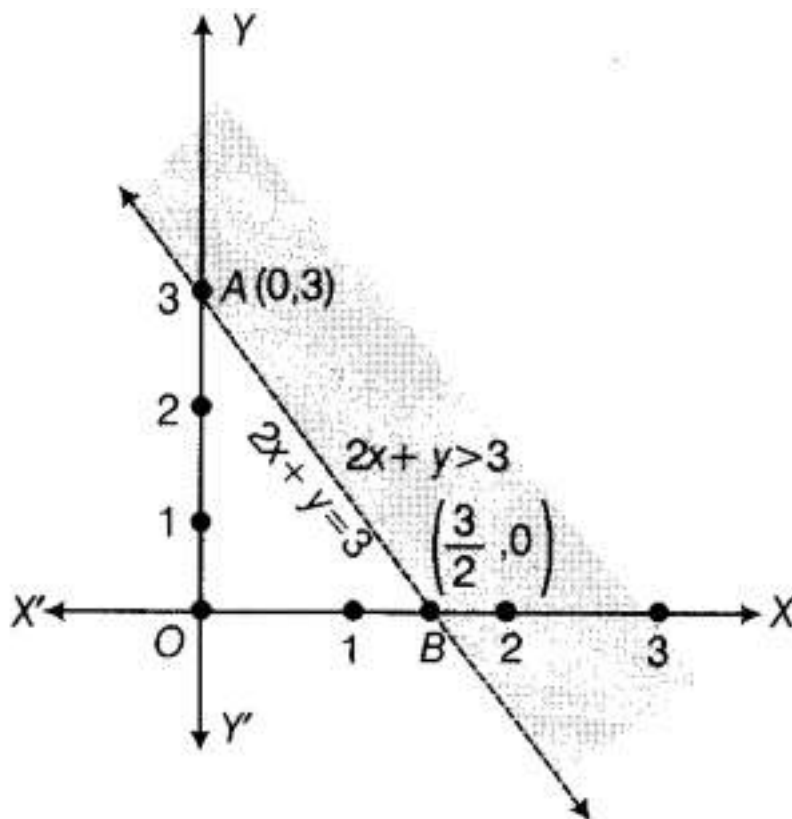
Step III Now, join the points obtained in **step II**. Since given inequality has the sign $>$, so we join the points by a dotted line.

Step IV Now, take a point not lying on the line say (0, 0) to check whether it satisfies the given inequality or not.

On putting $x = 0$ and $y = 0$ in given inequality, we get,

$$2(0) + 0 > 3 \Rightarrow 0 > 3 \text{ which is not correct.}$$

Since, (0, 0) does not satisfy the given inequality, so we shade the portion which does not contain (0, 0) i.e., the region above the dotted line AB. The shaded region gives the solution set.



OR

Here $\frac{x+3}{x-2} \geq 4, x \neq 2$

$$\Rightarrow \frac{x+3}{x-2} - 4 \geq 0$$

$$\Rightarrow \frac{x+3-4x+8}{x-2} \geq 0$$

$$\Rightarrow \frac{-3x+11}{x-2} \geq 0$$

$$\Rightarrow -3x + 11 \geq 0$$

$$\Rightarrow -3x \geq -11$$

Dividing both sides by -3

$$\therefore \frac{-3x}{-3} \leq \frac{-11}{-3}$$

$$\Rightarrow x \leq \frac{11}{3}$$

Thus the solution set of given inequation is $\left(-\infty, \frac{11}{3}\right] - \{2\}$.

32. Suppose $P(n)$ be the statement given by

$$P(n) : \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

$$\text{We have, } P(1) : \text{LHS} = \frac{1}{1.3} = \frac{1}{3}$$

$$\text{RHS} = \frac{1}{(2 \times 1 + 1)} = \frac{1}{3}$$

So, $P(1)$ is true.

Suppose $P(m)$ be true.

$$\Rightarrow \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2m-1)(2m+1)} = \frac{m}{2m+1} \dots (i)$$

We shall now show that $P(m+1)$ is true. For this we shall that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2m-1)(2m+1)} + \frac{1}{(2m+1)(2m+3)} = \frac{m+1}{2m+3}$$

Considering LHS

$$= \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2m-1)(2m+1)} + \frac{1}{(2m+1)(2m+3)}$$

$$= \frac{m}{2m+1} + \frac{1}{(2m+1)(2m+3)} \text{ [Using (i)]}$$

$$= \frac{2m^2 + 3m + 1}{(2m+1)(2m+3)} = \frac{(2m+1)(m+1)}{(2m+1)(2m+3)} = \frac{m+1}{2m+3} = \text{RHS}$$

$\therefore P(m+1)$ is true

$\Rightarrow P(m)$ is true $\therefore P(m+1)$ is true

\therefore by the principle of mathematical induction, the given result is true for all $n \in N$.

33. We have,

$$\text{LHS} = \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$$

$$\Rightarrow \text{LHS} = \cos 60^\circ (\cos 20^\circ \cos 40^\circ) \cos 80^\circ$$

$$\Rightarrow \text{LHS} = \frac{1}{2} \times \frac{1}{2} (2 \cos 20^\circ \cos 40^\circ) \cos 80^\circ \left[\because \cos \frac{\pi}{3} = \frac{1}{2} \right]$$

$$\begin{aligned}
&\Rightarrow \text{LHS} = \frac{1}{4} [\{\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)\} \cos 80^\circ] [\because 2 \cos A \cos B = \cos(A+B) + \cos(A-B)] \\
&\Rightarrow \text{LHS} = \frac{1}{4} \{(\cos 60^\circ + \cos 20^\circ) \cos 80^\circ\} \\
&\Rightarrow \text{LHS} = \frac{1}{4} \left\{ \left(\frac{1}{2} + \cos 20^\circ \right) \cos 80^\circ \right\} \\
&\Rightarrow \text{LHS} = \frac{1}{4} \left\{ \frac{1}{2} \cos 80^\circ + \cos 80^\circ \cos 20^\circ \right\} \\
&\Rightarrow \text{LHS} = \frac{1}{8} \{\cos 80^\circ + 2 \cos 80^\circ \cos 20^\circ\} \\
&\Rightarrow \text{LHS} = \frac{1}{8} [\cos 80^\circ + \{\cos(80^\circ + 20^\circ) + \cos(80^\circ - 20^\circ)\}] \\
&\Rightarrow \text{LHS} = \frac{1}{8} \{\cos 80^\circ + \cos 100^\circ + \cos 60^\circ\} \\
&\Rightarrow \text{LHS} = \frac{1}{8} \{\cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos 60^\circ\} \\
&\Rightarrow \text{LHS} = \frac{1}{8} \{\cos 80^\circ - \cos 80^\circ + \cos 60^\circ\} [\because \cos(180^\circ - x) = -\cos x] \\
&\Rightarrow \text{LHS} = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16} = \text{RHS}
\end{aligned}$$

OR

$$\text{Given, } \sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$$

$$\Rightarrow (\sin x - \cos x) - 3(\sin 2x - \cos 2x) + (\sin 3x - \cos 3x) = 0$$

$$\Rightarrow [\cos\left(\frac{\pi}{2} - x\right) - \cos x] - 3[\cos\left(\frac{\pi}{2} - 2x\right) - \cos 2x] + [\cos\left(\frac{\pi}{2} - 3x\right) - \cos 3x] = 0$$

$$[\because \sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)]$$

$$\Rightarrow -2 \sin\left(\frac{\frac{\pi}{2} - x + x}{2}\right) \sin\left(\frac{\frac{\pi}{2} - x - x}{2}\right) - 3[-2 \sin\left(\frac{\frac{\pi}{2} - 2x + 2x}{2}\right) \sin\left(\frac{\frac{\pi}{2} - 2x - 2x}{2}\right)] + [-2 \sin\left(\frac{\frac{\pi}{2} - 3x + 3x}{2}\right) \sin\left(\frac{\frac{\pi}{2} - 3x - 3x}{2}\right)] = 0$$

$$[\because \cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)]$$

$$\Rightarrow -2 \sin\left(\frac{\pi}{4}\right) \sin\left(\frac{\frac{\pi}{2} - 2x}{2}\right) + 3[2 \sin\left(\frac{\pi}{4}\right) \sin\left(\frac{\frac{\pi}{2} - 4x}{2}\right)] - 2 \sin\left(\frac{\pi}{4}\right) \sin\left(\frac{\frac{\pi}{2} - 6x}{2}\right) = 0$$

$$\Rightarrow 2 \sin\left(\frac{\pi}{4}\right) [-\sin\left(\frac{\pi}{4} - x\right) + 3 \sin\left(\frac{\pi}{4} - 2x\right) - \sin\left(\frac{\pi}{4} - 3x\right)] = 0$$

$$\Rightarrow [-\sin\left\{-\left(x - \frac{\pi}{4}\right)\right\} + 3 \sin\left\{-\left(2x - \frac{\pi}{4}\right)\right\} - \sin\left\{-\left(3x - \frac{\pi}{4}\right)\right\}] = 0$$

$$\Rightarrow [\sin\left(x - \frac{\pi}{4}\right) - 3 \sin\left(2x - \frac{\pi}{4}\right) + \sin\left(3x - \frac{\pi}{4}\right)] = 0$$

$$[\because \sin(-\theta) = -\sin \theta]$$

$$\Rightarrow [\sin\left(x - \frac{\pi}{4}\right) + \sin\left(3x - \frac{\pi}{4}\right)] - 3 \sin\left(2x - \frac{\pi}{4}\right) = 0$$

$$\Rightarrow 2 \sin\left(\frac{x - \frac{\pi}{4} + 3x - \frac{\pi}{4}}{2}\right) \cos\left(\frac{x - \frac{\pi}{4} - 3x + \frac{\pi}{4}}{2}\right) - 3 \sin\left(2x - \frac{\pi}{4}\right) = 0$$

$$\begin{aligned}
& [\because \sin(x+y) = 2\sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)] \\
& \Rightarrow 2\sin\left(\frac{4x-\pi}{2}\right) \cos\left(\frac{-2x}{2}\right) - 3\left(2x - \frac{\pi}{4}\right) = 0 \\
& \Rightarrow 2\sin\left(2x - \frac{\pi}{4}\right) \cos(-x) - 3\sin\left(2x - \frac{\pi}{4}\right) = 0 \\
& \Rightarrow 2\sin\left(2x - \frac{\pi}{4}\right) \cos x - 3\sin\left(2x - \frac{\pi}{4}\right) = 0 \\
& [\because \cos(-x) = \cos x] \\
& \Rightarrow \sin\left(2x - \frac{\pi}{4}\right) [2\cos x - 3] = 0 \\
& \Rightarrow \sin\left(2x - \frac{\pi}{4}\right) = 0 \text{ as } \cos x \neq \frac{3}{2} \\
& [\because \cos x \text{ cannot be greater than } 1] \\
& \therefore \sin\left(2x - \frac{\pi}{4}\right) = \sin 0 \\
& \Rightarrow 2x - \frac{\pi}{4} = n\pi \\
& \Rightarrow 2x = n\pi + \frac{\pi}{4} \\
& \Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}, n \in \mathbb{Z}
\end{aligned}$$

34. $a + b = 6\sqrt{ab}$

$$\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

by C and D

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$$

$$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{2}{1}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$$

again by C and D

$$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{a}{b} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)^2} \text{ (on squaring both sides)}$$

$$\frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$\frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$a : b = (3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$$

35. Let the equation of the circle passing through the given points be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \dots(i)$$

Since, circle passes through the point (2, - 6).

So, put $x = 2, y = - 6$ in Eq. (i), we get

$$4 + 36 + 4g - 12f + c = 0$$

$$\Rightarrow 4g - 12f + c = - 40 \dots(ii)$$

Also, circle passes through the point (6, 4).

So, put $x = 6, y = 4$ in Eq. (i), we get

$$36 + 16 + 12g + 8f + c = 0$$

$$\Rightarrow 12g + 8f + c = - 52 \dots(iii)$$

Also, circle passes through the point (- 3, 1).

So, put $x = - 3$ and $y = 1$ in Eq. (i), we get

$$9 + 1 - 6g + 2f + c = 0$$

$$\Rightarrow - 6g + 2f + c = - 10 \dots(iv)$$

On subtracting Eq. (iii) from Eq. (ii), we get

$$- 8g - 20f = 12 \dots(v)$$

On subtracting Eq. (iv) from Eq. (iii), we get

$$18g + 6f = - 42 \dots(vi)$$

On solving Eqs. (v) and (vi) for g and f , we get

$$g = - \frac{32}{13}, f = \frac{5}{13}$$

On putting the values of g and f in Eq. (ii), we get

$$c = - \frac{332}{13}$$

Now, on putting $g = - \frac{32}{13}, f = \frac{5}{13}$ and $c = - \frac{332}{13}$ in Eq. (i), we get

$$x^2 + y^2 - \frac{64}{13}x + \frac{10}{13}y - \frac{332}{13} = 0$$

$$\Rightarrow 13x^2 + 13y^2 - 64x + 10y - 332 = 0$$

which is the required equation of circle.

OR

Here, the equation of circle is $x^2 + y^2 + 4x + 6y + 11 = 0$

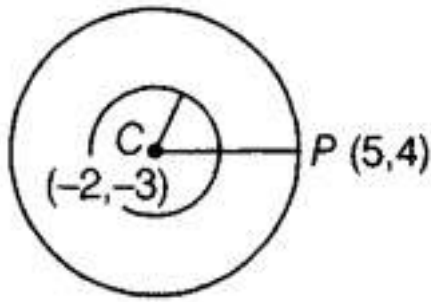
$$\Rightarrow (x^2 + 4x) + (y^2 + 6y) = - 11$$

On adding 4 and 9 both sides to make perfect squares, we get

$$(x^2 + 4x + 4) + (y^2 + 6y + 9) = - 11 + 4 + 9$$

$$\Rightarrow (x + 2)^2 + (y + 3)^2 = (\sqrt{2})^2 \dots(i)$$

Its centre is (- 2, - 3)



The required circle is concentric with circle 1, therefore its centre is $(-2, -3)$. Since, it passes through $(5, 4)$, therefore radius is

$$r = CP = \sqrt{(5 + 2)^2 + (4 + 3)^2} \quad [\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}]$$

$$= \sqrt{49 + 49} = 7\sqrt{2}$$

Hence, the equation of required circle having centre $(-2, -3)$ and radius $7\sqrt{2}$ is,

$$(x + 2)^2 + (y + 3)^2 = (7\sqrt{2})^2$$

$$\Rightarrow x^2 + 4x + 4 + y^2 + 6y + 9 = 98$$

$$\Rightarrow x^2 + 4x + y^2 + 6y - 85 = 0$$

36. To calculate the coefficient of variation we need to make the following table,

| Income | f_i | x_i | $u_i = \frac{x_i - A}{h} = \frac{x_i - 2750}{700}$ | $f_i u_i$ | u_i^2 | $f_i u_i^2$ |
|-----------|------------|-------|--|-----------|---------|-------------|
| 1000-1700 | 12 | 1350 | -2 | -24 | 4 | 48 |
| 1700-2400 | 18 | 2050 | -1 | -18 | 1 | 18 |
| 2400-3100 | 20 | 2750 | 0 | 0 | 0 | 0 |
| 3100-3800 | 25 | 3450 | 1 | 25 | 1 | 25 |
| 3800-4500 | 35 | 4150 | 2 | 70 | 4 | 140 |
| 4500-5200 | 10 | 4850 | 3 | 30 | 9 | 90 |
| | 120 | | | 83 | | 321 |

Here, $N = 120$, $A = 2750$, $\sum f_i u_i = 83$, $\sum f_i u_i^2 = 321$ and $h = 700$

First we need to find the mean of the given data,

$$\therefore \text{Mean} = \bar{X} = A + h \left(\frac{1}{N} \sum f_i u_i \right)$$

$$\Rightarrow \bar{X} = 2750 + 700 \left(\frac{83}{120} \right) = 3234.17$$

Next, we need to find standard deviation from the data.

$$\text{var}(x) = h^2 \left[\frac{1}{N} \sum f_i u_i^2 - \left(\frac{1}{N} \sum f_i u_i \right)^2 \right] = 490000 \left[\frac{321}{120} - \left(\frac{83}{120} \right)^2 \right] = 1076332.64$$

$$\therefore \text{S.D.} = \sqrt{\text{var}(x)} = \sqrt{1076332.64} = 1037.46$$

Now,

$$\text{Coefficient of variation (in percentage)} = \frac{S.D.}{\bar{X}} \times 100 = \frac{1037.46}{3234.17} \times 100 = 32.08\%$$