CBSE Sample Paper -02 (solved)
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours   Maximum Marks: 90

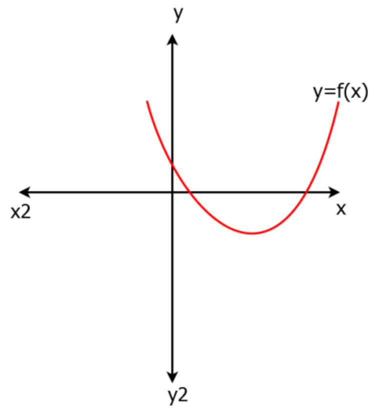
General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions. These are MCQs. Choose the correct option.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

SECTION – A

1. Without actually performing long division, state whether $\frac{13}{3125}$ will have terminating or non-terminating repeating decimal expansion. Also find the number of decimal places after which the decimal expansion terminates.

2. Identify the given graph corresponds to a linear polynomial or a quadratic polynomial.



3. For what value of k, will the following system of equations has a unique solution?

$x + 2y = 5$

$3x + ky = 15$

4. Evaluate tan5°tan25°tan30°tan65°tan85°.

5. Express sin81° + tan81° in terms of trigonometric ratios of angles between 0° and 45°.

SECTION – B

6. Find the HCF of 96 and 404 by prime factorisation method. Hence, find their LCM.

7. If $\tan^2\theta = 1 - a^2$, prove that $\sec\theta + \tan^3\theta\operatorname{cosec}\theta = \left(2 - a^2\right)^{3/2}$.

8. Sum of two numbers if 35 and their difference is 13. Find the numbers.

9. The number of students absent in a school was recorded every day for 147 days and the raw data was presented in the form of the following frequency table.

No. of students absent	5	6	7	8	9	10	11	12	13	15	18	20
No. of days	1	5	11	14	16	13	10	70	4	1	1	1

Obtain the median and describe what information it conveys.

10. A man goes 10 m due east and then 24 m due north. Find the distance from the starting point.

SECTION – C

11. Show that there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

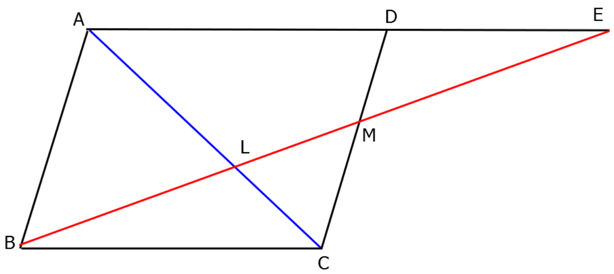
12. ABC is a right triangle right angled at C and AC = $\sqrt{3}$ BC. Prove that $\angle$ABC = 60°.

13. The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km, the charge paid is Rs 75 and for a journey of 15 km, the charge paid is Rs 110. What will a person have to pay for travelling a distance of 25 km?

14. If $a\sec\theta + b\tan\theta + c = 0$ and $p\sec\theta + q\tan\theta + r = 0$, prove that

$(br - qc)^2 - (pc - ar)^2 = (aq - bp)^2$

15. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD produced in E. Prove that EL = 2BL.



16. Find the zeros of the polynomial $f(u) = 4u^2 + 8u$ and verify the relationship between the zeros and its coefficients.

17. If $\sin\theta = \dfrac{a^2 - b^2}{a^2 + b^2}$, find the values of other five trigonometric ratios.

18. The following table gives weekly wages of workers in a certain organization. The frequency of class 49-52 is missing. It is known that the mean of the frequency distribution is 47.2. Find the missing frequency.

Weekly wages (Rs)	40-43	43-46	46-49	49-52	52-55
Number of workers	31	58	60	?	27

19. Solve: $ax + by = c$

$bx + ay = 1 + c$

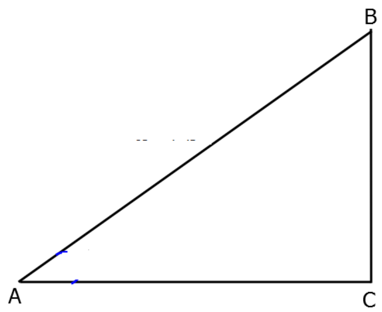
20. Without using trigonometric tables, evaluate

$$\frac{2}{3}\operatorname{cosec}^2 58° - \frac{2}{3}\cot 58° \tan 32° - \frac{5}{3}\tan 13° \tan 37° \tan 45° \tan 53° \tan 77°$$

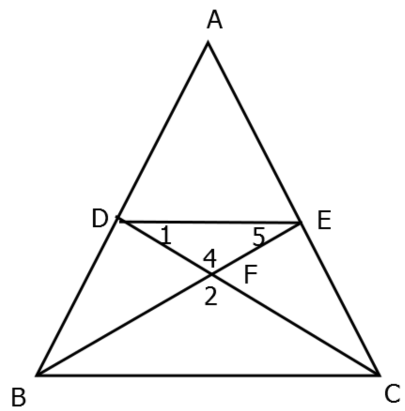
SECTION – D

21. Let a, b, c and p be rational numbers such that p is not a perfect cube. If $a + bp^{\frac{1}{3}} + cp^{\frac{2}{3}} = 0$, then prove that $a = b = c = 0$.

22. In a ΔABC, right angled at C, if tanA = $\frac{1}{\sqrt{3}}$, find the value of sinAcosB + cosAsinB.



23. In the given figure, DE || BC and AD : DB = 5 : 4. Find $\frac{\text{Area}(\Delta DEF)}{\text{Area}(\Delta CFB)}$.



24. Find the mean marks of students from the following cumulative frequency distribution:

Marks	Number of students	Marks	Number of students
0 and above	80	60 and above	28
10 and above	77	70 and above	16
20 and above	72	80 and above	10
30 and above	65	90 and above	8
40 and above	55	100 and above	0
50 and above	43		

25. If cosecθ – sinθ = l and secθ – cosθ = m, prove that $l^2m^2\left(l^2+m^2+3\right)=1$.

26. Find the values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

27. Draw the graphs of the following equations on the same graph paper.

$2x + y = 2$; $\quad 2x + y = 6$

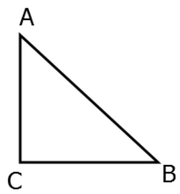
Find the coordinates of the vertices of the trapezium formed by these lines. Also, find the area of the trapezium formed.

28. Prove that if in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

29. In a ΔABC, right angled at C and ∠A = ∠B,

(i) Is cosA = cosB? (ii) Is tanA = tanB?

What about other trigonometric ratios for ∠A and ∠B. Will they be equal?



30. A sweet seller has 420 kaju burfis and 130 badam burfis. She wants to stack them in such a way that each stack has the same number and they take up the least area of the tray. What is the number of burfis that can be placed in each stack for this purpose?

31. Rohan's mother decided to distribute 900 bananas among patients of a hospital on her birthday. If the female patients are twice the male patients and the male patients are thrice the child patients in the hospital, each patient will get only one apple.

(i) Find the number of child patients, male patients and female patients in the hospital.

(ii) Which values are depicted by Rohan's father in the question?

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SECTION – A

1. **Solution:**

We have,

$$\frac{13}{3125} = \frac{13}{2^0 \times 5^5}$$

This shows that the prime factorization of the denominator of $\frac{13}{3125}$ is of the form $2^m \times 5^n$.

Hence, it has terminating decimal expansion which terminates after 5 places of decimals.

2. **Solution:**

We observe that the graph $y = f(x)$ is a parabola opening upwards. Therefore, $f(x)$ is a quadratic polynomial in which coefficient of x^2 is positive.

3. **Solution:**

The given system of equations will have a unique solution if

$$\frac{1}{3} \neq \frac{2}{k} \quad \Rightarrow \quad k \neq 6$$

4. **Solution:**

We have,

$\tan 5° \tan 25° \tan 30° \tan 65° \tan 85°$

$= (\tan 5° \tan 85°)(\tan 25° \tan 65°)\tan 30°$

$= (\tan 5° \cot 5°)(\tan 25° \cot 25°)\tan 30°$ $\left[\begin{array}{l} \because \tan 85° = \tan(90° - 5°) = \cot 5° \\ \tan 65° = \tan(90° - 25°) = \cot 25° \end{array}\right]$

$= 1 \times 1 \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

5. **Solution:**

We have,

$\text{Sin}81° + \tan 81°$

$= \sin(90° - 9°) + \tan(90° - 9°)$

$= \cos 9° + \cot 9°$ $[\because \sin(90° - \theta) = \cos\theta$ and $\tan(90° - \theta) = \cot\theta)$

SECTION – B

6. **Solution:**

We have,

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$404 = 2 \times 2 \times 101 = 2^2 \times 101$

$\therefore \quad HCF = 2^2 = 2 \times 2 = 4$

Now, HCF × LCM = Product of the numbers

$\Rightarrow \quad 4 \times LCM = 96 \times 404$

$\Rightarrow \quad LCM = \frac{96 \times 404}{4} = 96 \times 101 = 9696$

7. **Solution:**

We have,

$$\sec\theta + \tan^3\theta \operatorname{cosec}\theta$$

$$= \sec\theta \left\{ \frac{\sec\theta + \tan^3\theta \operatorname{cosec}\theta}{\sec\theta} \right\} \qquad \text{(Multiplying and dividing by } \sec\theta\text{)}$$

$$= \sec\theta \left\{ 1 + \tan^3\theta . \frac{\cos\theta}{\sin\theta} \right\}$$

$$= \sec\theta \left\{ 1 + \tan^3\theta \times \cot\theta \right\}$$

$$= \sqrt{1 + \tan^2\theta} \left\{ 1 + \tan^2\theta \right\}$$

$$= \left(1 + \tan^2\theta \right)^{3/2}$$

$$= \left\{ 1 + \left(1 - a^2 \right) \right\}^{3/2} = \left(2 - a^2 \right)^{3/2}$$

8. **Solution:**

Let the two numbers be x and y. Then,

$x + y = 35$

$x - y = 13$

Adding equations (i) and (ii), we get

$2x = 48 \quad \Rightarrow \quad x = 24$

Subtracting equation (ii) from equation (i), we get

$2y = 22 \quad \Rightarrow \quad y = 11$

Hence, the two numbers are 24 and 11.

9. **Solution:**

Calculation of median

x_i	5	6	7	8	9	10	11	12	13	15	18	20
f_i	1	5	11	14	16	13	10	70	4	1	1	1
cf	1	6	17	31	47	60	70	140	144	145	146	147

We have,

$$N = 147 \qquad \Rightarrow \qquad \frac{N}{2} = \frac{147}{2} = 73.5$$

The cumulative frequency just greater than $\frac{N}{2}$ is 140 and the corresponding value of variable x is 12. Thus, the median = 12. This means that for about half the number of days, more than 12 students were absent.

10. **Solution:**

Let the initial position of the man be O and his final position be B. Since the man goes 10 m due east and then 24 m due north. Therefore, ΔAOB is a right angled triangle right-angled at A such that OA = 10 m and AB = 24 m.

By Pythagoras theorem, we have

$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow \quad OB^2 = 10^2 + 24^2 = 100 + 576 = 676$$

$$\Rightarrow \quad OB = \sqrt{676} = 26 \text{ m}$$

SECTION – C

11. **Solution:**

If possible, let there be a positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational equal to $\frac{a}{b}$ (say), where a, b are positive integers. Then,

$$\frac{a}{b} = \sqrt{n-1}+\sqrt{n+1} \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{b}{a} = \frac{1}{\sqrt{n-1}+\sqrt{n+1}}$$

$$= \frac{\sqrt{n+1}-\sqrt{n-1}}{\left\{\sqrt{n+1}+\sqrt{n-1}\right\}\left\{\sqrt{n+1}-\sqrt{n-1}\right\}}$$

$$= \frac{\sqrt{n+1}-\sqrt{n-1}}{(n+1)-(n-1)} = \frac{\sqrt{n+1}-\sqrt{n-1}}{2}$$

$\Rightarrow \quad \frac{2b}{a} = \sqrt{n+1}-\sqrt{n-1}$ ...(ii)

Adding (i) and (ii) and subtracting (ii) from (i), we get

$$2\sqrt{n+1} = \frac{a}{b}+\frac{2b}{a} \text{ and } 2\sqrt{n-1} = \frac{a}{b}-\frac{2b}{a}$$

$\Rightarrow \quad \sqrt{n+1} = \frac{a^2+2b^2}{2ab}$ and $\sqrt{n-1} = \frac{a^2-2b^2}{2ab}$

$\Rightarrow \quad \sqrt{n+1}$ and $\sqrt{n-1}$ are rationals $\left[\begin{array}{l} \because a,b \text{ are integers} \\ \therefore \frac{a^2+2b^2}{2ab} \text{ and } \frac{a^2-2b^2}{2ab} \text{ are rationals.} \end{array}\right]$

$\Rightarrow \quad$ (n + 1) and (n – 1) are perfect squares of positive integers.

This is not possible as any two perfect squares differ at least by 3. Thus, there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

12. **Solution:**

Let D be the mid-point of AB. Join CD. Since ABC is a right triangle right angled at C, therefore

$AB^2 = AC^2 + BC^2$

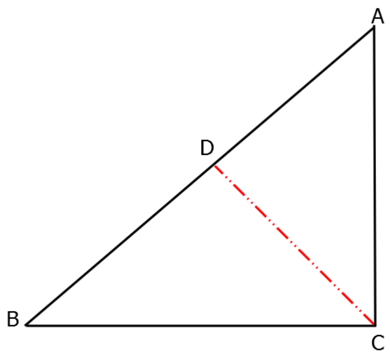
$\Rightarrow \quad AB^2 = \left(\sqrt{3}BC\right)^2 + BC^2$

$\Rightarrow \quad AB^2 = 3BC^2 + BC^2$

$\Rightarrow \quad AB^2 = 4BC^2$

$\Rightarrow \quad AB = 2BC$ ...(i)

But, $BD = \frac{1}{2}AB \quad \Rightarrow \quad AB = 2BD$ ...(ii)



From (i) and (ii), we have BC = BD.

We know that the mid-point of the hypotenuse of a right triangle is equidistant from the vertices.

$\therefore \quad CD = AD = BD \quad \Rightarrow \quad CD = BD$

Thus, in ΔABC, we have

BD = CD = BC

$\Rightarrow$ ΔBCD is an equilateral triangle.

$\Rightarrow \quad \angle ABC = 60°$

13. **Solution:**

Let the fixed charges of taxi be Rs x per km and the running charges be Rs y km/hr.

According to the given condition, we have

$x + 10y = 75$ ...(i)

$x + 15y = 110$ ...(ii)

Subtracting equation (ii) from equation (i), we get

$-5y = -35 \quad \Rightarrow \quad y = 7$

Putting $y = 7$ in equation (i), we get $x = 5$.

$\therefore$ Total charges from travelling a distance of 25 km

$= x + 25y$

$= 5 + 25 \times 7 = \text{Rs } 180$

14. **Solution:**

$a\sec\theta + b\tan\theta + c = 0$

$p\sec\theta + q\tan\theta + r = 0$

Solving these two equations for $\sec\theta$ and $\tan\theta$ by the cross-multiplication, we get

$$\frac{\sec\theta}{br-qc} = \frac{\tan\theta}{cp-ar} = \frac{1}{aq-bp}$$

$\Rightarrow$ $\sec\theta = \dfrac{br-cq}{aq-bp}$ and $\tan\theta = \dfrac{cp-ar}{aq-bp}$

Now, $\sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow \quad \left(\frac{br-cq}{aq-bp}\right)^2 - \left(\frac{cp-ar}{aq-bp}\right)^2 = 1$$

$$\Rightarrow \quad (br-cq)^2 - (cp-ar)^2 = (aq-bp)^2$$

15. **Solution:**

In ΔBMC and ΔEMD, we have

MC = MD [$\because$ M is the mid-point of CD]

$\angle$CMB = $\angle$EMD [Vertically opposite angles]

And, $\angle$MBC = $\angle$MED [Alternate angles]

So, by AAS-criterion of congruence, we have

$\Delta BMC \cong \Delta EMD$

$\Rightarrow$ BC = DE ...(i)

Also, AD = BC [$\because$ ABCD is a parallelogram] ...(ii)

AD + DE = BC + BC

$\Rightarrow$ AE = 2BC ...(iii)

Now, in ΔAEL and ΔCBL, we have

$\angle ALE = \angle CLB$ [Vertically opposite angles]

$\angle EAL = \angle BCL$ [Alternate angles]

So, by AA-criterion of similarity of triangles, we have

$\Delta AEL \sim \Delta CBL$

$\Rightarrow \quad \frac{EL}{BL} = \frac{AE}{CB}$

$\Rightarrow \quad \frac{EL}{BL} = \frac{2BC}{BC}$ [Using equation (iii)]

$\Rightarrow \quad \frac{EL}{BL} = 2 \quad \Rightarrow \quad EL = 2BL$

16. **Solution:**

We have,

$f(u) = 4u^2 + 8u$

$= 4u(u + 2)$

The zeros of $f(u)$ are given by

$f(u) = 0$

$\Rightarrow \quad 4u(u + 2) = 0$

$\Rightarrow \quad u = 0$ or $u + 2 = 0$

$\Rightarrow \quad u = 0$ or $u = -2$

Hence, the zeros of f(u) are:

$\alpha = 0$ and $\beta = -2$

Now, $\alpha + \beta = 0 + (-2) = -2$ and $\alpha\beta = 0 \times -2 = 0$

Also, $-\frac{\text{Coefficient of } u}{\text{Coefficient of } u^2} = -\frac{8}{4} = -2$

And, $\frac{\text{Constant term}}{\text{Coefficient of } u^2} = \frac{0}{2} = 0$

$\therefore$ Sum of the zeros $= -\frac{\text{Coefficient of } u}{\text{Coefficient of } u^2}$

And, Product of the zeros = $\dfrac{\text{Constant term}}{\text{Coefficient of } u^2}$

17. **Solution:**

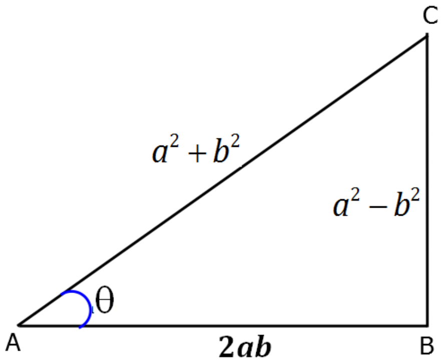
We have,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a^2 - b^2}{a^2 + b^2}$$

So, draw a right triangle right angled at B such that

Perpendicular = $a^2 - b^2$, Hypotenuse = $a^2 + b^2$ and $\angle$BAC = θ

By Pythagoras theorem, we have



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad \left(a^2+b^2\right)^2 = AB^2 + \left(a^2-b^2\right)^2$$

$$\Rightarrow \quad AB^2 = \left(a^2+b^2\right)^2 - \left(a^2-b^2\right)^2$$

$$= \left(a^4+b^4+2a^2b^2\right) - \left(a^4+b^4-2a^2b^2\right)$$

$$= 4a^2b^2 = \left(2ab\right)^2$$

$$\Rightarrow \quad AB = 2ab$$

When we consider the trigonometric ratios of $\angle$BAC = θ, we have

Base = AB = 2ab, Perpendicular = BC = $a^2 - b^2$ and Hypotenuse = AC = $a^2 + b^2$

$$\therefore \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{2ab}{a^2+b^2}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a^2-b^2}{2ab}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{a^2+b^2}{a^2-b^2}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{a^2+b^2}{2ab}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{2ab}{a^2-b^2}$$

18. **Solution:**

Let the missing frequency be *f*, the assumed mean be $A = 47$ and $h = 3$.

Calculation of mean

Class intervals	Mid-values (x_i)	f_i	$d_i = x_i - 47.5$	$u_i = \frac{x_i - 47.5}{3}$	f_iu_i
40-43	41.5	31	−6	−2	−62
43-46	44.5	58	−3	−1	−58
46-49	47.5	60	0	0	0
49-52	50.5	f	3	1	f
52-55	53.5	27	6	2	54
		$N = \sum f_i = 176 + f$			$\sum f_iu_i = f - 66$

We have,

$\overline{X}$ = 47.2, A = 47.5 and h = 3

$\therefore \quad \overline{X} = A + h\left\{\frac{1}{N}\sum f_iu_i\right\}$

$\Rightarrow \quad 47.2 = 47.5 + 3 \times \left\{\frac{f-66}{176+f}\right\}$

$\Rightarrow \quad -0.3 = 3 \times \left\{\frac{f-66}{176+f}\right\}$

$\Rightarrow \quad \frac{-1}{10} = \frac{f-66}{176+f}$

$\Rightarrow \quad -176 - f = 10f - 660$

$\Rightarrow \quad 11f = 484 \quad \Rightarrow \quad f = 44$

Hence, the missing frequency is 44.

19. **Solution:**

The given system of equations may be written as

$ax + by - c = 0$

$bx + ay - (1 + c) = 0$

By cross multiplication, we have

$$\frac{x}{b \times -(1+c) - a \times -c} = \frac{-y}{a \times -(1+c) - b \times -c} = \frac{1}{a \times a - b \times b}$$

$$\Rightarrow \quad \frac{x}{-b(1+c) + ac} = \frac{-y}{-a(1+c) + bc} = \frac{1}{a^2 - b^2}$$

$\Rightarrow \quad \frac{x}{ac-bc-b}=\frac{y}{ac-bc+a}=\frac{1}{a^2-b^2}$

$\Rightarrow \quad \frac{x}{c(a-b)-b}=\frac{y}{c(a-b)+a}=\frac{1}{(a-b)(a+b)}$

$\Rightarrow \quad x=\frac{c(a-b)-b}{(a-b)(a+b)}$ and $y=\frac{c(a-b)+a}{(a-b)(a+b)}$

$\Rightarrow \quad x=\frac{c}{a+b}-\frac{b}{(a-b)(a+b)}$ and $y=\frac{c}{a+b}+\frac{a}{(a-b)(a+b)}$

Hence, the solution of the given system of equation is

$$x=\frac{c}{a+b}-\frac{b}{a^2-b^2} \text{ and } y=\frac{c}{a+b}+\frac{a}{a^2-b^2}$$

20. **Solution:**

$$\frac{2}{3}\operatorname{cosec}^2 58°-\frac{2}{3}\cot 58°\tan 32°-\frac{5}{3}\tan 13°\tan 37°\tan 45°\tan 53°\tan 77°$$

$$=\frac{2}{3}\operatorname{cosec}^2 58°-\frac{2}{3}\cot 58°\tan(90°-58°)-\frac{5}{3}\tan 13°\tan 37°\tan 45° \tan(90°-37°)\tan(90°-13°)$$

$$=\frac{2}{3}\operatorname{cosec}^2 58°-\frac{2}{3}\cot^2 58°-\frac{5}{3}\tan 13°\tan 37°\tan 45°\cot 37°\cot 13°$$

$$=\frac{2}{3}(\operatorname{cosec}^2 58°-\cot^2 58°)-\frac{5}{3}\tan 13°\tan 37°\times 1\times\frac{1}{\tan 37°}\times\frac{1}{\tan 13°}$$

$$=\frac{2}{3}\times 1-\frac{5}{3}$$

$$=\frac{2}{3}-\frac{5}{3}=\frac{2-5}{3}=\frac{-3}{3}=-1$$

SECTION – D

21. **Solution:**

We have,

$$a+bp^{\frac{1}{3}}+cp^{\frac{2}{3}}=0 \qquad \text{...(i)}$$

Multiplying both sides by $p^{\frac{1}{3}}$, we get

$$ap^{\frac{1}{3}}+bp^{\frac{2}{3}}+cp=0 \qquad \text{...(ii)}$$

Multiplying (i) by b and (ii) by c and subtracting, we get

$$\left(ab+b^2p^{\frac{1}{3}}+bcp^{\frac{2}{3}}\right)-\left(acp^{\frac{1}{3}}+bcp^{\frac{2}{3}}+c^2p\right)=0$$

$\Rightarrow \quad \left(b^2-ac\right)p^{\frac{1}{3}}+ab-c^2p=0 \qquad [\because p^{\frac{1}{3}}$ is irrational]

$\Rightarrow \quad b^2-ac=0$ and $ab-c^2p=0$

$\Rightarrow \quad b^2=ac$ and $ab=c^2p$

$\Rightarrow \quad b^2=ac$ and $a^2b^2=c^4p^2$

$\Rightarrow \quad a^2\left(ac\right)=c^4p^2 \qquad \left[\text{Putting } b^2=ac \text{ in } a^2b^2=c^4p^2\right]$

$\Rightarrow \quad a^3c-c^4p^2=0$

$\Rightarrow \quad \left(a^3-c^3p^2\right)c=0$

$\Rightarrow \quad a^3-c^3p^2=0$ or $c=0$

Now, $\quad a^3-c^3p^2=0$

$\Rightarrow \quad p^2=\dfrac{a^3}{c^3}$

$\Rightarrow \quad \left(p^2\right)^{\frac{1}{3}}=\left(\dfrac{a^3}{c^3}\right)^{\frac{1}{3}}$

$\Rightarrow \quad \left(p^{\frac{1}{3}}\right)^2=\left\{\left(\dfrac{a}{c}\right)^3\right\}^{\frac{1}{3}}$

$\Rightarrow \quad \left(p^{\frac{1}{3}}\right)^2=\left\{\left(\dfrac{a}{c}\right)^3\right\}^{\frac{1}{3}}$

$\Rightarrow \quad \left(p^{\frac{1}{3}}\right)^2=\dfrac{a}{c}$

This is not possible as $p^{\frac{1}{3}}$ is irrational and $\dfrac{a}{c}$ is rational.

$\therefore \quad a^3-c^3p^2\neq 0$ and hence $c = 0$

Putting $c = 0$ in $b^2-ac=0$, we get $b = 0$.

Putting b = 0 and c = 0 in $a+bp^{\frac{1}{3}}+cp^{\frac{2}{3}}=0$, we get a = 0.

Hence, $a = b = c = 0$.

22. **Solution:**

Let us draw a ΔABC, right angled at C in which tanA = $\frac{1}{\sqrt{3}}$.

Now, tanA = $\frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{BC}{AC}=\frac{1}{\sqrt{3}} \qquad \left[\because \tan A=\frac{BC}{AC}\right]$

$\Rightarrow \quad$ BC = x and AC = $\sqrt{3}x$ ...(i)

By Pythagoras theorem, we have

$AB^2 = AC^2 + BC^2$

$= \left(\sqrt{3}x\right)^2+x^2$

$= 3x^2 + x^2 = 4x^2$

$\Rightarrow \quad AB = 2x$

To find the trigonometric ratios of ∠A, we have

Base = AC = $\sqrt{3}x$, Perpendicular = BC = x and Hypotenuse = AB = $2x$

$\therefore \quad \sin A = \frac{BC}{AB}=\frac{x}{2x}=\frac{1}{2}$ and $\cos A = \frac{AC}{AB}=\frac{\sqrt{3}x}{2x}=\frac{\sqrt{3}}{2}$

When we consider the trigonometric ratios of ∠B, we have

Base = BC = x, Perpendicular = AC = $\sqrt{3}x$ and Hypotenuse = AB = $2x$

$\therefore \quad \cos B = \frac{BC}{AB}=\frac{x}{2x}=\frac{1}{2}$ and $\sin B = \frac{AC}{AB}=\frac{\sqrt{3}x}{2x}=\frac{\sqrt{3}}{2}$

$\therefore \quad \sin A\cos B + \cos A\sin B = \frac{1}{2}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}=\frac{1}{4}+\frac{3}{4}=1$

23. **Solution:**

In ΔABC, we have,

DE || BC

$\Rightarrow \quad$ ∠ADE = ∠ABC and ∠AED = ∠ACB [Corresponding angles]

Thus, in triangles ADE and ABC, we have

$\angle A = \angle A$ [Common]

$\angle ADE = \angle ABC$

And, $\angle AED = \angle ACB$

$\therefore$ $\Delta AED \sim \Delta ABC$ [By AAA similarity]

$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$$

We have,

$$\frac{AD}{DB} = \frac{5}{4}$$

$$\Rightarrow \quad \frac{DB}{AD} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{DB}{AD} + 1 = \frac{4}{5} + 1$$

$$\Rightarrow \quad \frac{DB+AD}{AD} = \frac{4+5}{5}$$

$$\Rightarrow \quad \frac{AB}{AD} = \frac{9}{5} \quad \Rightarrow \quad \frac{AD}{AB} = \frac{5}{9}$$

$$\therefore \quad \frac{DE}{BC} = \frac{5}{9}$$

In ΔDFE and ΔCFB, we have

$\angle 1 = \angle 3$ [Alternate interior angles]

$\angle 2 = \angle 4$ [Vertically opposite angles]

Therefore, by AA similarity criterion, we have

$\Delta DFE \sim \Delta CFB$

$$\Rightarrow \quad \frac{\text{Area}(\Delta DEF)}{\text{Area}(\Delta CFB)} = \frac{DE^2}{BC^2} = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$$

24. **Solution:**

Here, we have the cumulative frequency distribution. So, first we convert it into an ordinary frequency distribution. We observe that there are 80 students getting marks greater than or equal to 0 and 77 students have secured 10 and more marks. Therefore, the number of students getting marks between 0 and 10 is 80 – 77 = 3.

Similarly, the number of students getting marks between 10 and 20 is 77 – 72 = 5 and so on.

Thus, we obtain the following frequency distribution:

Marks	Number of students	Marks	Number of students
0-10	3	50-60	15
10-20	5	60-70	12
20-30	7	70-80	6
30-40	10	80-90	2
40-50	12	90-100	8

Now, we compute arithmetic mean by taking 55 as the assumed mean.

Computation of mean

Marks (x_i)	Mid-value	Frequency (f_i)	$u_i = \frac{x_i - 55}{10}$	$f_i u_i$
0-10	5	3	–5	–15
10-20	15	5	–4	–20
20-30	25	7	–3	–21
30-40	35	10	–2	–20
40-50	45	12	–1	–12
50-60	55	15	0	0
60-70	65	12	1	12
70-80	75	6	2	12
80-90	85	2	3	6
90-100	95	8	4	32
Total		$\sum f_i = 80$		$\sum f_i u_i = -26$

We have,

$N = \sum f_i = 80, \sum f_i u_i = -26, A = 55$ and $h = 10$

$\therefore \quad \overline{X} = A + h\left\{\frac{1}{N}\sum f_i u_i\right\}$

$= 55 + 10 \times \frac{-26}{80} = 55 - 3.25 = 51.75$ marks

25. **Solution:**

We have,

LHS $= l^2 m^2 \left(l^2 + m^2 + 3\right) = 1$

$= (\text{cosec}\theta - \sin\theta)^2(\sec\theta - \cos\theta)^2\{(\text{cosec}\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3\}$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2 \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 \left\{\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 + 3\right\}$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2 \left\{\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2 + 3\right\}$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^2 \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 \left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right\}$$

$$= \frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^4\theta}{\cos^2\theta} \left\{\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right\}$$

$$= \cos^2\theta \times \sin^2\theta \left(\frac{\cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta}{\cos^2\theta\sin^2\theta}\right)$$

$$= \cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta$$

$$= \left\{\left(\cos^2\theta\right)^3 + \left(\sin^2\theta\right)^3\right\} + 3\cos^2\theta\sin^2\theta$$

$$= \left\{\left(\cos^2\theta + \sin^2\theta\right)^3 - 3\cos^2\theta\sin^2\theta\left(\cos^2\theta + \sin^2\theta\right)\right\} + 3\cos^2\theta\sin^2\theta$$

$$\left[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)\right]$$

$$= \left(1 - 3\cos^2\theta\sin^2\theta\right) + 3\cos^2\theta\sin^2\theta \qquad \left[\because \cos^2\theta + \sin^2\theta = 1\right]$$

= 1 = RHS

26. **Solution:**

If $x^4 + x^3 + 8x^2 + ax + b$ is exactly divisible by $x^2 + 1$, then the remainder should be zero.

On dividing, we get

$$\begin{array}{r|l}
 & x^2 + x + 7 \\
x^2 + 1 & x^4 + x^3 + 8x^2 + ax + b \\
 & x^4 \qquad + x^2 \\
 & - \qquad\quad - \\
\hline
 & \quad x^3 + 7x^2 + ax + b \\
 & \quad x^3 \qquad\quad + x \\
 & \quad + \qquad\quad - \\
\hline
 & \qquad 7x^2 + x(a-1) + b \\
 & \qquad 7x^2 \qquad\qquad + 7 \\
 & \qquad - \qquad\qquad - \\
\hline
\end{array}$$

$\therefore$ Quotient = $x^2 + x + 7$ and Remainder = $x(a - 1) + (b - 7)$

Now, remainder = 0

$\Rightarrow$ $x(a - 1) + (b - 7) = 0$

$\Rightarrow$ $x(a - 1) + (b - 7) = 0x + 0$

$\Rightarrow$ $a - 1 = 0$ and $b - 7 = 0$

$\Rightarrow$ $a = 1$ and $b = 7$

27. **Solution:**

Graph of the equation $2x + y = 2$:

When $y = 0$, we have $x = 1$

When $x = 0$, we have $y = 2$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 2$.

x	1	0
y	0	2

Graph of the equation $2x + y = 6$:

When $y = 0$, we get $x = 3$

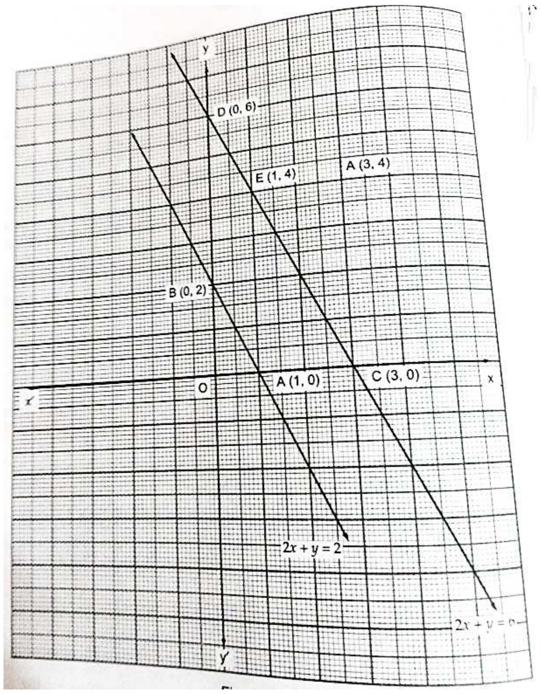
When $x = 0$, we get $y = 6$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 6$.

x	3	0
y	0	6

Plotting points A(1, 0) and B (0, 2) on the graph paper on a suitable scale and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 2$ as shown in the graph.

Plotting points C(3, 0) and D(0, 6) on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented



by the equation $2x + y = 6$ as shown in the graph.

Clearly, lines AB and CD form trapezium ACDB.

Also, area of trapezium ACDB = Area of ΔOCD – Area of ΔOAB

$$= \frac{1}{2}(OC \times OD) - \frac{1}{2}(OA \times OB)$$

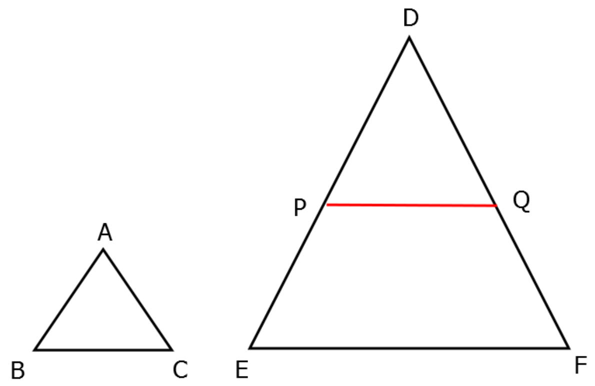
$$= \frac{1}{2}(3 \times 6) - \frac{1}{2}(1 \times 2) = 8 \text{ sq.units}$$

28. **Solution:**

Given: Two triangles ABC and DEF such that $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$.

To prove: $\Delta ABC \sim \Delta DEF$

Construction: Mark points P and Q on DE and DF, respectively such that DP = AB and DQ = AC. Join PQ.



Proof: In triangles ABC and DPQ, we have

$AB = DP$, $\angle A = \angle D$ and $AC = DQ$

Therefore, by SAS criterion of congruence, we have

$\Delta ABC \cong \Delta DPQ$ ...(i)

Now, $\frac{AB}{DE} = \frac{AC}{DF}$

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ [$\because$ AB = DP and AC = DQ]

$\Rightarrow PQ \parallel EF$ [By the converse of Thale's theorem]

$\Rightarrow \angle DPQ = \angle E$ and $\angle DQP = \angle F$ [Corresponding angles]

Thus, in triangles DPQ and DEF, we have

$\angle DPQ = \angle E$ and $\angle DQP = \angle F$

Therefore, by AAA criterion of similarity, we hve

$\Delta DPQ \sim \Delta DEF$ ...(ii)

From (i) and (ii), we get

$\Delta ABC \cong \Delta DPQ$ and $\Delta DPQ \sim \Delta DEF$

$\Rightarrow$ $\Delta ABC \sim \Delta DPQ$ and $\Delta DPQ \sim \Delta DEF$

$\Rightarrow$ $\Delta ABC \sim \Delta DEF$

29. **Solution:**

We have,

$\angle A = \angle B$

$\Rightarrow$ BC = AC [$\because$ Sides opposite to equal angles are equal]

Let BC = AC = x (say)

Using Pythagoras theorem in ΔACB, we have

$AB^2 = AC^2 + BC^2$

$= x^2 + x^2$

$\Rightarrow$ $AB = \sqrt{2}x$

(i) We have,

$$\cos A = \frac{AC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

$$\cos B = \frac{BC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

$\therefore$ cosA = cosB

(ii) We have,

$$\tan A = \frac{BC}{AC} = \frac{x}{x} = 1$$

$$\tan B = \frac{AC}{BC} = \frac{x}{x} = 1$$

$\therefore$ tanA = tanB

Now, $\sin A = \frac{BC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$ and $\sin B = \frac{AC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

$\therefore$ sinA = sinB

cotA = $\frac{AC}{BC} = \frac{x}{x} = 1$ and cotB = $\frac{BC}{AC} = \frac{x}{x} = 1$

$\therefore$ cotA = cotB

secA = $\frac{AB}{AC} = \frac{\sqrt{2}x}{x} = \sqrt{2}$ and secB = $\frac{AB}{BC} = \frac{\sqrt{2}x}{x} = \sqrt{2}$

$\therefore$ secA = secB

cosecA = $\frac{AB}{BC} = \frac{\sqrt{2}x}{x} = \sqrt{2}$ and cosecB = $\frac{AB}{AC} = \frac{\sqrt{2}x}{x} = \sqrt{2}$

$\therefore$ cosecA = cosecB

30. **Solution:**

The area of the tray that is used up in stacking the burfis will be least if the seet seller stacks maximum number of burfis in each stack. Since each stack must have the same number of burfis, therefore, the number of stacks will be least if the number of burfis in each stack is equal to the HCF of 420 and 130.

In order to find the HCF of 420 and 130, let us apply Euclid's division lemma to 420 and 130 to get

420 = 130 × 3 + 130 ...(i)

$$\left[\begin{array}{r} 3 \\ 130\overline{)\ 420} \\ \underline{-390} \\ 30 \end{array}\right]$$

Let us now consider the divisor 130 and the remainder 30 and apply division lemma to get

130 = 30 × 4 + 10 ...(ii)

$$\left[\begin{array}{r} 4 \\ 30\overline{)\ 130} \\ \underline{-120} \\ 10 \end{array}\right]$$

Considering now divisor 30 and the remainder 10 and apply division lemma, we get

30 = 3 × 10 + 0 ...(iii)

$$\left[\begin{array}{r} 3 \\ 10\overline{)\ 30} \\ \underline{-30} \\ 0 \end{array}\right]$$

Since, the remainder at this stage is zero. Therefore, last divisor 10 is the HCF of 420 and 130. Hence, the sweet seller can make stacks of 10 burfis of each kind to cover the least area of the tray.

31. **Solution:**

(i) Let the number of child patients in the hospital be x.

Then, the number of male patients = 3x

And, the number of female patients = 2(3x) = 6x

According to the question,

$6x + 3x + x = 900$

$\Rightarrow \quad 10x = 900$

$\Rightarrow \quad x = \frac{900}{10} = 90$

Thus, the number of child patients in the hospital is 90.

And, the number of male patients = 3 × 90 = 270

The number of female patients = 2 × 270 = 540

(ii) The values depicted by Rohan's father in the question are charity and empathy.