

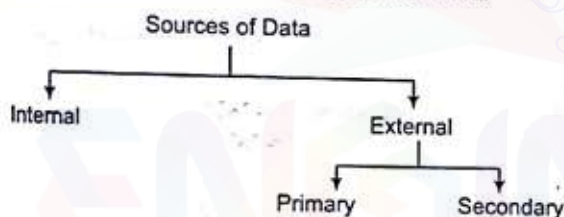
# Statistical Data Analysis

Statistics is the branch of Mathematics which deals with the collection, analysis and interpretation of numerical data.

## Collection of Data

Collection of data is the first step in statistics towards achieving the goal on conclusion.

There are different sources for collection of data.



On the basis of collection, data are of two types

- 1. Primary Data** It is the data collected actually in the process of investigation by the investigator. It is original and first hand information.
- 2. Secondary Data** It is the data which is already collected by other persons. e.g., as investigator collects data related to industries through the government publications.

## Presentation of Data

**Raw or Ungrouped Data** When the data are presented in random and is not prepared according to some order. It does not give us a clear picture of the class.

**Grouped Data** When the data is arranged in any manner like ascending or descending etc. It can also be presented in the form of a table called frequency distribution table.

## Types of Frequency Distribution

Frequency distribution are of two types.

- 1. Discrete Frequency Distribution** A frequency distribution is called a discrete frequency distribution, if

data are presented in a way that exact measurements of the units are clearly shown.

Marks	No. of Students (Frequency)
40	7
60	3
80	3
100	2
Total	15

- 2. Continuous Frequency Distribution** A frequency distribution in which data are arranged in classes or groups which are not exactly measurable.

Marks	No. of Students (Frequency)
10-20	1
20-30	2
30-40	3
40-50	4
50-60	6
60-70	7
Total	23

**Frequency** Number of observations falling in a particular class is called the frequency of that class.

As in the above table there are 7 students who scored 40 marks, so frequency of students having 40 marks is 7.

**Class Frequency** The frequency of a class in a continuous frequency distribution is called as class frequency.

As the class frequency of class 50 to 60 is 6 and of 60-70 is 7.

- The sum (total) of the frequency is known as total frequency. Here in above table total frequency is 23.

**Class Marks** It is the mid-point of the class interval class mark

$$= \frac{\text{Lower limit of class} + \text{Upper limit of class}}{2}$$

**Range** It is the difference between the two extreme observations in an arranged data.

Range = Maximum observation - Minimum observation  
e.g., range of the observations 4, 7, 8, 10, 12 = 12 - 4 = 8



**Cumulative Frequency** The cumulative frequency of a class interval is the sum of frequencies of all classes upto that class (including the frequency of that particular class).

**Example 1.** Consider the table given below

Marks	Number of Students (Frequency)	Cumulative Frequency
0-10	13	13
10-20	7	20
20-30	5	25
30-40	4	29
40-50	1	30
50-60	7	37
60-70	3	40
70-80	4	44
80-90	5	49
90-100	1	50
	50	

Then, the value of the following

(i) Frequency of class 10-20

(ii) Class size

(iii) Mid-value of 60-70

(iv) Total frequencies

(a) 10, 65, 50, 60 (b) 7, 10, 65, 60

(c) 50, 65, 10, 10 (d) 7, 10, 65, 50

**Sol.** (d) (a) Here, frequency of class 10-20 is 7.

(b) Class size = Upper limit - Lower limit = 30 - 20 = 10

(c) Mid value =  $\frac{\text{Upper limit} + \text{Lower limit}}{2} = \frac{60 + 70}{2} = 65$

(d) Total frequencies = 50

**Example 2.** The class mark of the interval 12.5-17.5 is

(a) 5 (b) 12.5 (c) 15 (d) 17.5

**Sol.** (c) Class mark =  $\frac{\text{Lower limit} + \text{Upper limit}}{2}$

$$= \frac{12.5 + 17.5}{2} = \frac{30}{2} = 15$$

**Example 3.** The class marks of a distribution are 54, 64, 74, 84, 94 and 104. Then, the class size is

(a) 5 (b) 10 (c) 54 (d) 104

**Sol.** (b) [Class size is the difference between the class marks of two adjacent classes]

Class size = 74 - 64 = 10

## Measures of Central Tendency

Generally average of a distribution is a measure of the middle or expected value of distribution. Such values are known as measures of central tendency. These are as follows

### Arithmetic Mean

1. **Arithmetic Mean of Ungrouped or Individual Series** If  $x_1, x_2, x_3, \dots, x_n$  are  $n$  observations, then

$$(a) \text{ Direct method } \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$(b) \text{ Shortcut method } \bar{x} = A + \frac{1}{n} \sum_{i=1}^n d_i$$

where, A = Assumed mean and  $d_i = x_i - A$

2. **Mean of Grouped or Continuous Series** If  $x_1, x_2, x_3, \dots, x_n$  are  $n$  observations whose corresponding frequencies are  $f_1, f_2, f_3, \dots, f_n$ , then

(a) Direct method

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + \dots + x_n f_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

$$(b) \text{ Shortcut method } \bar{x} = A + \frac{\sum_{i=1}^n f_i d_i}{\sum_{i=1}^n f_i}$$

where A = Assumed mean and  $d_i = x_i - A$

**Example 4.** The arithmetic mean of the marks from the following table is

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	12	18	27	20	17	6

(a) 20

(b) 28

(c) 2800

(d) 100

**Sol.** (b)

Marks	Class mark $x$	$f$	$fx$
0-10	5	12	60
10-20	15	18	270
20-30	25	27	675
30-40	35	20	700
40-50	45	17	765
50-60	55	6	330
		$\Sigma f = 100$	$\Sigma fx = 2800$

$$\therefore \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{2800}{100} = 28$$

### Weighted Arithmetic Mean

If corresponding weight of  $x_1, x_2, \dots, x_n$  are  $w_1, w_2, \dots, w_n$  respectively, then

Weighted arithmetic mean

$$= \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n} = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$$

**Example 5.** The weighted arithmetic mean of the first  $n$  natural numbers, the weights being the corresponding numbers, is

$$(a) \frac{n+1}{2}$$

$$(b) \frac{n+2}{2}$$

$$(c) \frac{2n+1}{3}$$

(d) None of these



Sol. (c) First  $n$  natural numbers are  $1, 2, 3, \dots, n$ ; whose corresponding

$$\left\{ \begin{array}{l} \therefore \Sigma n^2 = \frac{n(n+1)(2n+1)}{6} \\ \Sigma n = \frac{n(n+1)}{2} \end{array} \right\}$$

$$\therefore \text{Weight arithmetic mean} = \frac{1 \times 1 + 2 \times 2 + \dots + n \times n}{1 + 2 + \dots + n}$$

$$= \frac{1^2 + 2^2 + \dots + n^2}{1 + 2 + \dots + n} = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{2n+1}{3}$$

## Combined Arithmetic Mean

If two sets of observations are given, then the combined mean of both the sets can be calculated by the following formula.

$$\bar{x}_{12} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

where  $\bar{x}_{12}$  = mean of sets of observations

$\bar{x}_1$  = mean of first set of observations

$n_1$  = number of observations in first set

$\bar{x}_2$  = mean of second set of observations

$n_2$  = number of observations in second set

## Properties of Mean

1. Algebraic sum of the deviations of a set of values from their arithmetic mean is zero.
2. The sum of the squares of the deviations of a set of values is minimum when taken about mean.
3. Arithmetic mean is affected by the change or shifting of origin and change of scale.

**Example 6.** The average salary of male employees in a firm was ₹ 5200 and that of females was ₹ 4200. The mean salary of all the employees was ₹ 5000. The percentage of male and female employees are, respectively.

(a) 80 and 20 (b) 20 and 80 (c) 60 and 40 (d) 52 and 48

Sol. (a) Let  $x_1 = 5200$ ,  $x_2 = 4200$ ,  $\bar{x} = 5000$

$$\text{Also, we know that, } \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$\Rightarrow 5000(n_1 + n_2) = 5200n_1 + 4200n_2 \Rightarrow \frac{n_1}{n_2} = \frac{4}{1}$$

$\therefore$  The percentage of male employees in the firm

$$= \frac{4}{4+1} \times 100 = 80$$

and the percentage of female employees in the firm =  $\frac{1}{4+1} \times 100 = 20$

## Median

**1. Median of an Individual Series** If number of observations is  $n$ . Arrange the observations in ascending or descending order,

(a) If  $n$  is an odd number, then

$$\text{Median} = \text{Value of } \left( \frac{n+1}{2} \right) \text{th term}$$

(b) If  $n$  is an even number, then

$$\text{Median} = \frac{\text{Value of } \left( \frac{n}{2} \right) \text{th term} + \text{Value of } \left( \frac{n}{2} + 1 \right) \text{th term}}{2}$$

**2. Median of a Discrete Frequency Series** First arrange the data in ascending or descending order and find cumulative frequency. Now, find  $\frac{N}{2}$ , where  $N = \Sigma f_i$ .

See the cumulative frequency just greater than  $\frac{N}{2}$ . The corresponding value of  $x$  is median.

**3. Median of a Continuous Series** In this case, the class corresponding to the cumulative frequency just greater than  $\frac{N}{2}$  is called the median class and the value of median is obtained by the following formula.

$$\text{Median} = l + \left( \frac{\frac{N}{2} - c}{f} \right) \times h$$

where,  $l$  = lower limit of median class

$f$  = frequency of median class

$h$  = size of median class

$c$  = commulative frequency of class before median class

**Example 7.** From the data given, the median of the average deposit balance of saving for the branch during March 1982 is

Average Deposit Balance (in ₹)	Number of Deposit
Less than 100	26
100-200	68
200-300	145
300-400	242
400-500	188
500-600	65
600-700	16

(a) 356

(b) 300

(c) 56.2

(d) 356.2

Sol. (d)

Average Deposit Balance (in ₹)	$f$	Cumulative Frequency $cf$
Less than 100	26	26
100-200	68	94
200-300	145	239
300-400	242	481
400-500	188	669
500-600	65	734
600-700	16	750

$$\frac{N}{2} = \frac{750}{2} = 375$$

The frequency just greater than 375 is 481.

$\therefore$  Median class is 300-400.



$$\begin{aligned}\text{Median} &= l + \frac{\left(\frac{N}{2} - c\right)}{f} \times h = 300 + \frac{375 - 239}{242} \times 100 \\ &= 300 + 56.2 = 356.2\end{aligned}$$

## Mode

- Mode of Zan Individual Series** The value which is repeated maximum number of times is called mode of the series.
- Mode of a Discrete Frequency Series** In this case, mode is the value of the variate corresponding to the maximum frequency.
- Mode of a Continuous Series** The class which has maximum frequency is called modal class or group. The mode is given by the formula

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

where,  $l$  = lower limit of modal class

$h$  = size of modal class

$f_1$  = frequency of modal class

$f_0$  = frequency of the class preceding the modal class

$f_2$  = frequency of the class succeeding the modal class

□ Relation among Mean, Median and Mode  
Mode = 3 (Median) - 2 (Mean)

## Some Useful Formulae

- If  $x_1, x_2, \dots, x_n$  are  $n$  observations with their mean  $M$  then deviation  $i_i$  is given by  $i_i = |x_i - M|$ .
- Mean deviation for individual series is given by  $\frac{\sum |i_i|}{n}$ .
- Mean deviation for discrete series is given by  $\frac{\sum f |i_i|}{n}$ .
- Standard deviation ( $\sigma$ ) is given by  $\sqrt{\frac{\sum |i_i|^2}{n}}$ .
- Coefficient of variation =  $\frac{\text{Standard deviation}}{\text{Mean}} \times 100$ .
- Coefficient of mean deviation =  $\frac{\text{Mean deviation}}{\text{Mean or median}} \times 100$ .

**Example 8.** The mode of the following distribution is  
(a) 46 (b) 6.66 (c) 46.67 (d) None of these

Class Interval	Frequency
0-10	5
10-20	8
20-30	7
30-40	12
40-50	28
50-60	20
60-70	10
70-80	10

**Sol.** (c) Here, maximum frequency is 28. Thus, the class 40-50 is the modal class.

$$\therefore \text{Mode} = 40 + \frac{10(28 - 12)}{(2 \times 28 - 12 - 20)} = 40 + 6.666 = 46.67 \text{ (approx)}$$

## Geometric Mean

If  $a, G$  and  $b$  are in GP, then the geometric mean between  $a$  and  $b$  is,  $G = \sqrt{ab}$ .

## To Insert $n$ Geometric Means Between Two given Numbers

Let  $a$  and  $b$  two numbers, there can be  $n$  geometric means  $G_1, G_2, \dots, G_n$  such that  $a, G_1, G_2, \dots, G_n, b$  form a GP.

In general,  $G_k = a \left(\frac{b}{a}\right)^{\frac{k}{n+1}}, \forall k = 1, 2, 3, \dots, n$

- The geometric mean of  $n$  positive numbers  $a_1, a_2, a_3, \dots, a_n$  is GM.

$$\Rightarrow (a_1 a_2 a_3 \dots a_n)^{\frac{1}{n}}$$

## Harmonic Mean

The harmonic mean of  $n$  positive numbers  $a_1, a_2, \dots, a_n$  is

$$\frac{1}{H} = \frac{1}{n} \left( \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_n} \right)$$

where  $H$  denotes the harmonic mean.

## Relation among Arithmetic Mean, Geometric Mean and Harmonic Mean

$$G^2 = A \times H,$$

where  $G$  = geometric mean

$A$  = arithmetic mean

$H$  = harmonic mean and  $A \geq G \geq H$

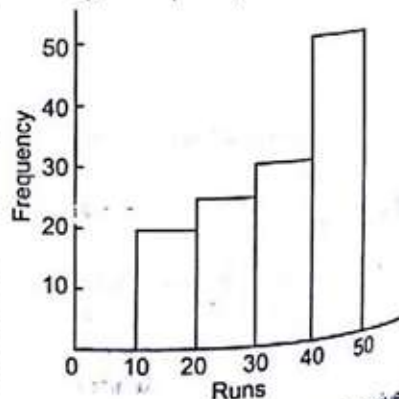
## Graphical Representation of Data

There are many ways of picturing a frequency distribution of continuous type.

- Histogram
- Frequency polygon
- Ogives (Cumulative frequency curves)
- Percentile curve

### Histogram

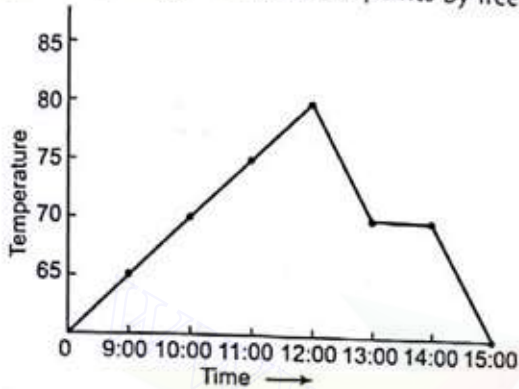
A histogram is a set of adjacent rectangles whose area are proportional to the frequencies of a given continuous frequency distribution. There being no gap between any two successive rectangles.





**Frequency Polygon** A frequency polygon can be drawn joining the mid-points of the respective tops of the rectangle of a histogram in the case of equal class intervals.

A frequency polygon for a grouped data can also be drawn independently by plotting the mid-points of the all classes along x-axis and frequencies along y-axis and joining the plotted points by straight line. Join the points by free hand.



**Ogive (Cumulative Frequency Curve)** When we plot the upper class limits along x-axis and cumulative frequencies along y-axis. And on joining them we get a curve called an ogive.

We have two types of ogive curves

- less than ogive (i.e., the rising curve)
- more than ogive (i.e., a falling curve)

## Bar Chart

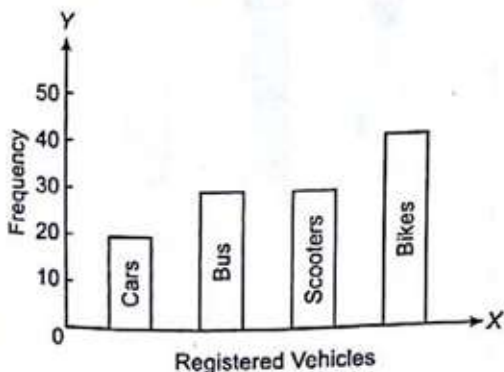
**In a Bar graph**

- The width of each bar can be any, but widths of all the bars is same.
- Space between these bars should be same.

The width of the bar has no special meaning except to make it pictorially more attractive.

**Illustration** Represent the following data by a bar graph.

Registration of vehicles in 2000	Car	Bus	Scooters	Bikes
No. of vehicles	20	30	30	40



## Pie-Diagram (or Pie-Chart)

**In a Pie-graph**

- Data is represented by sectors of a circle.
- Each part of data makes a certain central angle.
- Sum of all the angles of sector is  $360^\circ$ .

**Example** The pie-chart of the data given above as,

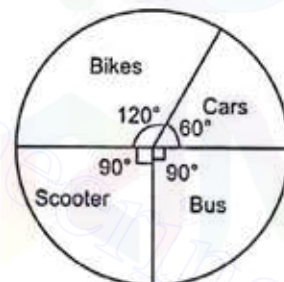
Divide the central angle i.e.,  $360^\circ$  in the proportion of numbers 20, 30, 30, 40.

i.e., 360 should be divided in ratio

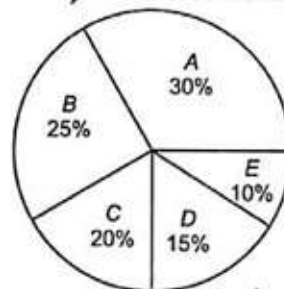
$$20 : 30 : 30 : 40 \text{ or } 2 : 3 : 3 : 4$$

Registered Vehicle	Number of Vehicle	Central Angle
Car	20	$\frac{2}{12} \times 360^\circ = 60^\circ$
Bus	30	$\frac{3}{12} \times 360^\circ = 90^\circ$
Scooter	30	$\frac{3}{12} \times 360^\circ = 90^\circ$
Bikes	40	$\frac{4}{12} \times 360^\circ = 120^\circ$

So, pie-chart is



**Example 9.** The following diagram show the expenditure of a family on various items A, B, C, D and E.



Study the diagram carefully and answer the following questions.

- The angle of pie-diagram showing the expenditure incurred on item A is
  - $30^\circ$
  - $35^\circ$
  - $108^\circ$
  - None of these

**Sol.** (c) Angle for 100% =  $360^\circ$

$$\therefore \text{Angle for expenditure of A} = 30\% \text{ of } 360^\circ = \frac{30}{100} \times 360 = 108^\circ$$



- (ii) Which two expenditures together will form an angle of  $90^\circ$  at the centre of pie-diagram?

(a) B and O (b) C and A  
(c) D and E (d) None of these

Sol. (c)  $\because 100\% = 360^\circ$

$\therefore$  An angle of  $90^\circ$  at the centre of pie-diagram in percentage is  

$$= \frac{90 \times 100}{360} = 25\%$$

Expenditure of items D and E makes upto  $(15 + 10) = 25\%$ .

So, expenditure of D and E together will form an angle of  $90^\circ$  at the centre of pie-diagram.

- (iii) If the income of the family is ₹ 3000 per month, then expenditure of item C will be

(a) ₹ 400 (b) ₹ 500 (c) ₹ 600 (d) ₹ 800

Sol. (c) Expenditure on item C

$$= 20\% \text{ of } ₹ 3000 = \frac{20}{100} \times 3000 = ₹ 600$$

**Example 10.** Study the table given below and answer the question that follow.  
 Number of students from various schools playing various games

Games	Schools				
	A	B	C	D	E
Football	125	250	100	175	250
Basket ball	175	200	195	245	225
Cricket	250	200	225	215	200
Tennis	240	210	200	130	165
Badminton	75	125	55	45	100

- (i) If 20% of the students playing football from school A also play badminton, what should be the total number of students playing badminton from school A?  
 (a) 25 (b) 100 (c) 75 (d) 105
- (ii) The number of students playing basket ball from school C is approximately what per cent of the students playing basket ball from school E?  
 (a) 87% (b) 93% (c) 95% (d) 97%
- (iii) What is the difference between the average number of students playing cricket from all the schools and the average number of students playing tennis from all the schools?  
 (a) 12 (b) 27 (c) 29 (d) 31
- (iv) The number of students playing football from school D is what per cent of the total number of students playing all the given games from the school? (Rounded off to 2 digits after decimal)  
 (a) 20.66% (b) 20.93% (c) 21.33% (d) 21.6%
- (v) What is the difference between the average number of students playing all the given games from school B and the number of students playing badminton from that school?  
 (a) 72 (b) 75 (c) 95 (d) 102

Sol. (i) (b) Total number of students playing badminton from school A  

$$= (75 + 20\% \text{ of } 125) = \left(75 + \frac{20}{100} \times 125\right)$$

$$= (75 + 25) = 100$$

(ii) (a) Required percentage  $= \left(\frac{195}{225} \times 100\right)\% = 87\%$  (approx)

- (iii) (c) Average number of students playing cricket from all the schools

$$= \frac{1}{5} (250 + 200 + 225 + 215 + 200) = \frac{1090}{5} = 218$$

Average number of students playing tennis from all the schools

$$= \frac{1}{5} (240 + 210 + 200 + 130 + 165) = \frac{945}{5} = 189$$

$$\text{Required difference} = (218 - 189) = 29$$

- (iv) (d) Number of students playing football from school D = 175

$$\text{Number of students playing all the games from school D} = (175 + 245 + 215 + 130 + 45) = 810$$

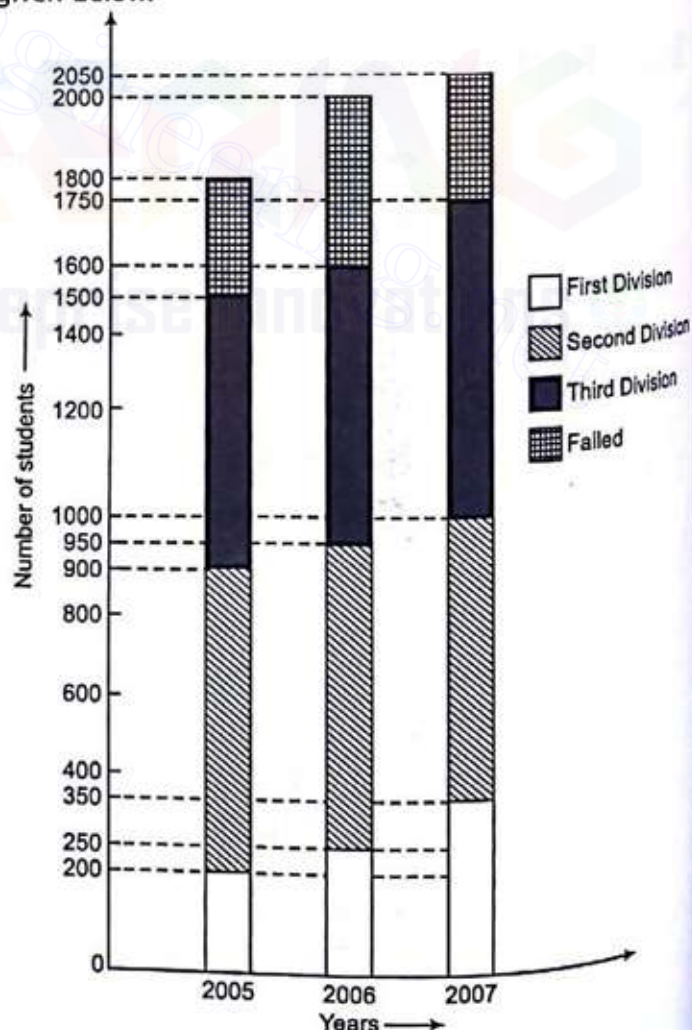
$$\text{Required percentage} = \left(\frac{175}{810} \times 100\right)\% = 21.6\%$$

- (v) (a) Average number of students playing all the games from school B

$$= \frac{1}{5} (250 + 200 + 200 + 210 + 125) = \frac{985}{5} = 197$$

$$\text{Required difference} = (197 - 125) = 72$$

**Example 11.** The subdivided bar diagram given below depicts the result of B.Sc. students of a college for three years. Study the bar diagram and answer the questions given below.





(i) What per cent of the students passed in first division in 2005?

(a) 50%

(b)  $11\frac{1}{9}\%$

(c)  $22\frac{1}{3}\%$

(d) None of the above

(ii) What was the pass percentage in 2005?

(a) 64.33%

(b) 73.33%

(c) 83.33%

(d) 85%

(iii) In which year the college had the best result for B.Sc.?

(a) 2005

(b) 2006

(c) 2007

(d) None of these

(iv) What is the number of third divisions in 2007?

(a) 550

(b) 750

(c) 350

(d) 1000

(v) What is the percentage of students in 2007 over 2005?

(a) 113.8%

(b) 114.2%

(c) 115.1%

(d) 115.3%

(vi) What is the aggregate pass percentage during 3 yr?

(a) 73.5%

(b) 77.6%

(c) 81.2%

(d) 82.9%

Sol. (i) (b) Percentage of first divisioners

$$= \left( \frac{200}{1800} \times 100 \right) \% = \frac{100}{9} \% = 11\frac{1}{9} \%$$

(ii) (c) Pass percentage in 2005

$$= \left( \frac{1500}{1800} \times 100 \right) \%$$

$$= \frac{250}{3} \% = 83.33\%$$

(iii) (c) Pass percentage in 2005 =  $\left( \frac{1500}{1800} \times 100 \right) \% = 83.33\%$

$$\text{Pass percentage in 2006} = \left( \frac{1600}{2000} \times 100 \right) \% = 80\%$$

$$\text{Pass percentage in 2007} = \left( \frac{1750}{2050} \times 100 \right) \% = 85.3\%$$

The college had best result for B.Sc. in 2007.

(iv) (b) Number of third divisions in 2007

$$= (1750 - 1000) = 750$$

(v) (a) Required percentage

$$= \left( \frac{2050}{1800} \times 100 \right) \% = 113.8\%$$

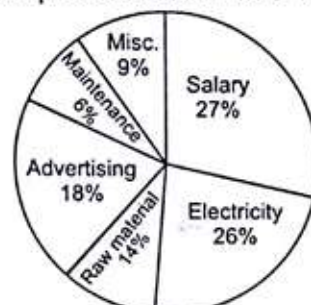
(vi) (d) Required percentage

$$= \left( \frac{(1500 + 1600 + 1750)}{(1800 + 2000 + 2050)} \times 100 \right) \%$$

$$= \left( \frac{4850}{5850} \times 100 \right) \% = 82.9\%$$

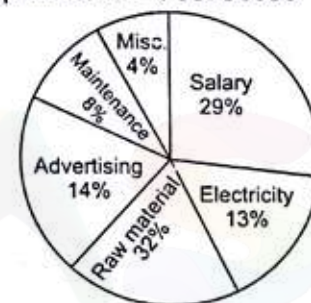
**Example 12.** Study the following graph carefully to answer the questions given below.

Expenditure Incurred by two Companies A and B in a Year



Company A

Total expenditure = ₹ 35750000



Company B

Total expenditure = ₹ 40550000

(i) What is the amount spent by company A on electricity?

(a) ₹ 9295000

(b) ₹ 9396000

(c) ₹ 9495000

(d) ₹ 9565000

(ii) How much have both companies together spent on raw material?

(a) ₹ 15282000

(b) ₹ 17981000

(c) ₹ 15381000

(d) ₹ 17881000

(iii) What is the ratio of amount spent by company A on advertising and electricity, respectively?

(a) 13 : 9

(b) 14 : 9

(c) 9 : 13

(d) 9 : 14

(iv) On which item is there least expenditure? (irrespective of company A or B)

(a) maintenance of A

(b) maintenance of B

(c) miscellaneous of B

(d) miscellaneous of A

(v) What is the total amount spent by company B on raw material, maintenance and salary together?

(a) ₹ 27979500

(b) ₹ 29779500

(c) ₹ 30979500

(d) ₹ 31333500

Sol. (i) (a) Amount spent on Electricity by company A

$$= 26\% \text{ of } ₹ 35750000 = ₹ \left( 35750000 \times \frac{26}{100} \right)$$

$$= ₹ 9295000$$

(ii) (b) Amount spent by A on raw material

$$= 14\% \text{ of } ₹ 35750000 = ₹ \left( 35750000 \times \frac{14}{100} \right) = ₹ 5005000$$

Amount spent by B on raw material



$$= 32\% \text{ of } ₹ 40550000 = ₹ \left( 40550000 \times \frac{32}{100} \right) = ₹ 12976000$$

$$\text{Required amount} = ₹ (5005000 + 12976000) = ₹ 17981000$$

- (iii) (c) Ratio of amounts spent by A on advertising and electricity = (18% of ₹ 35750000) : (26% of ₹ 35750000)

$$= \frac{18}{100} \times 35750000 : \frac{26}{100} \times 35750000 = 18:26 = 9:13$$

- (iv) (c) Expenditure by A on maintenance

$$= 6\% \text{ of } ₹ 35750000 = ₹ \left( 35750000 \times \frac{6}{100} \right) = ₹ 2145000$$

Expenditure by B on miscellaneous

$$= 4\% \text{ of } ₹ 40550000 = ₹ \left( 40550000 \times \frac{4}{100} \right) = ₹ 1622000$$

So, it is least on miscellaneous.

- (v) (a) Required amount = (32 + 8 + 29)% of ₹ 40550000

$$= ₹ \left( 40550000 \times \frac{69}{100} \right)$$

$$= ₹ 27979500$$

## Exercise

- Class size is the difference between the class marks of two
  - adjacent classes
  - classes
  - alternate classes
  - None of these
- In bar chart, the gap between the bars is always
  - same
  - zero
  - varies
  - can't say
- Histogram is
  - one dimensional
  - two dimensional
  - three dimensional
  - None of these
- Joining the mid-points of the respective tops in histogram, we get
  - bar chart
  - pie chart
  - ogive
  - frequency polygon
- The most stable measure of central tendency is
  - the median
  - the mean
  - the mode
  - None of these
- The measure which takes into account all the data items is
  - mean
  - frequency
  - mode
  - median
- Consider the following statement
  - A bar graph is two dimensional.
  - In a histogram, the height as well as the width of each rectangle matters.
  - In a bar graph, not only height, but also width of each rectangle matters.
  - In a bar graph, height of each rectangle matters and not its width of these statements
    - I and IV are true
    - All are true
    - III and IV are true
    - II and IV are true
- In a bar chart, what matter?
  - length of y-axis
  - length of x-axis
  - width of the bar
  - length of the bar
- Frequency polygon can be drawn after drawing
  - ogive
  - bar chart
  - histogram
  - None of these
- Pie chart is drawn with the help of
  - radius of the circle
  - chords of the circle
  - sectors of the circle
  - diameter of the circle
- The total expenditure incurred by an industry under different heads is best presented as a
  - pie diagram
  - histogram
  - frequency polygon
  - bar chart
- Which one of the following is a source of data for primary investigations?
  - News Papers
  - Magazines
  - Questionnaires
  - All of these
- Pie diagrams used to represent statistical data are
  - one dimensional
  - two dimensional
  - three dimensional
  - None of these
- An ogive is used to determine
  - mean
  - median
  - GM
  - HM
- Let the observations at hand be arranged in increasing order. Which one of the following measures will not be affected when the smallest and the largest observations are removed? (CDS 2011 II)
  - Mean
  - Median
  - Mode
  - Standard deviation
- If the population figures are given for each State of India, then the data can be classified as (CDS 2011 II)
  - qualitative
  - quantitative
  - chronological
  - geographical
- The mid-value of a class interval is 42. If the class size is 10, then the upper and lower limits of the class are
  - 37.5 and 47.5
  - 47 and 37
  - 37 and 47
  - 47.5 and 37.5
- The actual lower class limits of the following classes 10-19, 20-29, 30-39 and 40-49 are
  - 9.5, 19, 29 and 39.5
  - 10, 20, 30 and 40
  - 9.5, 19.5, 29.5 and 39.5
  - 18.5, 28.5, 38.5 and 48.5
- State which of the following variable are discrete
  - Number of children in a family
  - Wages of workers
  - The ages of students
  - Weights of a set of a students
    - I and II
    - I, II and III
    - All of these
    - None of these
- If the mean of five observations  $x, x + 2, x + 4, x + 6$  and  $x + 8$  is 11, then the mean of first three observations is
  - 9
  - 11
  - 13
  - None of these
- The combined mean of three groups is 12 and the combined mean of first two groups is 3. If the first, second and third groups have 2, 3 and 5 items respectively, then mean of third group is
  - 10
  - 21
  - 12
  - 18



22. 10 is the mean of a set of 7 observations and 5 is the mean of a set of 3 observations. The mean of the combined set is given by  
(a) 15 (b) 10 (c) 8.5 (d) 7.5
23. In a class of 50 students, 10 have failed and their average marks are 28. The total marks obtained by the entire class are 2800. The average marks of those who have passed are  
(a) 43 (b) 53  
(c) 63 (d) 70
24. Consider the following statements in respect of the set  $S = \{1, 2, 3, \dots, n\}$ .  
I.  $(n+1)/2$  is the median of the numbers in  $S$ .  
II.  $n$  is the mode of the numbers in  $S$ .  
Which of the above statements is/are correct?  
(a) Only I (b) Only II (c) Both I and II (d) Neither I nor II
25. Which of the following statements about the median is true?  
(a) It is not affected by extreme values  
(b) It can be found even, if some items are not known  
(c) It is useful when the data cannot be measured quantitatively  
(d) All of the above
26. The middle item of the series arranged in ascending or descending order is called  
(a) mean (b) median  
(c) mode (d) standard deviation
27. For any set of variables  
(a) arithmetic mean < geometric mean > harmonic mean  
(b) arithmetic mean > geometric mean < harmonic mean  
(c) arithmetic mean < geometric mean < harmonic mean  
(d) arithmetic mean > geometric mean > harmonic mean
28. The arithmetic mean and geometric mean of two numbers are 14 and 12, respectively. What is the harmonic mean of the numbers? (CDS 2011 II)  
(a) 10 (b) 13  
(c)  $32/3$  (d)  $72/7$

**Directions (Q. Nos. 29-30)** The itemwise expenditure of a Non-Government Organisation for the year 2008-2009 is given below

Item	Expenditure (in ₹ Lakh)
Salary of employees	6
Social welfare activities	7
Office contingency	3
Vehicle maintenance	4
Rent and hire charges	2.5
Miscellaneous expenses	1.5

The above data are represented by a pie diagram.

29. What is the sectorial angle of the largest sector? (CDS 2011 I)  
(a)  $120^\circ$  (b)  $105^\circ$  (c)  $90^\circ$  (d)  $85^\circ$
30. What is the difference in the sectorial angles of the largest and the smallest sectors?  
(a)  $90^\circ$  (b)  $85^\circ$  (c)  $82.5^\circ$  (d)  $77.5^\circ$

**Directions (Q. Nos. 31-32)** The following table gives the frequency distribution of life length in hours of 100 electric bulbs having median life 20 h.

Life of bulbs (in h)	Number of bulbs
8-13	7
13-18	x
18-23	40
23-28	y
28-33	10
33-38	2

31. What is the missing frequency x? (CDS 2011 I)  
(a) 31 (b) 27 (c) 24 (d) 14
32. What is the missing frequency y?  
(a) 27 (b) 24 (c) 14 (d) 11
33. Consider the following statements in respect of a histogram  
I. The histogram consists of vertical rectangular bars with a common base such that there is no gap between consecutive bars.  
II. The height of the rectangle is determined by the frequency of the class it represents.  
Which of the statements given above is/are correct? (CDS 2011 I)  
(a) Only I (b) Only II  
(c) Both I and II (d) Neither I nor II
34. The arithmetic mean of 10 numbers was computed as 7.6. It was later discovered that a number 8 was wrongly read as 3 during the computation. What should be the correct mean? (CDS 2011 I)  
(a) 7.1 (b) 7.6 (c) 8.1 (d) 8.6
35. For a set of positive numbers, consider the following statements.  
I. If each number is reduced by 2, then the geometric mean of the set may not always exist.  
II. If each number is increased by 2, then the geometric mean of the set is increased by 2.  
Which of the above statements is/are correct? (CDS 2010 II)  
(a) Only I (b) Only II  
(c) Both I and II (d) Neither I nor II

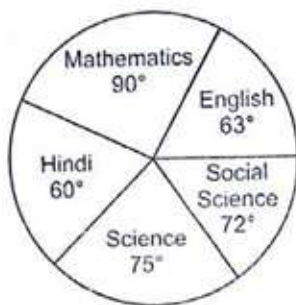
36. Consider the following frequency distribution.

Class	0-10	0-20	0-30	0-40	0-50
Frequency	3	8	14	14	20

What is the above frequency distribution known as? (CDS 2010 II)

- (a) Cumulative distribution in more than type  
(b) Cumulative distribution in less than type  
(c) Continuous frequency distribution  
(d) None of the above
37. The following pie chart shows the marks obtained by a student in an examination whose scored 540 marks in all. The subject in which the student scored 108 marks is





- (a) Science (b) Hindi  
(c) English (d) Social Science

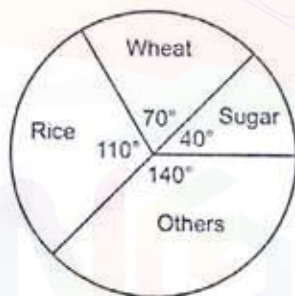
38. A distribution consists of three components with frequencies 45, 40 and 15 having their means 2, 2.5 and 2 respectively. The mean of the combined distribution is

- (a) 2.1 (b) 2.2 (c) 2.3 (d) 2.4

39. Out of 100 numbers, 20 were 4s, 40 were 5s, 30 were 6s and the remaining were 7s. The arithmetic mean of the number is

- (a) 5.3 (b) 5.4 (c) 6.1 (d) 6.5

**Directions (Q. Nos. 40-42)** Study the following pie diagram and answer the questions.



40. The percentage of wheat production is  
(a) 35% (b) 70% (c) 19.44% (d) 20%
41. If the sugar production is 9000 kg, then the wheat production is  
(a) 20000 kg (b) 24000 kg  
(c) 15750 kg (d) None of these
42. If the total production is 180000 kg, the difference in sugar and wheat production is  
(a) 100000 (b) 150000  
(c) 200000 (d) None of these

43. Consider the following statements in respect of histogram.

- I. Histogram is an equivalent graphical representation of the frequency distribution.  
II. Histogram is suitable for continuous random variables, where the total frequency of an interval is evenly distributed over the interval.

Which of the statements given above is/are correct?

- (a) Only I (b) Only II (c) Both I and II (d) Neither I nor II

44. Let  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  are  $n$  pairs of positive numbers. The arithmetic mean and geometric mean of any set of positive numbers  $(c_1, c_2, \dots, c_n)$  are denoted by  $M(c_i), G(c_i)$  respectively.

Consider the following

I.  $M(x_i + y_i) = M(x_i) + M(y_i)$

II.  $G(x_i y_i) = G(x_i) G(y_i)$

Which of the above statements is/are correct?

- (a) Only I (b) Only II (c) Both I and II (d) Neither I nor II

45. What is the weighted mean of first 10 natural numbers whose weights are equal to the corresponding number?

- (a) 7 (b) 5.5 (c) 5 (d) 4.5

**Directions (Q. Nos. 46-47)** The average age of 6 persons living in a house is 23.5 yr. Three of them are majors and their average age is 42 yr. The difference in ages of the three minor children is same.

46. What is the mean of the ages of minor children?

- (a) 3 yr (b) 4 yr (c) 5 yr (d) 6 yr

47. What is the median of the ages of minor children?

- (a) 3 yr (b) 5 yr (c) 7 yr  
(d) Cannot be determined due to insufficient data

48. A new frequency distribution is constructed by doubling each frequency of the original distribution keeping the other entries intact. The following measures are computed for both the tables

I. Arithmetic mean

II. Median

III. Harmonic mean

Which of the following statements with reference to above is correct?

- (a) Corresponding values of I and II only are equal in both the distributions  
(b) Corresponding values of I and III only are equal in both the distributions  
(c) Corresponding values of II and III only are equal in both the distributions  
(d) Corresponding values of I, II and III are equal in both the distributions

**Directions (Q. Nos. 49-52)** The arithmetic mean, geometric mean and median of 06 positive numbers  $a, a, b, b, c, c$  where  $a < b < c$  are  $\frac{7}{3}, 2, 2$  respectively.

49. What is the sum of the squares of all the six numbers?

- (a) 40 (b) 42 (c) 45 (d) 48

50. What is the value of  $c$ ?

- (a) 1 (b) 2 (c) 3 (d) 4

51. What is the mode?

- (a) 1 (b) 2 (c) 1, 2 and 4 (d) No mode

52. The following table shows the percentage of male and female coffee drinkers and non-coffee drinkers in two Towns A and B.

Attributes	Town A		Town B	
	Male	Female	Male	Female
Coffee drinkers	40%	5%	25%	15%
Non-coffee drinkers	20%	35%	30%	30%



If the total population of the Towns A and B are 10000 and 20000 respectively, then what is the total number of female coffee drinkers in both towns? (CDS 2009 II)

- (a) 8000 (b) 6000 (c) 3500 (d) 2500

53. Examples of data are given below

- I. Information on households collected by an investigator by door-to-door visits.
- II. Data on the percentage of literates, sex-wise, for the different districts of a state collected from records of the census of India.
- III. General information about families, collected by telephonic interviews.

Which one of the following in respect of the above is correct? (CDS 2009 II)

- (a) I and II are primary data  
(b) I and III are primary data  
(c) II and III are primary data  
(d) I, II and III are primary data

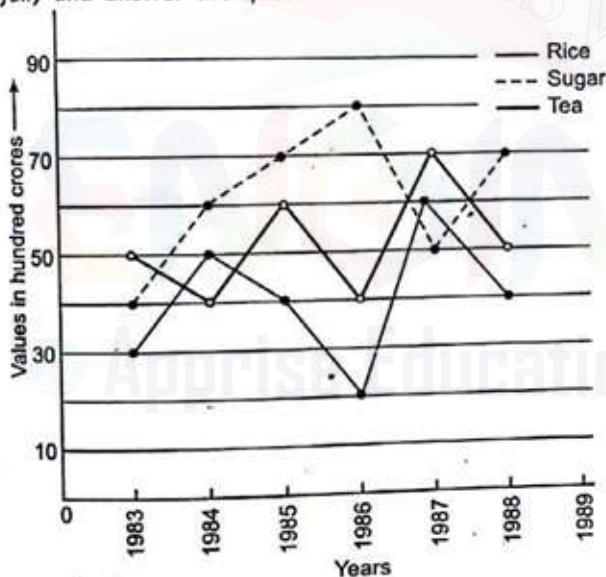
54. Consider the following pairs of numbers

- I. (8, 12), II. (9, 11), III. (6, 24)

Which pairs of number have the same harmonic means? (CDS 2009 II)

- (a) I and II (b) II and III (c) I and III (d) I, II and III

Directions (Q. Nos. 55-59) Study the following graph carefully and answer the questions.



55. In which year the export value of tea is maximum among the given years?

- (a) 1983 (b) 1985 (c) 1986 (d) 1987

56. What was the difference in the export value of rice and tea in the year 1986?

- (a) ₹ 20 crore (b) ₹ 2000 crore  
(c) ₹ 2 crore (d) ₹ 200 crore

57. What was the per cent decrease in export value of sugar from 1986 to 1987?

- (a) 37.5 (b) 63.5  
(c)  $26\frac{2}{3}$  (d) 30

58. What was the per cent increase in the export value of tea from 1986 to 1987?

- (a) 75 (b) 25 (c) 125 (d) 50

59. In which of the following years was the total export value of rice, tea and sugar together the minimum among the given years?

- (a) 1984 (b) 1986 (c) 1983 (d) 1988

60. If the values  $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots, \frac{1}{n}$  occur at frequencies 1, 2, 3, 4, 5, 6, ..., n respectively, in a frequency distribution, then the mean is

- (a) 1 (b) n (c)  $\frac{1}{n}$  (d)  $\frac{2}{n+1}$

61. If the geometric mean of three observations 40, 50 and x is 10, then the value of x is

- (a)  $\frac{1}{2}$  (b) 4 (c) 6 (d) 2

62. The mean of a set of number is  $\bar{X}$ . If each number is divided by 3, then the new mean is

- (a)  $\bar{X}$  (b)  $\bar{X} + 3$  (c)  $3\bar{X}$  (d)  $\frac{\bar{X}}{3}$

63. Which one of the following represents statistical data? (CDS 2009 II)

- (a) The names of all owners of shops located in a shopping complex  
(b) A list giving the names of all States of India  
(c) A list of all European countries and their respective capital cities  
(d) The volume of a rainfall in certain geographical area, recorded every month for 24 consecutive months

64. Consider the following types of data

- I. Marks of students who appeared for a test of 100 marks.
- II. Collar sizes of 200 shirts sold in a week.
- III. Monthly incomes of 250 employees of a factory.

For which of the above data, mode is a suitable measure of central tendency? (CDS 2009 II)

- (a) I and II (b) Only II  
(c) I and III (d) I, II and III

65. Suppose, X is some statistical variable with mean  $\mu$ . Let  $x_1, x_2, \dots, x_n$  be its deviations from mean with the respective frequencies  $f_1, f_2, \dots, f_n$ . What is the value of the sum  $x_1f_1 + x_2f_2 + \dots + x_nf_n$ ? (CDS 2008 II)

- (a) 0 (b) 1  
(c)  $\mu$  (d)  $\mu + 1$

66. Which one of the following statements is not correct with reference to a histogram? (CDS 2008 II)

- (a) Frequency curve is obtained by joining the mid-points of the top of the adjacent rectangles with smooth curves  
(b) Histogram is drawn for continuous data  
(c) The height of the bar is proportional to the frequency of that class  
(d) Mode of the distribution can be obtained from the histogram



67.	X	0	1	2	3	4
	Frequency	4	$f$	9	$g$	4

The table above gives the frequency distribution of a discrete variable  $X$  with two missing frequencies. If the total frequency is 25 and the arithmetic mean of  $X$  is 2, then what is the value of the missing frequency  $f$ ?

(CDS 2008 II)

- (a) 4 (b) 5 (c) 6 (d) 7

68. The yield of paddy per plot of one acre were obtained from a number of plots from two different districts in a State and are summarized in the following table.

Yield of paddy per plot in quintals	District A No. of plots	District B No. of plots
38.0-41.0	25	14
41.0-44.0	36	29
44.0-47.0	59	35
47.0-50.0	30	54
50.0-53.0	25	41

Which one of the following statements is correct?

(CDS 2008 II)

- (a) The mode for district A is higher than the mode for district B  
 (b) The mode for district B is higher than the mode for district A  
 (c) Both the distributions are symmetric  
 (d) Both the distributions have the same mean

69. Average score of 50 students in a class is 44. Later on it was found that the score 23 was incorrectly recorded as 73. The correct average score is

- (a) 42 (b) 43 (c) 45 (d) 46

70. Mode is approximately given by

- (a) 2 median - 3 mean (b) 3 median - 2 mean  
 (c) 2 median + 3 mean (d) 3 median + 2 mean

71. An ogive is a graphical representation of

- (a) an ungrouped data  
 (b) a discrete series  
 (c) a continuous frequency distribution  
 (d) a cumulative frequency distribution

72. In the "less than" type of ogive the cumulative frequency is plotted against

- (a) the lower limit of the concerned class interval  
 (b) the upper limit of the concerned class interval  
 (c) the mid-value of the concerned class interval  
 (d) any value of the concerned class interval

73. Which of the following pairs is not correctly matched?

- (a) Circles—Two dimensional diagram  
 (b) Multi-bars—One dimensional diagram  
 (c) Pictograms—Abstract representation of data  
 (d) Ogive—Used to determine median

74. A candidate obtains the marks as given below  
 English : 60, Hindi : 75, Mathematics : 63,  
 Physics : 59 and Chemistry : 55.

In the weights of 1, 2, 1, 3, 3, respectively are allotted to the subjects, the candidates weighted mean is

- (a) 62.4 (b) 61.5 (c) 61 (d) 60.5

75. A, B and C are three sets of values of  $x$

A : 2, 3, 7, 1, 3, 2, 3

B : 7, 5, 9, 12, 5, 3, 8

C : 4, 4, 11, 7, 2, 3, 4

Select the correct statement from among the following

- (a) Mean of A is equal to mode of C  
 (b) Mean of C is equal to median of B  
 (c) Median of B is equal to mode of A  
 (d) Median, mean and mode of A are same

76. A student obtains 75%, 80% and 85% marks in three subjects. If the marks of any other subject are added, then their average cannot be less than

- (a) 60% (b) 65% (c) 70% (d) 80%

77. If the mean and median of a set of numbers are 8.9 and 9 respectively, then the mode will be

- (a) 7.2 (b) 8.2 (c) 9.2 (d) 10.2

78. The arithmetic mean of the data with observations  $a, a + d, a + 2d, \dots, a + 2md$  is

- (a)  $a + md$  (b)  $a + (m-1)d$   
 (c)  $a + \frac{1}{2}md$  (d)  $a + \frac{1}{2}(m+1)d$

79. The mean of  $n$  observations is  $\bar{x}$ . If one observation  $x_{n+1}$  is added, then the mean remains the same. The value of  $x_{n+1}$  is

- (a) 0 (b) 1 (c)  $n$  (d)  $\bar{x}$

80. If every number of a finite set is increased by any number  $k$ , the measure of central tendency should also increase by  $k$ . Which one of the following measures of central tendency does not have this property?

- (a) Arithmetic mean (CDS 2008 II)  
 (b) Median  
 (c) Mid-range i.e., the arithmetic mean of the largest and the smallest numbers  
 (d) Geometric mean

81. If the median of the distribution (arranged in ascending order) 1, 3, 5, 7, 9,  $x$ , 15, 17 is 8, what is the value of  $x$ ? (CDS 2008 II)

- (a) 11 (b) 13  
 (c)  $9 < x < 15$  (d)  $9 \leq x \leq 15$

82. If the width of each of the ten classes in a frequency distribution is 2.5 and the lower class boundary of the lowest class is 5.1, then the upper class boundary of the highest class is

- (a) 30.1 (b) 30 (c) 31.1 (d) 27.56

83. The average of the values

2, 4, 6, 8, 10, ...,  $2n$  is

- (a)  $n$  (b)  $n+1$  (c)  $2n$  (d)  $2(n+1)$

84. The mean of the squares of the numbers 1, 2, 3, 4, ...,  $n$  is

- (a)  $\frac{(n+1)(2n+1)}{2}$  (b)  $\frac{1}{6}n(2n+1)$   
 (c)  $\frac{1}{6}(n+1)(2n+1)$  (d)  $2n+1$



85. If the sizes in a frequency distribution are given in an ascending order of magnitude, then the median  $M$  is given by

$$(a) M = l + \frac{(N/2 - C) \times i}{f} \quad (b) M = l + \frac{(N/2 + C) \times i}{f}$$

$$(c) M = l - \frac{(N/2 - C) \times i}{f} \quad (d) M = l - \frac{(N/2 + C) \times i}{f}$$

86. The average value of the median of 2, 8, 3, 7, 4, 6, 7 and the mode of 2, 9, 3, 4, 9, 6, 9 is  
(a) 9 (b) 8 (c) 7.5 (d) 6

87. The median of the following incomplete frequency distribution is 4.

$x$	1	2	3	4	5	6	7	8
Frequency	2	3	4	1	2	4	2	—

The frequency of 8 is

- (a) 1 (b) 2 (c) 3 (d) 4
88. The geometric mean of two numbers is 8 and their harmonic mean is 6.4. The numbers are  
(a) 2, 8 (b) 4, 16 (c) 4, 8 (d) 2, 16
89. If the sum of 11 consecutive natural numbers is 2761, then the middle number is  
(a) 252 (b) 251 (c) 250 (d) 249
90. A boy draws  $n$  squares with side 1, 2, 3, 4, 5, ...,  $n$  inches. The average area covered by these  $n$  squares (in sq inches) will be

$$(a) \frac{1}{2}(n+1) \quad (b) \left(\frac{2n+1}{3}\right)\left(\frac{n+1}{2}\right)^{-1}$$

$$(c) \frac{1}{6}(n+1)(2n+1) \quad (d) \frac{1}{2}\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3}\right)^{-1}$$

91. When 10 is subtracted from each of the given observations, the mean is reduced to 60%. If 5 is added to all the given observations, the mean will be  
(a) 25 (b) 30 (c) 60 (d) 65
92. A data has highest value 120 and the lowest value 71. A frequency distribution in descending order with seven classes is to be constructed.

The limits of the second class interval shall be

- (a) 77 and 78 (b) 78 and 85  
(c) 85 and 113 (d) 113 and 120
93. If the sizes in a frequency distribution are given in descending order of magnitude, then the median is given by

$$(a) M = l' - \frac{\left(\frac{N}{2} + C'\right) \times i}{f} \quad (b) M = l' - \frac{\left(\frac{N}{2} - C'\right) \times i}{f}$$

$$(c) M = l' - \frac{\left(\frac{N}{2} + C\right) \times i}{f} \quad (d) M = l' - \frac{\left(\frac{N}{2} - C\right) \times i}{f}$$

94. If mean of  $y$  and  $\frac{1}{y}$  is  $M$ , then what is the mean of  $y^3$  and  $\frac{1}{y^3}$ ? (CDS 2007 II)

$$(a) \frac{M(M^2 - 3)}{3} \quad (b) M^3 \quad (c) M^3 - 3 \quad (d) M(4M^2 - 3)$$

95. For the following frequency distribution

Class interval	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	10	15	30	80	40	20

If  $m$  is the value of mode, then which one of the following is correct? (CDS 2007 II)

- (a)  $5 < m < 10$  (b)  $10 < m < 15$   
(c)  $15 < m < 20$  (d)  $20 < m < 25$

96. The cumulative frequency curve of a frequency distribution with 6 classes and total frequency 60 is a straight line. Consider the following statements

- I. The first and the last classes have a frequency of 10 each.  
II. Both the middle classes have a total frequency of 30.

III. The frequency distribution does not have a mode.

Which of the above are correct? (CDS 2007 II)

- (a) I and II (b) I and III  
(c) II and III (d) I, II and III

97. Square diagrams are drawn to represent the following data

Country	Pakistan	India	Myanmar	China
Labour	36	81	25	100
Production (in ₹)				

Using the scale  $1 \text{ cm}^2 = ₹ 25$  what is the length of the representative square for India? (CDS 2007 II)

- (a) 1.8 cm (b) 1.2 cm  
(c) 1 cm (d) 2 cm

98. The heights of 50 students are tabulated as below

Class interval (in cm)	Frequency
145-146	1
147-148	2
149-150	4
151-152	5
153-154	8
155-156	15
157-158	9
159-160	6

Which one of the following is correct about the distribution? (CDS 2007 I)

- (a) It is positively skewed (b) It is negatively skewed  
(c) It is symmetric (d) It is extremely asymmetric

99. Assertion (A) The mean of first seven prime numbers is greater than their median.

Reason (R) Mean is always greater than median.

(CDS 2007 I)

- (a) A and R are correct and R is correct explanation of A.  
(b) A and R are correct but R is not correct explanation of A.  
(c) A is correct but R is wrong.  
(d) A is wrong but R is correct.



## Answers

1. (a)	2. (a)	3. (b)	4. (d)	5. (b)	6. (a)	7. (a)	8. (c)	9. (c)	10. (c)
11. (a)	12. (c)	13. (b)	14. (b)	15. (b)	16. (a)	17. (c)	18. (c)	19. (b)	20. (a)
21. (b)	22. (c)	23. (c)	24. (d)	25. (d)	26. (b)	27. (d)	28. (d)	29. (b)	30. (c)
31. (b)	32. (c)	33. (c)	34. (c)	35. (a)	36. (b)	37. (d)	38. (b)	39. (a)	40. (c)
41. (c)	42. (d)	43. (c)	44. (c)	45. (a)	46. (c)	47. (b)	48. (d)	49. (b)	50. (d)
51. (d)	52. (c)	53. (b)	54. (c)	55. (d)	56. (b)	57. (a)	58. (a)	59. (c)	60. (d)
61. (a)	62. (d)	63. (d)	64. (b)	65. (a)	66. (c)	67. (a)	68. (b)	69. (b)	70. (b)
71. (d)	72. (b)	73. (c)	74. (b)	75. (d)	76. (a)	77. (c)	78. (a)	79. (d)	80. (d)
81. (d)	82. (a)	83. (b)	84. (c)	85. (b)	86. (c)	87. (a)	88. (b)	89. (b)	90. (c)
91. (a)	92. (b)	93. (c)	94. (d)	95. (c)	96. (b)	97. (a)	98. (b)	99. (c)	

## Hints and Solutions

- Class size is the difference between the class marks of two adjacent classes.
- Clearly, same.
- Clearly, histogram is a two dimensional figure.
- Frequency polygon.
- Mean
- Mean
- Width of the rectangle only makes it pictorially more attractive.
- Frequency polygon can be drawn by joining the mid-points of the respective tops in histogram.
- Pie chart is drawn with the help of sectors of the circle the angle of sector depends upon the percentage of the quantity showing.
- Primary investigation is done by questionnaires.
- A ogive is used to determine the median.
- In an increasing order arrangements of observations, the median will not be affected when the smallest and the largest observations are removed.
- If the population figures are given for each State of India, then data can be classified as qualitative.
- Let the lower limit be  $x$ . Then, the upper limit of class interval  $= x + 10$

$$\therefore \frac{x + (x + 10)}{2} = 42$$

$$2x + 10 = 84$$

$$2x = 74 \Rightarrow x = 37$$

$\therefore$  Lower limit = 37 and Upper limit =  $37 + 10 = 47$

- True lower class limit are obtained by subtracting 0.5 from the lower limit, so clearly 9.5, 19.5, 29.5 and 39.5.
- A discrete frequency distribution is such a distribution in which data are presented in a way that exact measurements of the units are clearly shown. So, clearly weights of a set of students is continuous, while other three as discrete.
- Here, sum of 5 observations  $= x + (x + 2) + (x + 4) + (x + 6) + (x + 8) = 5x + 20$   
 $\therefore$  Mean of total observations  $= \frac{5x + 20}{5} = 11$

$$5x + 20 = 55 \Rightarrow x = 7$$

$$\therefore \text{Mean of first three} = \frac{x + (x + 2) + (x + 4)}{3}$$

$$= \frac{3x + 6}{3} = \frac{27}{3} = 9$$

( $\because x = 7$ )

- Total sum of items  $= 2 \times 12 + 3 \times 12 + 5 \times 12 = 120$   
 Total sum of items of first two group  $= (2 + 3) \times 3 = 15$   
 Total sum of 5 items of group third  $= 120 - 15 = 105$   
 $\therefore$  Mean of third group  $= \frac{105}{5} = 21$
- We have,  $n_1 = 7, \bar{x}_1 = 10$  and  $n_2 = 3, \bar{x}_2 = 5$   
 Combined mean  $= \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \frac{7 \times 10 + 3 \times 5}{7 + 3} = \frac{85}{10} = 8.5$
- Total number of students passed  $= 50 - 10 = 40$   
 Total marks obtained by failed students  $= 28 \times 10 = 280$   
 $\therefore$  Total marks of 40 students  $= 2800 - 280 = 2520$   
 $\therefore$  Required average of passed students  $= \frac{2520}{40} = 63$   

$$p = \frac{150}{10} = 15$$

- Given,  $S = (1, 2, 3, \dots, n)$

I. Here,  $n$  is not given whether it is odd or even. So, we cannot define median.

II. Here,  $n$  is not the mode of the numbers, because all elements in  $S$  have same frequency.

Hence, both statements are not correct.

- Clearly, all the statements are true about median.
- Median is the middle item of the series arranged in ascending or descending order.
- Arithmetic mean  $>$  Geometric mean  $>$  Harmonic mean
- Given, Arithmetic mean,  $A = 14$   
 Geometric mean,  $G = 12$   
 $\therefore G^2 = AH$   
 $\therefore (12)^2 = 14 \times H \Rightarrow H = \frac{12 \times 12}{14} = \frac{72}{7}$



29. Here, total expenditure =  $6 + 7 + 3 + 4 + 25 + 15 = ₹ 24$  lakh

$$\text{Sectorial angle of largest sector} = \frac{360^\circ}{24} \times 7 = 105^\circ$$

30. Difference in the sectorial angles of the largest and the smallest sectors

$$= 105^\circ - \frac{360^\circ}{24} \times 1.5 = 105^\circ - 22.5^\circ = 82.5^\circ$$

**Solutions (Q.Nos. 31-32)** Number of total bulbs = 100 (given)

$$7 + x + 40 + y + 10 + 2 = 100$$

$$\Rightarrow x + y = 41 \quad \dots(i)$$

Life of bulbs (in h)	Number of bulbs	Cumulative frequency
8-13	7	7
13-18	$x$	$7 + x$
18-23	40	$47 + x$
23-28	$y$	$47 + x + y$
28-33	10	$57 + x + y$
33-38	2	$59 + x + y$

$$\therefore \text{Number of bulbs, } N = 100 \Rightarrow \frac{N}{2} = 50$$

Since, the median life is 20 h, so median interval will be (18-23).

$$\text{Here, } l = 18, \frac{N}{2} = 50, c = 7 + x, f = 40$$

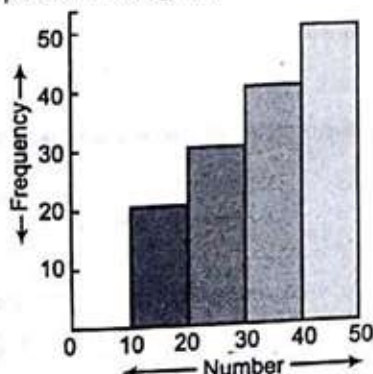
$$31. \therefore \text{Median} = l + \frac{\left(\frac{N}{2} - c\right)}{f} \times h = 18 + \frac{(50 - (7 + x))}{40} \times 5$$

$$\therefore 20 = 18 + \frac{5}{40} (43 - x) \quad (\text{given})$$

$$\Rightarrow x = 43 - 16 \Rightarrow x = 27$$

32. Putting  $x = 27$  in Eq. (i), we get  $y = 14$

33.  $\therefore$  The histogram consists of vertical rectangular bars with a common base. Such that there is no gap between consecutive bars and also the height of the rectangle is determined by the frequency of the class it represents. Hence, both statements are true in respect of a histogram.



$$34. \text{Correct mean} = \frac{10 \times 7.6 + 8 - 3}{10} = \frac{81}{10} = 8.1$$

35. I. Let the number be 1, 3, 5. If each number is reduced by 2, then new number will be -1, 1, 3.

$$\therefore G = \sqrt[3]{-1 \times 1 \times 3} = \sqrt[3]{-3} \text{ does not exist.}$$

II. If we increased each observation by 2, then the geometric mean of the set is not increased by 2.

36. The given frequency distribution is known as cumulative distribution.

37. Central angle for 540 mark =  $360^\circ$ .

$$\therefore \text{Central angle for 108 marks} = \frac{108}{540} \times 360 = 72^\circ$$

Here, the student scored 108 marks in social science.

$$38. \text{Required mean} = \frac{45 \times 2 + 40 \times 2.5 + 15 \times 2}{100} = \frac{220}{100} = 2.2$$

39. We can tabulate the information as

Numbers $x$	Frequency $f$	$f \times x$
4	20	80
5	40	200
6	30	180
7	10	70
	$\Sigma f = 100$	$\Sigma fx = 530$

$$\therefore \text{Arithmetic mean} = \frac{\Sigma f x_i}{\Sigma f_i} = \frac{530}{100} = 5.3$$

40. As,  $360^\circ = 100\%$  product

$$\therefore 70^\circ = \left( \frac{100 \times 70}{360} \right) \% = 19.44\%$$

41. If  $40^\circ = 9000$  kg

$$\therefore 70^\circ = \frac{9000 \times 70}{40} = 15750 \text{ kg}$$

42. If  $360^\circ = 180000$  kg

$$\text{Differ in angle } (70^\circ - 40^\circ) = 30^\circ = \frac{180000 \times 30}{360} = 15000 \text{ kg}$$

43. We know that histogram is an equivalent graphical representation of the frequency distribution and is suitable for continuous random variables where, the total frequency of an interval is evenly distributed over the interval. Hence, both the given statements are correct.

44. Given,  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  are  $n$  pairs of positive numbers. The arithmetic mean and geometric mean of any set of positive numbers  $(C_1, C_2, \dots, C_n)$  and denoted by  $M(C_i)$ ,  $G(C_i)$  respectively.

$$\text{I. } M(x_n + y_n) = \frac{(x_1 + y_1) + (x_2 + y_2) + \dots + (x_n + y_n)}{n} = \frac{(x_1 + x_2 + \dots + x_n) + (y_1 + y_2 + \dots + y_n)}{n}$$

$$= M(x_n) + M(y_n)$$

$$\Rightarrow M(x_n + y_n) = M(x_n) + M(y_n)$$

$$\text{II. } G(x_n y_n) = \{(x_1 y_1)(x_2 y_2)(x_3 y_3) \dots (x_n y_n)\}^{1/n}$$

$$= (x_1 \cdot x_2 \cdot \dots \cdot x_n)^{1/n} \cdot (y_1 \cdot y_2 \cdot \dots \cdot y_n)^{1/n} = G(x_n) \cdot G(y_n)$$

$$\Rightarrow G(x_n \cdot y_n) = G(x_n) \cdot G(y_n)$$

Hence, both statements are true.

$$45. \therefore \text{Weighted mean} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_{10} x_{10}}{w_1 + w_2 + \dots + w_{10}}$$

$$= \frac{1 \cdot 1 + 2 \cdot 2 + \dots + 10 \cdot 10}{1 + 2 + 3 + \dots + 10} = \frac{1^2 + 2^2 + \dots + 10^2}{1 + 2 + \dots + 10}$$



$$\therefore \Sigma n^2 = \frac{n(n+1)(2n+1)}{6} \text{ and } \Sigma n = \frac{n(n+1)}{2}$$

$$= \frac{10(10+1)(20+1)}{6} = \frac{10 \times 11 \times 21}{6 \times 55} = 7$$

**Solutions (Q. Nos. 46-47)**

Total age of six persons =  $235 \times 6 = 141$  yr

Total age of three major persons =  $42 \times 3 = 126$  yr

$\therefore$  Remaining age of three minor children =  $141 - 126 = 15$  yr

Since, the difference in ages of the three minor children is same.

Therefore, we take ages may be

5, 5, 5; 3, 5, 7; 2, 5, 8 and 1, 5, 9

So, in all the cases median will be 5 yr.

46. Mean age of minor children =  $\frac{15}{3} = 5$  yr

47. Median age of minor children is 5 yr.

48. If we double each value of original frequency distribution, then mean, median and harmonic mean remain same. Hence, option (d) is correct.

Since, in case observation since arithmetic mean, median and harmonic mean is dependent of change of origin but if we multiply the frequency of same quantity, then these are independent.

**Solutions (Q. Nos. 49-51)**

Since,  $a < b < c$

Therefore, series in increasing order  $a, a, b, b, c, c$

$$\therefore \text{Median} = \frac{\left(\frac{6}{2}\right)\text{th term} + \left(\frac{6}{2} + 1\right)\text{th term}}{2}$$

$$= \frac{3\text{rd term} + 4\text{th term}}{2} \Rightarrow 2 = \frac{b+b}{2} = b$$

$$\text{Also, arithmetic mean} = \frac{a+a+b+b+c+c}{6} \Rightarrow \frac{7}{3} = \frac{a+b+c}{3}$$

$$\Rightarrow a+c = 7-2 = 5 \quad \dots(i)$$

And geometric mean =  $(a^2 \times b^2 \times c^2)^{1/6}$

$$\Rightarrow 2 = (abc)^{1/3} \Rightarrow abc = 8$$

$$\Rightarrow ac = \frac{8}{2} = 4 \quad \dots(ii)$$

$\therefore$  From Eqs. (i) and (ii), we get  
 $a=1, c=4$  and  $b=2$

49. Required sum =  $2(a)^2 + 2(b)^2 + 2(c)^2$

$$= 2(1)^2 + 2(2)^2 + 2(4)^2$$

$$= 2 + 8 + 32 = 42$$

50. The value of  $c$  is 4.

51. Mode = 3 (median) - 2 (mean) =  $3(2) - 2\left(\frac{7}{3}\right) = \frac{18-14}{3} = \frac{4}{3}$

52. Total number of female coffee drinkers  
= 5% of 10000 + 15% of 20000 =  $500 + 3000 = 3500$

53. Statements I and III are primary data.

54. I.  $HM = \frac{2 \times 8 \times 12}{8+12} = \frac{2 \times 8 \times 12}{20} = \frac{48}{5} = 9.6$

$\left( \because \text{harmonic mean of } a \text{ and } b = \frac{2ab}{a+b} \right)$

II.  $HM = \frac{2 \times 9 \times 11}{9+11} = \frac{2 \times 9 \times 11}{20} = 9.9$

III.  $HM = \frac{2 \times 6 \times 24}{6+24} = \frac{2 \times 6 \times 24}{30} = \frac{48}{5} = 9.6$

Hence, 1st and 3rd pairs have same harmonic means.

55. The highest peak of tea export graph corresponds to the year 1987. Here, export is maximum in 1987.

56. The difference =  $(40 - 20)$  hundred crores = 2000 crores

57. Decrease of in export value of sugar from 1986 to 1987

$$= \frac{8000 - 5000}{8000} \times 100\% = 37.5\%$$

58. Increase percentage =  $\left( \frac{7000 - 4000}{4000} \times 100 \right)\%$

$$= \frac{3000}{4000} \times 100 = 75\%$$

59. Total export in 1983 =  $(30 + 40 + 50) = 120$  hundred crores

Total export in 1984 =  $(40 + 50 + 60) = 150$  hundred crores

Total export in 1985 =  $(40 + 60 + 70) = 170$  hundred crores

Total export in 1986 =  $(20 + 40 + 80) = 140$  hundred crores

Total export in 1987 =  $(50 + 60 + 70) = 180$  hundred crores

Total export in 1988 =  $(40 + 50 + 70) = 160$  hundred crores

Comparison show that the total export is minimum in 1983.

60. Here,  $\Sigma f_i = (1+2+3+\dots+n) = \frac{n(n+1)}{2}$

$$\Sigma f_i \times x_i = \left( 1 \times 1 + 2 \times \frac{1}{2} + 3 \times \frac{1}{3} + \dots + n \times \frac{1}{n} \right)$$

$$= (1+1+1+\dots n \text{ times}) = n$$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{n}{\frac{1}{2}n(n+1)} = \frac{2}{n+1}$$

61. Geometric mean of 40, 50 and  $x = (40 \times 50 \times x)^{1/3}$

$$\Rightarrow (40 \times 50 \times x)^{1/3} = 10 \quad \text{(given)}$$

$$40 \times 50 \times x = 10^3$$

$$x = \frac{1000}{40 \times 50} = \frac{1}{2}$$

62. Let  $x_1, x_2, x_3, \dots, x_n$  be  $n$  numbers, then  $\bar{X} = \frac{1}{n} \sum_{i=1}^n x_i$ .

If each number is divisible by 3, then the new mean,

$$\bar{z} = \frac{1}{n} \sum_{i=1}^n \left( \frac{x_i}{3} \right) = \frac{1}{3n} \sum_{i=1}^n (x_i) = \frac{1}{3} \left( \frac{1}{n} \sum_{i=1}^n x_i \right) = \frac{1}{3} \bar{X}$$

63. The volume of rainfall in certain geographical area, recorded every month for 24 consecutive months.

64. Collar sizes of 200 shirts sold in a week, mode is a suitable measure of central tendency.



65. We know that, the sum of deviation from their mean is zero.  
 $\therefore x_1f_1 + x_2f_2 + \dots + x_nf_n = 0$

66. The height of the bar is not proportional to the frequency of the class.

67. Arithmetic mean

$$\bar{X} = 2 = \frac{4 \times 0 + 1 \times f + 2 \times 9 + 3 \times g + 4 \times 4}{25} \Rightarrow 50 = 34 + f + 3g$$

$$\Rightarrow f + 3g - 16 = 0 \quad \dots(i)$$

$$\text{Also given, } 4 + f + 9 + g + 4 = 25$$

$$\Rightarrow f + g - 8 = 0 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get  
 $f = g = 4$

68. For district A, as the class 44.0-47.0 has maximum frequency, so it is the modal class.

$$\therefore l = 44, f_1 = 59, f_0 = 36, f_2 = 30, h = 3$$

$$\therefore \text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 44 + \frac{59 - 36}{2 \times 59 - 36 - 30} \times 3$$

$$= 44 + \frac{23 \times 3}{52} = 44 + 1.33 = 45.33$$

For district B, as the class 47.0-50.0 has the maximum frequency, so it is the modal class.

$$\therefore l = 47, f_1 = 54, f_0 = 35, f_2 = 41, h = 3$$

$$\therefore \text{Mode} = 47 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 47 + \frac{54 - 35}{2 \times 54 - 35 - 41} \times 3$$

$$= 47 + \frac{19 \times 3}{32} = 47 + 1.78 = 48.78$$

Thus, the mode for district B is higher than the mode for district A.

69. Total score =  $44 \times 50 = 2200$

$$\text{Correct score} = (2200 - 73 + 23) = 2150$$

$$\therefore \text{Correct average score} = \frac{2150}{50} = 43$$

70. Clearly, 3 median - 2 mean.

71. Clearly, a cumulative frequency distribution.

72. In the less than type of ogive the cumulative frequency is plotted against the upper limit of the concerned class interval.

73. Clearly, (c) is not correctly matched.

74. Weighted mean

$$= \frac{60 \times 1 + 75 \times 2 + 63 \times 1 + 59 \times 3 + 55 \times 3}{1 + 2 + 1 + 3 + 3} = \frac{615}{10} = 61.5$$

75. Mean of A =  $\frac{2 + 3 + 7 + 1 + 3 + 2 + 3}{7} = \frac{21}{7} = 3$

Mode of A = 3, has maximum frequency

Arrange data in ascending order, we have 1, 2, 2, 3, 3, 3, 7.

$$\text{Median of A} = \left(\frac{7+1}{2}\right)\text{th value} = 4\text{th value} = 3$$

Here, mean, median and mode are equal.

76. Total marks in 3 subjects =  $(75 + 80 + 85) = 240$  out of 300.

In any other subject the marks are atleast 0.

$$\text{Average of 4 subjects} = \frac{240}{4} = 60\%$$

$\therefore$  Average cannot be less than 60%.

77. By relation

$$\text{Mode} = (3 \text{ median}) - (2 \text{ mean})$$

$$= (3 \times 9 - 2 \times 8.9) = 27 - 17.8 = 9.2$$

78. Here, given series is an AP.

$$T_n = a + (n-1)d \Rightarrow a + 2md = a + (n-1)d \Rightarrow n = (2m+1)$$

So, sum of  $(2m+1)$  sums of AP is

$$S_{2m+1} = \frac{2m+1}{2} [2a + (2m+1-1)d] \quad [\because S_n = \frac{n}{2} [2a + (n-1)d]]$$

$$= (2m+1)(a+md)$$

$$\text{Average} = \frac{(2m+1)(a+md)}{(2m+1)} = (a+md)$$

79. Total sum of given  $n$  observations =  $n\bar{x}$

$$\text{Sum of } (n+1) \text{ observation} = n\bar{x} + x_{n+1}$$

$$\frac{n\bar{x} + x_{n+1}}{n+1} = \bar{x} \quad (\text{given})$$

$$\Rightarrow n\bar{x} + x_{n+1} = (n+1)\bar{x}$$

$$\Rightarrow x_{n+1} = \bar{x}$$

80. If every number of a finite set is increased by any number  $k$ , the measure of central tendency should also increase by  $k$ . Geometric mean does not have this property.

81. Given distribution is 1, 3, 5, 7, 9,  $x$ , 15, 17.

Number of terms = 8 (even)

$$\therefore \text{Median} = \frac{\text{Value of } \frac{8}{2} \text{th term} + \text{Value of } \left(\frac{8}{2} + 1\right) \text{th term}}{2}$$

$$= \frac{\text{Value of 4th term} + \text{Value of 5th term}}{2}$$

$$= \frac{7 + 9}{2} = \frac{16}{2} = 8$$

Also, the distribution is arranged in ascending order, then  $9 \leq x \leq 15$

82. Required class boundary = lower class boundary of lowest class  
 $+ (\text{width of class}) \times \text{number of class}$   
 $= 5.1 + 25 \times 10 = 30.1$

83. Required average =  $\frac{2 + 4 + 6 + \dots + 2n}{n}$

$$[\because \text{sum of natural number} = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}]$$

$$= \frac{2(1 + 2 + 3 + \dots + n)}{n} = \frac{2 \cdot \frac{n(n+1)}{2}}{n} = (n+1)$$

84. As sum of squares of natural numbers =  $\frac{n(n+1)(2n+1)}{6}$

$$\therefore \text{Required average} = \frac{1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2}{n}$$

$$= \frac{\frac{1}{6} n(n+1)(2n+1)}{n} = \frac{1}{6} (n+1)(2n+1)$$



85. Clearly (b) is true.

86. Arranging observation in ascending order 2, 3, 4, 6, 7, 7, 8, so median = 6

And mode of 2, 9, 3, 4, 9, 6, 9 is 9.

$$\therefore \text{Average} = \frac{1}{2}(6+9) = 7.5$$

87. Let the frequency of 8 be  $P$ .

$x$	Frequency ( $f$ )	Cumulative Frequency
1	2	2
2	3	5
3	4	9
4	1	10
5	2	12
6	4	16
7	2	18
8	$P$	$18+P$

Since, the median is 4.

This value is corresponding to the cumulative frequency 10.

$$\therefore \frac{18+P+1}{2} = 10 \Rightarrow P = 1$$

88. Let the number be  $a$  and  $b$ .

Here,  $\sqrt{ab} = 8$  (given)

Harmonic mean of  $a$  and  $b = \frac{2}{\frac{1}{a} + \frac{1}{b}} = 6.4$  (given)

$$\frac{2ab}{a+b} = 6.4 \Rightarrow ab = 64 \text{ and } \frac{2ab}{a+b} = 6.4$$

$$a+b = \frac{2 \times 64}{6.4} = 20$$

$$\Rightarrow a-b = \sqrt{(a+b)^2 - 4ab} = \sqrt{(20)^2 - 4(64)} \\ = \sqrt{400 - 256} = \sqrt{144} = 12$$

$$a+b = 20 \text{ and } a-b = 12$$

Solving above equations, we get

$$a = 16 \text{ and } b = 4$$

89. Let the numbers be  $(n+1), (n+2), (n+3), \dots, (n+11)$

and the middle number is  $(n+6)$

$$\text{Sum of number} = (n+1) + (n+2) + (n+3) + \dots + (n+11) \\ = 11 \times n + (1+2+3+\dots+11) = 11n + 66$$

$$\Rightarrow 11n + 66 = 2761 \Rightarrow 11n = 2761 - 66 = 2695$$

$$n = \frac{2695}{11} = 245$$

$$\text{Middle number} = (n+6) = (245+6) = 251$$

90. Number of square =  $n$

$$\text{Total area (in sq inch)} = 1^2 + 2^2 + 3^2 + \dots + n^2 \\ = \frac{1}{6}(n)(n+1)(2n+1)$$

$$\text{Sum of square of } n \text{ number} = \frac{1}{6}n(n+1)(2n+1)$$

$$\text{Average area} = \frac{\frac{1}{6}n(n+1)(2n+1)}{n} = \frac{1}{6}(n+1)(2n+1)$$

91. Let the mean of observations  $x_1, x_2, x_3, \dots, x_n$  be  $\bar{x}$ .

On subtracting 10 from each, the mean would be  $(\bar{x} - 10)$   
 $(\bar{x} - 10) = 60\% \text{ of } \bar{x}$

$$\Rightarrow (\bar{x} - 10) = \frac{60}{100} \bar{x} = \frac{3}{5} \bar{x}$$

$$\left(1 - \frac{3}{5}\right) \bar{x} = 10 \Rightarrow \frac{2}{5} \bar{x} = 10$$

$$\bar{x} = \frac{5 \times 10}{2} = 25$$

Now, 5 is added to all the given observations, then new mean becomes  $= \bar{x} + 5 = 25 + 5 = 30$

92. Range of data =  $120 - 71 = 49$

$$\therefore \text{Class size} = \frac{\text{range}}{\text{number of classes}} \\ = \frac{49}{7} = 7$$

The class are 71-78, 78-85, 85-92, 92-97 etc.

So, limits of second class interval are 78 and 85.

93. If the sizes in a frequency distribution are given in descending order of magnitude, then we use following the formula for finding median:

$$M_e = l' - \frac{(N/2 - c')}{f} \times h$$

Here,  $l'$  = upperclass of interval

$c'$  = New cumulative frequency of class before median class.

94. Here,  $M = \frac{y + \frac{1}{y}}{2}$  ... (i)

$$\text{New mean} = \frac{y^3 + \frac{1}{y^3}}{2} = \frac{\left(y + \frac{1}{y}\right)^3 - 3\left(y + \frac{1}{y}\right)}{2}$$

$$= \frac{\left(y + \frac{1}{y}\right) \left[\left(y + \frac{1}{y}\right)^2 - 3\right]}{2} \quad [\text{from Eq. (i)}] \\ = M[(2M)^2 - 3] = M(4M^2 - 3)$$

95. Maximum frequency is 80, hence mode will be between 15 to 20.

96. Since, the frequency in a straight line, so we take all classes have equal frequency, i.e., 10.

I. It is true that first and last class have 10 frequency.

II. Both the middle classes have  $10 + 10 = 20$

III. Since, all have equal frequency, so we cannot determined the mode.

97. ₹ 25 =  $1 \text{ cm}^2$

$$\therefore ₹ 1 = \frac{1}{25} \text{ cm}^2$$

$$\therefore ₹ 81 = \frac{81}{25} \text{ cm}^2 = \text{Area of square}$$

$$\text{Side of square} = \sqrt{\frac{81}{25}} = \frac{9}{5} \\ = 1.8 \text{ cm}$$



98.

CI	x	f	d	fd	cf
144.5-146.5	145.5	1	-8	-8	1
146.5-148.5	147.5	2	-6	-12	3
148.5-150.5	149.5	4	-4	-16	7
150.5-152.5	151.5	5	-2	-10	12
152.5-154.5	153.5	8	0	0	20
154.5-156.5	155.5	15	2	30	35
156.5-158.5	157.5	9	4	36	44
158.5-160.5	159.5	6	6	36	50
		$\Sigma f = 50$		$\Sigma fd = 56$	

A = Assumed mean = 153.5

$$\text{Mean} = A + \frac{\Sigma fd}{\Sigma f} = 153.5 + \frac{56}{50} = 153.5 + 1.12 = 154.62$$

Since,  $\frac{n}{2} = \frac{50}{2} = 25$

∴ Median group is 154.5 - 156.5

∴  $l = 154.5, h = 2, c = 20, f = 15,$

$$\text{Hence median} = l + \frac{\frac{n}{2} - c}{f} \times h = 154.5 + \frac{25 - 20}{15} \times 2$$

$$= 154.5 + \frac{2}{3} = 155.16$$

∴ Since, maximum frequency is 15, then modal group is 155.5 - 156.5.

Then,  $l = 154.5, f_1 = 15, f_0 = 8, f_2 = 9, h = 2$

$$\begin{aligned} \therefore M_0 &= l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h \\ &= 154.5 + \frac{(15 - 8)}{(30 - 8 - 9)} \times 2 = 154.5 + \frac{14}{13} = 155.58 \end{aligned}$$

Here,

and

Mean < Mode

Mean < Median

∴ It is negatively skewed.

99. (A) First seven prime numbers are 2, 3, 5, 7, 11, 13, 17.

$$\text{Mean} = \frac{2 + 3 + 5 + 7 + 11 + 13 + 17}{7} = \frac{58}{7} = 8.28$$

$$\text{Median} = \text{Value of } \left( \frac{7+1}{2} \right) \text{th term} = \text{Value of 4th term} = 7$$

Hence, A and R are individually true and R is correct explanation of A.