**Class- X Session- 2022-23**

**Subject- Mathematics (Standard)**

**Sample Question Paper - 30**

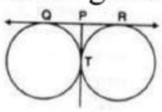
**Time Allowed: 3 Hrs.** **Maximum Marks : 80**

**General Instructions:**

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

## Section A

1. In the given figure if QP = 4.5 cm, then the measure of QR is equal to **[1]**




a) 15 cm b) 9 cm

c) 18 cm d) 13.5 cm

2. A card is drawn at random from a pack of 52 cards. The probability that the card is drawn is a black king. **[1]**

a) $\frac{11}{13}$ b) $\frac{3}{13}$

c) $\frac{1}{26}$ d) $\frac{1}{52}$

3. The distance between the points A (0, 6) and B (0, –2) is **[1]**

a) 8 b) 4

c) 6 d) 2

4. If the point P(2, 1) lies on the line segment joining points A(4, 2) and B(8, 4), then **[1]**

a) $AP = \frac{1}{4}AB$ b) $AP = \frac{1}{2}AB$

c) $AP = \frac{1}{3}AB$ d) AP = PB

5. If the line segment joining the points (3, -4) and (1, 2) is trisected at points P (a, -2) and $Q\left(\frac{5}{3}, b\right)$. Then, **[1]**

a) $a = \frac{8}{3}, b = \frac{2}{3}$

b) $a = \frac{7}{3}, b = 0$

c) $a = \frac{2}{3}, b = \frac{1}{3}$

d) $a = \frac{1}{3}, b = 1$

6. The lines represented by $3x + y - 12 = 0$ and $x - 3y + 6 = 0$ intersects the x – axis at **[1]**

a) ( – 6, 0) and (4, 0)

b) ( – 6, 0) and ( – 4, 0)

c) (6, 0) and ( – 4, 0)

d) (6, 0) and (4, 0)

7. The probability that a non-leap year has 53 Sundays, is **[1]**

a) $\frac{1}{7}$

b) $\frac{2}{7}$

c) $\frac{6}{7}$

d) $\frac{5}{7}$

8. If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, then the probability that it is odd and is a multiple of 3 is **[1]**

a) $\frac{1}{9}$

b) $\frac{2}{9}$

c) $\frac{2}{3}$

d) $\frac{1}{3}$

9. The volume of a cylinder of radius r is 1/4 of the volume of a rectangular box with a square base of side length x. If the cylinder and the box have equal heights, what is r in terms of x? **[1]**

a) $\frac{x}{2\sqrt{\pi}}$

b) $\frac{x^2}{2\pi}$

c) $\frac{\pi}{2\sqrt{x}}$

d) $\frac{\sqrt{2x}}{\pi}$

10. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is **[1]**

a) $\frac{1}{2}$

b) -2

c) $\frac{1}{4}$

d) 2

11. $3x^2 + 4x + 5 = 0$ have **[1]**

a) Real roots

b) Real and Equal roots

c) No Real roots

d) Real and Distinct roots

12. The number 1.732 is **[1]**

a) a whole number

b) an integer

c) a rational number

d) an irrational number

13. The value of $\sin 45° + \cos 45°$ is **[1]**

a) $\sqrt{2}$

b) $\frac{1}{\sqrt{2}}$

c) 1

d) $\frac{1}{\sqrt{3}}$

14. The distance of the point P (2, 3) from the x-axis is **[1]**

a) 3

b) 1

c) 2

d) 5

15. The angle of elevation of the sun when the shadow of a pole 'h' metres high is $\frac{h}{\sqrt{3}}$ metres long is **[1]**

a) 45°

b) 30°

c) 60°

d) none of these

16. If HCF (26,169) = 13, then LCM (26,169) = **[1]**

a) 13

b) 26

c) 52

d) 338

17. Median = ? **[1]**

a) $l+\left\{h \times \frac{\left(cf-\frac{N}{2}\right)}{f}\right\}$

b) $l-\left\{h \times \frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

c) $l+\left\{h \times \frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

d) none of these

18. If the system of equations
3x + y = 1 and
(2k - 1) x + (k - 1) y = 2k + 1
is inconsistent, then k = **[1]**

a) -1

b) 1

c) 2

d) 0

19. **Assertion (A):** No two positive numbers can have 18 as their H.C.F and 380 as their L.C.M. **[1]**
**Reason (R):** L.C.M. is always completely divisible by H.C.F.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Two similar triangles are always congruent. **[1]**
**Reason (R):** If the areas of two similar triangles are equal then the triangles are

congruent.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

## Section B

21. A lot consists of 144 ballpoint pens of which 20 are defective and others good. Tanvy will buy a pen if it is good, but will not buy it if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that **[2]**

i. she will buy it,

ii. she will not buy it?

22. Does the pair of the linear equation have no solution? Justify your answer. **[2]**
x = 2y, y = 2x

23. Find the points on the x-axis, each of which is at a distance of 10 units from the point A(11, -8). **[2]**

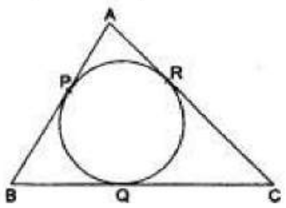
OR

Find the co-ordinates of the points which divide the line segment joining the points (–4, 0) and (0, 6) in four equal parts.

21.

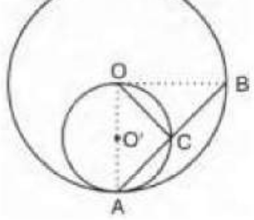
24. Form a quadratic polynomial whose zeroes are $\frac{3-\sqrt{3}}{5}$ and $\frac{3+\sqrt{3}}{5}$. **[2]**

25. A circle is inscribed in $\triangle ABC$ touching AB, BC and AC at P, Q and R respectively. If AB = 10 cm, AR = 7 cm and CR = 5 cm, find the length of BC. **[2]**




OR

In Figure, circles C(O,r) and C (O' r/2) touch internally at a point A and AB is a chord of the circle C(O,r) intersecting C (O', r/2) at C. Prove that AC = CB.




## Section C

26. Prove that: $\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}=2.$ **[3]**

27. Given the linear equation 2x + 3y - 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is: **[3]**

i. intersecting lines

ii. parallel lines

iii. coincident lines.

28. Prove that $4-5\sqrt{2}$ is an irrational number. **[3]**

OR

Prove that $\sqrt{5}+\sqrt{3}$ is irrational.

29. In a $\triangle ABC$, D and E are points on the sides AB and AC respectively. Show that DE || BC : AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm, and EC = 5.5 cm. **[3]**

30. The angle of elevation of a jet fighter from point A on ground is $60^{\text{o}}$. After flying 10 seconds, the angle changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying. **[3]**

31. Prove that the tangent drawn at the midpoint of an arc of a circle is parallel to the chord joining the end points of the arc. **[3]**

OR

ABC is a right triangle in which ∠B = $90^{\text{o}}$. If AB = 8 cm and BC = 6 cm, find the diameter of the circle inscribed in the triangle.

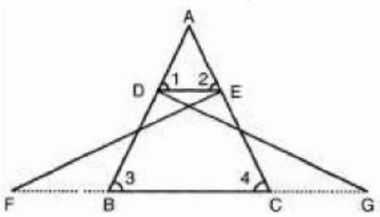
**Section D**

32. A shopkeeper buys a number of books for Rs.1200. If he had bought 10 more books for the same amount, each book would have cost him Rs.20 less. Find how many books did he buy? **[5]**

OR

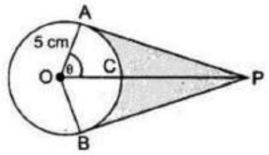
A journey of 192 km from a town A to town B takes 2 hours more by an ordinary passenger train than a super fast train. If the speed of the faster train is 16 km/h more, find the speed of the faster and the passenger train.

33. In the following figure, $\triangle FEC \cong \triangle GBD$ and $\angle 1 = \angle 2$ Prove that $\triangle ADE \cong \triangle ABC$. **[5]**




34. An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm **[5]**

from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area.




OR

Find the area of the segment of a circle of radius 12 cm whose corresponding sector central angle 60°. (Use $\pi = 3.14$).

35. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes. **[5]**

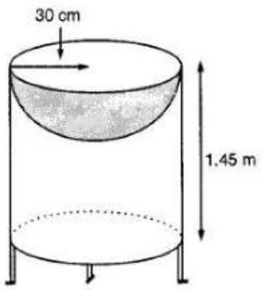
| Number of mangoes | 50-52 | 53-55 | 56-58 | 59-61 | 62-64 |
|---|---|---|---|---|---|
| Number of boxes | 15 | 110 | 135 | 115 | 25 |

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

## Section E

36. **Read the text carefully and answer the questions:** **[4]**

Mayank a student of class $7^{th}$ loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class $10^{th}$ helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.




(i) Find the curved surface area of the hemisphere.

(ii) Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)

(iii) What is total cost for making the bird bath?

**OR**

Mayank and his brother thought of increasing the radius of hemisphere to 35 cm with same material so that birds get more space, then what is the new height of cylinder?

37. **Read the text carefully and answer the questions:** **[4]**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.

(i) If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it

(ii) Find the total money he saved.

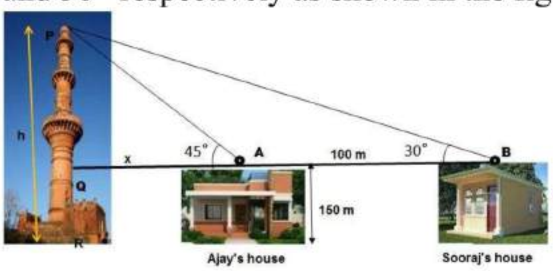
**OR**

How many coins are there in piggy bank on 15th day?

(iii) How much money Akshar saves in 10 days?

38. **Read the text carefully and answer the questions:** **[4]**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are $45^{\circ}$ and $30^{\circ}$ respectively as shown in the figure.




(i) Find the height of the tower.

(ii) What is the distance between the tower and the house of Sooraj?

**OR**

Find the distance between top of tower and top of Ajay's house?

(iii) Find the distance between top of the tower and top of Sooraj's house?

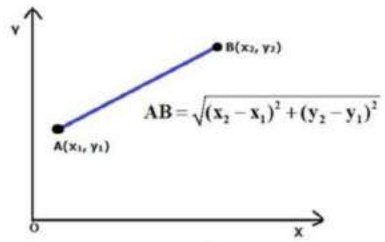
# SOLUTION

## Section A

1. **(b)** 9 cm
   **Explanation:** Here QP = PT = 4.5 cm [Tangents to a circle from an external point P]
   Also PT = PR = 4.5 cm [Tangents to a circle from an external point P]
   $\therefore$ QR = QP + PQ= 4.5 + 4.5 = 9 cm

2. **(c)** $\frac{1}{26}$
   **Explanation:** Total number of outcomes = 52
   Favourable outcomes, in this case, = 2 {2 black kings}
   $\therefore$ P (black king) = $\frac{Favourable\ outcomes}{Total\ outcomes} = \frac{2}{52} = \frac{1}{26}$

3. **(a)** 8
   **Explanation:** By using the distance formula:




$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Lets calculate the distance between the points $(x_1, y_1)$ and $(x_2, y_2)$

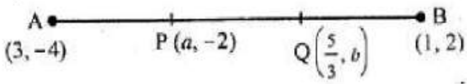
We have;
$x_1 = 0, x_2 = 0$
$y_1 = 6, y_2 = -2$
$d^2 = (0 - 0)^2 + (-2 - 6)^2$
$d = \sqrt{(0)^2 + (-8)^2}$
$d = \sqrt{64}$
d = 8 units
So, the distance between A (0, 6) and B (0, - 2) = 8

4. **(b)** $AP = \frac{1}{2}AB$

**Explanation:** $AP = \sqrt{(2-4)^2 + (1-2)^2}$
$= \sqrt{4+1} = \sqrt{5} =$ units
$AB = \sqrt{(8-4)^2 + (4-2)^2}$
$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ units
Here $AB = 2 \times AP$
$\therefore AP = \frac{1}{2}AB$

5. **(b)** $a = \frac{7}{3}, b = 0$

**Explanation:** The points P(a, -2) and Q $\left(\frac{5}{3}, b\right)$ trisect the line segment joining the points A(3, -4) and B(1, 2)
$\because$ P divides AB in the ratio 1 : 2




Then, $a = \frac{1\times1+2\times3}{1+2} = \frac{1+6}{3} = \frac{7}{3}$
and Q divides AB in the ratio 2 : 1, then
$b = \frac{2\times2+1\times(-4)}{2+1} = \frac{4-4}{3} = 0$
$\therefore a = \frac{7}{3}, b = 0$

6. **(a)** $(-6, 0)$ and (4, 0)

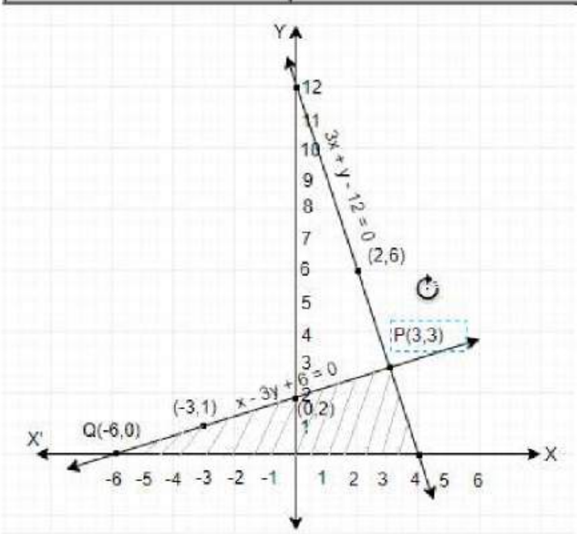
**Explanation:** Here are the two solutions of each of the given equations.
$3x + y - 12 = 0,$

| $x$ | 4 | 3 | 2 |
|---|---|---|---|
| $y$ | 0 | 3 | 6 |

$x - 3y + 6 = 0$

| $x$ | -6 | 0 | -3 |
|---|---|---|---|

| $y$ | 0 | 2 | 1 |
|---|---|---|---|




The triangle $\triangle$PQR is formed by the given two lines and x-axis. Therefore, both lines intersect the x-axis at (–6, 0) and at (4, 0).

7. **(a)** $\frac{1}{7}$

   **Explanation:** In a non leap years, number of days = 365 i.e. 52 weeks + 1 day
   $\therefore$ Probability of being 53 Sundays $= \frac{m}{n} = \frac{1}{\text{No. of day in a week}} = \frac{1}{7}$

8. **(b)** $\frac{2}{9}$

   **Explanation:** Total numbers of digits for 1 to 9(n) = 9
   Number divisible by 3(m) = 3, 6, 9
   Odd numbers out of 3, 6, 9 = 3, 9
   $\therefore$ Probability $= \frac{m}{n} = \frac{2}{9}$

9. **(a)** $\frac{x}{2\sqrt{\pi}}$

   **Explanation:** Let $V_1$ be the volume of the cylinder with radius r and height h, then
   $V_1 = \pi r^2 h$ .... (i)
   Now, let $V_2$ be the volume of the box, then
   $V_2 = x^2 h$
   It is given that $V_1 = 1/4\ V_2$. Therefore,
   $\pi r^2 h = \frac{1}{4}x^2 h$
   $\Rightarrow r^2 = \frac{x^2}{4\pi} \Rightarrow r = \frac{x}{2\sqrt{\pi}}$

10. **(d)** 2

    **Explanation:** If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$ then, substituting the value of $\frac{1}{2}$ in place of x should give us the value of k.
    Given, $x^2 + kx - \frac{5}{4} = 0$ where, $x = \frac{1}{2}$
    $\Rightarrow \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$
    $\Rightarrow \frac{k}{2} = \frac{5}{4} - \frac{1}{4}$
    $\therefore k = 2$

11. **(c)** No Real roots

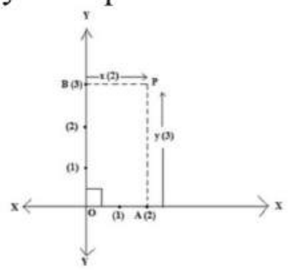
    **Explanation:** $D = b^2 - 4ac$

$D = 4^2 - 4 \times 3 \times 5$
$D = 16 - 60$
$D = -44$
$D < 0$. Hence No Real roots.

12. **(c)** a rational number
**Explanation:** Clearly, 1.732 is a terminating decimal.
Hence, it is a rational number.

13. **(a)** $\sqrt{2}$
**Explanation:** Given: $\sin 45^\circ + \cos 45^\circ$
$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$
$= \frac{2}{\sqrt{2}} = \sqrt{2}$

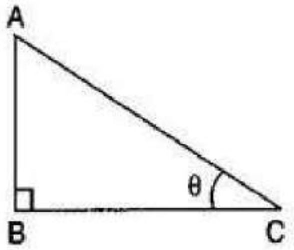
14. **(a)** 3
**Explanation:** We know that,
x, y is any point on the Cartesian plane in first quadrant.
Then,
x = Perpendicular distance from Y - axis and
y = Perpendicular distance from X - axis



So, the distance of the point P (2, 3) from the X - axis = 3

15. **(c)** $60^\circ$

**Explanation:**




Given: Height of the pole = AB = $h$ meters And the length of the shadow of the pole = BC = $\frac{h}{\sqrt{3}}$ meters $\therefore \tan\theta = \frac{h}{\frac{h}{\sqrt{3}}}$
$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$

16. **(d)** 338
**Explanation:** HCF (26, 169) = 13
We have to find the value for LCM (26, 169)
We know that the product of numbers is equal to the product of their HCF and LCM.
Therefore,
13(LCM) = 26(169)
$\text{LCM} = \frac{26(169)}{13}$
LCM = 338

17. **(c)** $l+\left\{h \times \frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

18. **(c)** 2

**Explanation:** The given system of equations is inconsistent,

3x + y = 1

(2k - 1)x + (k - 1)y = 2k + 1

If the system of equations is inconsistent, we have

$\frac{3}{2k-1}=\frac{1}{k-1}=\frac{1}{2k+1}$

Take,

$\frac{3}{2k-1}=\frac{1}{k-1}$

$\Rightarrow 3k-3=2k-1$

$\Rightarrow k=2$

19. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** 380 is not divisible by 18.

20. **(d)** A is false but R is true.

**Explanation:** Two similar triangles are not congruent generally. So, A is false but R is true.

## Section B

21. Total number of pens = 144

Number of defective pens = 20

Number of good pens = 144 – 20 = 124

i. Let $E_1$ be the event of buying a pen.

Tanvy will buy a pen, if it is a good pen.

number of good pens = 124

Therefore, P(buying a pen) = $P(E_1)=\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}=\frac{124}{144}=\frac{31}{36}$

ii. Let $E_2$ be the event of getting a defective pen.

Tanvy will not buy a pen, if it is a defective pen.

number of defective pens = 20.

Therefore, P(not buying a pen) = $P(E_2)=\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}=\frac{20}{144}=\frac{5}{36}$

22. No.

The Condition for no solution is : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (parallel lines)

Given pair of equations,

x = 2y and y = 2x

or x - 2y = 0 and 2x - y = 0;

Comparing with ax + by + c = 0;

Here, $a_1 = 1$, $b_1 = -2$, $c_1 = 0$;

And $a_2 = 2$, $b_2 = -1$, $c_2 = 0$;

$a_1 / a_2 = 1/2$

$b_1 / b_2 = -2/-1 = 2$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has unique solution.

23. Let P(x, 0) be the point on the x - axis. Then, as per the question, we have

AP = 10

$\Rightarrow \sqrt{(x-11)^2 + (0+8)^2} = 10$

Squaring both sides,

$\Rightarrow (x - 11)^2 + 8^2 = 100$

$\Rightarrow (x - 11)^2 = 100 - 64 = 36$

$\Rightarrow x - 11 = \pm 6$

$\Rightarrow x = 11 \pm 6$

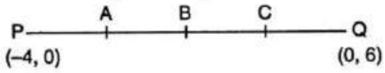
$\Rightarrow x = 11 - 6, 11 + 6$

$\Rightarrow x = 5, 17$

Hence, the points on the x - axis are (5, 0) and (17, 0).

OR

Let the given points be denoted by P and Q.




Co-ordinate of B(mid-point of PQ) are: $\left(\frac{-4+0}{2}, \frac{0+6}{2}\right)$ i.e. (–2, 3)

Co-ordinates of A (mid-point of PB) are: $\left(\frac{-4-2}{2}, \frac{0+3}{2}\right)$ i.e. $\left(-3, \frac{3}{2}\right)$

Co-ordinates of C (mid-point of BQ) are: $\left(\frac{-2+0}{2}, \frac{6+3}{2}\right)$ i.e. $\left(-1, \frac{9}{2}\right)$ .

Hence, the co-ordinates of the required mid-points are $\left(-1, \frac{9}{2}\right)$, (–2, 3) and $\left(-3, \frac{3}{2}\right)$

24. Let $\alpha = \frac{3-\sqrt{3}}{5}$ and $\beta = \frac{3+\sqrt{3}}{5}$

Given $\alpha+\beta = \frac{3-\sqrt{3}}{5} + \frac{3+\sqrt{3}}{5} = \frac{6}{5}$

Product of zeroes,

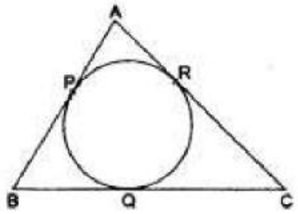
$\alpha\beta = \left(\frac{3-\sqrt{3}}{5}\right) \times \left(\frac{3+\sqrt{3}}{5}\right) = \frac{3^2-(\sqrt{3})^2}{5\times5} = \frac{6}{25}$

Polynomial

$p(x) = x^2 - (\alpha+\beta)x) + \alpha\beta$

$= x^2 - \frac{6}{5}x + \frac{6}{25}$

$p(x) = 25x^2 - 30x + 6$

25. 


In the given figure it is shown that a circle is inscribed in Δ ABC, such that the circle touches the sides AB, BC & AC at points P, Q & R respectively.

Also, given AR = 7 cm , CR = 5 cm , AB = 10 cm. .....(1)

We know that, tangents drawn to a circle from an external point are equal in length.

So, AP = AR, BP = BQ & CR = CQ ..........(2)

From (1) & (2), AP = AR = 7 cm, CR = CQ =5 cm.....(3)

Now, from figure, BP = AB - AP = 10 - 7 = 3 cm.

∴ BP = BQ = 3cm [ from (2) ].....(4)

Again from figure,

BC = (BQ + QC)

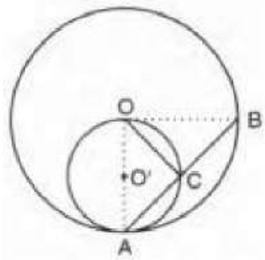
⇒ BC = 3 + 5 [ from (3) & (4) ]

⇒ BC = 8

∴ The length of BC is 8 cm.

OR

Join OA, OC and OB. Clearly, $\angle OCA$ is the angle in a semi-circle




$\therefore \angle OCA = 90^{\circ}$

In right triangles OCA and OCB, we have

OA = OB = r

$\angle OCA = \angle OCB = 90^{\circ}$

and, OC = OC

So, by RHS-criterion of congruence, we get

$\triangle OCA \cong \triangle OCB$

$\Rightarrow$ AC = CB

## Section C

26. LHS = $\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

$= \frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\sin\theta\cos\theta)}{(\cos\theta+\sin\theta)}+\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{(\cos\theta-\sin\theta)}$

$= (1 - sin\theta cos\theta) + (1 + sin\theta cos\theta)$

$= 1 + 1 - sin\theta cos\theta + sin\theta cos\theta$

= 2 = RHS

27. Given, linear equation is 2x + 3y - 8 = 0 ...(i)

    Given: 2x + 3y - 8 = 0 ..... (i)

    i. For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

       $\therefore$ Any line intersecting with eq (i) may be taken as 3x + 2y - 9 = 0 or 3x + 2y - 7 = 0

    ii. For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

       $\therefore$ Any line parallel with eq(i) may be taken as 6x + 9y + 7 = 0 or 2x +3y - 2 = 0

    iii. For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

       $\therefore$ Any line coincident with eq (i) may be taken as 4x + 6y - 16 = 0 or 6x + 9y - 24 = 0

28. Let us assume that $4 - 5\sqrt{2}$ is rational. Then, there must exist positive co-primes between a and b such that

    $4 - 5\sqrt{2} = \frac{a}{b}$

    $\Rightarrow 5\sqrt{2} = \frac{a}{b} - 4$

    $\Rightarrow \sqrt{2} = \frac{\frac{a}{b}-4}{5}$

    $\Rightarrow \sqrt{2} = \frac{a-4b}{5b}$

    Since a,b are integers, $\therefore \frac{a-4b}{5b}$ is a rational number. Therefore, it follows that $\sqrt{2}$ is a rational number, which is a contradiction as $\sqrt{2}$ is an irrational number.

$\therefore$ Our supposition is wrong.
Hence $4-5\sqrt{2}$ is irrational.

OR

Let $\sqrt{5}+\sqrt{3}$ be rational number equal to $\frac{a}{b}$. there exist co-prime integers a and b such that

$\sqrt{5}+\sqrt{3}=\frac{a}{b}$

$\Rightarrow \quad \sqrt{5}=\frac{a}{b}-\sqrt{3}$

$\Rightarrow \quad (\sqrt{5})^2=\left(\frac{a}{b}-\sqrt{3}\right)^2$ [Squaring both sides] we get,

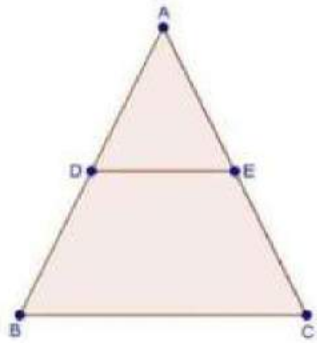
$\Rightarrow \quad 5=\frac{a^2}{b^2}-\frac{2a\sqrt{3}}{b}+3$

$\Rightarrow \quad 2=\frac{a^2}{b^2}-\frac{2\sqrt{3}a}{b}$

$\sqrt{3}=(a^2-2b^2)\frac{b}{2ab}$

Since a,b are integers, therefore $(a^2-2b^2)\frac{b}{2ab}$ is a rational number which is a contradiction as $\sqrt{3}$ is an irrational number.
Hence, $\sqrt{5}+\sqrt{3}$ is irrational.

29. We have




AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm.

Now, $\frac{AD}{BD}=\frac{5.7}{9.5}$

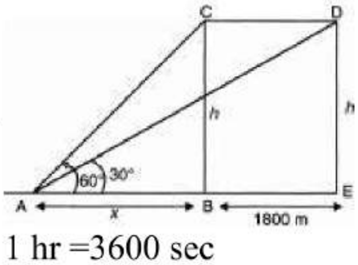
$=\frac{57}{95}$

$\Rightarrow \frac{AD}{BD}=\frac{3}{5}$

And, $\frac{AE}{EC}=\frac{3.3}{5.5}=\frac{33}{5.5}$

$\Rightarrow \frac{AE}{EC}=\frac{3}{5}$

Thus, DE divides sides AB and AC of $\triangle$ABC in the same ratio.
Therefore, by the converse of basic proportionality theorem,
We have,
DE || BC.

30. 


1 hr =3600 sec
Hence in 3600 sec distance travelled by plane = 648 km = 648000 m
In 10 sec distance travelled by plane = $\frac{648000}{3600}\times 10=1800$ m

So BE = CD= 1800 m

In $\triangle$ABC,

$\frac{h}{x} = \tan 60^\circ$

$\frac{h}{x} = \sqrt{3}$

$\Rightarrow h = x\sqrt{3}$......(i)

In $\triangle$ADE we have

$\frac{h}{x+1800} = \tan 30^\circ$

$\frac{h}{x+1800} = \frac{1}{\sqrt{3}}$

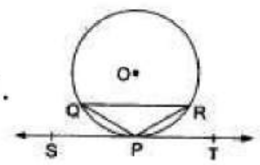
$\Rightarrow \quad h = \frac{x+1800}{\sqrt{3}}$.........(ii)

From equation (i) and (ii) we get

$x\sqrt{3} = \frac{x+1800}{\sqrt{3}}$

3x = x + 1800

x = 900 m So h = 900√3 meter

31. 


Point P is the midpoint of arc $\widehat{QR}$ of a circle with centre O.

ST is the tangent to the circle at point P.

TO prove :Chord QR$\|$ST

Proof: P is the midpoint of $\widehat{QR}$

$\Rightarrow \widehat{QP} = \widehat{PR}$

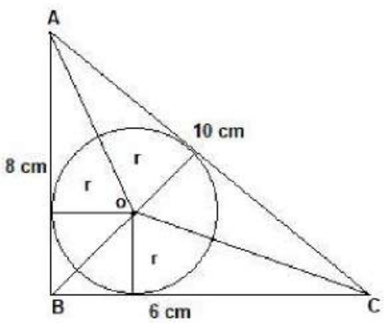
$\Rightarrow$ chord QP = chord PR [$\because$ in a circle, if two arcs are equal, then their corresponding chords are equal]

$\therefore \quad \angle PQR = \angle PRQ$

$\Rightarrow \angle TPR = \angle PRQ$ [as , $\angle PQR = \angle TPR$, angles in alternate segments]

$\Rightarrow \quad QR \| ST$, [$\because \angle TPR$ and $\angle PRQ$ are alternate interior $angles$]

OR



By pythagoras theorem
AC = $\sqrt{AB^2 + BC^2}$
= $\sqrt{8^2 + 6^2}$
= $\sqrt{64 + 36}$
= $\sqrt{100}$ = 10 cm
Area of $\triangle$ ABC = Area of $\triangle$ AOB + Area of $\triangle$BOC + Area of $\triangle$ AOC
$\frac{1}{2} \times b \times h = \frac{1}{2} \times b_1 \times h_1 + \frac{1}{2} \times b_2 \times h_2 \times \frac{1}{2} \times b_3 \times h_3$
$\frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 8 \times r + \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r$
24 = 4r + 3r + 5r
24 = 12 r
r = 2 cm
Hence the radius is 2 cm.

**Section D**

32. Let number of books the shopkeeper buys = $x$
Price of each book = Rs $\frac{1200}{x}$
cost of each book when x +10 books are bought = RS $\frac{1200}{x+10}$
According to given question,
$\frac{1200}{x} - \frac{1200}{x+10} = 20$
$1200(\frac{1}{x} - \frac{1}{x+10}) = 20$
$(\frac{1}{x} - \frac{1}{x+10}) = \frac{20}{1200}$
$\frac{(x+10)-x}{x(x+10)} = \frac{1}{60}$
$x$ +10- $x = \frac{x^2+10x}{60}$
$600 = x^2 + 10x$
$x^2 + 10x - 600 = 0$
Here, it is quadratic equation
$x^2 + 30x - 20x - 600 = 0$
$x(x+30) - 20(x+30) = 0$
$(x+30)(x-20) = 0$

either

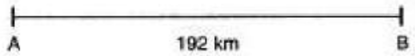
$(x+30) = 0$ or $(x-20) = 0$

$x = -30$ or $x = 20$

$x = -30$, is not possible because the number of books can't be negative.

so, number of books $= x = 20$.

OR




Let speed of passenger train be x km/h

$\therefore$ speed of superfast train = (x + 16) km/h

By question, $T_{\text{passenger}} = \frac{192}{x}$ and $\mathrm{T}_{\text{superfast}} = \frac{192}{(x+16)}$

or, $\frac{192}{x} - \frac{192}{x+16} = 2$

or, $192(x+16) - 192x = 2\left(x^2 + 16x\right)$

or, $192x + 192 \times 16 - 192x = 2\left(x^2 + 16x\right)$

192x + 3072 - 192x $= 2\left(x^2 + 16x\right)$( divide throughout by 2, we get,

96x + 1536 - 96x $= \left(x^2 + 16x\right)$

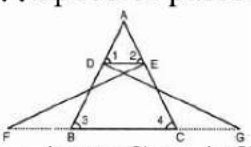
or x(x + 48) - 32(x + 48) = 0

or, (x - 32) (x + 48) = 0

or, x = 32 or - 48

Since speed can't be negative, therefore - 48 is not possible.

$\therefore$ Speed of passenger train = 32 km/h and Speed of fast train = 48 km/h

33. 


$\because \triangle FEC \cong \triangle GBD$

or, EC = BD ......(i)

It is given that $\angle 1 = \angle 2$

or, AE = AD ( $\because$ Isosceles triangle property)...(ii)

From ,eqns. (i) and (ii),

$\frac{AE}{EC} = \frac{AD}{DB}$

or, DE || BC, ( $\because$ converse of B.PT)

or, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ ( $\because$ Corresponding angles)

Thus in $\triangle ADE$ and $\triangle ABC$ ,

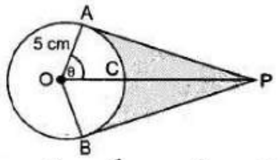
$\angle \mathrm{A} = \angle \mathrm{A}$

$\angle 1 = \angle 3$

$\angle 2 = \angle 4$

$\triangle ADE \sim \triangle ABC$ ( $\because$ AAA criterion of similarity)

$\triangle ADE \sim \triangle ABC$ Hence proved

34. 


$\cos\theta = \frac{1}{2}$ or, $\theta = 60^\circ$

Reflex $\angle AOB = 120^\circ$

$\therefore$ ADB $= \frac{2 \times 3.14 \times 5 \times 240}{360} = 20.93$ cm

Hence length of elastic in contact = 20.93 cm

Now, AP = $5\sqrt{3}$cm

a ($\triangle$OAP) = $\frac{1}{2}\times$ base $\times$ height = $\frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$

Area ($\triangle$OAP + $\triangle$OBP) = 2$\times$ $\frac{25\sqrt{3}}{2} = 25\sqrt{3}$ = 43.25 cm$^2$

Area of sector OACB = $\frac{\theta}{360} \times \pi r^2$

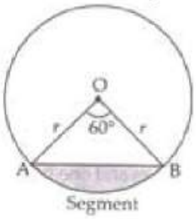
$= \frac{25\times3.14\times120}{360}$ = 26.16 cm$^2$

Shaded Area = 43.25 - 26.16 = 17.09 cm$^2$

OR

Area of minor segment = Area of sector – Area of $\triangle$OAB

In $\triangle$OAB,




$\theta = 60°$

OA = OB = r = 12 cm

$\angle$B = $\angle$A = x [$\angle$s opp. to equal sides are equal]

$\Rightarrow \angle A + \angle B + \angle O = 180°$

$\Rightarrow$ x + x + 60° = 180°

$\Rightarrow$ 2x = 180° – 60°

$\Rightarrow$ x = $\frac{120^\circ}{2}$ = 60°

$\therefore$ $\triangle$OAB is equilateral $\triangle$ with each side (a) = 12 cm

Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of $\triangle$OAB

$= \frac{\pi r^2\theta}{360^\circ} - \frac{\sqrt{3}}{4}a^2$

$= \frac{3.14\times12\times12\times60^\circ}{360^\circ} - \frac{\sqrt{3}}{4} \times 12 \times 12$

$= 6.28 \times 12 - 36\sqrt{3}$

$\therefore$ Area of minor segment = $(75.36 - 36\sqrt{3})$ cm$^2$.

35. Since value of number of mangoes and number of boxes are large numerically. So we use step-deviation method

| True Class Interval | No. of boxes($f_i$) | Class mark($x_i$) | $u_i = \frac{x_i - a}{h}$ | $f_iu_i$ |
|---|---|---|---|---|
| 49.5-52.5 | 15 | 51 | -2 | -30 |
| 52.5-55.5 | 110 | 54 | -1 | -110 |
| 55.5-58.5 | 135 | 57 | 0 | 0 |
| 58.5-61.5 | 115 | 60 | 1 | 115 |
| 61.5-64.5 | 25 | 63 | 2 | 50 |
| | $\sum f_i = 400$ | | | $\sum f_iu_i = 25$ |

Let assumed mean (a) = 57,

h = 3 ,

$\therefore \overline{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{25}{400} = 0.0625$ (approx.)

Using formula, Mean $(\overline{x})$ = a + h$\overline{u}$

= 57 + 3 (0.0625

= 57 + 0.1875

= 57.1875

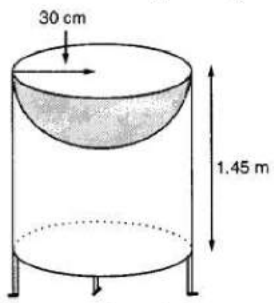
= 57.19 (approx)

Therefore, the mean number of mangoes is 57.19

## Section E

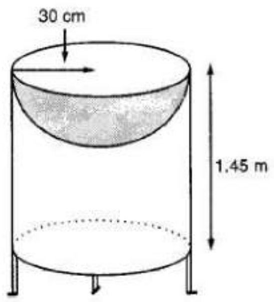
36. **Read the text carefully and answer the questions:**

Mayank a student of class $7^{th}$ loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class $10^{th}$ helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.




(i) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.

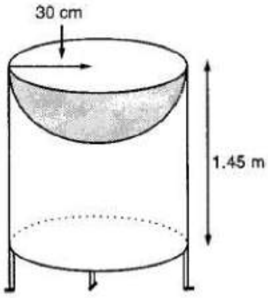



Curved surface area of the hemisphere = $2\pi r^2$

$= 2 \times 3.14 \times 30^2 = 0.56\ m^2$

(ii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.




Let S be the total surface area of the birdbath.

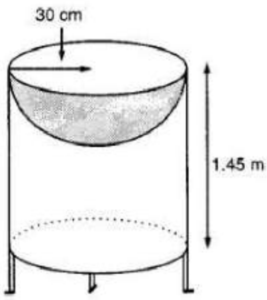
S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$\Rightarrow S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$\Rightarrow S = 2 \times \frac{22}{7} \times 30(145 + 30) = 33000 \text{ cm}^2 = 3.3 \text{ m}^2$

(iii)Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



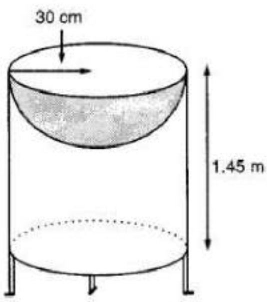

Total Cost of material = Total surface area $\times$ cost per sq $\text{m}^2$

$= 3.3 \times 40 =$ ₹132

OR

Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.




r = 35 cm = $\frac{35}{100}$ m

We know that S.A = 3.3 $\text{m}^2$

S = $2\pi r(r + h)$

$\Rightarrow 3.3 = 2 \times \frac{22}{7} \times \frac{35}{100}\left(\frac{35}{100} + h\right)$

$\Rightarrow 3.3 = \frac{22}{10}\left(\frac{35}{100} + h\right)$

$\Rightarrow \frac{33}{22} = \frac{35}{100} + h$

$\Rightarrow h = \frac{3}{2} - \frac{7}{20} = \frac{23}{20} = 1.15 \text{ m}$

37. **Read the text carefully and answer the questions:**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.

(i) Child's Day wise are,

$\underbrace{5}_{1\text{ coin}}, \underbrace{10}_{2\text{ coins}}, \underbrace{15}_{3\text{ coins}}, \underbrace{20}_{4\text{ coins}}, \underbrace{25}_{5\text{ coins}}$ .... to $\underbrace{n\text{ days}}_{n\text{ coins}}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n - 1)1] = 190$

$\Rightarrow n(n + 1) = 380 \Rightarrow n^2 + n - 380 = 0$

$\Rightarrow (n + 20)(n - 19) = 0 \Rightarrow (n + 20)(n - 19) = 0$

$\Rightarrow n = -20$ or $n = 19 \Rightarrow n = -20$ or $n = 19$

But number of coins cannot be negative

$\therefore n = 19$ (rejecting $n = -20$)

So, number of days = 19

(ii) Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms

$= \frac{19}{2}[2 \times 5 + (19 - 1)5]$

$= \frac{19}{2}[100] = \frac{1900}{2} = 950$

and total money she shaved = ₹950

OR

Number of coins in piggy bank on 15th day

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 5 + (15 - 1) \times 5]$

$\Rightarrow S_{15} = \frac{15}{2}[2 + 14]$

$\Rightarrow S_{15} = 120$

So, there are 120 coins on 15th day.

(iii) Money saved in 10 days

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 5 + (10 - 1) \times 5]$

$\Rightarrow S_{10} = 5[10 + 45\}]$

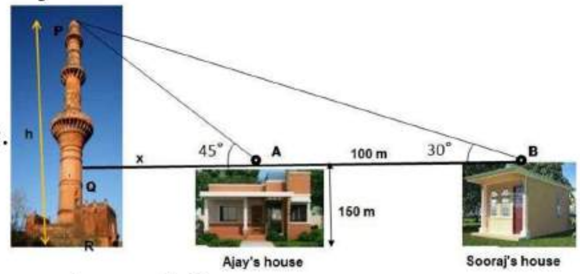
$\Rightarrow S_{10} = 275$

Money saved in 10 days = ₹275

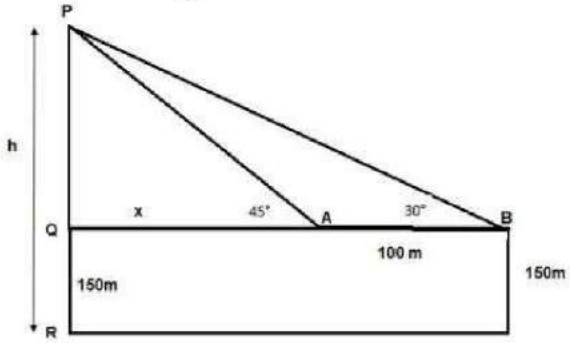
38. **Read the text carefully and answer the questions:**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are $45^{\circ}$ and $30^{\circ}$

respectively as shown in the figure.




(i) The above figure can be redrawn as shown below:




Let PQ = y

In $\triangle PQA$,

$\tan 45 = \frac{PQ}{QA} = \frac{y}{x}$

$1 = \frac{y}{x}$

x = y ...(i)

In $\triangle PQB$,

$\tan 30 = \frac{PQ}{QB} = \frac{PQ}{x+100} = \frac{y}{x+100} = \frac{x}{x+100}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+100}$

$x\sqrt{3} = x + 100$

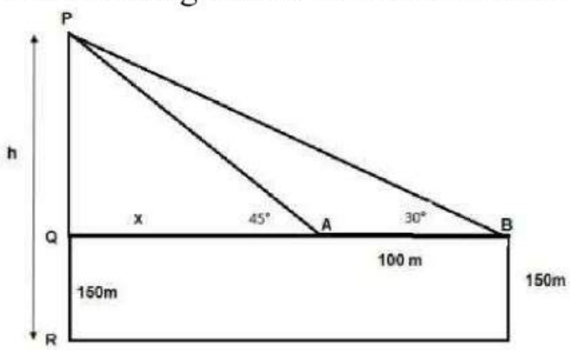
$x = \frac{100}{\sqrt{3}-1} = 136.61$ m

From the figure, height of tower h = PQ + QR

= x + 150 = 136.61 + 150

h = 286.61 m

(ii) The above figure can be redrawn as shown below:

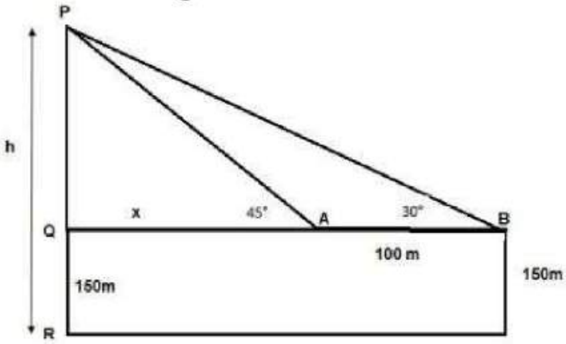



Distance of Sooraj's house from tower = QA + AB

= x + 100 = 136.61 + 100 = 236.61 m

OR

The above figure can be redrawn as shown below:




Distance between top of the tower and top of Ajay's house is PA

In $\triangle$PQA

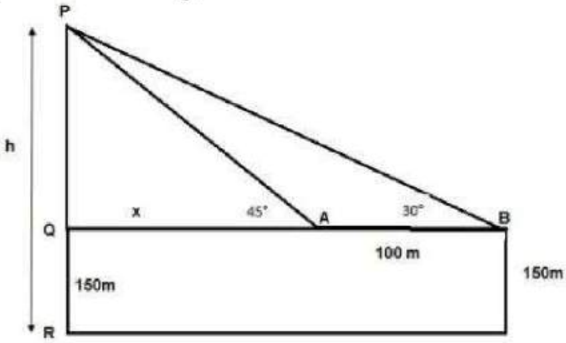
$\sin 45^{o} = \frac{PQ}{PA}$

$\Rightarrow PA = \frac{PQ}{\sin 45^{o}}$

$\Rightarrow$ PA = $\frac{y}{\frac{1}{\sqrt{2}}} = \sqrt{2} \times 136.61$

$\Rightarrow$ PA = 193.20 m

(iii)The above figure can be redrawn as shown below:




Distance between top of tower and Top of Sooraj's house is PB

In $\triangle$PQB

$\sin 30^{o} = \frac{PQ}{PB}$

$\Rightarrow PB = \frac{PQ}{\sin 30^{o}}$

$\Rightarrow$ PB = $\frac{y}{\frac{1}{2}} = 2 \times 136.61$

$\Rightarrow$ PB = 273.20 m