
Sample Paper-02
Mathematics
Class – XI

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- a) All questions are compulsory.
 - b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
 - c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
 - d) Use of calculators is not permitted.
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Section A

1. Identify a function $f(x)$ so that $f(x).f(y) = f(x+y)$
2. If $A = \{(x, y) : y = a^x, x \in R\}$ and $B = \{(x, y) : y = a^{-x}, x \in R\}$ then what is $(A \cap B)$
3. If R is a relation from a set A containing p elements to a set B containing q elements the find the number of subsets of $A \times B$
4. Check whether the given lines are parallel or perpendicular.
$$ax - by + c = 0 \quad \text{and} \quad \frac{ax}{2} - \frac{by}{2} + d = 0$$
5. Find the area of the triangle whose vertices are $(2,0), (5,3), (2,6)$
6. Write the equation of a circle with center $(0,0)$ and radius 5

Section B

7. Solve $\cos 3x = -\frac{1}{2}$
 8. Prove by mathematical induction that $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$
 9. Find the square root of $\sqrt{-8i}$
 10. Solve the inequality $\frac{2x+5}{x-2} \geq 3$
 11. Find the value of x if ${}^{12}C_x = {}^{12}C_{x+4}$
 12. Three cars are there in a race. Car A is 3 times as likely to win as car B. Car B is twice as likely to win as car C. What is the probability of winning each car.
 13. If $f(x)$ is a function that contains 3 in its domain and range and satisfy the relation
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$$f(f(x)).(1+f(x)) = -f(x) \text{ find } f(3)$$

14. If $\tan A = \frac{1}{3}$ and $\tan B = \frac{1}{2}$ prove that $\sin 2(A+B) = 1$

15. Find two numbers such that their arithmetic mean is 15 and Geometric mean is 9 without using the identity $(a+b)^2 = (a-b)^2 + 4ab$

16. Let $f: R \rightarrow R$ be a function given by $f(x) = x^2 + 2$ find $f^{-1}(27)$

17. Find the domain and range of the function $f(x) = \frac{x-a}{a+1-x}$ where a is a positive integer.

18. Find the limit of $\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x}$

19. Find the sign and value of the expression $\sin 75^\circ + \cos 75^\circ$

Section C

20. In how many ways can 3 students from Class 12, 4 from class 11, 4 from class 10 and 2 from class 9 be seated in a row so that those of the same classes sit together. Also find the number of ways they can be arranged in at a round table

21. A circle represented by the equation $(x-a)^2 + (y-b)^2 = r^2$

This makes two complete revolutions along the positive direction of the x axis. Find the equation of the circle in the new position

22. Show that the equation $x^2 + 4y^2 + 4x + 16y + 16 = 0$ represents an ellipse.

23. Calculate the mean deviation about the mean from the following data

x_i	2	15	17	23	27
f_i	12	6	12	9	5

24. If the ratio of the roots of the equation $x^2 + px + q = 0$ is the same as $x^2 + p_1x + q_1 = 0$ then prove that

$$p^2 q_1 = p_1^2 q$$

25. Prove that $a \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{4}} \cdot a^{\frac{1}{8}} \dots \dots \infty = a^2$

26. In a survey of 700 students in a medical college 200 went for regular entrance coaching, 295 attended only correspondence coaching, 115 attended both regular and correspondence coaching. Find how many got admission without any entrance coaching.

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ANSWERS

Section A

1. **Solution :**

$$f(x) = a^x$$

$$f(y) = a^y$$

$$f(x) \cdot f(y) = a^x \cdot a^y = a^{x+y} = f(x+y)$$

2. **Solution :**

When $x = 0, y = 1$ in both cases. Hence

$$(A \cap B) = \{0, 1\}$$

3. **Solution :** 2^{pq}

4. **Solution**

They are parallel since

$$\begin{vmatrix} a & -b \\ a & -b \\ 2 & 2 \end{vmatrix} = 0$$

5. **Solution**

Area of a triangle

$$\frac{1}{2} \begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} = 9$$

6. **Solution**

$$x^2 + y^2 = 25$$

Section B

7. **Solution:**

$$\cos 3x = \cos \frac{2\pi}{3}$$

$$3x = 2n\pi \pm \frac{2\pi}{3}$$

$$x = \frac{2n\pi}{3} \pm \frac{2\pi}{9}, n \in \mathbb{Z}$$

8. **Solution:**

Let $P(n)$ be the statement given by $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

$$P(1) = \frac{1(1+1)}{2}$$

=1, True

Let it be true for $n=m$

$$1 + 2 + 3 + \dots + m = \frac{m(m+1)}{2}$$

$$1 + 2 + 3 + \dots + m + (m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$P(m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$P(m+1) = \frac{m^2 + 3m + 2}{2}$$

$$P(m+1) = \frac{(m+1)(m+2)}{2}$$

Thus $P(m)$ is true $\Rightarrow P(m+1)$ is True

9. Solution:

$$\text{Let } \sqrt{z} = \sqrt{-8i}$$

$$\sqrt{z} = \pm \left\{ \frac{\sqrt{|z| - \operatorname{Re}(z)}}{\sqrt{2}} \right\} - i \left\{ \frac{\sqrt{|z| - \operatorname{Re}(z)}}{\sqrt{2}} \right\}, \operatorname{Im}(z) < 0$$

$$\sqrt{-8i} = \pm \left\{ \frac{\sqrt{8+0}}{\sqrt{2}} - i \frac{\sqrt{8-0}}{\sqrt{2}} \right\}, \operatorname{Im}(z) < 0$$

$$= \pm(2-2i)$$

10. Solution

$$\frac{2x+5}{x-2} - 3 \geq 0$$

$$= \frac{2x+5-3x+6}{x-2} \geq 0$$

$$= \frac{-x+11}{x-2} \geq 0$$

$$= \frac{x-11}{x-2} \leq 0$$

$$= (x-11)(x-2) \leq 0$$

$$x \in (2, 11]$$

11. Solution

$$x + x + 4 = 12$$

$$2x = 8$$

$$x = 4$$

12. Solution

Let p be the probability of winning Car C, $P(C)$

$$P(C) = p$$

$$P(B) = 2p$$

$$P(A) = 6p$$

$$P(A) + P(B) + P(C) = 1$$

$$p + 2p + 6p = 1$$

$$9p = 1$$

$$p = \frac{1}{9}$$

$$P(C) = \frac{1}{9}$$

$$P(B) = \frac{2}{9}$$

$$P(A) = \frac{6}{9}$$

13. Solution :

Let a satisfy the relation $f(a) = 3$

$$f(f(a)) \cdot (1 + f(a)) = -f(a)$$

$$f(3) \cdot (4) = -3$$

$$f(3) = -\frac{3}{4}$$

14. Solution:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\begin{aligned} &= \frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \cdot \frac{1}{2}} \\ &= 1 \end{aligned}$$

$$A + B = 45$$

$$2(A + B) = 90$$

$$\sin 90 = 1$$

15. Solution:

Form a quadratic equation sum of whose roots are 30 and product of the roots is 81

$$x^2 - x(30) + 81 = 0$$

$$x^2 - 3x - 27x + 81 = 0$$

$$x(x-3) - 27(x-3)$$

$$(x-3)(x-27) = 0$$

Hence the numbers are 3 and 27

16. Solution:

Let $f : R \rightarrow R$ be a function given by $f(x) = x^2 + 2$ find $f^{-1}(27)$

$$f(x) = x^2 + 2$$

$$x^2 + 2 = 27$$

$$x^2 = 25$$

$$x = \pm 5$$

$$f^{-1}(27) = \{-5, 5\}$$

17. Solution:

The function is defined for all values of x where the denominator is not equal to zero

$$a+1-x \neq 0$$

Hence domain =

$$R - \{(a+1)\}$$

Range of f

Let $y = f(x)$

$$y = \frac{x-a}{a+1-x}$$

$$(a+1)y - xy = x - a$$

$$x(y+1) = (a+1)y + a$$

$$x = \frac{(a+1)y + a}{y+1}$$

$$\text{Range of } f = R - \{-1\}$$

18. Solution

Rationalize the numerator

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{a+x} - \sqrt{a})(\sqrt{a+x} + \sqrt{a})}{x(\sqrt{a+x} + \sqrt{a})} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{a+x} + \sqrt{a})} \\ &= \frac{1}{2\sqrt{a}} \end{aligned}$$

19. Solution:

$$\begin{aligned} & \sin 75^\circ + \cos 75^\circ \\ &= \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin 75^\circ + \frac{1}{\sqrt{2}} \cos 75^\circ \right) \\ &= \sqrt{2} (\cos 45^\circ \sin 75^\circ + \sin 45^\circ \cos 75^\circ) \\ &= \sqrt{2} \sin(75^\circ + 45^\circ) \\ &= \sqrt{2} \sin 120^\circ \end{aligned}$$

Hence sign is positive and value is $\frac{\sqrt{2} \cdot \sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

Section C**20. Solution:**

There are 4 groups and four groups can be arranged in $4!$ ways. Class 12 can be arranged in $3!$ ways,

Class 11 can be arranged in $4!$ Class 10 can be arranged in $4!$. Class 9 can be arranged in $2!$ ways

Hence Total number of ways that they can be arranged in a row $4 \times 3 \times 4 \times 4 \times 2! = 165888$

In a circular seating arrangement the four groups can be arranged only in $3!$ ways only. Hence the

total number of ways that they can be seated at a round table = $3 \times 3 \times 4 \times 4 \times 2! = 41472$

21. Solution

The new coordinates of the centre in the new position are

$$(a + 4\pi r, b)$$

$$\{x - (a + 4\pi r)\}^2 + (y - b)^2 = r^2$$

22. Solution

$$x^2 + 4y^2 + 4x + 16y + 16 = 0$$

$$x^2 + 4x + 4 + 4y^2 + 16y + 16 = 4$$

$$(x + 2)^2 + 4(y + 2)^2 = 4$$

$$\frac{(x + 2)^2}{2^2} + \frac{(y + 2)^2}{1^2} = 1$$

This equation represents an ellipse.

23. Solution

x_i	f_i	$f_i x_i$	$ x_i - 15 $	$f_i x_i - 15 $
2	12	24	13	156
15	6	90	0	0
17	12	204	2	24
23	9	207	8	72

27	5	135	12	60
	$N = \sum f_i = 44$	$\sum f_i x_i = 660$		$f_i \sum x_i - 15 = 312$

$$\text{Mean} = \bar{X} = \frac{1}{N} (\sum f_i x_i) = \frac{660}{44} = 15$$

$$\text{Mean Deviation} = M.D = \frac{1}{N} (\sum f_i |x_i - 15|) = \frac{312}{44} = 7.0909$$

24. Solution

Let the ratios be

$$a : b$$

$$x^2 + px + q = 0$$

$$a\alpha + b\alpha = -p$$

$$a\beta + b\beta = -p_1$$

$$a\alpha \times b\alpha = q$$

$$a\beta \times b\beta = q_1$$

$$(a + b)\alpha = -p$$

$$(a + b)\beta = -p_1$$

$$ab\alpha^2 = q$$

$$ab\beta^2 = q_1$$

$$\frac{(a + b)^2 \alpha^2}{(a + b)^2 \beta^2} = \frac{p^2}{p_1^2}$$

$$\frac{\alpha^2}{\beta^2} = \frac{p^2}{p_1^2}$$

$$\frac{\alpha^2}{\beta^2} = \frac{q}{q_1}$$

$$\frac{p^2}{p_1^2} = \frac{q}{q_1}$$

$$p^2 q_1 = p_1^2 q$$

25. Solution :

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \infty = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

$$a \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{4}} \cdot a^{\frac{1}{8}} \dots \infty = a^2$$

26. Solution

It is given that

$$n(U) = 700, n(A) = 200, n(B) = 295, n(A \cap B) = 115$$

We need to find out

$$n(A' \cap B')$$

$$n(A' \cap B') = n(A \cup B)'$$

$$= n(U) - n(A \cup B)$$

$$= n(U) - \{n(A) + n(B) - n(A \cap B)\}$$

$$= 700 - \{200 + 295 - 115\}$$

$$= 320$$
