Real Numbers

- **Rational numbers:** Numbers which can be written in the form of $\frac{p}{q}(q \neq 0)$ where p and q are integers, are called rational numbers.

  Note: Every terminating decimal and non-terminating repeating decimal can be expressed as a rational number.

- **Irrational numbers:** Numbers which cannot be written in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$ are called irrational numbers. In other words, numbers which are not rational are called irrational numbers.
- **Real numbers:** The rational numbers and the irrational numbers together are called real numbers.

  Note: Any number that can be represented on a number line is called a real number.
- **Lemma:** A proven statement which is used to prove another statement is called a lemma.
- **Euclid's division lemma:** For any two positive integers 'a' and 'b; there exist whole numbers 'q' and 'r' such that $\mathrm{a} = \mathrm{bq} + \mathrm{r}, 0 \leq \mathrm{r} < \mathrm{b}$.

  Note: Euclid's division algorithm is stated only for positive integers, but can be extended for all negative integers.

- **Algorithm:** An' algorithm is a process of solving particular problems.
- **Euclid's division algorithm:** is used to find the greatest common divisor (G.C.D.) or Highest Common Factor (H.C.F.) of two numbers.
- **Finding H.C.F. using Euclid's division algorithm:** Suppose the two positive numbers are 'a' and 'b', such that a > b. Then the H.C.F. of 'a' and 'b' can be found by following the steps given:
- Apply the division lemma to find 'q' and 'r' where $a = bq + r, 0 \leq r < b$.
- If $\mathrm{r} = 0$, then H.C.F. is b. If $\mathrm{r} \neq 0$, then apply Euclid's lemma to find 'b' and 'r'.
- Continue steps (a) and (b) till $r = 0$. The divisor at this state will be H.C.F. (a, b). Also, H.C.F. (a, b)= H.C.F. (b, r).
- **Fundamental theorem of Arithmetic:** Every composite number can be expressed as a unique product of prime numbers. This is also called the unique prime factorization theorem.

Note: (i) The order in which the prime factors occur may differ. In general, any composite number x, can be expressed as a product of prime numbers as shown below.

$x = p_1 p_2 p_3 ....p_n$ where $p_1, p_2, p_3, ...p_n$ are primes in ascending order.

- If 'p' is a prime, 'a' is a positive integer, and if 'p' divides a^2, then 'p' divides 'a'. Also, if 'p' divides a^3, then 'p' divides 'a'.
- If 'a' is a terminating decimal, then 'a' can be expressed as $\frac{p}{q}(q \neq 0)$, where 'p' and 'q' are co primes and the prime factorization of q is of the form $2^m 5^n$, (where m and n are whole numbers.).
- If $\frac{p}{q}$ is a rational number and q is not of form $2^m 5^n (m, n \in W)$, then $\frac{p}{q}$ has a non-terminating repeating decimal expansion.
- H.C.F. of two numbers is the product of the smallest power of each common prime factor in the numbers.
- LC.M. of two numbers is the product of the greatest power of each prime factor involved in the numbers.
- For any two numbers 'a' and 'b', L.C.M. (a, b) x H.C.F. $(a, b) = a \times b$.

  That is, the product of two numbers is equal to the product of their LC.M. and H.C.F.
- but, $H.C.F.(p,q,r) \times L.C.M.(p,q,r) \neq pqr$, where p, q and r are positive integers.
- If H.C.F. (a, b) = 1, then 'a' and 'b' are said to be co-prime or relatively prime.
- If $a = bq + r$ and r < b, then H.C.F. (a, b) - H.C.F. (b, r).
- For all a, b, m e N, H.C.F. (am, bm) = m [H.C.F. (a, b)].

  In other words, if 'a', 'b' and 'm' are natural numbers, then H.C.F. (am, bm) is m times H.C.F. of 'a' and 'b'.
- H.C.F. of three numbers is the H.C.F. of the H.C.F. of any two numbers and the third number. i.e., H.C.F. (p, q, r) = H.C.F. [H.C.F. (p, q), r].
- L.C.M. of two co-prime numbers or two prime numbers is the product of the numbers.
- L.C.M. of fractions or rational numbers $= \frac{A}{B}$, where A = L.C.M. of numerators and B = H.C.F. of denominators, i.e., L.C.M. of $\frac{a}{b}$ and $\frac{c}{b} = \frac{L.C.M. of\ a\ and\ c}{H.C.F. of\ b\ and\ d} = \frac{L.C.M.(a,c)}{H.C.F.(b,d)}$

- H.C.F. of fractions or rational numbers $= \frac{p}{q}$ where P = H.C.F. of numerators and Q = L.C.M. of denominators, i.e., H.C.F. of $\frac{a}{b}$ and $\frac{c}{d} = \frac{H.C.F. of\ a\ and\ c}{L.C.M. of\ b\ and\ d} = \frac{H.C.F.(a,c)}{L.C.M.(b,d)}$