Exercise 13.1

Answer 1E.

We are suppose to find the domain of the vector function.

$$r(t) = <\sqrt{4-t^2}\,,\ e^{-3t},\ \ln(t+1)>$$

These expression are all defined when $t + 1 > 0$ and $t \leq 2$

So the domain of r is the interval (-1, 2]

Answer 2E.

The given vector function is

$$\vec{r}(t) = \frac{t-2}{t+2}\hat{i} + \sin t\ \hat{j} + \ln\left(9-t^2\right)\hat{k}$$

Then domain of vector function is the domain in which the component functions are defined.

Now the component functions are:

(i) $\frac{t-2}{t+2}$: which is not defined for $t = -2$

(ii) $\sin t$: defined every where

(iii) $\ln\left(9-t^2\right)$: which is not defined for $9-t^2 \leq 0$

It is defined only for $9-t^2 > 0$

i.e. $9 > t^2$

i.e. $-3 < t < 3$

Hence the domain of given vector function is

$$(-3,3)-\{-2\}$$

Or $\boxed{(-3,-2)\cup(-2,3)}$

Answer 3E.

To evaluate $\lim_{t\to 0}\left(e^{-3t}i + \frac{t^2}{\sin^2 t}j + \cos 2tk\right) = L(\text{1et})$

$$L = i\lim_{t\to 0}\left(e^{-3t}\right) + j\lim_{t\to 0}\left(\frac{t^2}{\sin^2 t}\right) + k\lim_{t\to 0}\left(\cos 2t\right)$$

$$= L_1 i + L_2 j + L_3 k \qquad \text{--- (1)}$$

Consider $L_1 = \lim_{t \to 0}\left(e^{-3t}\right)$

$$= e^{-3(0)}$$

$$= e^{0}$$

$$= 1$$

Now $L_2 = \lim_{t \to 0}\left(\frac{t^2}{\sin^2 t}\right)$

Substituting t with 0 in $\frac{t^2}{\sin^2 t}$, we obtain the indeterminate form $\frac{0}{0}$ (Since $\sin 0 = 0$)

So to evaluate L_2 we apply L' Hospital's rule as follows:

$$L_2 = \lim_{t \to 0} \frac{\frac{d}{dt}\left(t^2\right)}{\frac{d}{dt}\left(\sin^2 t\right)}$$

$$= \lim_{t \to 0} \frac{2t}{2\sin t \cos t}$$

$$= \lim_{t \to 0} \frac{2t}{\sin 2t}$$

$$= \lim_{t \to 0} \frac{\frac{d}{dt}(2t)}{\frac{d}{dt}(\sin 2t)}$$

$$= \lim_{t \to 0} \frac{2}{2\cos 2t}$$

$$= \lim_{t \to 0} \frac{1}{\cos 2t}$$

$$= \frac{1}{\cos 0}$$

$$= 1 \left(\text{since } \cos 0 = 1\right)$$

Consider $L_3 = \lim_{t \to 0}\left(\cos 2t\right)$

$$= \cos 0$$

$$= 1$$

Put L_1, L_2, L_3 in (1) we have

$$L = i(1) + j(1) + k(1)$$

Therefore $\lim_{t \to 0}\left(e^{-3t} i + \frac{t^2}{\sin^2 t} j + \cos 2t k\right) = i + j + k$

Answer 4E.

Need to find $\lim_{t\to1}\frac{\sin\pi t}{\ln t}$.

$$\lim_{t\to1}\frac{\sin\pi t}{\ln t}=\lim_{t\to1}\frac{\sin\pi t}{\ln t}$$
$$=\lim_{x\to0}\frac{\sin\pi(x+1)}{\ln(x+1)}$$
$$=\lim_{x\to0}\frac{-\sin\pi x}{\ln(x+1)}$$
$$=\lim_{x\to0}\frac{-\sin\pi x}{x\left[\frac{\ln(x+1)}{x}\right]}$$
$$=\frac{\left[\lim_{x\to0}\frac{-\sin\pi x}{x}\right]}{\left[\lim_{x\to0}\ln(1+x)^{\frac{1}{x}}\right]}$$
$$=\frac{-\pi}{1}$$
$$=-\pi$$

Plug $\lim_{t\to1}\frac{\sin\pi t}{\ln t}$ value in equation (2), then the limit value is,

$$\lim_{t\to1}\left(\frac{t^2-t}{t-1}\mathbf{i}+\sqrt{t+8}\,\mathbf{j}+\frac{\sin\pi t}{\ln t}\mathbf{k}\right)=\mathbf{i}+3\mathbf{j}-\pi\mathbf{k}.$$

Hence, $\lim_{t\to1}\left(\frac{t^2-t}{t-1}\mathbf{i}+\sqrt{t+8}\,\mathbf{j}+\frac{\sin\pi t}{\ln t}\mathbf{k}\right)=\boxed{\mathbf{i}+3\mathbf{j}-\pi\mathbf{k}}$.

Answer 5E.

Consider the expression

$$\lim_{t\to\infty}\left\langle\frac{1+t^2}{1-t^2},\tan^{-1}t,\frac{1-e^{-2t}}{t}\right\rangle$$

Recall that,

If the vector valued function is of the form $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, then its limit is given by

$$\lim_{t\to a}\mathbf{r}(t)=\left[\lim_{t\to a}f(t)\right]\mathbf{i}+\left[\lim_{t\to a}g(t)\right]\mathbf{j}+\left[\lim_{t\to a}h(t)\right]\mathbf{k}.$$

provided the limits of the functions $f(t), g(t)$, and $h(t)$ exists.

Here the functions,

$$f(t) = \frac{1+t^2}{1-t^2},\ g(t) = \tan^{-1} t, \text{ and } h(t) = \frac{1-e^{-2t}}{t}.$$

Determine the limits of the functions $f(t), g(t), \text{and } h(t)$

To find $\lim_{t\to\infty} f(t)$

$$\lim_{t\to\infty} f(t) = \lim_{t\to\infty}\left(\frac{1+t^2}{1-t^2}\right)$$

$$= \lim_{t\to\infty}\left(\frac{t^2\left(\frac{1}{t^2}+1\right)}{t^2\left(\frac{1}{t^2}-1\right)}\right)$$

$$= \lim_{t\to\infty}\left(\frac{\left(\frac{1}{t^2}+1\right)}{\left(\frac{1}{t^2}-1\right)}\right)$$

$$= \frac{\lim_{t\to\infty}\left(\frac{1}{t^2}+1\right)}{\lim_{t\to\infty}\left(\frac{1}{t^2}-1\right)}$$

$$= \frac{0+1}{0-1}$$

$$= -1$$

Therefore,

$$\boxed{\lim_{t\to\infty} f(t) = -1} \text{ ...... } (1)$$

To find the limit $\lim_{t\to\infty} g(t)$

$$\lim_{t\to\infty} g(t) = \lim_{t\to\infty} \tan^{-1} t$$

$$= \tan^{-1}(\infty)$$

$= \frac{\pi}{2}$ Since $\tan\left(\frac{\pi}{2}\right) = \infty$

Therefore

$$\boxed{\lim_{t\to\infty} g(t) = \frac{\pi}{2}} \text{ ...... } (2)$$

To find the limit $\lim_{t\to\infty} h(t)$

$$\lim_{t\to\infty} h(t) = \lim_{t\to\infty}\frac{1-e^{-2t}}{t}$$

$$= \left(\lim_{t\to\infty}\frac{1}{t}\right)\left(\lim_{t\to\infty}\left(1-e^{-2t}\right)\right)$$

$$= (0)(1-0)$$ Since $\lim_{x\to\infty} e^{-x} = 0$

$$= 0$$

Therefore

$$\boxed{\lim_{t\to\infty} h(t) = 0} \quad \cdots\cdots (3)$$

The limit of the vector $\left\langle \frac{1+t^2}{1-t^2}, \tan^{-1} t, \frac{1-e^{-2t}}{t} \right\rangle$ is

$$\lim_{t\to\infty}\left\langle \frac{1+t^2}{1-t^2}, \tan^{-1} t, \frac{1-e^{-2t}}{t} \right\rangle = \left\langle \lim_{t\to\infty}\left(\frac{1+t^2}{1-t^2}\right), \lim_{t\to\infty}\left(\tan^{-1} t\right), \lim_{t\to\infty}\left(\frac{1-e^{-2t}}{t}\right) \right\rangle$$

$= \left\langle -1, \frac{\pi}{2}, 0 \right\rangle$ From (1), (2), and (3)

Therefore the limit of the vector $\left\langle \frac{1+t^2}{1-t^2}, \tan^{-1} t, \frac{1-e^{-2t}}{t} \right\rangle$ is

$$\boxed{\lim_{t\to\infty}\left\langle \frac{1+t^2}{1-t^2}, \tan^{-1} t, \frac{1-e^{-2t}}{t} \right\rangle = \left\langle -1, \frac{\pi}{2}, 0 \right\rangle}$$

Answer 6E.

Consider the expression

$$\lim_{t\to\infty}\left\langle te^{-t}, \frac{t^3+t}{2t^3-1}, t\sin\left(\frac{1}{t}\right) \right\rangle$$

Recall that,

If the vector valued function is of the form $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, then its limit is given by

$$\lim_{t\to a}\mathbf{r}(t) = \left[\lim_{t\to a} f(t)\right]\mathbf{i} + \left[\lim_{t\to a} g(t)\right]\mathbf{j} + \left[\lim_{t\to a} h(t)\right]\mathbf{k}.$$

provided the limits of the functions $f(t), g(t)$, and $h(t)$ exists.

Here the functions,

$f(t) = te^{-t}$, $g(t) = \dfrac{t^3+t}{2t^3-1}$, and $h(t) = t\sin\left(\dfrac{1}{t}\right)$.

Determine the limits of the functions $f(t), g(t), \text{and } h(t)$

To find $\lim\limits_{t\to\infty} f(t)$

$$\lim_{t\to\infty} f(t) = \lim_{t\to\infty}\left(te^{-t}\right)$$

$$= \lim_{t\to\infty}(t)\cdot\lim_{t\to\infty}\left(e^{-t}\right)$$

$= \lim\limits_{t\to\infty}(t)\cdot(0)$ Since $\lim\limits_{x\to\infty} e^{-x} = 0$

$= 0$

Therefore,

$$\boxed{\lim_{t\to\infty} f(t) = 0} \quad \text{......} \quad (1)$$

To find the limit $\lim\limits_{t\to\infty} g(t)$

$$\lim_{t\to\infty} g(t) = \lim_{t\to\infty}\left(\frac{t^3+t}{2t^3-1}\right)$$

$$= \lim_{t\to\infty}\left(\frac{t^3\left(1+\frac{1}{t^2}\right)}{t^3\left(2-\frac{1}{t^2}\right)}\right)$$

$$= \lim_{t\to\infty}\left(\frac{\left(1+\frac{1}{t^2}\right)}{\left(2-\frac{1}{t^2}\right)}\right)$$

$$= \frac{\lim\limits_{t\to\infty}\left(1+\frac{1}{t^2}\right)}{\lim\limits_{t\to\infty}\left(2-\frac{1}{t^2}\right)}$$

$$= \frac{(1+0)}{(2-0)}$$

$$= \frac{1}{2}$$

Therefore

$$\boxed{\lim_{t\to\infty} g(t) = \frac{1}{2}} \quad \text{......} \quad (2)$$

To find the limit $\lim_{t \to \infty} h(t)$

$$\lim_{t \to \infty} h(t) = \lim_{t \to \infty}\left(t \sin\left(\frac{1}{t}\right)\right)$$

$$= \lim_{t \to \infty}\left(\frac{\sin\left(\frac{1}{t}\right)}{\left(\frac{1}{t}\right)}\right)$$

$=1$ Since $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

Therefore

$$\boxed{\lim_{t \to \infty} h(t) = 1} \quad \text{......} \quad (3)$$

The limit of the vector $\left\langle te^{-t}, \frac{t^3+t}{2t^3-1}, t\sin\left(\frac{1}{t}\right)\right\rangle$ is

$$\lim_{t \to \infty}\left\langle te^{-t}, \frac{t^3+t}{2t^3-1}, t\sin\left(\frac{1}{t}\right)\right\rangle = \left\langle \lim_{t \to \infty} te^{-t}, \lim_{t \to \infty}\frac{t^3+t}{2t^3-1}, \lim_{t \to \infty} t\sin\left(\frac{1}{t}\right)\right\rangle$$

$= \left\langle 0, \frac{1}{2}, 1\right\rangle$ From (1), (2), and (3)

Therefore the limit of the vector $\left\langle te^{-t}, \frac{t^3+t}{2t^3-1}, t\sin\left(\frac{1}{t}\right)\right\rangle$ is

$$\boxed{\lim_{t \to \infty}\left\langle te^{-t}, \frac{t^3+t}{2t^3-1}, t\sin\left(\frac{1}{t}\right)\right\rangle = \left\langle 0, \frac{1}{2}, 1\right\rangle}$$

Answer 7E.

Consider the vector equation:

$$\mathbf{r}(t) = \langle \sin t, t\rangle$$

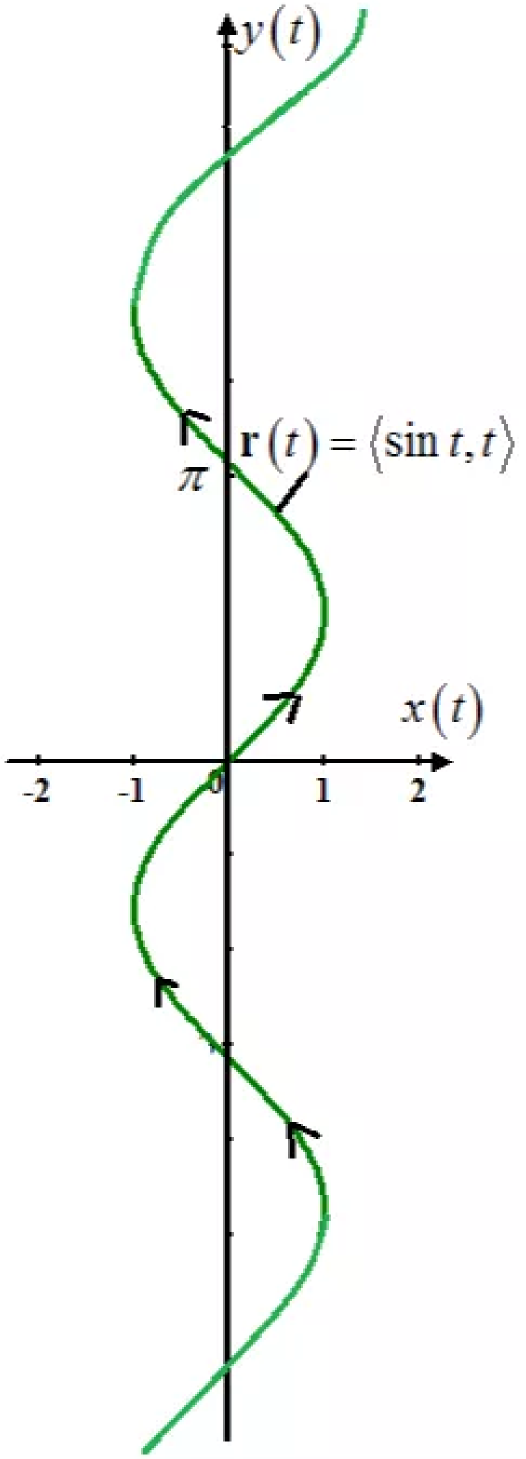
The parametric equations for the curve are $x(t) = \sin(t)$, and $y(t) = t$.

Eliminate the parameter t form parametric equations, the equation for the curve is $x = \sin y$.

The equation represents a sinusoidal wave moving upward direction.

That is, depend on variable $y(t)$, $x(t)$ is moving upward direction.

Sketch the vector function $\mathbf{r}(t) = \langle \sin t, t \rangle$.



Verification:

Use the maple software to plot the parametric equations:

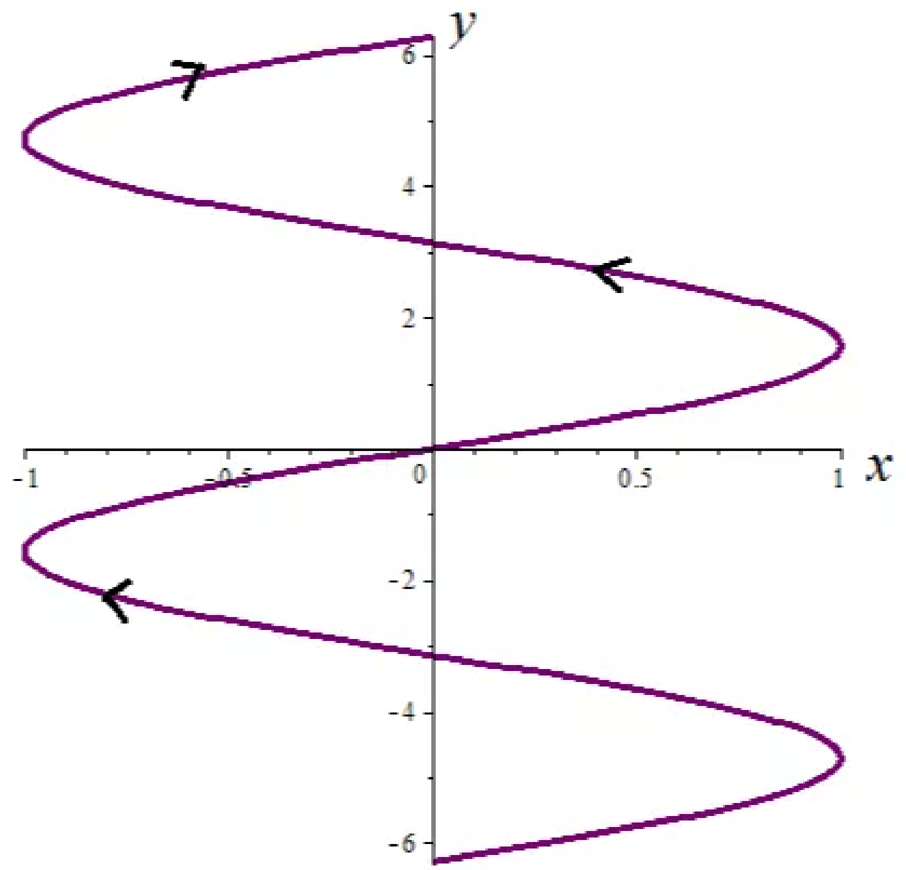
Load **with (plots)** package:

Enter the **Maple input** command:

Plot ([sin (t), t, t=-2*Pi...2*Pi])

Maple output:

plot([sin(*t*), *t*, *t* = −2 * Pi ..2 * Pi])



Answer 8E.

Consider the following vector equation:

$$\mathbf{r}(t) = \langle t^3, t^2 \rangle$$

The objective is to sketch the given vector equation and indicate the direction.

The equation to convert to parametric form is as follows:

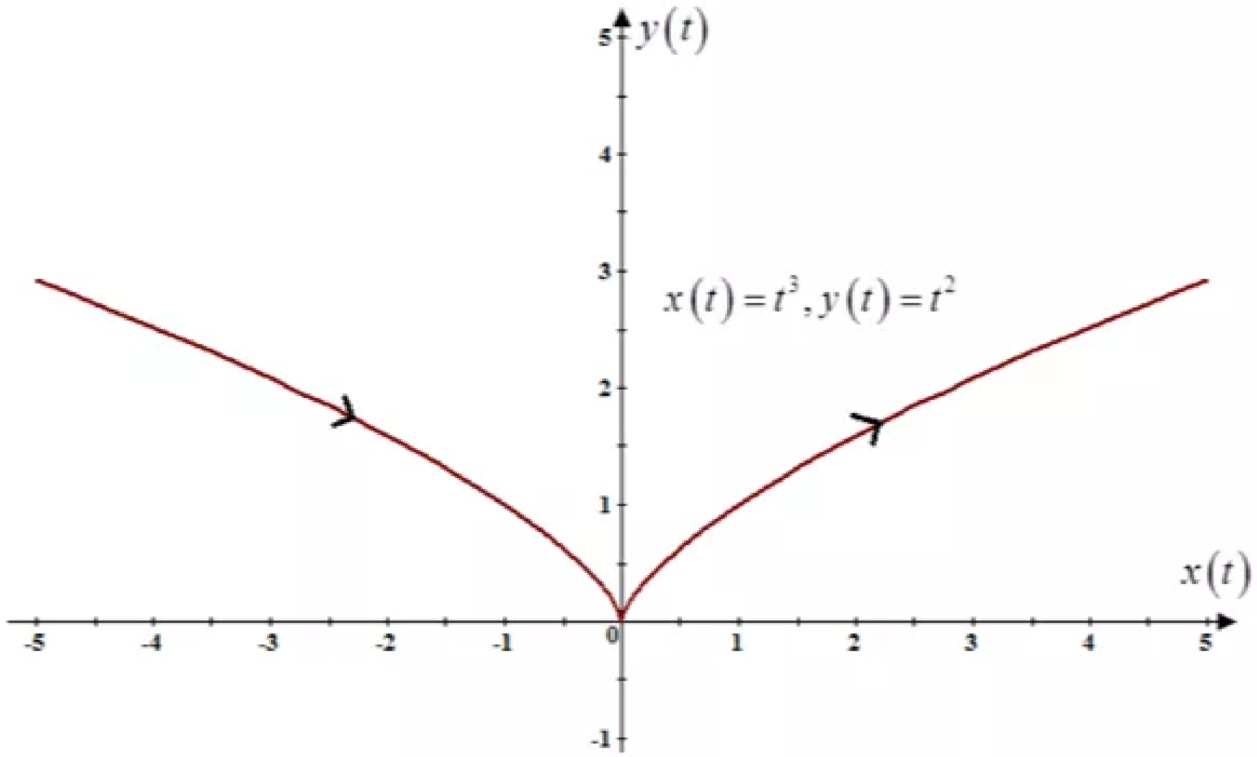
If $\mathbf{r}(t)=\langle f(t),g(t)\rangle$ then the parameterization of it is

$x=f(t)y=g(t)$

Compare $\mathbf{r}(t)=\langle t^3,t^2\rangle$ with $\mathbf{r}(\mathbf{t})=\langle f(t),g(t)\rangle$, to get:

$x(t)=t^3, y(t)=t^2$

The graph of the given parametric curves is shown below:



Here, the value of t increases as the function goes to the right.

Answer 9E.

Consider the following vector equation:

$\mathbf{r}(t)=\langle t,2-t,2t\rangle$

The objective is to sketch the curve of the given vector equation.

So the parametric equations of the vector function are:

$x(t)=t$

$y(t)=2-t$

$z(t)=2t$

The vector equation can be written as follows:

$$\mathbf{r}(t) = \langle 0,2,0 \rangle + t\langle 1,-1,2 \rangle$$

Recollect that the vector equation of a line passing through **a** and parallel to **b** is $\mathbf{r} = \mathbf{a} + t\mathbf{b}.$

Therefore, the given vector equation represents the equation of a line that passes through the point $\langle 0,2,0 \rangle$ and is parallel to the vector $\langle 1,-1,2 \rangle$.

Now, find the point where the *xz*-plane occurs.

On the *xz*-plane, $y = 0.$

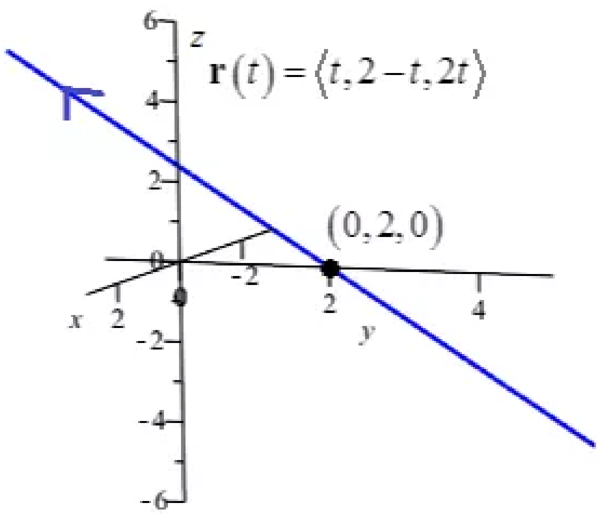
That is $t-2=0$ or $t=2.$

Thus, the corresponding point is,

$$\mathbf{r}(2) = \langle 2,0,4 \rangle.$$

The direction of the line is upwards as t increases.

Sketch of the vector function $\mathbf{r}(t) = \langle t, 2-t, 2t \rangle$



Answer 10E.

Consider the vector equation of the curve:

$$\mathbf{r}(t)=\langle \sin \pi t,\ t, \cos \pi t\rangle.$$

The parametric equations of the curve:

$$x=\sin \pi t, \quad y=t, \quad z=\cos \pi t.$$

Since

$$\begin{aligned} x^2+z^2 &= \sin^2 \pi t+\cos^2 \pi t \\ &= 1 \end{aligned}$$

The curve must lie on the circular cylinder $x^2+z^2=1$.

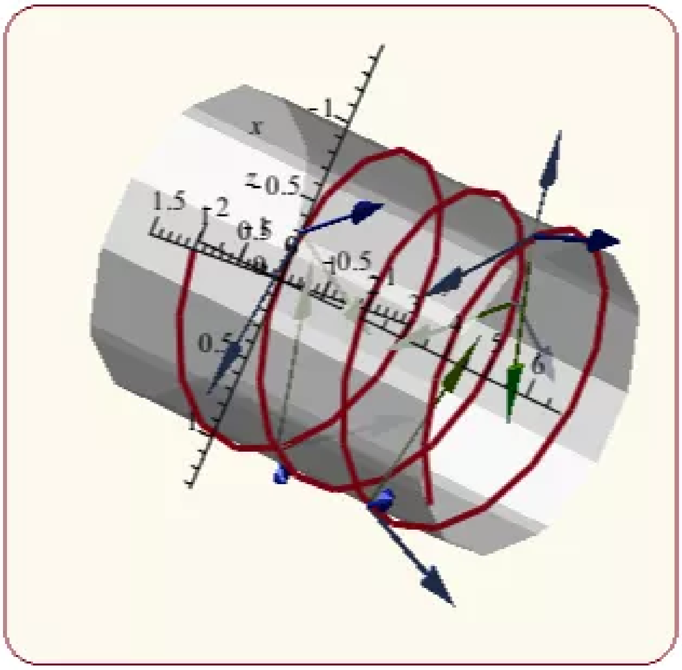
The point (x,y,z) lies directly above the point which moves counterclockwise around the circle $x^2+z^2=1$ in the $xz-$plane.

(The projection of the curve onto the $xz-$plane has vector equation $\mathbf{r}(t)=\langle \sin \pi t,\ t, \cos \pi t\rangle$).

Since $y=t$, then the curve spirals right around the cylinder as t increases.

Sketch the curve of the vector function $\mathbf{r}(t)=\langle \sin \pi t,\ t, \cos \pi t\rangle$.

$$\mathbf{r}(t)=\langle \sin \pi t, t, \cos \pi t\rangle$$



Answer 11E.

Consider the vector, $\mathbf{r}(t) = \langle 1, \cos t, 2\sin t \rangle$

The vector equation is equivalent to the parametric equations $x = 1, y = \cos t, z = 2\sin t$

The curve is an ellipse on the plane $x = 1$, because

$$y = \cos t, z = 2\sin t$$

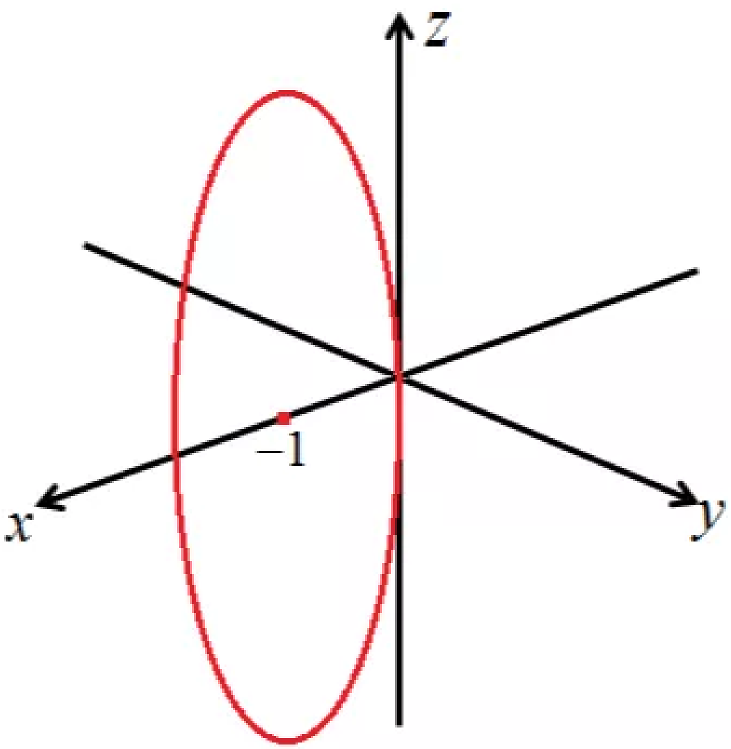
$$y = \cos t, \frac{z}{2} = \sin t$$

$$y^2 = \cos^2 t, \left(\frac{z}{2}\right)^2 = \sin^2 t$$

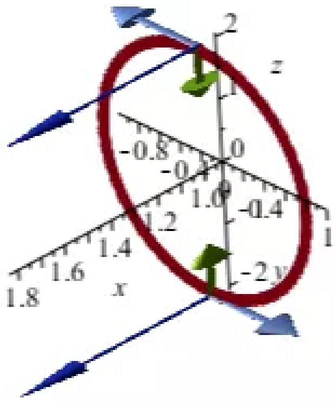
$$y^2 = \cos^2 t, \frac{z^2}{4} = \sin^2 t$$

$$y^2 + \frac{z^2}{4} = \cos^2 t + \sin^2 t$$
$$= 1$$

This an ellipse parallel to $yz-$plane, this is shown below



The sketch of curve represented by the vector, $\mathbf{r}(t) = \langle 1, \cos t, 2\sin t \rangle$ is shown below.



Answer 12E.

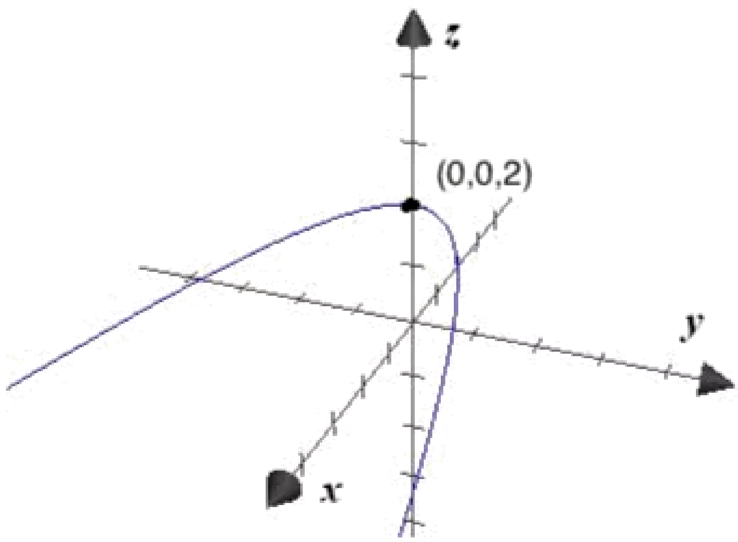
Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases.

$$\vec{r}(t) = t^2\vec{i} + t\vec{j} + 2\vec{k}$$

The corresponding parametric equations are $x = t^2$, $y = t$, and $z = 2$. By substitution, $x = y^2$, which is the equation of a parabola on the xy-plane.

But since $z = 2$, the given vector equation actually describes a parabola that lies on the plane $z = 2$.

As t increases, so does y. The arrows shown indicate this.



Answer 13E.

Consider the vector equation of a curve is $\mathbf{r}(t)=t^2\mathbf{i}+t^4\mathbf{j}+t^6\mathbf{k}$.

Compare $\mathbf{r}(t)=\langle t^2,t^4,t^6\rangle$ with $\mathbf{r}(t)=\langle x(t),y(t),z(t)\rangle$, to get the following equations.

$x=t^2, y=t^4, z=t^6$

The parametric equations are $x=t^2, y=t^4, z=t^6$.

Substitute $x=t^2$ in $y=(t^2)^2$ to get

$y=x^2$

Substitute $x=t^2$ in $z=(t^2)^3$ to get

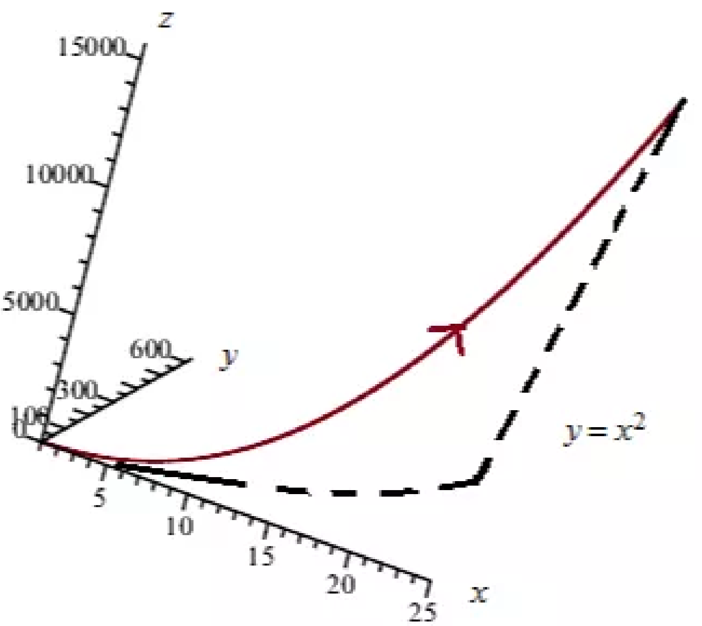
$z=x^3$

Since $y=x^2$, the curve must lie on parabola.

The projection on to the $xy-$ plane will travel along the curve $y=x^2$ in the first quadrant.

$z=t^6=(t^2)^3$ Or $z=x^3$ means it is always positive in the $z-$ direction, goes to 0 and gets large quickly.

Sketch the graph of the curve $\mathbf{r}(t)=t^2\mathbf{i}+t^4\mathbf{j}+t^6\mathbf{k}$ is shown below:



Answer 14E.

Consider the vector function,

$$\mathbf{r}(t) = \langle at\cos 3t, b\sin^3 t, c\cos^3 t \rangle.$$

Find the derivative of the vector function.

The derivative of each component individually to obtain

$$\begin{aligned}\mathbf{r}'(t) &= \frac{d}{dt}\langle at\cos 3t, b\sin^3 t, c\cos^3 t \rangle \\ &= \frac{d}{dt}(at\cos 3t)\mathbf{i} + \frac{d}{dt}(b\sin^3 t)\mathbf{j} + \frac{d}{dt}(c\cos^3 t)\mathbf{k} \\ &= (a\cos 3t - 3at\sin 3t)\mathbf{i} + (3b\cos t\sin^2 t)\mathbf{j} - (3c\sin t\cos^2 t)\mathbf{k}\end{aligned}$$

Therefore, the derivative of $\mathbf{r}(t)$ is,

$$\boxed{\mathbf{r}'(t) = (a\cos 3t - 3at\sin 3t)\mathbf{i} + 3b\cos t\sin^2 t\mathbf{j} - 3c\sin t\cos^2 t\mathbf{k}}.$$

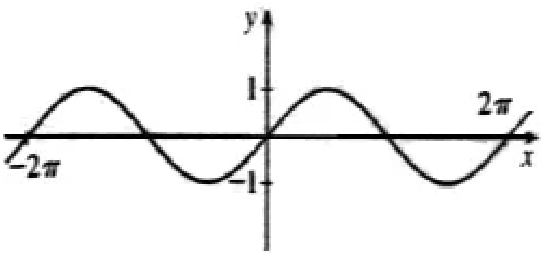
Answer 15E.

Consider the curve $\mathbf{r}(t) = \langle t, \sin t, 2\cos t \rangle$

The projections of the curve on the three coordinate planes

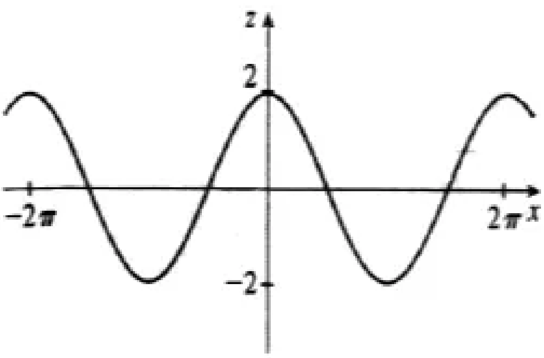
In order to sketch the projection of **r** (*t*) in the *xy*-plane we set $h(t) = 0$.

Then, $\mathbf{r}(t) = \langle t, \sin t \rangle$.



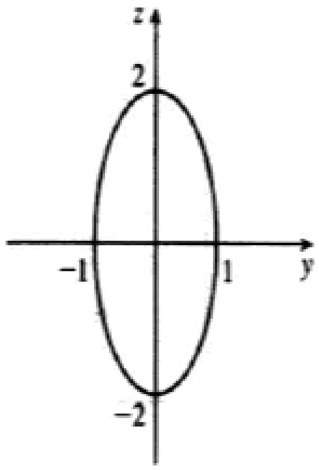
Similarly, the projection of **r** (*t*) in the *xz*-plane by setting $g(t) = 0$.

Then, $\mathbf{r}(t) = \langle t, 2\cos t \rangle$.

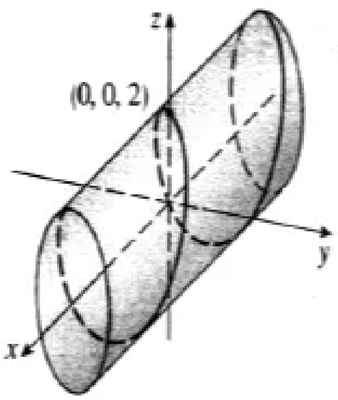


Now, set $f(t)=0$ and sketch the projection on *yz*-plane.

Then, $\mathbf{r}(t) = \langle \sin t, 2\cos t \rangle$



Let us sketch the curve represented by the given vector.



Answer 16E.

Consider the following vector equation:

$\mathbf{r}(t)=\langle t,t,t^2 \rangle$

The objective is to draw the projections of the curve on the three coordinate planes and use these projections to help sketch the curve.

The given vector equation is, $\mathbf{r}(t)=\langle t,t,t^2\rangle$

Compare the given vector equation with $\mathbf{r}(t)=\langle f(t),g(t),h(t)\rangle$

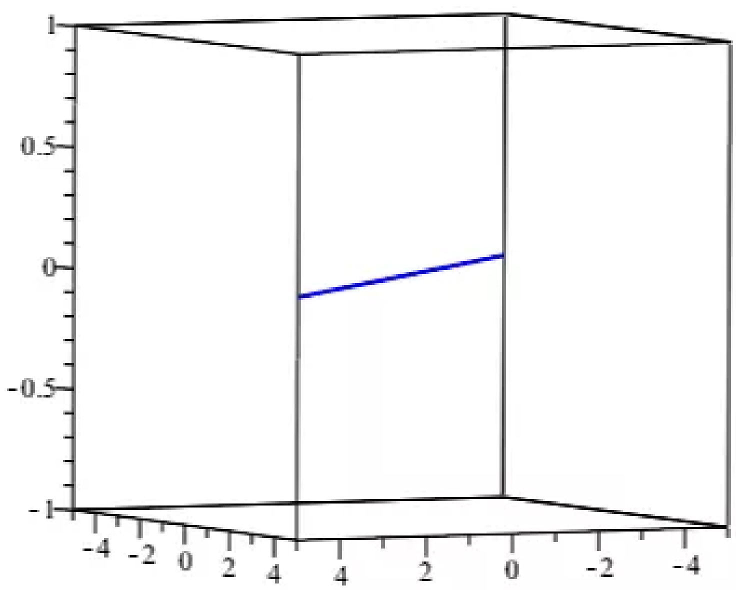
Where $f(t)=t, g(t)=t,$ and $h(t)=t^2$

To sketch the projection of $\mathbf{r}(t)$ in the xy-plane the z- coordinate is equals to zero.

That is $h(t)=0$.

Then, the vector equation is $\mathbf{r}(t) = \langle t, t, 0\rangle$.

Draw the following space curve:

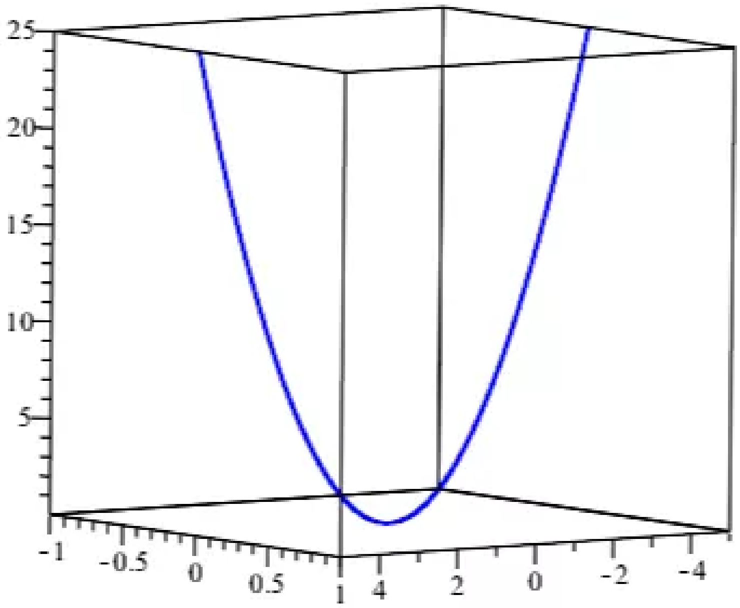


To sketch the projection of **r**(*t*) in the *xz*-plane the y- coordinate is equals to zero.

That is $g(t)=0$.

Then, the vector equation is $\mathbf{r}(t) = \langle t,0,t^2 \rangle$.

Draw the following space curves:

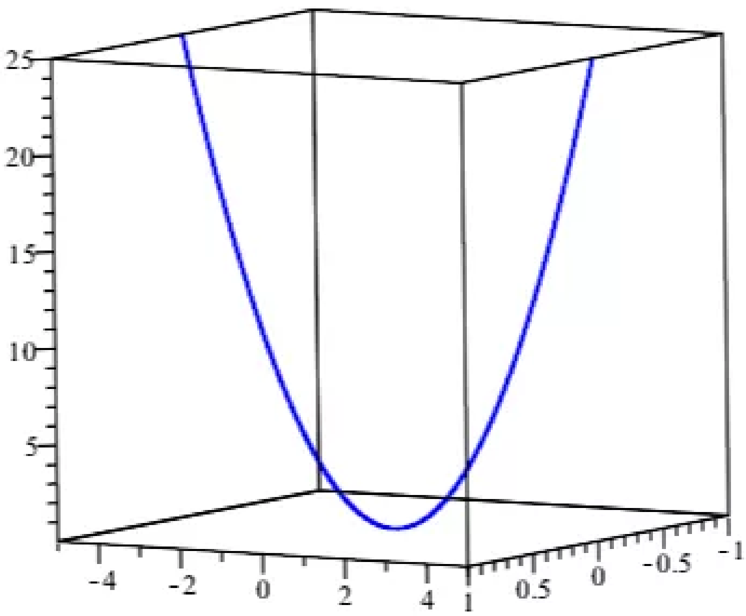


To sketch the projection of **r**(*t*) in the *yz*-plane the *x*- coordinate is equals to zero.

That is $f(t)=0$.

Then, the vector equation is $\mathbf{r}(t) = \langle 0,t,t^2 \rangle$.

Draw the following space curves:

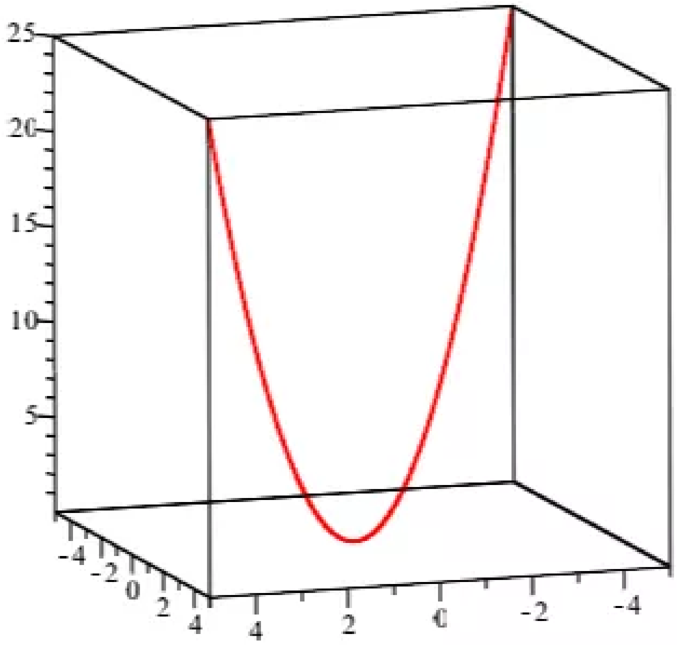


Now sketch the vector function use the above projections on three coordinate planes.

Sketch the vector function.

$\mathbf{r}(t)=\langle t,t,t^2\rangle$

Draw the following vector function:



Therefore, the sketched vector function is $\mathbf{r}(t)=\langle t,t,t^2\rangle$.

Answer 17E.

The two point vector form of a line that passes through the points is given by
$\mathbf{r}=(1-t)\mathbf{r}_0+t\mathbf{r}_1,\ 0\le t\le 1$.

Let P be coordinates of $\mathbf{r}_0$ and Q be that of $\mathbf{r}_1$. Then, $\mathbf{r}_0=2\mathbf{i}$ and $\mathbf{r}_1=6\mathbf{i}+2\mathbf{j}-2\mathbf{k}$.

Substitute the values in $\mathbf{r}=(1-t)\mathbf{r}_0+t\mathbf{r}_1$ and simplify.

$$\begin{aligned}\mathbf{r}&=\left[(1-t)(2)\mathbf{i}\right]+\left[(t)(6)\mathbf{i}+(t)(2)\mathbf{j}+(t)(-2)\mathbf{k}\right]\\&=(2-2t+6t)\mathbf{i}+(2t)\mathbf{j}+(-2t)\mathbf{k}\\&=(2+4t)\mathbf{i}+2t\mathbf{j}-2t\mathbf{k}\end{aligned}$$

Thus, the vector value function of the line segment joining PQ is
$\mathbf{r}(t)=(2+4t)\mathbf{i}+2t\mathbf{j}-2t\mathbf{k},\ 0\le t\le 1$.

The required set of parametric equations is $\boxed{x=2+4t,\ y=2t, \text{ and } z=-2t}$, $0\le t\le 1$.

Answer 18E.

The two point vector form of a line that passes through the points is given by
$\mathbf{r}=(1-t)\ \mathbf{r0}+t\mathbf{r}_1, 0\le t\le 1$.

Let P be coordinates of $\mathbf{r}_0$ and Q be that of $\mathbf{r}_1$. Then, $\mathbf{r}_0=-\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ and
$\mathbf{r}_1=-3\mathbf{i}+5\mathbf{j}+\mathbf{k}$.

Substitute the values in $\mathbf{r} = (1-t)\,\mathbf{r0} + t\mathbf{r}_1$ and simplify.

$$\mathbf{r} = \left[(1-t)(-1)\mathbf{i} + (1-t)(2)\mathbf{j} + (1-t)(-2)\mathbf{k}\right] + \left[(t)(-3)\mathbf{i} + (t)(5)\mathbf{j} + (t)(1)\mathbf{k}\right]$$
$$= \left[(-1+t)\mathbf{i} + (2-2t)\mathbf{j} + (-2+2t)\mathbf{k}\right] + \left[-3t\mathbf{i} + 5t\mathbf{j} + t\mathbf{k}\right]$$
$$= (-1-2t)\mathbf{i} + (2+3t)\mathbf{j} + (-2+3t)\mathbf{k}$$

Thus, the vector value function of the line segment joining PQ is
$\mathbf{r}(t) = (-1-2t)\mathbf{i} + (2+3t)\mathbf{j} + (-2+3t)\mathbf{k}$, $0 \le t \le 1$.

The required set of parametric equations is $\boxed{x = -1-2t,\ y = 2+3t, \text{ and } z = -2+3t}$, $0 \le t \le 1$.

Answer 19E.

The two point vector form of a line that passes through the points is given by
$\mathbf{r} = (1-t)\,\mathbf{r0} + t\mathbf{r}_1$, $0 \le t \le 1$.

Let P be coordinates of $\mathbf{r}_0$ and Q be that of $\mathbf{r}_1$. Then, $\mathbf{r}_0 = 0\mathbf{i} - \mathbf{j} + \mathbf{k}$
and $\mathbf{r}_1 = \frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{4}\mathbf{k}$.

Substitute the values in $\mathbf{r} = (1-t)\,\mathbf{r0} + t\mathbf{r}_1$ and simplify.

$$\mathbf{r} = \left[(1-t)(0)\mathbf{i} + (1-t)(-1)\mathbf{j} + (1-t)(1)\mathbf{k}\right] + \left[(t)\left(\frac{1}{2}\right)\mathbf{i} + (t)\left(\frac{1}{3}\right)\mathbf{j} + (t)\left(\frac{1}{4}\right)\mathbf{k}\right]$$
$$= \left[0\mathbf{i} + (-1+t)\mathbf{j} + (1-t)\mathbf{k}\right] + \left[\frac{1}{2}t\mathbf{i} + \frac{1}{3}t\mathbf{j} + \frac{1}{4}t\mathbf{k}\right]$$
$$= \frac{1}{2}t\mathbf{i} + \left(-1 + \frac{4}{3}t\right)\mathbf{j} + \left(1 - \frac{3}{4}t\right)\mathbf{k}$$

Thus, the vector value function of the line segment joining PQ is

$$\boxed{\mathbf{r}(t) = \frac{1}{2}t\mathbf{i} + \left(-1 + \frac{4}{3}t\right)\mathbf{j} + \left(1 - \frac{3}{4}t\right)\mathbf{k}},\ 0 \le t \le 1$$

The required set of parametric equations is $\boxed{x = \frac{1}{2}t,\ y = -1 + \frac{4}{3}t, \text{ and } z = 1 - \frac{3}{4}t}$,
$0 \le t \le 1$

Answer 20E.

The two point vector form of a line that passes through the points is given by
$\mathbf{r} = (1-t)\mathbf{r}_0 + t\mathbf{r}_1,\ 0 \le t \le 1.$

Let P be coordinates of $\mathbf{r}_0$ and Q be that of $\mathbf{r}_1$. Then, $\mathbf{r}_0 = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and
$\mathbf{r}_1 = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$.

Substitute the values in $\mathbf{r} = (1-t)\mathbf{r}_0 + t\mathbf{r}_1$ and simplify.

$$\begin{aligned}\mathbf{r} &= \left[(1-t)(a)\mathbf{i} + (1-t)(b)\mathbf{j} + (1-t)(c)\mathbf{k}\right] + \left[(t)(u)\mathbf{i} + (t)(v)\mathbf{j} + (t)(w)\mathbf{k}\right] \\ &= (a - at + ut)\mathbf{i} + (b - bt + vt)\mathbf{j} + (c - ct + wt)\mathbf{k}\end{aligned}$$

Thus, the vector value function of the line segment joining PQ is
$\mathbf{r}(t) = (a - at + ut)\mathbf{i} + (b - bt + vt)\mathbf{j} + (c - ct + wt)\mathbf{k},\ 0 \le t \le 1.$

The required set of parametric equations is $\boxed{x = a - at + ut, y = b - bt + vt}$, and
$\boxed{z = c - ct + wt}$.

Answer 21E.

Consider the parametric equations,

$$x = t\cos t, y = t, z = t\sin t, t \ge 0. \quad \text{......} \ (1)$$

The objective is to graph this parametric equation by using computers.

Consider the curve,

$$x = t\cos t, y = t, z = t\sin t, t \ge 0$$

Compute,

$$\begin{aligned}x^2 + z^2 &= t^2\cos^2 t + t^2\sin^2 t \\ &= t^2\left(\cos^2 t + \sin^2 t\right) \\ &= t^2\end{aligned}$$

So, the curve must lie on circular cylinder $x^2 + z^2 = t^2$.

Any point on the curve (x, y, z) lies above the point $(x, 0, z)$.

Thus, the point (x, y, z) moves along circular cylinder in counter clock wise direction in the $xz-$ plane.

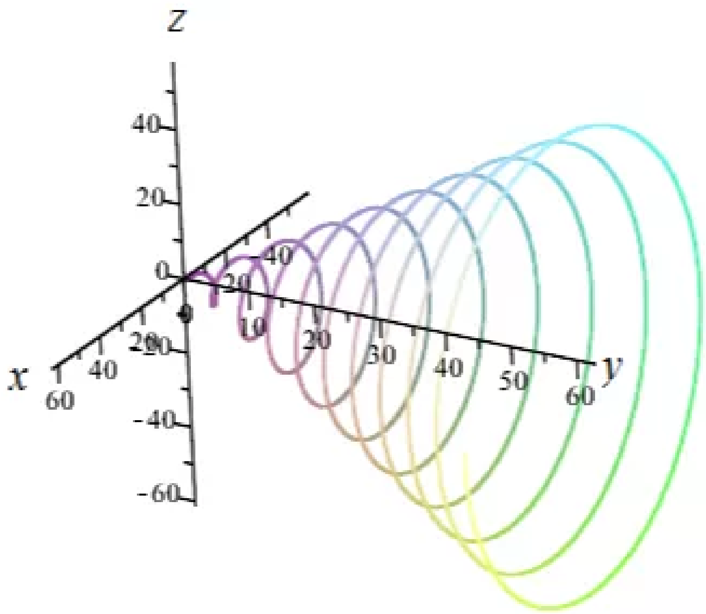
Here, $y = t$ represents the curve spirals around the y-axis as t increases.

The sketch of the parametric equations (1) by using maple software is shown below.

Maple input: > $with(plots);$

> $spacecurve([t\cdot\cos(t), t, t\cdot\sin(t)], t = 0..20\cdot\text{Pi}, numpoints = 500, axes = normal);$

Maple output:



This figure is matched with the figure mentioned in option II.

Hence, the parametric equation (1) is matched with the figure II.

Answer 22E.

Consider the parametric equations $x = \cos t, y = \sin t, z = \frac{1}{1+t^2}$

Match the parametric equations with the graphs

$x^2 + y^2 = \cos^2 t + \sin^2 t$

$x^2 + y^2 = 1$ is a circle and $z = \frac{1}{1+t^2}$,independent of both *x* and *y*.

Let $t = 0$ then *z* takes 1

Whether *t* is positive or negative, follow that $x^2 + y^2 = 1$ and $z \geq 1$

This is a path on the cylinder of radius 1 lies above *z* = 1.

Let $z = \frac{1}{1+t^2}$ lies between 0 and 1.

As t increases from 0 to infinity

So circular behavior on the xy plane and our vertical behavior stops a finite amount so namely 1.

Or In other words

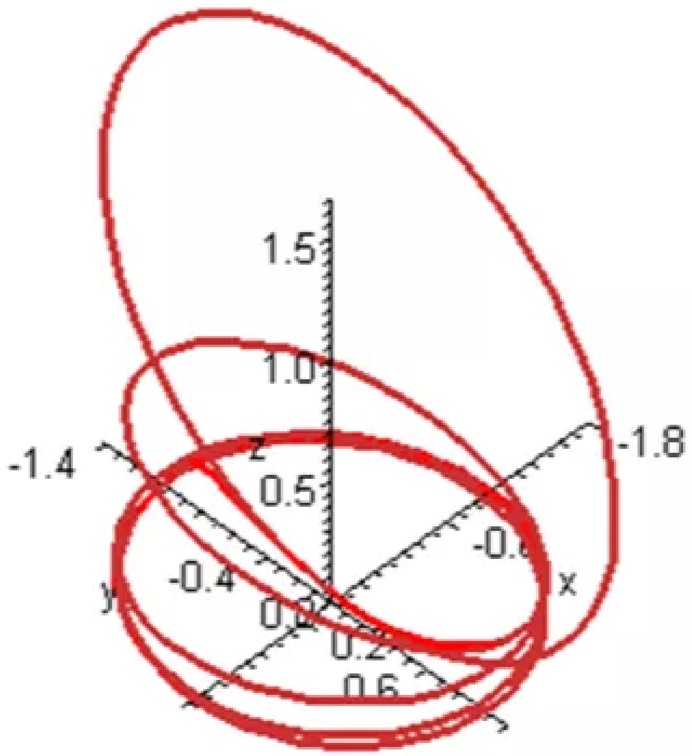
Since the equation $z = \frac{1}{1+t^2}$, looking for a graph whose z value is always decreasing, never negative, and never quite reaches zero.

Since $x = \cos(t)$ and $y = \sin(t)$ are the equations of a circle, looking for a graph whose projection onto the $z = 0$ plane is just a circle.

Putting these two facts together, looking for a graph that features circles getting closer and closer to the plane $z = 0$.

So our answer must be $\boxed{\text{VI}}$.

The graph of the curve is



Answer 23E.

Consider the parametric equations

$$x = t, y = \frac{1}{1+t^2}, z = t^2$$

The surface determined by these curves can be followed as

$$z = x^2, y = \frac{1}{1+x^2}$$

We know that $y = \frac{1}{1+x^2}$ always lies in the positive axis of *y*.

The function y takes a maximum of 1 when $t = 0$.

Also, for the positive or negative values of *t*, we follow that *y* is positive.

Therefore, the surface lies in the positive side of the *xz* – plane. ...... (1)

Also, for the positive or negative values of *t*, we follow that $t^2 = z$ is positive.

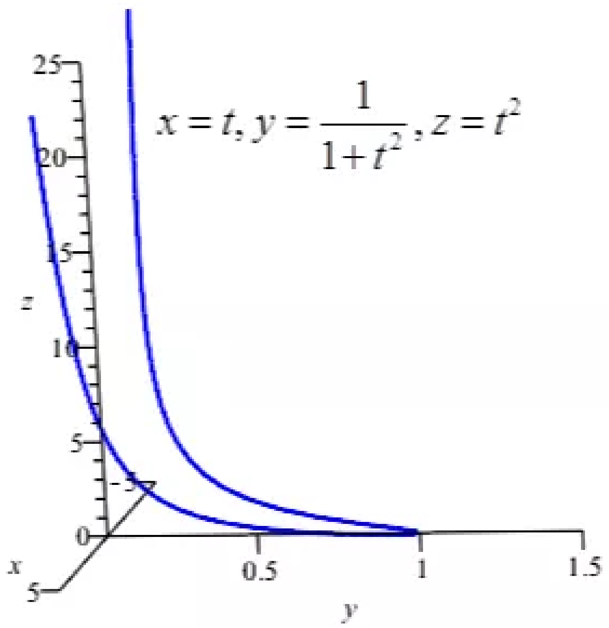
Therefore, the surface lies on the positive part of *z* axis.

In other words, the surface lies in the positive side of *xy* - plane. ...... (2)

The shape $z = x^2$ is the parabola in *xz* plane. ...... (3)

Putting the above three observations together, we get the parabolic path on both sides of *x* - axis and on the positive sides of *y* and *z*.

Sketch the graph of the parametric equation $x = t, y = \frac{1}{1+t^2}, z = t^2$



Hence the graph of the parametric equation matches with option $\boxed{\text{V}}$.

Answer 24E.

Consider the following parametric equations:

$$x = \cos t, y = \sin t, z = \cos 2t$$

The objective is to match the parametric equations with these graphs and give reasons for your choice.

The parametric equations are,

$$x = \cos t, y = \sin t, z = \cos 2t$$

The projection on the xy -plane is $\langle \cos t, \sin t, 0 \rangle$

Since,

$$\begin{aligned} x^2 + y^2 &= \cos^2 t + \sin^2 t \\ &= 1 \end{aligned}$$

The projection is a circle in the xy -plane.

The projection on to the xz -plane is traced by the curve $\langle \cos t, 0, \cos 2t \rangle$.

Therefore, $x = \cos t, y = \sin t$

Now express z in terms of x.

$$\begin{aligned} z &= \cos 2t \\ &= 2\cos^2 t - 1 \\ &= 2x^2 - 1 \end{aligned}$$

The projection on to the xz -plane is parabola.

The projection on to the yz -plane is traced by the curve $\langle 0, \sin t, \cos 2t \rangle$.

Therefore, $y = \sin t, z = \cos 2t$

Now express z in terms of y.

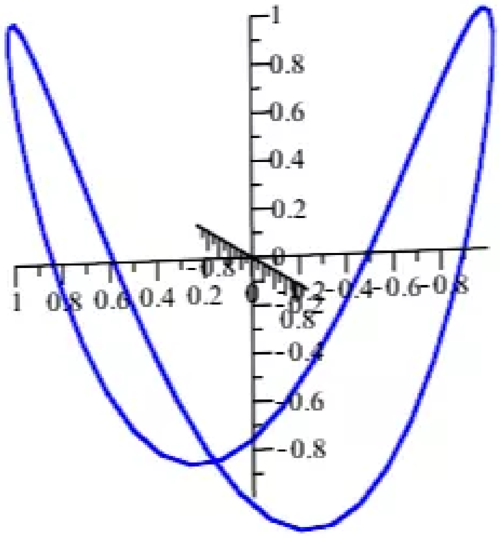
$$\begin{aligned} z &= \cos 2t \\ &= 1 - 2\sin^2 t \\ &= 1 - 2y^2 \end{aligned}$$

The projection on to the yz -plane is again parabola.

Observe that when $t=0$ then $x=1$, and $y=0$

So z take maximum at $t=0$.

Sketch the following graph:



Therefore, the given parametric equations match the figure $\boxed{\text{I}}$.

Answer 25E.

Consider the parametric equations

$$x=\cos 8t, y=\sin 8t, z=e^{0.8t}, t\geq 0$$

Determine the surface upon which the position function of the particle is given by these parametric equations.

Consider

$$\begin{aligned} x^2+y^2 &= (\cos 8t)^2+(\sin 8t)^2 \\ &= \cos^2 8t+\sin^2 8t \\ &= 1 \end{aligned}$$

The equation $x^2 + y^2 = 1$ represents a circle with center at origin and radius 1.

And $z(t) = e^{0.8t}$ travels independent of both *x* and *y*.

Observe that, when *t* = 0, *z* takes 1

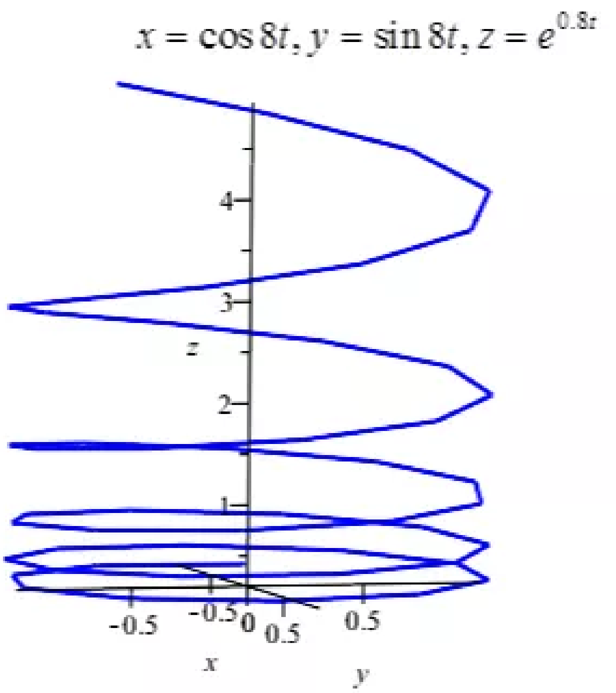
Whether t is positive, $x^2 + y^2 = 1$, and $z(t) \geq 1$

Clearly, this is a path on the cylinder of radius 1 lies above *z* = 1.

So, this is a cylindrical path above the plane $z = 1$ or the plane parallel to *xy* - plane.

As *t* increases from 0 to infinity, the function $z = e^{0.8t}$ increases rapidly.

Sketch of the parametric equation $x = \cos 8t, y = \sin 8t, z = e^{0.8t}$



Hence the graph of the parametric equation matches with the graph $\boxed{\text{IV}}$.

Answer 26E.

Consider the parametric equations

$$x=\cos^2 t, y=\sin^2 t, z=t$$

Observe that

$$\begin{aligned} x+y &= \cos^2 t+\sin^2 t \\ &= 1 \end{aligned}$$

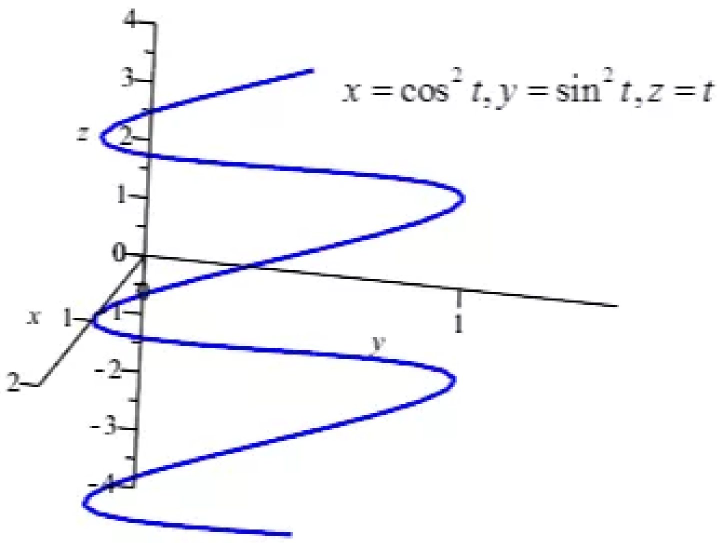
So the equation of the surface obtained by the parametric equation is

$$x+y=1, z=t$$

As t varies the sum of the coefficients of x and y varies as the sine or cosine curve perpendicular to xy – plane.

So, the path is an oscillating path perpendicular to xy – plane.

Sketch of the graph of the parametric equation $x=\cos^2 t, y=\sin^2 t, z=t$ is



Hence the graph of the parametric equation matches with graph $\boxed{\text{III}}$

Answer 27E.

Consider the following parametric equations:

$x = t\cos t, y = t\sin t, z = t.$

The objective is to show that the curve with parametric equations lies on the curve $z^2 = x^2 + y^2$.

Consider,

$$\begin{aligned} x^2 + y^2 &= (t\cos t)^2 + (t\sin t)^2 \\ &= t^2\cos^2 t + t^2\sin^2 t \\ &= t^2(\cos^2 t + \sin^2 t) \\ &= t^2 \\ &= z^2 \end{aligned}$$

So, $x^2 + y^2 = z^2$.

Thus, the curve with parametric equations $x = t\cos t, y = t\sin t$ and $z = t$ lies on the cone $z^2 = x^2 + y^2$.

Use the Maple software and upload the following package.

> *with*(*plots*);

Maple Input:

A:=spacecurve([t*cos(t),t*sin(t),t],t=-2..Pi..2*Pi,color='red')

> *A* := *spacecurve*([*t*·cos(*t*), *t*·sin(*t*), *t*], *t* = −2·Pi ..2·Pi, *color* ='*red*')

PLOT3D(...)

Maple input:

B:=spacecurve([z^2=x^2+y^2,x=-5..5,y=-5..5,z=-5..5,color=blue)

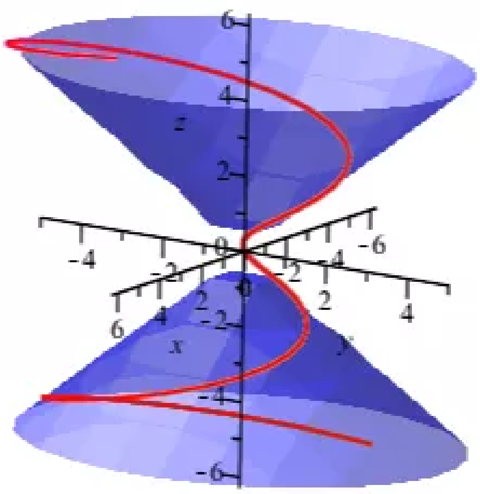
> *B* := *implicitplot3d*($z^2 = x^2 + y^2$, *x* = −5 ..5, *y* = −5 ..5, *z* = −5 ..5, *color* = *blue*)

PLOT3D(...)

display({A,B});

Maple output:

> *display*({ *A*, *B* });



Answer 28E.

The parametric equations of the curve C are,

$x=\sin t, y=\cos t,\ z=\sin^2 t$

The objective is to show C is the curve of intersection of the circular cylinder $x^2+y^2=1$ and the parabolic cylinder $z=x^2$.

Consider the expression,

$x^2+y^2=\sin^2 t+\cos^2 t$ Since $x=\sin t, y=\cos t$

$=1$

And

$$\begin{aligned} z &= \sin^2 t \\ &= (\sin t)^2 \\ &= x^2 \end{aligned}$$

Therefore, C is the intersection of the surfaces $x^2+y^2=1$ and $z=x^2$.

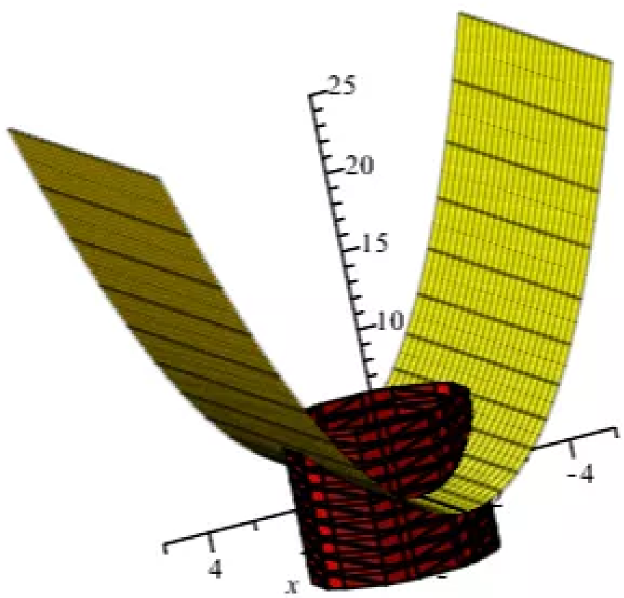
Again substitute $x^2=z$ in the equation $x^2+y^2=1$.

$z+y^2=1$.

This equation represents a parabola.

Therefore, the curve C represents a parabola.

The graph of intersection of the surfaces $z=x^2$ and $x^2+y^2=1$ is shown below:



Answer 29E.

Consider the vector function $\mathbf{r}(t) = t\mathbf{i} + (2t - t^2)\mathbf{k}$ and the paraboloid $z = x^2 + y^2$.

To determine the point of intersection of the curve $\mathbf{r}(t)$ and the paraboloid, find the point such that, the point lies on a line as well on the plane.

Let any point on curve be $P = (t, 0, 2t - t^2)$.

Substitute the point $(t, 0, 2t - t^2)$ in paraboloid $z = x^2 + y^2$.

$$2t - t^2 = (t)^2$$
$$2t - 2t^2 = 0$$
$$2t(1 - t) = 0$$

Use the zero product rule, $ab = 0$ then $a = 0$ or $b = 0$.

Then,

$$t = 0 \text{ or } 1 - t = 0$$
$$t = 0 \text{ or } t = 1$$

To find the point of intersection of the curve and paraboloid, substitute $t = 0$ in $(t, 0, 2t - t^2)$.

$$P_1 = ((0), 0, 2(0) - (0)^2)$$
$$= (0, 0, 0)$$

Clearly, the point $(0,0,0)$ is on curve for $t = 0$.

To check the point $(0,0,0)$ is the point of intersection a curve and the paraboloid, substitute the point on $(0,0,0)$ in $z = x^2 + y^2$.

$$(0) \stackrel{?}{=} (0)^2 + (0)^2$$
$$0 \stackrel{?}{=} 0 \qquad \text{True}$$

Therefore, the point $(0,0,0)$ satisfies the paraboloid, $z = x^2 + y^2$.

Hence, one of the points of intersection is $\boxed{(0,0,0)}$.

To find the point of intersection of the curve and paraboloid, substitute $t=1$ in $\left(t,0,2t-t^2\right)$.

$$P_2=\left((1),0,2(1)-(1)^2\right)$$
$$=(1,0,1)$$

Clearly, the point $(1,0,1)$ is on curve for $t=0$.

To check the point $(1,0,1)$ is the point of intersection a curve and the paraboloid, substitute the point on $(1,0,1)$ in $z=x^2+y^2$.

$$(1)\overset{?}{=}(1)^2+(0)^2$$
$$1\overset{?}{=}1 \quad \text{True}$$

Therefore, the point $(1,0,1)$ satisfies the paraboloid, $z=x^2+y^2$.

Hence, one of the points of intersection is $\boxed{(1,0,1)}$.

Therefore, the curve $\mathbf{r}(t)=t\mathbf{i}+\left(2t-t^2\right)\mathbf{k}$ intersects the paraboloid $z=x^2+y^2$ at the points $\boxed{(0,0,0) \text{ and } (1,0,1)}$.

Answer 30E.

Substitute the expressions for *x*, *y*, and *z* from the parametric equations (3) into the equation (2):

$x^2+y^2+z^2=5$ Write the equation of the sphere (2)

$(\sin t)^2+(\cos t)^2+t^2=5$. Put $x=\sin t, y=\cos t, z=t$ from (3)

Solve this equation for *t* as follows:

$$(\sin t)^2+(\cos t)^2+t^2=5$$
$$\sin^2 t+\cos^2 t+t^2=5$$
$$\left(\sin^2 t+\cos^2 t\right)+t^2=5$$
$$1+t^2=5$$
$$t^2=4$$
$$t=\pm 2.$$

Hence, the point of intersection occurs when the parameter value is $t=\pm 2$.

Substitute the parameter value; $t = 2$, into the parametric equations (3), and simplify as follows:

$x = \sin t, \quad y = \cos t, \quad z = t$ Write the equation (1)

$x = \sin 2, \quad y = \cos 2, \quad z = 2$ Substitute $t = 2$

$x \approx 0.91, \quad y \approx -0.42, \quad z = 2$. Evaluate values

So, the point of intersection is about $\boxed{(0.91, -0.42, 2)}$.

Substitute the parameter value; $t = -2$, into the parametric equations (3), and simplify as follows:

$x = \sin t, \quad y = \cos t, \quad z = t$ Write the equation (1)

$x = \sin(-2), \quad y = \cos(-2), \quad z = -2$ Substitute $t = -2$

$x \approx -0.91, \quad y \approx -0.42, \quad z = -2$. Evaluate values

So, the point of intersection is about $\boxed{(-0.91, -0.42, -2)}$.

Therefore, the points, at which the helix (1) intersects the sphere (2), are $\boxed{(0.91, -0.42, 2)}$ and $\boxed{(-0.91, -0.42, -2)}$.

Answer 31E.

Consider the curve, $\mathbf{r}(t) = \langle \cos t \sin 2t, \sin t \sin 2t, \cos 2t \rangle$.

The objective is to sketch the graph of the curve.

The parametric equations are,

$$x = \cos t \sin 2t$$
$$y = \sin t \sin 2t$$
$$z = \cos 2t$$

To graph this, use maple software.

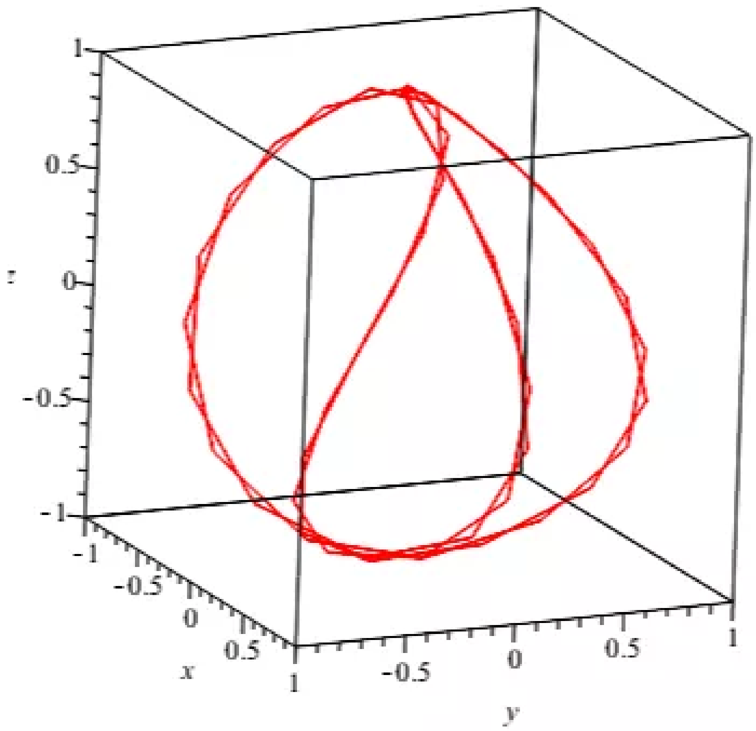
Maple input:

with(plots);

spacecurve([cos(t)*sin(2*t), sin(t)*sin(2*t), cos(2*t)], t = -2*Pi .. 2*Pi, color = red);

Maple output:

spacecurve([cos(*t*)·sin(2·*t*), sin(*t*)·sin(2·*t*), cos(2·*t*)], *t* = −2·Pi..2·Pi, *color* = *red*)



Answer 32E.

Consider the following vector equation:

$$\mathbf{r}(t)=\langle t^2, \ln(t), t\rangle$$

Use the maple commands to graph the curve with the vector equation.

Use the computer to plot the curve with parametric equations $x=t^2$, $y=\ln(t)$ and $z=t$.

Domain is $t>0$.

The result is shown in below figure, but it's hard to see the true nature of the curve from that graph alone. Most three-dimensional computer graphing programs allow the user to enclose a curve or surface in a box instead of displaying the coordinate axes.

Use the following input and output maple command to draw the curve with the given vector equation:

Input command:

With (plots);

Output command:

with(*plots*);

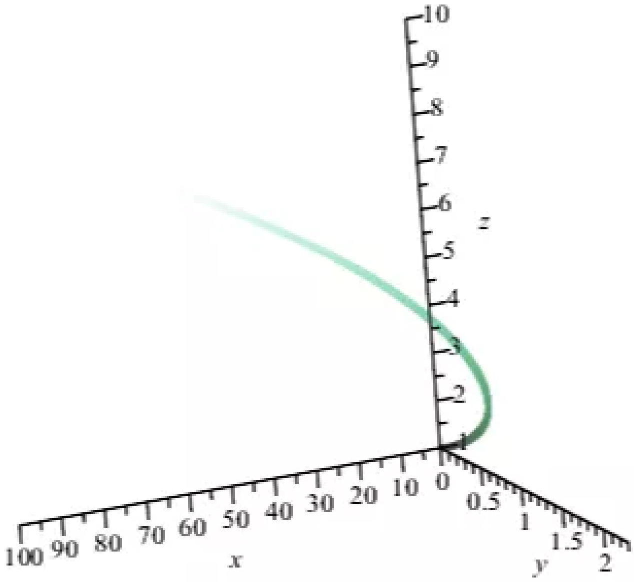
[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

Input command:

spacecurve ([t^2, ln (t), t], t=1 ..10)

Output command:

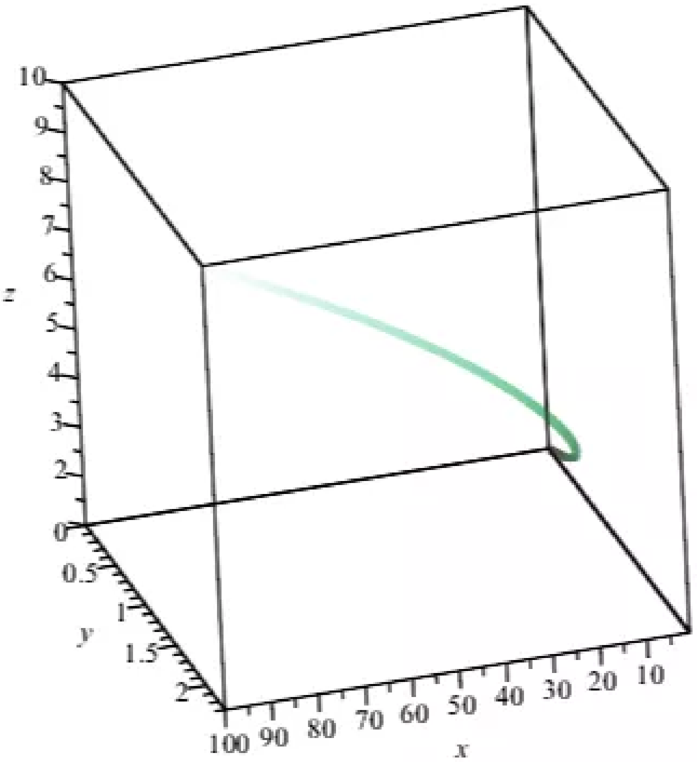
$spacecurve([t^2, \ln(t), t], t = 1..10)$



When you look at the same curve in a box in below figure, you have a much clearer picture of the curve. You can see that it climbs from a lower corner of the box to the upper corner nearest us, and it twists as it climbs.

Output command:

$spacecurve([t^2, \ln(t), t], t = 1..10)$



The true nature of the curve and it is correct view of the curve.

Answer 33E.

Consider the curves

$\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$ and $\mathbf{r}_2(t) = \langle \sin t, \sin 2t, t \rangle$

Suppose that, the two curves are intersect at the origin.

It is needed to find the angle of intersection correct to the nearest degree.

Recall that, the angle between two curves is equal to the angle between their tangents at the point of intersection.

The vector $\mathbf{r}'(t)$ is called the tangent vector to the curve defined by $\mathbf{r}$ at the point P, provided that $\mathbf{r}'(t)$ exists and $\mathbf{r}'(t) \neq 0$ and unit tangent vector to curve $\mathbf{r}(t)$ is given by

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

As $\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$, $\mathbf{r}_1'(t) = \langle 1, 2t, 3t^2 \rangle$

And then $\mathbf{r}_1'(0) = \langle 1, 2(0), 3(0)^2 \rangle$

$= \langle 1, 0, 0 \rangle$

And $|\mathbf{r}_1'(0)| = \sqrt{1^2 + 0^2 + 0^2}$

$= 1$

Unit tangent vector at origin to the curve $\mathbf{r}_1(t)$ is given by

$$\mathbf{T}_1(0) = \frac{\mathbf{r}_1'(0)}{|\mathbf{r}_1'(0)|}$$

$$= \frac{\langle 1, 0, 0 \rangle}{1}$$

$= \langle 1, 0, 0 \rangle$ ...... (1)

As $\mathbf{r}_2(t) = \langle \sin t, \sin 2t, t \rangle$, $\mathbf{r}_2'(t) = \langle \cos t, 2\cos 2t, 1 \rangle$

And then $\mathbf{r}_2'(0) = \langle \cos(0), 2\cos 2(0), 1 \rangle$

$= \langle 1, 2(1), 1 \rangle$

$= \langle 1, 2, 1 \rangle$

And $|\mathbf{r}_2'(0)| = \sqrt{1^2 + 2^2 + 1^2}$

$= \sqrt{6}$

Unit tangent vector at origin to the curve $\mathbf{r}_2(t)$ is given by

$$\mathbf{T}_2(0) = \frac{\mathbf{r}_2'(0)}{|\mathbf{r}_2'(0)|}$$

$$= \frac{\langle 1, 2, 1 \rangle}{\sqrt{6}}$$

$= \langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$ ...... (2)

Let θ be the angle between tangents (1) and (2)

Then, $\cos\theta = \dfrac{\mathbf{T}_1(t)\cdot\mathbf{T}_2(t)}{|\mathbf{T}_1(t)||\mathbf{T}_2(t)|}$

$$= \frac{\langle 1,0,0\rangle.\langle \frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}}\rangle}{\sqrt{1^2+0^2+0^2}\sqrt{\left(\frac{1}{\sqrt{6}}\right)^2+\left(\frac{2}{\sqrt{6}}\right)^2+\left(\frac{1}{\sqrt{6}}\right)^2}}$$

$$= \frac{\langle 1,0,0\rangle.\langle \frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}}\rangle}{(1)\sqrt{\frac{1}{6}+\frac{4}{6}+\frac{1}{6}}}$$

$$= \frac{\langle 1,0,0\rangle.\langle \frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}}\rangle}{(1)(1)}$$

$$= 1\left(\frac{1}{\sqrt{6}}\right)+0\left(\frac{2}{\sqrt{6}}\right)+0\left(\frac{1}{\sqrt{6}}\right)$$

$$= \frac{1}{\sqrt{6}}$$

Thus, $\cos\theta = 1/\sqrt{6}$

And then $\theta = \cos^{-1}\left(1/\sqrt{6}\right) \approx 66°$

Hence, angle between two given curves is $\boxed{66°}$.

Answer 34E.

Consider the curves

$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2\rangle$ and $\mathbf{r}_2(s) = \langle 3-s, s-2, s^2\rangle$

It is needed to find the intersection point of the two curves.

Parametric equations of the given curves are

$x = t,\ y = 1-t,\ z = 3+t^2$ and $x = 3-s,\ y = s-2,\ z = s^2$

If the given two curves intersect, then they have a common point.

This means that for some value of s and t,

$t = 3 - s$

$1 - t = s - 2$

$3 + t^2 = s^2$

As $t = 3 - s$ and $3 + t^2 = s^2$, we have

$$3 + (3 - s)^2 = s^2$$
$$3 + 9 + s^2 - 6s = s^2$$
$$12 + s^2 - 6s = s^2$$
$$12 - 6s = 0$$

$s = 2$

Substitute $s = 2$ in $t = 3 - s$, to get

$t = 3 - 2$
$= 1$

Thus, we get $s = 2$ and $t = 1$

These values satisfy equation $1 - t = s - 2$ as well.

So, the two curves intersect and we can get the point of intersection of the curves by replacing $t = 1$ in $\mathbf{r}_1(t)$ (or) $s = 2$ in $\mathbf{r}_2(s)$.

Point of intersection of the curves is

$(t, 1 - t, 3 + t^2) = (1, 1 - 1, 3 + (1)^2)$ Replace t by 1

$= (1, 0, 4)$

Thus, Point of intersection of the curves is $\boxed{(1,0,4)}$.

As the two curves are intersecting at the point $(1,0,4)$, it is needed to find the angle of intersection correct to the nearest degree.

Recall that, the angle between two curves is equal to the angle between their tangents at the point of intersection.

The vector $\mathbf{r}'(t)$ is called the tangent vector to the curve defined by $\mathbf{r}$ at the point P, provided that $\mathbf{r}'(t)$ exists and $\mathbf{r}'(t) \neq 0$ and unit tangent vector to curve $\mathbf{r}(t)$ is given by

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

As $\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle$, $\mathbf{r}_1'(t) = \langle 1, -1, 2t \rangle$

And then $\mathbf{r}_1'(1) = \langle 1, -1, 2(1) \rangle$

$= \langle 1, -1, 2 \rangle$

And $|\mathbf{r}_1'(0)| = \sqrt{1^2 + (-1)^2 + (2)^2}$

$= \sqrt{1+1+4}$

$= \sqrt{6}$

Unit tangent vector at the point $(1,0,4)$ to the curve $\mathbf{r}_1(t)$ is given by

$$\mathbf{T}_1(1) = \frac{\mathbf{r}_1'(1)}{|\mathbf{r}_1'(1)|}$$

$$= \frac{\langle 1, -1, 2 \rangle}{\sqrt{6}}$$

$$= \langle \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \rangle \quad \ldots\ldots (1)$$

As $\mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$, $\mathbf{r}_2'(s) = \langle -1, 1, 2s \rangle$

And then $\mathbf{r}_2'(2) = \langle -1, 1, 2(2) \rangle$

$= \langle -1, 1, 4 \rangle$

And $|\mathbf{r}_2'(2)| = \sqrt{(-1)^2 + (1)^2 + (4)^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

Unit tangent vector at the point $(1,0,4)$ to the curve $\mathbf{r}_2(s)$ is given by

$$\mathbf{T}_2(2) = \frac{\mathbf{r}_2'(2)}{|\mathbf{r}_2'(2)|}$$

$$= \frac{\langle -1, 1, 4 \rangle}{\sqrt{18}}$$

$$= \langle -\frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}} \rangle \quad \ldots\ldots (2)$$

Let θ be the angle between tangents (1) and (2)

Then, $\cos\theta = \dfrac{\mathbf{T}_1(1)\cdot\mathbf{T}_2(2)}{|\mathbf{T}_1(1)||\mathbf{T}_2(2)|}$

$$= \frac{\langle \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\rangle . \langle -\frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}\rangle}{\sqrt{\frac{1}{6}+\frac{1}{6}+\frac{4}{6}}\sqrt{\frac{1}{18}+\frac{1}{18}+\frac{16}{18}}}$$

$$= \frac{\langle \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\rangle . \langle -\frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}\rangle}{\sqrt{\frac{6}{6}}\cdot\sqrt{\frac{18}{18}}}$$

$$= \frac{-\frac{1}{\sqrt{6}}\cdot\frac{1}{\sqrt{18}} - \frac{1}{\sqrt{6}}\cdot\frac{1}{\sqrt{18}} + \frac{2}{\sqrt{6}}\cdot\frac{4}{\sqrt{18}}}{\sqrt{\frac{6}{6}}\cdot\sqrt{\frac{18}{18}}}$$

Continuation to the above

$$\cos\theta = \frac{-\frac{2}{6\sqrt{3}}+\frac{8}{6\sqrt{3}}}{1\cdot 1}$$

$$= \frac{6}{6\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

Thus, $\cos\theta = 1/\sqrt{3}$

And then $\theta = \cos^{-1}\left(1/\sqrt{3}\right) = 54.73561^\circ \approx 55^\circ$

Since the angle between curves is equal to the angle between tangents at the point of intersection, so the angle between given curves is $\boxed{55^\circ}$.

Answer 35E.

Consider the vector equation

$$\mathbf{r}(t) = \langle \cos 2t, \cos 3t, \cos 4t \rangle$$

Use the maple commands to sketch the vector equation.

To draw the vector equation, first load vector calculus package in maple.

> with(VectorCalculus);

> pv: = PositionVector ([cos(2t),cos(3t),cos(4t);

> PlotPositionVector (pv, t = -3..3);

Input the commands individually and observe the outputs

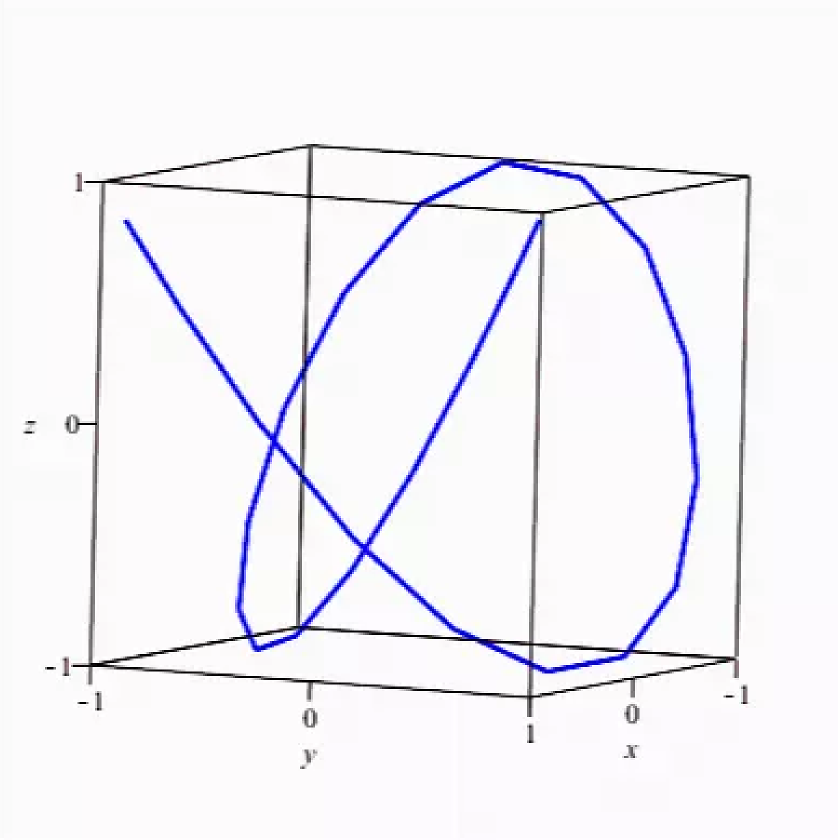
```
> with(VectorCalculus);
```

[&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, Compatibility, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series]

```
> pv := PositionVector( [cos(2 t), cos(3 t), cos(4 t)], cartesian[x, y, z] );
```

$$pv := \begin{bmatrix} \cos(2t) \\ \cos(3t) \\ \cos(4t) \end{bmatrix}$$

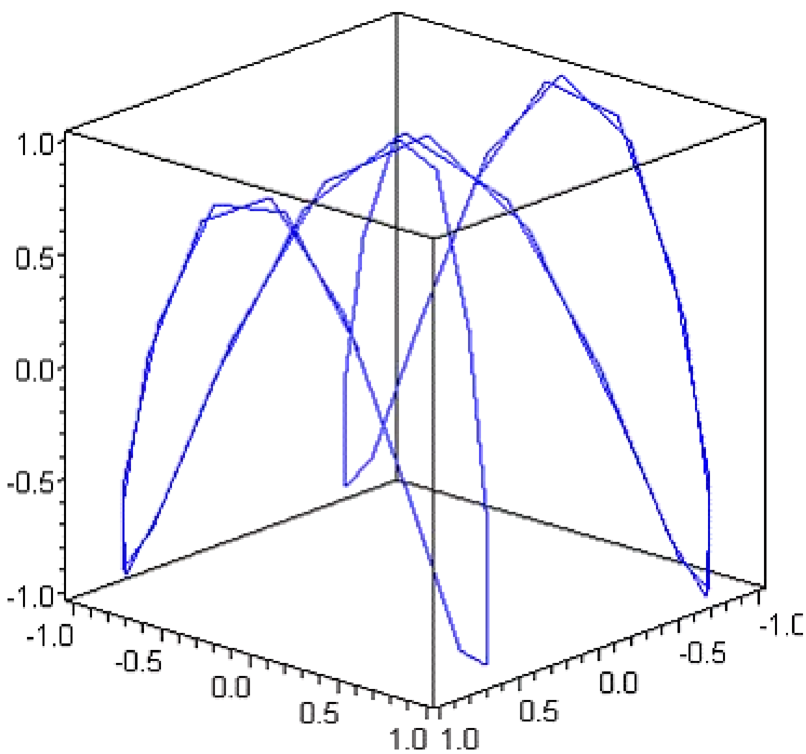
```
> PlotPositionVector( pv, t = -3 ..3);
```



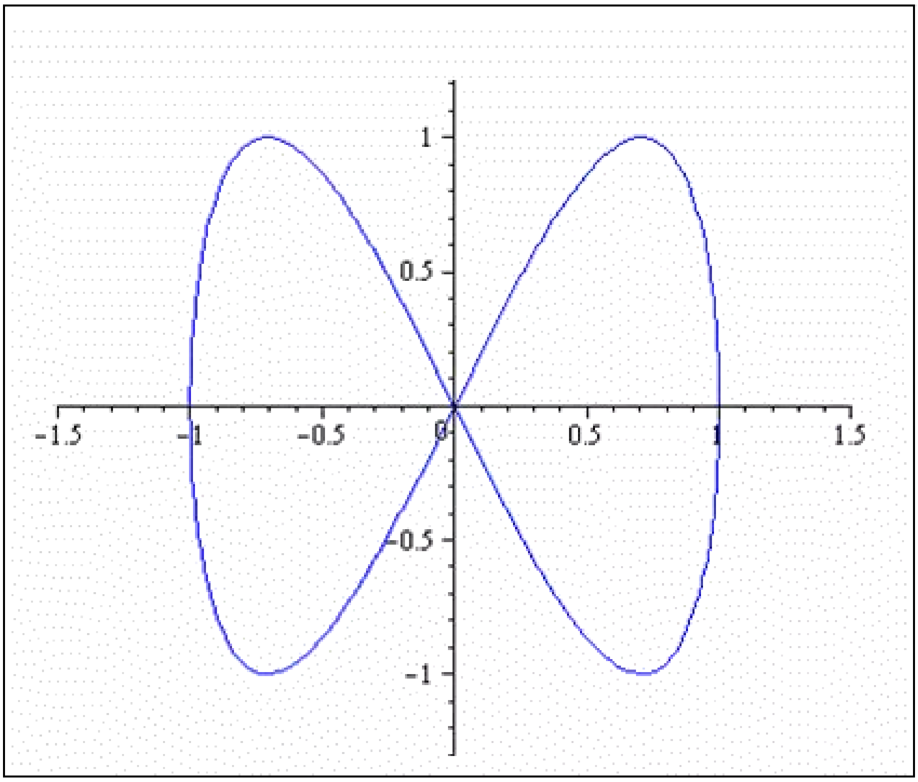
Hence sketch of the graph of the vector equation $\mathbf{r}(t)=\langle \cos 2t, \cos 3t, \cos 4t \rangle$

Answer 36E.

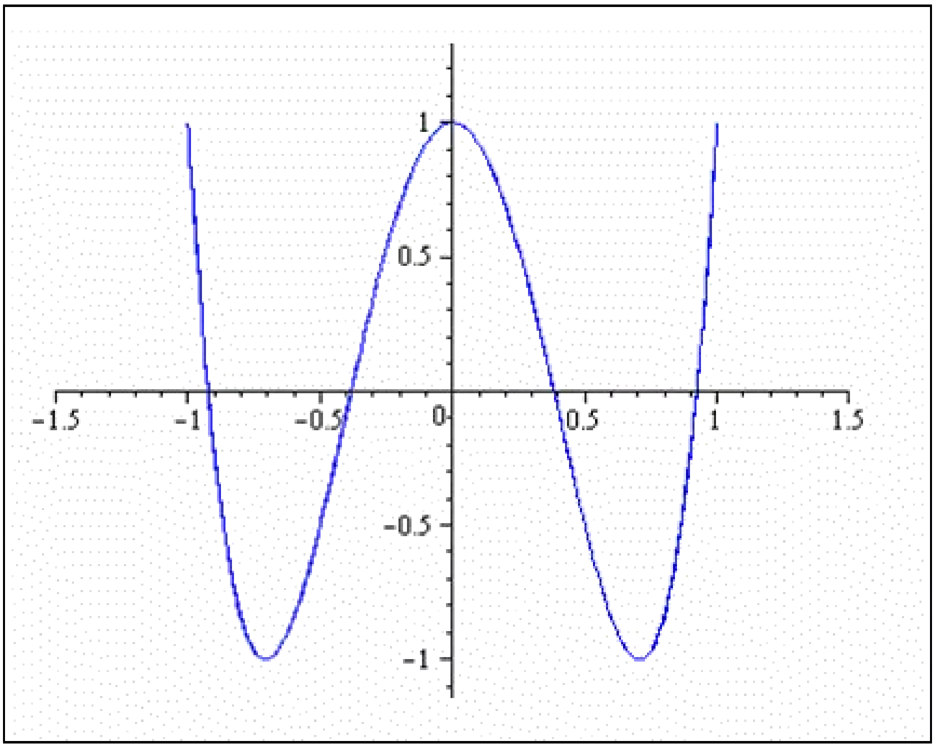
Let us start by sketching $\mathbf{r}(t) = \langle \sin t, \sin 2t, \cos 4t \rangle$.

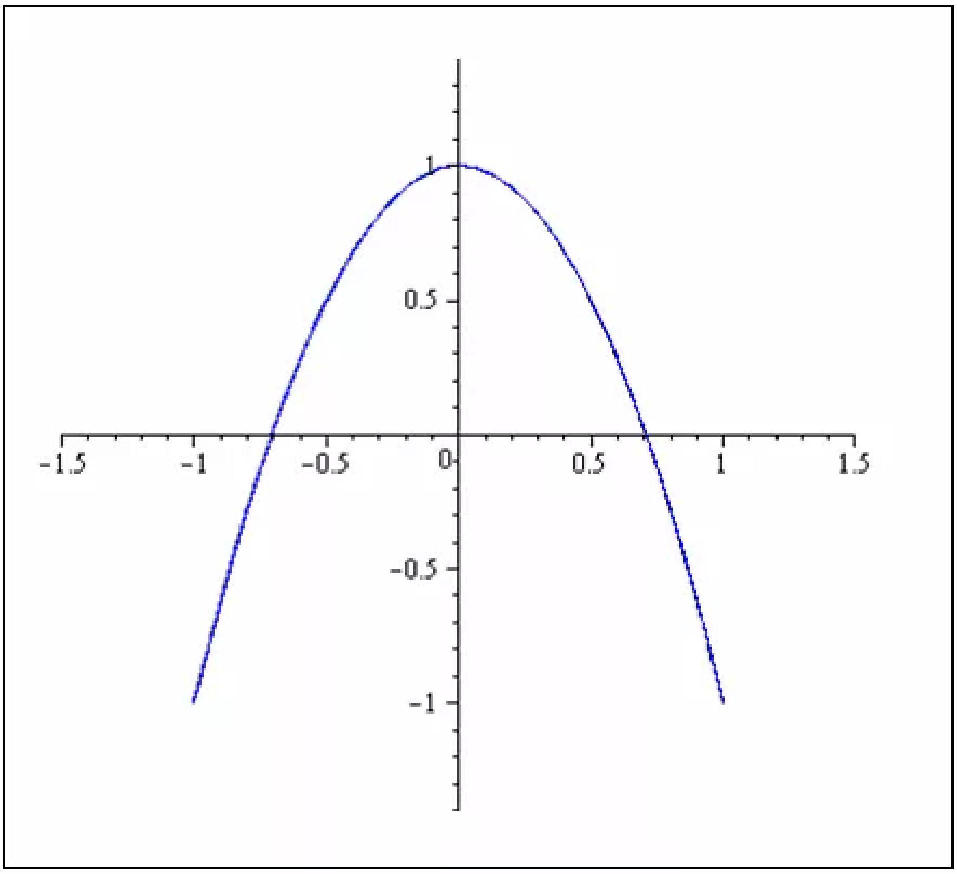


In order to sketch the projection of $\mathbf{r}(t)$ in the xy-plane we set $z = 0$. Then, $\mathbf{r}(t) = \langle \sin t, \sin 2t \rangle$.



Similarly, the projection of $\mathbf{r}(t)$ in the xz-plane is obtained by setting $y = 0$.





From the projections of the curve, we can say that the top view of the curve shows a butterfly curve. The projection of the curve on the *yz*-plane, shows that the front view of the curve is a parabola.

Answer 37E.

Consider the following parametric equations;

$$x = (1+\cos 16t)\cos t$$
$$y = (1+\cos 16t)\sin t$$
$$z = 1+\cos 16t$$

The objective is to graph the curve and then to show that the curve lies on a cone.

Eliminate the variable *t* from the above parametric equations to obtain the equation to the surface in rectangular coordinates.

Let us square, all the three equations, we get

$$\left.\begin{aligned} x^2 &= (1+\cos 16t)^2 \cos^2 t \quad &\text{......}(1)\\ y^2 &= (1+\cos 16t)^2 \sin^2 t \quad &\text{......}(2)\\ z^2 &= (1+\cos 16t)^2 \quad &\text{......}(3)\end{aligned}\right\}$$

Add (1) and $(2) \Rightarrow$

$$\begin{aligned} x^2+y^2 &= (1+\cos 16t)^2 \cos^2 t + (1+\cos 16t)^2 \sin^2 t\\ &= (1+\cos 16t)^2 \left(\cos^2 t + \sin^2 t\right)\\ &= (1+\cos 16t)^2\\ &= z^2 \end{aligned}$$

Therefore the equation in rectangular coordinates is $x^2+y^2=z^2$

Hence, this equation represents the cone.

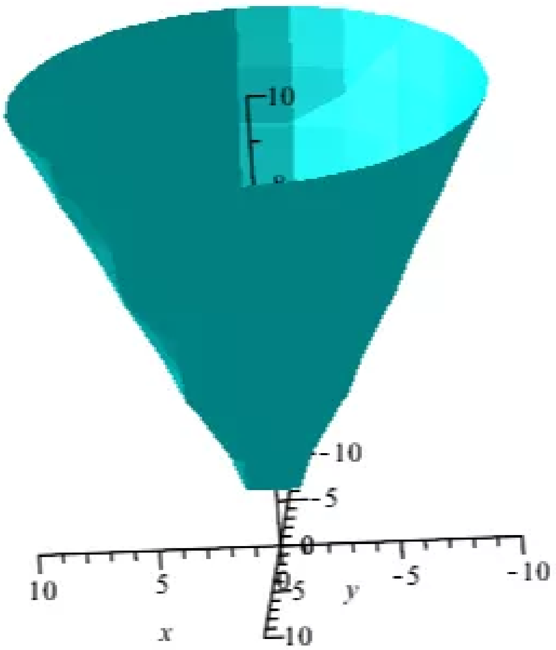
The graph of the surface can be obtained using the following maple command.

Input:

with(plots);

implicitplot3d(x^2+y^2 = z^2, x = -10 .. 10, y = -10 .. 10, z = 0 .. 10, style = surface, color = cyan, axes = normal);

Output:



From the above diagram it can be said the space curve obtained from the given parametric equations lies inside this cone.

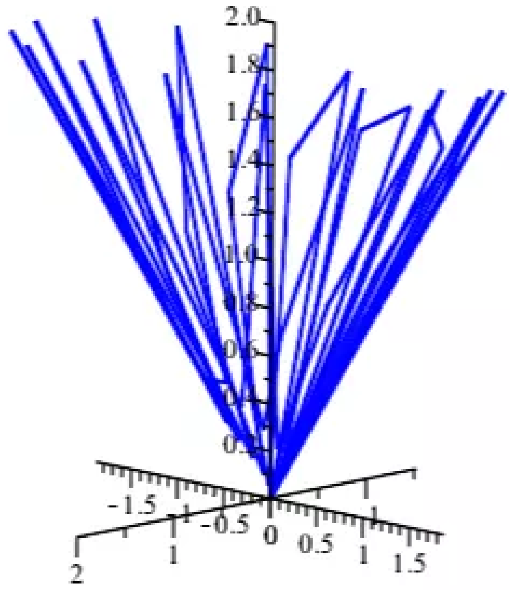
Use the following maple command to plot the space curve with the parametric equations:

$x=(1+\cos 16t)\cos t, y=(1+\cos 16t)\cos t, z=1+\cos 16t$.

Input:

spacecurve([(1+cos(16*t))*cos(t), (1+cos(16*t))*sin(t), 1+cos(16*t)], t = 0 .. 4*Pi, color = blue, axes = normal);

Output:



Answer 38E.

Consider the following definite integral:

$$\int_0^1\left(\frac{4}{1+t^2}\hat{j}+\frac{2t}{1+t^2}\hat{k}\right)dt$$

Use the following identity to evaluate the integral:

$$\int[f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx$$

The integral can be written as follows:

$$\int_0^1\left(\frac{4}{1+t^2}\hat{j}+\frac{2t}{1+t^2}\hat{k}\right)dt=\left(\int_0^1\frac{4}{1+t^2}dt\right)\hat{j}+\left(\int_0^1\frac{2t}{1+t^2}dt\right)\hat{k} \quad \text{...... (1)}$$

Use the following identity to evaluate the second integral:

$$\int\frac{1}{1+t^2}dt=\tan^{-1}(t)+C$$

Therefore, the second integral can be written as follows:

$$\int\frac{2t}{1+t^2}dt=\int\frac{1}{u}du \text{ Let } u=1+t^2\text{, then } du=2tdt$$

$$=\log|u|$$

$$=\log\left|1+t^2\right|+C$$

Now, equation (1) can be written as follows:

$$\int_0^1\left(\frac{4}{1+t^2}\hat{j}+\frac{2t}{1+t^2}\hat{k}\right)dt$$

$$=\left(\int_0^1\frac{4}{1+t^2}dt\right)\hat{j}+\left(\int_0^1\frac{2t}{1+t^2}dt\right)\hat{k}$$

$$=4\tan^{-1}t\Big]_0^1\hat{j}+\ln\left(1+t^2\right)\Big]_0^1\hat{k}$$

$$=4\left[\tan^{-1}(1)-\tan^{-1}(0)\right]\hat{j}+\left[\ln(2)-\ln(1)\right]\hat{k}$$

$$=4\left(\frac{\pi}{4}\right)\hat{j}+\ln(2)\hat{k} \text{ Since } \ln(1)=0 \text{ and } \tan^{-1}(\tan\theta)=\theta,\frac{-\pi}{2}<\theta<\frac{\pi}{2}$$

$$=\pi\hat{j}+\ln 2\hat{k}$$

Therefore,

$$\int_0^1\left(\frac{4}{1+t^2}\hat{j}+\frac{2t}{1+t^2}\hat{k}\right)dt=\boxed{\pi\hat{j}+\ln 2\hat{k}}.$$

Answer 39E.

Consider the following parametric equations of the curve:

$x = t^2,\ y = 1 - 3t\ , z = 1 + t^3$.

Suppose that the curve passes through the point $(1,4,0)$. Then, equate the parametric equations $x = t^2,\ y = 1 - 3t\ , z = 1 + t^3$ to $(1,4,0)$.

$$1 = t^2$$
$$4 = 1 - 3t$$
$$0 = 1 + t^3$$

All these equations are have unique solution, $t = -1$.

Therefore, the curve passes through the point $(1,4,0)$.

Now, suppose that the curve passes through the point $(9,-8,28)$. Then, equate the parametric equations $x = t^2,\ y = 1 - 3t\ , z = 1 + t^3$ to $(9,-8,28)$.

$$9 = t^2$$
$$-8 = 1 - 3t$$
$$28 = 1 + t^3$$

All these equations are have unique solution, $t = 3$.

Therefore, the curve passes through the point $(9,-8,28)$.

Suppose the curve passes through the point $(4,7,-6)$. Then, equate the parametric equations: $x = t^2,\ y = 1 - 3t\ , z = 1 + t^3$ to $(4,7,-6)$.

$$4 = t^2$$
$$7 = 1 - 3t$$
$$-6 = 1 + t^3$$

From the equation: $t^2 = 4$, $t = \pm 2$. But, $t = \pm 2$ does not satisfies the equation $-6 = 1 + t^3$.

So, these equations do not have unique solution.

Therefore, the curve does not pass through the point $(4,7,-6)$.

Hence, it is shown that the indicated curve passes through the points $(1,4,0)$ and $(9,-8,28)$, but not through the point $(4,7,-6)$.

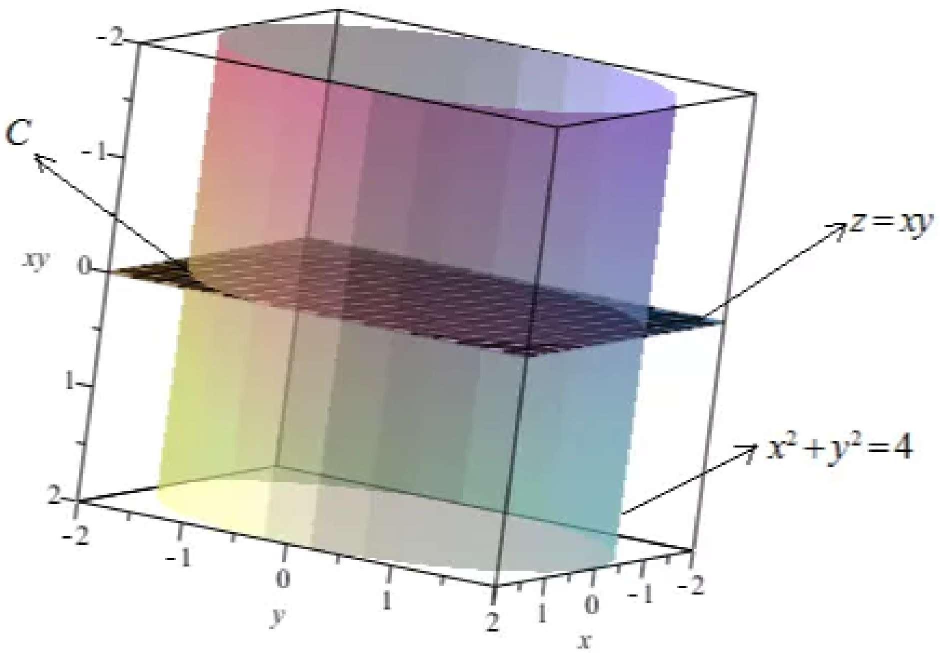
Answer 40E.

Consider the two surfaces, the cylinder $x^2+y^2=4$ and the surface $z=xy$

It is needed to find a vector function that represents the curve of the intersection of the two surfaces.

The following figure shows how the plane and the cylinder intersect and it shows the curve of intersection which is a circle.

$z=0.$



The projection of C onto the $xy-$plane is the circle $x^2+y^2=4, z=0$

Therefore, the parametric equations for this projection (circle) are

$x=2\cos t \quad y=2\sin t \quad 0\le t\le 2\pi$ Here t is a parameter.

From the equation of the plane $z=xy$, we have

$z=(2\cos t)(2\sin t)$

$=2(2\sin t\cos t)$ Use the trigonometric identity $\sin 2t=2\sin t\cos t$

$=2\sin 2t$

Therefore, the parametric equations for C are

$x=2\cos t \quad y=2\sin t \quad z=2\sin 2t \quad 0\le t\le 2\pi$

Hence, the corresponding vector equation (parametrization) of C is

$\mathbf{r}(t)=(2\cos t)\mathbf{i}+(2\sin t)\mathbf{j}+(2\sin 2t)\mathbf{k} \quad 0\le t\le 2\pi$

Answer 41E.

The equations of two surfaces are the cone, $z=\sqrt{x^2+y^2}$ and the plane $z=1+y$.

The objective is to find vector function for curve of intersection of two surfaces.

The curve of intersection will be obtained by equating the equations.

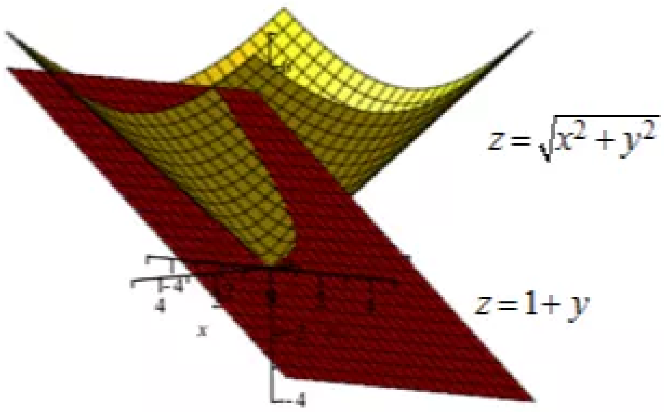
$$\sqrt{x^2+y^2}=1+y$$

Square it on both sides.

$$\begin{aligned} x^2+y^2 &= (1+y)^2 \\ &= 1+2y+y^2 \\ x^2 &= 1+2y \\ y &= \frac{x^2-1}{2} \end{aligned}$$

It represents the equation of parabola.

The curve of intersection of surfaces is shown below:



Let $x = t$, where t is a parameter.

Substitute $x = t$ in $y = \frac{x^2 - 1}{2}$.

$$y = \frac{t^2 - 1}{2}$$

Finally substitute $y = \frac{t^2 - 1}{2}$ in $z = 1 + y$

$$z = 1 + \frac{t^2 - 1}{2}$$
$$= \frac{t^2 + 1}{2}$$

The vector function that represents curve of intersection of two surfaces is,

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$
$$= t\mathbf{i} + \left(\frac{t^2 - 1}{2}\right)\mathbf{j} + \left(\frac{t^2 + 1}{2}\right)\mathbf{k}$$

Therefore, required vector function is $\boxed{\mathbf{r} = t\mathbf{i} + \left(\frac{t^2 - 1}{2}\right)\mathbf{j} + \left(\frac{t^2 + 1}{2}\right)\mathbf{k}}$.

Answer 42E.

The two given surfaces are:

Paraboloid: $z = 4x^2 + y^2$

Parabolic cylinder: $y = x^2$

Take $x = t$

Then $y = t^2$

And $z = 4(t)^2 + (t^2)^2$

$= 4t^2 + t^4$

Therefore the vector function of intersection of the given curve is

$< x, y, z > = < t,\ t^2,\ 4t^2 + t^4 >$

Or $\boxed{\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^2(4 + t^2)\hat{k}}$

Answer 43E.

We have $z = x^2 - y^2$ and $x^2 + y^2 = 1$.

From $x^2 + y^2 = 1$, we get the parametric equations for x as $\cos t$ and y as $\sin t$, where $0 \le t \le 2\pi$.

Also, we have $z = \cos^2 t - \sin^2 t$ or $z = \cos 2t$.

Therefore, the vector function that represents the curve of intersection is

$\boxed{\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \cos 2t\mathbf{k}}$.

Answer 44E.

We have $x^2+y^2+4z^2=4$ and $x^2+z^2=1$.

From $x^2+z^2=1$, we get the parametric equations for x as $\cos t$ and z as $\sin t$, where $0 \le t \le \frac{\pi}{2}$.

Substitute the known values in $x^2+y^2+4z^2=4$ and solve for y.

$$
\begin{aligned}
(\cos t)^2 + y^2 + 4(\sin t)^2 &= 4 \\
y^2 &= 4 - \cos^2 t - 4\sin^2 t \\
y^2 &= 3 - 3\sin^2 t \\
y^2 &= 3(\cos^2 t) \\
y &= \sqrt{3}\cos t
\end{aligned}
$$

Therefore, the vector function that represents the curve of intersection is

$\mathbf{r}(t) = \cos t\mathbf{i} + \sqrt{3}\cos t\mathbf{j} + \sin t\mathbf{k}$

Answer 45E.

Consider the following vectors;

$\mathbf{r}'(t) = 2t\mathbf{i} + 3t^2\mathbf{j} + \sqrt{t}\mathbf{k}$, and $\mathbf{r}(1) = \mathbf{i} + \mathbf{j}$

Determine, $\mathbf{r}(t)$.

To find $\mathbf{r}(t)$, use the following formula;

$$
\begin{aligned}
\mathbf{r}(t) &= \int \mathbf{r}'(t)\,dt \\
&= \int \left(2t\mathbf{i} + 3t^2\mathbf{j} + \sqrt{t}\mathbf{k}\right) dt \\
&= 2\left(\frac{t^2}{2}\right)\mathbf{i} + 3\left(\frac{t^3}{3}\right)\mathbf{j} + \frac{2}{3}t^{\frac{3}{2}}\mathbf{k} + c
\end{aligned}
$$

Here c is a constant of integration.

$\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + \frac{2}{3}t^{\frac{3}{2}}\mathbf{k} + c$ ...... (1)

Now, substitute $\mathbf{r}(1)=\mathbf{i}+\mathbf{j}$ in (1), we get

$$\mathbf{i}+\mathbf{j}=(1)^2\mathbf{i}+(1)^3\mathbf{j}+\frac{2}{3}(1)^{3/2}\mathbf{k}+c$$

$$c=-\frac{2}{3}\mathbf{k}$$

The substitute this value in (1), we get

$$\mathbf{r}(t)=t^2\mathbf{i}+t^3\mathbf{j}+\frac{2}{3}\left(t^{3/2}\right)\mathbf{k}-\frac{2}{3}\mathbf{k}$$

$$=t^2\mathbf{i}+t^3\mathbf{j}+\frac{2}{3}\left(t^{3/2}-1\right)\mathbf{k}$$

Hence, the result is $\boxed{\mathbf{r}(t)=t^2\mathbf{i}+t^3\mathbf{j}+\frac{2}{3}\left(t^{3/2}-1\right)\mathbf{k}}$.

Answer 46E.

Consider the vectors $\mathbf{r}'(t)=t\mathbf{i}+e^t\mathbf{j}+te^t\mathbf{k}$, and $\mathbf{r}(0)=\mathbf{i}+\mathbf{j}+\mathbf{k}$

Find the position vector $\mathbf{r}(t)$.

$$\int u\,dv=uv-\int v\,du$$

Then,

$$\mathbf{r}(t)=\int \mathbf{r}'(t)\,dt$$

$$=\int\left(t\mathbf{i}+e^t\mathbf{j}+te^t\mathbf{k}\right)dt$$

$$=\frac{t^2}{2}\mathbf{i}+e^t\mathbf{j}+\int te^t\mathbf{k}\,dt$$

$$=\frac{t^2}{2}\mathbf{i}+e^t\mathbf{j}+\left(t\int e^t dt-\int (t)'\left(\int e^t dt\right)dt\right)\mathbf{k} \qquad \text{(Method of by parts)}$$

$$=\frac{t^2}{2}\mathbf{i}+e^t\mathbf{j}+\left(te^t-\left(\int e^t dt\right)\right)\mathbf{k}$$

$$=\frac{t^2}{2}\mathbf{i}+e^t\mathbf{j}+\left(te^t-e^t\right)\mathbf{k}+\mathbf{c}$$

$$\mathbf{r}(t)=\frac{t^2}{2}\mathbf{i}+e^t\mathbf{j}+\left(te^t-e^t\right)\mathbf{k}+\mathbf{c} \quad \text{.......(1)}$$

Now, plug in $\mathbf{r}(0)=\mathbf{i}+\mathbf{j}+\mathbf{k}$ in (1),

$$\mathbf{i}+\mathbf{j}+\mathbf{k}=\frac{0^2}{2}\mathbf{i}+e^0\mathbf{j}+\left(0e^0-e^0\right)\mathbf{k}+\mathbf{c}$$
$$\mathbf{i}+\mathbf{j}+\mathbf{k}=\mathbf{j}-\mathbf{k}+\mathbf{c}$$
$$\mathbf{c}=\mathbf{i}+2\mathbf{k}$$

Plug in this into (1),

$$\mathbf{r}(t)=\frac{t^2}{2}\mathbf{i}+e^t\mathbf{j}+\left(te^t-e^t\right)\mathbf{k}+\mathbf{i}+2\mathbf{k}$$

Thus, $\boxed{\mathbf{r}(t)=\left(\frac{t^2}{2}+1\right)\mathbf{i}+e^t\mathbf{j}+\left(te^t-e^t+2\right)\mathbf{k}}$.

Answer 47E.

The trajectories of two particles are given by:

$r_1(t)=<t^2, 7t-12, t^2>$

And $r_2(t)=<4t-3,\ t^2,\ 5t-6>$

Then the parametric equations are:

$x=t^2,\ \ y=7t-12,\ \ z=t^2$

And $x=4t-3,\ \ y=t^2,\ \ z=5t-6$

Let us suppose the two particles collide. Then for some value of t,

$t^2=4t-3$

$7t-12=t^2$

And $t^2=5t-6$

Answer 48E.

$\vec{r}_1(t)=<t,t^2,t^3>$

And $\vec{r}_2(t)=<1+2t,\ 1+6t,\ 1+14t>$

The particle collide when $\vec{r}_1(t)=\vec{r}_2(t)$

i.e. $<t,t^2,t^3>=<1+2t,\ 1+6t,\ 1+14t>$

i.e. $t=1+2t$

$t^2=1+6t$

$t^3=1+14t$

The first of these equations gives $t = -1$ which does not satisfy the other equations. Therefore we can say that the two particles do not collide.

For the paths to intersect, $\vec{r}_1(t)=\vec{r}_2(s)$

i.e. $<t,t^2,t^3>=<1+2s,\ 1+6s,\ 1+14s>$

i.e. $t=1+2s$

$t^2=1+6s$

$t^3=1+14s$

On solving first two equations $s = 0, \frac{1}{2}$ and $t = 1, 2$

Clearly these two pairs satisfy the third equation

Thus the paths intersect twice at:

$(1,1,1)$, for $s = 0,\ t = 1$

And $(2,4,8)$, for $s = \frac{1}{2}, 2$

Answer 49E.

Let $\vec{u}(t) = < f_1(t), g_1(t), h_1(t) >$

And $\vec{v}(t) = < f_2(t), g_2(t), h_2(t) >$

(A)

Consider $\lim_{t\to a}\left[\vec{u}(t) + \vec{v}(t)\right]$

$$= \lim_{t\to a}\left[< f_1(t), g_1(t), h_1(t) > + < f_2(t), g_2(t), h_2(t)\right]$$

$$= \lim_{t\to a}\left[< f_1(t) + f_2(t), g_1(t) + g_2(t), h_1(t) + h_2(t) >\right]$$

$$= <\lim_{t\to a}\left[f_1(t) + f_2(t)\right], \lim_{t\to a}\left[g_1(t) + g_2(t)\right], \lim_{t\to a}\left[h_1(t) + h_2(t)\right]$$

$$= <\lim_{t\to a} f_1(t) + \lim_{t\to a} f_2(t), \lim_{t\to a} g_1(t) + \lim_{t\to a} g_2(t),$$
$$\lim_{t\to a} h_1(t) + \lim_{t\to a} h_2(t) >$$

$$= <\lim_{t\to a} f_1(t), \lim_{t\to a} g_1(t), \lim_{t\to a} h_1(t) >$$
$$+ <\lim_{t\to a} f_2(t), \lim_{t\to a} g_2(t), \lim_{t\to a} h_2(t) >$$

$$= \lim_{t\to a} < f_1(t), g_1(t), h_1(t) >$$
$$+ \lim_{t\to a} < f_2(t), g_2(t), h_2(t) >$$

$$= \lim_{t\to a} \vec{u}(t) + \lim_{t\to a} \vec{v}(t)$$

Hence $\lim_{t\to a}\left[\vec{u}(t) + \vec{v}(t)\right] = \lim_{t\to a} \vec{u}(t) + \lim_{t\to a} \vec{v}(t)$

(B)

Consider $\lim_{t\to a} c\vec{u}(t)$

$$= \lim_{t\to a} c < f_1(t), g_1(t), h_1(t) >$$

$$= \lim_{t\to a} < c f_1(t), c g_1(t), c h_1(t) >$$

$$= < \lim_{t\to a} c f_1(t), \lim_{t\to a} c g_1(t), \lim_{t\to a} c h_1(t) >$$

$$= < c\lim_{t\to a} f_1(t), c\lim_{t\to a} g_1(t), c\lim_{t\to a} h_1(t) >$$

$$= c < \lim_{t\to a} f_1(t), \lim_{t\to a} g_1(t), \lim_{t\to a} h_1(t) >$$

$$= c\lim_{t\to a} < f_1(t), g_1(t), h_1(t) >$$

$$= c\lim_{t\to a} \vec{u}(t)$$

Hence $\lim_{t\to a} c\vec{u}(t) = c\lim_{t\to a} \vec{u}(t)$

(C)

$$\lim_{t\to a} \left[\vec{u}(t).\vec{v}(t)\right]$$

$$= \lim_{t\to a} \left[< f_1(t), g_1(t), h_1(t) > . < f_2(t), g_2(t), h_2(t) > \right]$$

$$= \lim_{t\to a} \left[< f_1(t) f_2(t), g_1(t) g_2(t), h_1(t) h_2(t) > \right]$$

$$= < \lim_{t\to a} f_1(t) f_2(t), \lim_{t\to a} g_1(t) g_2(t), \lim_{t\to a} h_1(t) h_2(t) >$$

$$= < \lim_{t\to a} f_1(t) \lim_{t\to a} f_2(t), \lim_{t\to a} g_1(t) \lim_{t\to a} g_2(t), \lim_{t\to a} h_1(t) \lim_{t\to a} h_2(t) >$$

$$= < \lim_{t\to a} f_1(t), \lim_{t\to a} g_1(t), \lim_{t\to a} h_1(t) > . < \lim_{t\to a} f_2(t), \lim_{t\to a} g_2(t), \lim_{t\to a} h_2(t) >$$

$$= \lim_{t\to a} < f_1(t), g_1(t), h_1(t) > . \lim_{t\to a} < f_2(t), g_2(t), h_2(t) >$$

$$= \lim_{t\to a} \vec{u}(t) . \lim_{t\to a} \vec{v}(t)$$

Hence $\lim_{t\to a} \left[\vec{u}(t).\vec{v}(t)\right] = \lim_{t\to a} \vec{u}(t) . \lim_{t\to a} \vec{v}(t)$

(D)

Consider $\lim_{t\to a}\left[\vec{u}(t)\times\vec{v}(t)\right]$

$$= \lim_{t\to a}\left[< f_1(t), g_1(t), h_1(t) > \times < f_2(t), g_2(t), h_2(t) > \right]$$

$$= \lim_{t\to a}\left[\begin{array}{r} < g_1(t)h_2(t) - h_1(t)g_2(t), f_2(t)h_1(t) - f_1(t)h_2(t), f_1(t)g_2(t) - \\ f_2(t)g_1(t) > \end{array} \right]$$

$$= < \lim_{t\to a} g_1(t)h_2(t) - \lim_{t\to a} h_1(t)g_2(t), \lim_{t\to a} f_2(t)g_1(t) - \lim_{t\to a} f_1(t)h_2(t)$$
$$, \lim_{t\to a} f_1(t)g_2(t) - \lim_{t\to a} f_2(t)g_1(t) >$$

$$= < \lim_{t\to a} g_1(t)\lim_{t\to a} h_2(t) - \lim_{t\to a} h_1(t)\lim_{t\to a} g_2(t),$$
$$\lim_{t\to a} f_2(t)\lim_{t\to a} h_1(t) - \lim_{t\to a} f_1(t)\lim_{t\to a} h_2(t),$$
$$\lim_{t\to a} f_1(t)\lim_{t\to a} g_2(t) - \lim_{t\to a} f_2(t)\lim_{t\to a} g_1(t) >$$

$$= < \lim_{t\to a} f_1(t), \lim_{t\to a} g_1(t), \lim_{t\to a} h_1(t) > \times < \lim_{t\to a} f_2(t), \lim_{t\to a} g_2(t), \lim_{t\to a} h_2(t) >$$

$$= \lim_{t\to a} < f_1(t), g_1(t), h_1(t) > \times \lim_{t\to a} < f_2(t), g_2(t), h_2(t) >$$

$$= \lim_{t\to a}\vec{u}(t)\times\lim_{t\to a}\vec{v}(t)$$

Hence $\lim_{t\to a}\left[\vec{u}(t)\times\vec{v}(t)\right] = \lim_{t\to a}\vec{u}(t)\times\lim_{t\to a}\vec{v}(t)$