UPSEE 2013 - Solved Paper

Physics

Question 1

The resolving power of telescope depends on

Options:

A. focal length of eye lens

B. focal length of objective lens

C. length of the telescope

D. diameter of the objective lens

Answer: D

Solution:

The resolving power of a telescope is given by
$R = \frac{d}{1.22\lambda}$
where d is the aperture or diameter of the objective.
So, the resolving power of a telescope depends on the diameter of the objective lens.

Question 2

A ray is incident at an angle of incidence *i* on one surface of a prism of small angle *A* and emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, the angle of incidence *i* is nearly equal to

Options:

A. A/μ

B. $A/2\mu$

C. μ / A

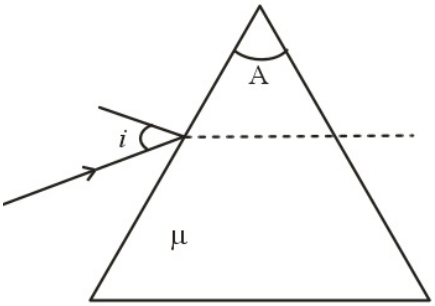
D. μA

Answer: D

Solution:

Given that the angle of the prism is, A. and refractive index of the

prism is μ.



As we know that the angle of deviation of the prism is

$\delta = (\mu - 1) A$ .....(i)

For the small angle of the prism,

given that the ray emerge normally so $\angle e = 0$

$\angle A + \angle \delta = \angle i + \angle e$

$\angle \delta = \angle i + \angle A$ .....(ii)

By Equation (i) and (ii)

$(\mu - 1)A = i - A \Rightarrow i = \mu A$

Question 3

With the propagation of a longitudinal wave through a material medium the quantities transmitted in the propagation direction are

Options:

A. energy, momentum and mass

B. energy

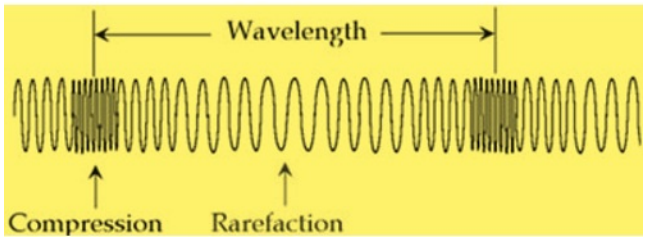
C. energy and mass

D. energy and linear momentum

Answer: D

Solution:

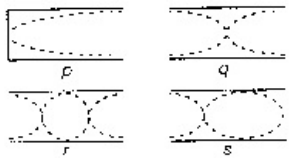
In longitudinal waves, the displacement of the medium is parallel to the propagation of the wave.



In the longitudinal wave the transmit energy and momentum is in the direction of propagation of wave.

Question 4

The vibrating of four air columns are represented in the figure. The ratio of frequencies $n_p : n_q : n_r : n_s$ is



Options:

A. 12 : 6 : 3 : 5

B. 1 : 2 : 4 : 3

C. 4 : 2 : 3 : 1

D. 6 : 2 : 3 : 4

Answer: B

Solution:

From the given figure

For Closed pipe p

In the closed pipe *P,* length of the pipe is, $L = \frac{\lambda_p}{4}$,

$\therefore \lambda_p = 4L$

Frequency in pipe P is,

$\Rightarrow \quad n_p = \frac{V}{\lambda_p} = \frac{V}{4L}$

For open pipe q

In the pipe q length of the pipe is, $L = \frac{\lambda_q}{2}$,

$\therefore \lambda_q = 2L$

Frequency in pipe q is

$\Rightarrow \quad n_q = \frac{V}{\lambda_q} = \frac{V}{2L}$

For open pipe r

In the pipe r length of the pipe is, $L = \lambda$

$\therefore \lambda_r = L$

Frequency in pipe r is

$\Rightarrow \quad n_r = \frac{V}{\lambda_r} = \frac{V}{L}$

For open pipe s

In the pipe s the length of the pipe is, $L = \frac{3\lambda_s}{4}$,

$\therefore \lambda_s = \frac{4L}{3}$

Frequency in the pipe s is

$\Rightarrow n_s = \frac{V}{\lambda_s} = \frac{3V}{4L}$

The ratio of frequency in the pipe, p, q, r, and s is

$n_p : n_q : n_r : n_s$

$\frac{V}{4L} : \frac{V}{2L} : \frac{V}{L} : \frac{3V}{4L}$

$\Rightarrow \frac{1}{4} : \frac{1}{2} : \frac{1}{1} : \frac{3}{4}$

$\Rightarrow 1:2:4:3$

Question 5

A source of sound emitting a tone of frequency 200 Hz moves towards an observer with a velocity *v* equal to the velocity of sound. If the observer also moves away from then source with the same velocity *v*, the apparent frequency heard by the observer is

Options:

A. 0 Hz

B. 100 Hz

C. 150 Hz

D. 200 Hz

Answer: D

Solution:

Given that the frequency of sound is, f = 200 Hz
The velocity of the source = Velocity of the sound = v
The velocity of the observer = velocity of the sound = v
As source and observer both are moving in the same direction with the same velocity, their relative velocity is zero.
Thus according to the Doppler effect, the apparent frequency is,
$\therefore f' = f = 200$ Hz
So apparent frequency heard by the observer is 200 Hz.

Question 6

If a gas has '*n*' degrees of freedom, the ratio of the specific heats γ of the gas is

Options:

A. $\frac{1+n}{2}$

B. $1+\frac{n}{2}$

C. $1+\frac{1}{n}$

D. $1+\frac{2}{n}$

Answer: D

Solution:

Let us consider 1 mole of an ideal gas at Kelvin temperature T. It has N molecules (Avogadro's number). The internal energy of an ideal gas is entirely kinetic. The average kinetic energy per molecule of a ideal gas is
$\frac{1}{2}nkT$,where n is degree of freedom, k is Boltzmann's constant)
Therefore the internal energy of N moles of an gas would be

$$E=N\left(\frac{1}{2}nkT\right) \qquad \left(\because k=\frac{R}{N}\right)$$

$$E=\frac{1}{2}nRT$$

Specific heat at constant volume is,

$$C_v=\frac{dE}{dT}=\frac{n}{2}R$$

According to Mayer's law, C_P - C_V = R
C_P = R + C_V = $R+\frac{n}{2}R=R\left(1+\frac{n}{2}\right)$
Specific heat at constant pressure is,

$$C_P=R\left(1+\frac{n}{2}\right)$$

Ratio of specific heat at constant pressure to constant volume is,

$$\frac{C_p}{C_v}=\gamma$$

$$\frac{C_p}{C_v}=\frac{\left(\frac{n}{2}+1\right)R}{\frac{n}{2}R}$$

$$\frac{C_p}{C_v}=\left(1+\frac{2}{n}\right)$$

Question 7

During an experiment, an ideal gas is found to obey an additional law

Vp^2=constant. The gas is initially at temperature T and volume V. What will be the temperature of the gas when it expand to a volume $2V$?

Options:

A. $\sqrt{3}T$

B. $T\sqrt{1/2}$

C. $T\sqrt{2}$

D. $T\sqrt{3}$

Answer: C

Solution:

Here it is given that Vp^2 = constant k (say)

So, $P=\sqrt{\frac{k}{V}}$

As we know that the ideal gas equation is, $pV = nRT$

$$\Rightarrow \sqrt{\frac{k}{V}}V = nRT$$

$$\Rightarrow \sqrt{V} = \frac{nR}{\sqrt{k}}T$$

Hence, $\sqrt{V} \propto T$

SO, $\sqrt{\frac{V_1}{V_2}} = \frac{T_1}{T_2}$

$$\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}}$$

$$T_2 = T_1\sqrt{\frac{V_2}{V_1}}$$

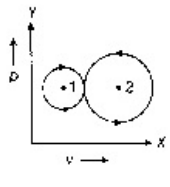
$$T_2 = T_1\sqrt{\frac{2V}{V}}$$

$$T_2 = T_1\sqrt{2}$$

So the temperature of the gas when it is expanded to 2V is $T\sqrt{2}$.

Question 8

In the indicator diagram shown in figure the net amount of work done is



Options:

A. negative

B. positive

C. zero

D. infinity

Answer: A

Solution:

From the figure the process 1 is clockwise, therefore W_1 is positive.
But in process 2 is anti-clockwise and work done W_2 is negative.
In the PV diagram, the work done is the area under the PV curve.
Since area of cycle 2 is > area of cycle 1
$W_2 > W_1$ and hence net work is done negative.

Question 9

A sphere, a cube and a thin circular plate have the same mass and are made of the same material. All of them are heated to the same temperature. The rate of cooling is

Options:

A. the maximum for the sphere and minimum for the plate

B. the maximum for the plate and minimum for the sphere

C. Both (A) and (B)

D. None of the above

Answer: B

Solution:

Given that each body made of the same material so the density of each material is same.
the density of the cube (ρ_{Cube}) = density of the square (ρ_{square}) = density of the circular plate ($\rho_{circular}$)
the mass of the cube (M_{Cube}) = mass of the square (M_{square}) = mass of the circular plate ($M_{circular}$)
Thus,
the volume of the cube (V_{Cube}) = volume of the square (V_{square}) =

volume of the circular plate ($V_{circular}$)

For the same volume, the surface area of the plate is the largest and of the sphere is the smallest.

Since the rate of loss of heat by radiation is proportional to surface area.

$$\frac{dH}{dt} \propto A$$

So the rate of cooling is maximum for plate and minimum for the sphere.

Question 10

When an ideal gas at pressure *P*, temperature *T* and volume V is isothermally compressed to V/n, its pressure becomes p_i. If the gas is compressed adiabatically to $\frac{V}{n}$, its pressure becomes P_a. The ratio p_i/p_a is

$\left(\gamma = \frac{C_p}{C_v}\right)$

Options:

A. 1

B. n

C. n^{γ}

D. $n^{(1-\gamma)}$

Answer: D

Solution:

As we know in the isothermal process temperature of the system is constant. So for an isothermal process,

PV= constant.

$P_iV_i = pV$

where, $V_i = \frac{V}{n}$

$$P_i \times \frac{V}{n} = pV$$

$$P_i = pn$$

In adiabatic, no heat is absorbed or radiate by the system, so for

adiabatic process

pV^{γ} =constant

$P_a(V_a)^{\gamma} = pV^{\gamma}$

where, $V_a = \left(\frac{V}{n}\right)$

$$P_a\left(\frac{V}{n}\right)^{\gamma} = pV^{\gamma}$$

$$P_a = pn^{\gamma}$$

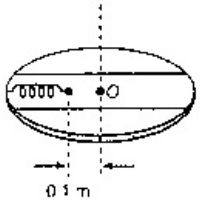
The ratio of p_i/p_a is

$$\frac{P_i}{P_a} = \frac{pn}{pn^{\gamma}}$$

$$\frac{P_i}{P_a} = n^{(1-\gamma)}$$

Question 11

A circular turntable of radius 0.5 m has a smooth groove as shown in the figure. A ball of mass 90 g is placed inside the groove along with a spring of spring constant 10^2 N/cm. The ball is at a distance of 0.1 m from the center when the turntable is at rest. On rotating the turntable with a constant angular frequency of 10^2 sec^{-1}, the ball moves away from the center by a distance nearly equal to



Options:

A. 10^{-1} m

B. 10^{-2} m

C. 10^{-3} m

D. 2×10^{-1} m

Answer: B

Solution:

The radius of the circular turntable is, r = 0.5 m
Mass of the ball is, m= 90 gm
spring constant is, k = 10^2 N/cm
The angular frequency of the table is, ω = 10^2 sec^{-1}
Distance of the ball from the centre of the table is, d = 0.1 m
Let the mass is moved by x distance. Hence, on balancing the centrifugal force, we get

$mw^2r = kx$

Here x *is the compression in the spring and* $r = x + 0.1$

Now, solving the above equation, we get (in SI units)

$9\times10^{-2}\times10^2\times10^2\times(0.1 + x) = 10^2\times10^2\times x$

so, $0.9\times10^{-2} + 9x\times10^{-2} = x$

$x = \frac{0.9\times10^{-2}}{(1 - 0.09)}$

Hence, $x = 0.00989\ m = 1\ cm = 10^{-2}\ m$

Question 12

A particular is moving in a vertical circle. The tension in the string when passing through two positions at angles 30^0 and 60^0 from vertical (lowest position) are T_1 and T_2 respectively

Options:

A. $T_1 = T_2$

B. $T_2 > T_1$

C. $T_1 > T_2$

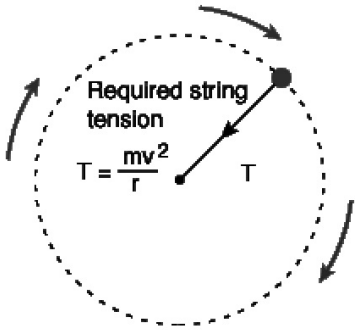
D. tension in the string always remain the same

Answer: C

Solution:

Tension in the string is,

$T = \frac{mv^2}{r} + mg\cos\theta$



For $\theta = 30^0$

$T_1 = \frac{mv^2}{r} + mg\cos 30^0$

$T_1 = \frac{mv^2}{r} + \frac{\sqrt{3}}{2}mg$ .....(1)

For $\theta = 60^0$

$T_2 = \frac{mv^2}{r} + mg\cos 60^0$

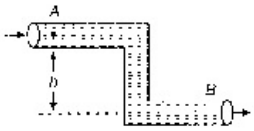
$\therefore \frac{mv^2}{r} + \frac{1}{2} mg$ .....(2)

From equation (1) and (2)

$T_1 > T_2$

Question 13

In this figure, an ideal liquid flows through the tube having uniform area of cross-section and is held in vertical plane. Find the speed of liquid at *A* and *B* and also find the pressure difference between these points



Options:

A. $2\rho gh$

B. ρgh

C. $\frac{3}{2}\rho gh$

D. zero

Answer: B

Solution:

As area of cross-section is uniform therefore according to equation of continuity speed of liquid is same at all points.

i.e., $V_A = V_B$

But during motion of liquid from *A* to *B* the potential energy decreases.

So according to Bernoulli's theorem,

$$P_1 + \frac{1}{2}\rho(v_1)^2 + \rho g h_1 = P_1 + \frac{1}{2}\rho(v_1)^2 + \rho g h_1$$

$$P_A + \frac{1}{2}\rho(v_A)^2 + \rho g h_A = P_B + \frac{1}{2}\rho(v_B)^2 + \rho g h_B$$

$$P_A + \rho g h_A = P_B + \rho g h_B \qquad (v_A = v_B)$$

$$P_B - P_A = \rho g\left(h_A - h_B\right)$$

$$P_B - P_A = \rho g h$$

Question 14

A sphere of radius *R* and density ρ_1 is dropped in a liquid of density σ. Its terminal velocity is v_1. If another sphere of radius *R* and density ρ_2 is dropped in the same liquid, its terminal velocity will be

Options:

A. $\left(\frac{\rho_2-\sigma}{\rho_1-\sigma}\right)v_1$

B. $\left(\frac{\rho_1-\sigma}{\rho_2-\sigma}\right)v_1$

C. $\left(\frac{\rho_1}{\rho_2}\right)v_1$

D. $\left(\frac{\rho_2}{\rho_1}\right)v_1$

Answer: A

Solution:

According to Stoke's theorem

$6\pi\eta Rv_1 = \frac{4}{3}\pi R^3(\rho_1-\sigma)$ .....(i)

If another ball of density ρ_2 is dropped into the same liquid

$6\pi\eta Rv_2 = \frac{4}{3}\pi R^3(\rho_2-\sigma)$ .....(ii)

Eq. (ii) dividing by Eq. (i) we get

$$\frac{6\pi\eta RV_2}{6\pi\eta RV_1}=\frac{\frac{4}{3}\pi R^3(\rho_2-\sigma)}{\frac{4}{3}\pi R^3(\rho_1-\sigma)}$$

$$\frac{V_2}{V_1}=\frac{(\rho_2-\sigma)}{(\rho_1-\sigma)}$$

$$V_2=\frac{(\rho_2-\sigma)}{(\rho_1-\sigma)}V_1$$

Question 15

A force *F* is required to break a wire of length *l* and radius *r*. What force is required to break a wire, of same material having twice the length and six times the radius?

Options:

A. F

B. $3F$

C. $9F$

D. $36F$

Answer: D

Solution:

Breaking force does not depend upon length.
Breaking force = Breaking stress × area of cross-section
And breaking stress is constant for a given material
$\because$ Breaking force $\propto$ area of cross-section
$F \propto A$

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$

$$\frac{F_2}{F_1} = \frac{\pi(R_2)^2}{\pi(R_1)^2}$$

$$\frac{F_2}{F_1} = \frac{(6R)^2}{(R)^2}$$

$$\frac{F_2}{F_1} = 36$$

$$F_2 = 36F_1$$

So force needed to break the wire is, 36F

Question 16

In a wire stretched by hanging a weight from its end, the elastic potential energy per unit volume in terms of the longitudinal strain σ and modulus of elasticity *Y* is

Options:

A. $Y\sigma^2/2$

B. $Y\sigma/2$

C. 2 $Y\sigma^2/2$

D. $Y^2\sigma/2$

Answer: A

Solution:

Given that the longitudinal strain of wire is, σ
Modulus of elasticity is, Y
Elastic potential energy per unit volume

E = $\frac{1}{2}$ × stress × strain

E = $\frac{1}{2}$ × [Young's modulus×strain] × strain

$E = \frac{1}{2} \times Y\sigma^2$

$E = \frac{Y\sigma^2}{2}$

Question 17

A projectile is fixed from level ground at an angle θ above the horizontal. The elevation angle Φ of the highest point as seen from the launch point is related to θ by the relation.

Options:

A. tan Φ = $\frac{1}{4}$ tan θ

B. tan Φ = tan θ

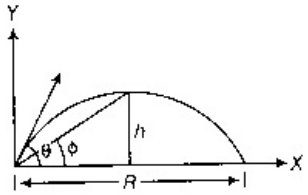
C. tan Φ = $\frac{1}{2}$ tan θ

D. tan Φ= 2tanθ

Answer: C

Solution:

From figure, tan Φ = $\frac{h}{(R/2)}$



Height (H) in a projectile motion = $\frac{u^2 \sin^2\theta}{2g}$

Range (R) of a projectile motion = $\frac{u^2 \sin 2\theta}{g}$, where θ is the angle by which the body is projected.

From Right angle triangle, tanΦ can be written as below:

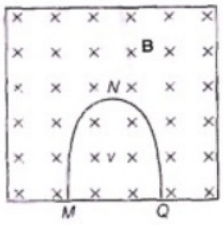
$\therefore \tan\phi = \frac{u^2 \sin^2\theta/2g}{u^2 \sin 2\theta/2g} = \frac{\sin^2\theta}{\sin 2\theta}$

$= \frac{\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\sin\theta}{2\cos\theta} = \frac{1}{2}\tan\theta$

Question 18

A thin semicircular conducting ring of radius *R* is falling with its plane

vertical in a horizontal magnetic field *B*. At the position *MNQ*, the speed of the ring is *v* and the potential difference across the ring is



Options:

A. zero

B. $\frac{1}{2}$ $Bv\pi R^2$ and M is at higher potential

C. $\pi RBv$and Q is at higher potential

D. $2RBv$and Q is at higher potential

Answer: D

Solution:

The induced emf in the conductor moving with velocity v is,
e = l(v× B)
The length of the MQ is, MQ = 2R
Since velocity is perpendicular to B. So
e = 2RvB
So the direction of induced emf will be from Q to M. So Q is at higher potential.

Question 19

A body of mass 2 kg moving with a velocity $(\hat{i}+2\hat{j}-3\hat{k})$ m/s collides with another body of mass 3 kg moving with a velocity $(2\hat{i}+\hat{j}+\hat{k})$ m/s. If they stick together, the velocity in m/s of the composite body is

Options:

A. $\frac{1}{5}(8\hat{i}+7\hat{j}-3k)$

B. $\frac{1}{5}(4\hat{i}+\hat{j}-3\hat{k})$

C. $\frac{1}{5}(5\hat{i}+\hat{j}-\hat{k})$

D. $\frac{1}{5}(-4\hat{i}+8\hat{j}-3\hat{k})$

Answer: A

Solution:

Let us assume that after the collision the velocity of the combined mass is, $\left(V_x\hat{i}+V_y\hat{j}+V_z\hat{k}\right)$

Initial momentum of the system is

$P_i=2\left(\hat{i}+2\hat{j}-3\hat{k}\right)+3\left(2\hat{i}+\hat{j}+\hat{k}\right)$

$P_i=8\hat{i}+7\hat{j}-3\hat{k}$

Final momentum of the system after the collision is

$P_f=5\left(V_x\hat{i}+V_y\hat{j}+V_z\hat{k}\right)$

According to the conservation of momentum

Initial momentum before collision = Final momentum after the collision

$8\hat{i}+7\hat{j}-3\hat{k}=5\left(V_x\hat{i}+V_y\hat{j}+V_z\hat{k}\right)$

$\left(V_x\hat{i}+V_y\hat{j}+V_z\hat{k}\right)=\frac{8}{5}\hat{i}+\frac{7}{5}\hat{j}-\frac{3}{5}\hat{k}$

$\left(V_x\hat{i}+V_y\hat{j}+V_z\hat{k}\right)=\frac{1}{5}\left(8\hat{i}+7\hat{j}-3\hat{k}\right)$

Question 20

A circular disc rolls down an inclined plane. The ratio of the rotational kinetic energy to the total kinetic energy is

Options:

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

Answer: B

Solution:

As we know that the moment of inertia of circular disc is, $I=\frac{MR^2}{2}$

Rational kinetic energy, $K_r=\frac{1}{2}I\omega^2$

$$K_r = \frac{1}{2} \times \frac{MR^2}{2} \omega^2 = \frac{1}{4} Mv^2 \quad (\because v = R\omega)$$

Translational kinetic energy

$$K_T = \frac{1}{2} Mv^2$$

Total kinetic energy= $K_T + K_R$

$$K = \frac{1}{2} Mv^2 + \frac{1}{4} Mv^2$$

$$K = \frac{3}{4} Mv^2$$

The ratio of the rotational kinetic energy to the total kinetic energy is

$$\frac{K_r}{K} = \frac{\frac{1}{4} Mv^2}{\frac{3}{4} Mv^2}$$

$$\frac{K_r}{K} = \frac{1}{3}$$

Question 21

A stone of mass *m* is tied to a string and is moved in a vertical circle of radius *'r'* making *'n'* revolution per minute. The total tension in the string when the stone is at its lowest point is

Options:

A. mg

B. $m(g+\pi nr^2)$

C. $m(g+\pi nr)$

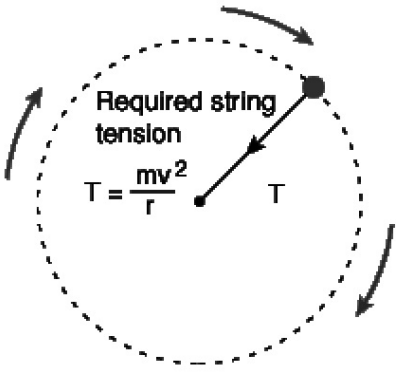
D. $m\{g+(\pi^2 n^2 r/900)\}$

Answer: D

Solution:

Weight of the stone is, W = mg

Centrifugal force on the wire is, F = $m\omega^2 r$



Tension in string when it reach lower-most point

$T = mg + m\omega^2 r$

$T = m(g + 4\pi^2 n^2 r)$ [as $\omega = 2\pi n$]

$$T = m\left\{g + 4\pi^2\left(\frac{n}{60}\right)^2 r\right\}$$

$$T = m\left\{g + \left(\frac{\pi^2 n^2 r}{900}\right)\right\}$$

Question 22

The angle between two vectors A and B is θ. Vector R is the resultant of the two vectors. If R makes an angle $\frac{\theta}{2}$ with A, then

Options:

A. $A = 2B$

B. $A = \frac{B}{2}$

C. $A = B$

D. $AB = 1$

Answer: C

Solution:

Given that the R makes an angle $\frac{\theta}{2}$ with A. Then

$$\text{Tan}\frac{\theta}{2} = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\frac{\sin(\theta/2)}{\cos(\theta/2)} = \frac{2B\sin(\theta/2)\cos(\theta/2)}{A + B\cos\theta}$$

$$\frac{1}{\cos(\theta/2)} = \frac{2B\cos(\theta/2)}{A + B\cos\theta}$$

$$A + B\cos\theta = 2B\cos^2(\theta/2)$$

$$A + B\left[2\cos^2\left(\frac{\theta}{2}\right) - 1\right] = 2B\cos^2\left(\frac{\theta}{2}\right)$$

A - B = 0

$A = B$

Question 23

A point moves with uniform acceleration and v_1, v_2 and v_3 denote the average velocities in the three successive intervals of time t_1, t_2, and t_3. Which of the following relation is correct?

Options:

A. $(v_1\text{-}v_2):(v_2\text{–}v_3)=(t_1\text{-}t_2):(t_2+t_3)$

B. $(v_1\text{-}v_2):(v_2\text{–}v_3)=(t_1 + t_2):(t_2+t_3)$

C. $(v_1\text{-}v_2):(v_2\text{–}v_3)=(t_1\text{-}t_2):(t_1\text{–}t_3)$

D. $(v_1\text{-}v_2):(v_2\text{–}v_3)= (t_1\text{-}t_2):(t_2 - t_3)$

Answer: B

Solution:

Let u be the initial velocity,

$v_1 = u + at_1$ .....(1)

$v_2 = u + a(t_1 + t_2)$ .....(2)

$v_3 = u + a(t_1 + t_2 + t_3)$ .....(3)

Average velocity, $V_1 = \frac{u + v_1}{2} = \frac{u + (u + at_1)}{2} = u + \frac{1}{2}at_1$

Average velocity, $V_2 = \frac{V_1 + V_2}{2} = u + at_1 + \frac{1}{2}at_2$

Average velocity, $V_3 = \frac{V_2 + V_3}{2} = u + at_1 + at_2 + \frac{1}{2}at_3$

$(V_1 - V_2) = -\frac{1}{2}a(t_1 + t_2)$

$(V_2 - V_3) = -\frac{1}{2}a(t_2 + t_3)$

$\therefore (V_1\text{-}V_2) : (V_2\text{-}V_3) = (t_1 + t_2) : (t_2 + t_3)$

Question 24

A particle is dropped from a height *H*. The de-Broglie wavelength of the particle as a function of height is proportional to

Options:

A. H

B. $H^{1/2}$

C. H^0

D. $H^{-1/2}$

Answer: D

Solution:

The initial velocity of the particle is, u = 0

The particle is dropped from the height H

So, according to the law of motion

$v^2 = u^2 + 2gh$

$v^2 = 2gH$

$v = \sqrt{2gH}$

Velocity acquired by a particle while falling from a height H is $v = \sqrt{2gH}$

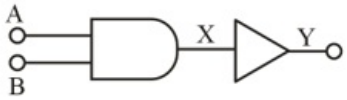
According to De-Broglie wavelength,

$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}}$

$\lambda \propto H^{-1/2}$

Question 25

Which one of the following represents correctly the truth table of the configuration shown in figure?



Options:

A.

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

B.

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

C.

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

D.

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

Answer: C

Solution:

Given that the input of the circuit is A and B, and the output is Y.

A	B	X	Y
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Question 26

The radius of Ge nuclide is measured to be twice the radius of $_4Be^9$. The number of nucleon in Ge are

Options:

A. 72

B. 73

C. 74

D. 75

Answer: A

Solution:

Given that $R_{Ge} = 2R_{Be}$

As we know that the radius of any nuclei is, $R = R_oA^{1/3}$

Where (R_0 is a constant, A is the atomic number)

$R_0(A_{Ge})^{1/3} = 2\ R_0(A_{Be})^{1/3}$

$$\left(\frac{A_{Ge}}{A_{Be}}\right)^{1/3} = 2$$

$A_{Ge} = 8A_{Be}$

$A_{Ge} = 8 \times 9 = 72$

Question 27

Which of the following has zero average value in a plane electromagnetic wave?

Options:

A. Kinetic energy

B. Magnetic field

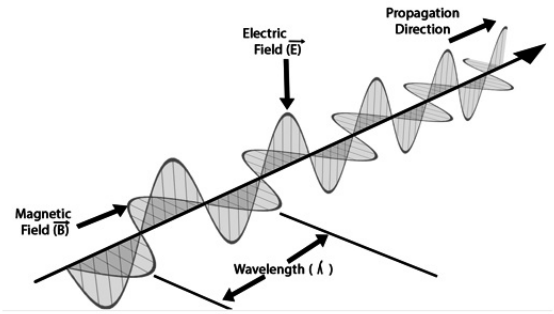
C. Electric field

D. Both (B) and (C)

Answer: D

Solution:

An electromagnetic wave which radiated by an accelerated charge propagates through space as coupled electric and magnetic field, oscillating perpendicular to each other and to the direction of propagation of the wave.



In an electromagnetic wave, the average value of an electric field or magnetic field is zero because both fields oscillate as sin and cosine waves which, over one period, average zero.

Question 28

Two identical cells of emf E and internal resistance r are connected in parallel with an external resistance R. To get maximum power developed across R, the value of R is

Options:

A. $R=r/2$

B. $R=r$

C. $r=r/3$

D. $R=2r$

Answer: A

Solution:

As two cell are connected in parallel combination. Then their internal resistance is

$$\frac{1}{r_{eq}}=\frac{1}{r}+\frac{1}{r}$$

$$\frac{1}{r_{eq}}=\frac{2}{r}$$

$$r_{eq}=\frac{r}{2}$$

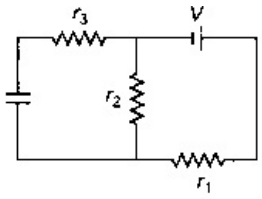
So, their internal resistance $= \frac{r}{2}$

From maximum power theorem

Internal resistance = external resistance = $\frac{r}{2} = R$

Question 29

In the circuit of given figure, the final voltage drop across the capacitor *C* is



Options:

A. $\frac{Vr_1}{r_1 + r_2}$

B. $\frac{Vr_2}{r_1 + r_2}$

C. $\frac{V(r_1 + r_2)}{r_2}$

D. $\frac{V(r_1 + r_2)}{r_1 + r_2 + r_3}$

Answer: B

Solution:

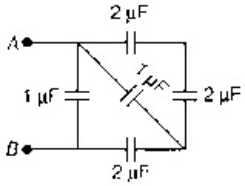
In steady state, the capacitor arm produce an infinite resistance. So, the potential difference across C is that across r_2.

So, current through $r_2 = \frac{V}{(r_1 + r_2)}$

Potential difference across $r_2 = \frac{V(r_2)}{(r_1 + r_2)}$

Question 30

The total capacitance of the system of capacitors in figure. Between *A* and *B* is



Options:

A. 1μF

B. 2μF

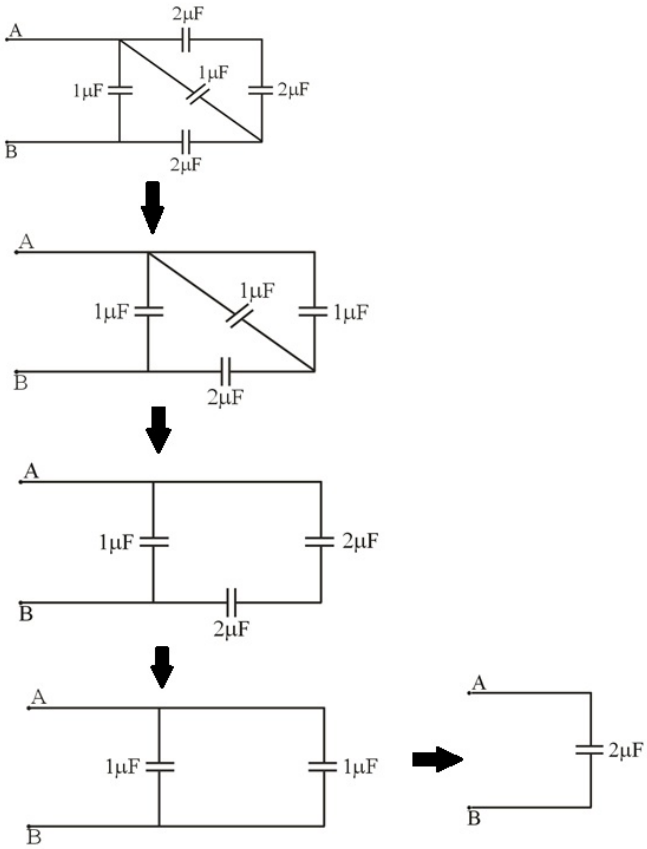
C. 3μF

D. 4μF

Answer: B

Solution:

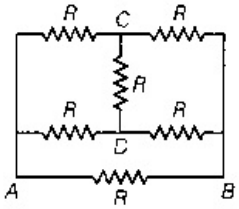
The simplified circuit is shown in



So the equivalent capacitance between A and B is 2μF.

Question 31

In the circuit shown in figure, the effective resistance between *A* and *B* is



Options:

A. $\frac{R}{2}$

B. R

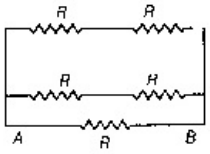
C. $2R$

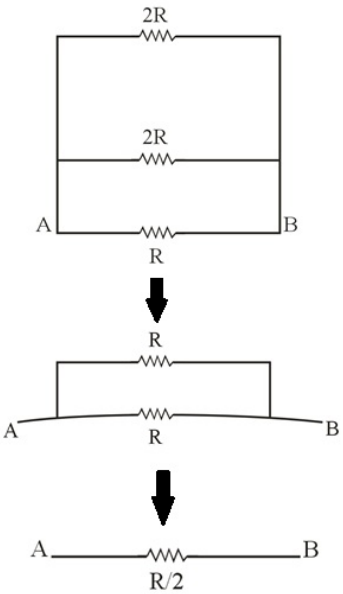
D. $4R$

Answer: A

Solution:

The upper part of the circuit is balanced Wheat stone's bridge. So, resistance R in branch CD is ineffective as no current will flow in this branch. The circuit, therefore, reduces to three parallel branches having resistance R, $R + R$ and $R + R$.



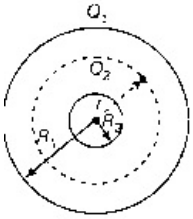


So the effective resistance between A and B is R/2.

Question 32

Two concentric, thin, metallic, spheres of radii R_1 and R_2 ($R_1 > R_2$) bear

charges Q_1 and Q_2 respectively. Then the potential at radius r between R_1 and R_2 will be$1/4\pi\varepsilon_0$ times



Options:

A. $\frac{Q_1 + Q_2}{r}$

B. $\frac{Q_1}{R_1} + \frac{Q_2}{r}$

C. $\frac{Q_1}{R_1} + \frac{Q_2}{R_2}$

D. $\frac{Q_1}{R_2} + \frac{Q_2}{R_1}$

Answer: B

Solution:

Potential at radius, r *is*

V_r = potential at P due to Q_1 + potential at P due to Q_2

$V_r = V_1 + V_2$

$$V_r = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q_1}{R_1} + \frac{Q_2}{r}\right]$$

Question 33

A charge situated at certain distance from an electric dipole in the end on position, experience a force F. If the distance of charge is doubled, the force acting on the charge will be

Options:

A. 2 F

B. $F/2$

C. $F/4$

D. $F/8$

Answer: D

Solution:

In case of electric dipole the force experienced by the charge q at distance r in end position is,

$$F \propto \frac{1}{r^3}$$

$$F \propto \frac{k}{r^3} \qquad \text{.....(i)}$$

If the distance between the charge is double then the force

$$F' = \frac{k}{(2r)^3} = \frac{k}{8r^3} \qquad \text{.....(ii)}$$

From Eqs. (i) and (ii), we get

$$F' = \frac{F}{8}$$

Question 34

At what angle (θ) with the horizontal should a body be projected so that its horizontal range equals the maximum height it attains?

Options:

A. $\theta = \tan^{-1}(\sqrt{2})$

B. $\theta = \tan^{-1}(2\sqrt{2})$

C. $\theta = \tan^{-1}(2\sqrt{3})$

D. $\theta = \tan^{-1}(4)$

Answer: D

Solution:

We know that in projectile motion range is

$$R = \frac{u^2 \sin 2\theta}{g}$$

maximum height in projectile motion is,

$$h_{max} = \frac{u^2 \sin^2\theta}{2g}$$

Given that the range equals to the maximum height. So $R = h_{\max}$

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$$\sin 2\theta = \frac{\sin^2\theta}{2}$$

$$2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$$

$$\cos\theta = \frac{\sin\theta}{4}$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1}(4)$$

Question 35

Dimensions $[ML^{-1}T^{-1}]$ are related to

Options:

A. torque

B. work

C. energy

D. coefficient of viscosity

Answer: D

Solution:

Physical Quantity	Formula	Dimension
Torque	distance ×Force (r×F)	$[ML^2T^{-2}]$
Work	Force ×displacement (F.d)	$[ML^2T^{-2}]$
Energy	Mass×gravity×height (mgh)	$[ML^2T^{-2}]$
Coefficient of viscosity	$\frac{\text{force / area}}{\text{velocity gradient}}$	$[ML^{-1}T^{-1}]$

Question 36

The dimension of $\frac{a}{b}$ in the equation $p = \frac{a - t^2}{bx}$, where *p* is pressure, *x* is distance and *t* is time is

Options:

A. $[LT^{-3}]$

B. $[ML^{-3}\ T^{-1}]$

C. $[M^2LT^{-3}]$

D. $[MT^{-2}]$

Answer: D

Solution:

Given that the pressure is, $P = \frac{a - t^2}{bx} = \frac{a}{bx} - \frac{t^2}{bx}$

According to the principal of homogeneity

Dimension of pressure = $\left[\frac{\text{force}}{\text{area}}\right]=\left[\frac{a}{bx}\right]$

$[ML^{-1}T^{-2}]=\left[\frac{a}{b}\right]\left[\frac{1}{L}\right]$, So

$\left[\frac{a}{b}\right]=[MT^{-2}]$

Question 37

If the distance between the earth and the sun is half its present values, the number of days in a year would have been

Options:

A. 730

B. 182.5

C. 129

D. 64.5

Answer: C

Solution:

According to Kepler's law of the period

$T^2 \propto R^3$

$T \propto R^{3/2}$

$\frac{T_1}{T_2}=\left(\frac{R_1}{R_2}\right)^{3/2}$

Now the distance between earth and the Sun is, $R_2=\frac{R_1}{2}$.

So time period is

$\frac{T_1}{T_2}=\left(\frac{2R_1}{R_1}\right)^{3/2}$

$\frac{T_1}{T_2}=(2)^{3/2}$

$T_2=\frac{T_1}{(2)^{3/2}}$

$T_2=\frac{365}{(2)^{3/2}}$

$T_2=\frac{365}{2.82}$

$T_2=129 \text{ days}$

Question 38

In a SHM, when the displacement is one half the amplitude, what fraction of the total energy is kinetic?

Options:

A. zero

B. 1/4

C. 1/2

D. 3/4

Answer: D

Solution:

Kinetic energy of a simple harmonic motion is,

K.E = $\frac{1}{2}m\omega^2(a^2\text{-}x^2)$

And at x = a/2

K.E = $\frac{1}{2}m\omega^2[a^2\text{-}(a/2)^2]$

K.E $= \frac{3}{4}\left[\frac{1}{2}m\omega^2a^2\right]$

Total energy in simple harmonic motion is, E $= \frac{1}{2}m\omega^2a^2$

$\therefore$ The fraction of total energy

$$\frac{K.E}{E} = \frac{\frac{3}{4}\left[\frac{1}{2}m\omega^2a^2\right]}{\frac{1}{2}m\omega^2a^2}$$

$$\frac{K.E}{E} = \frac{3}{4}$$

Question 39

A body is executing simple harmonic motion. At a displacement *x*, its potential energy is E_1 and at a displacement *y* its potential energy is E_2. The potential energy *E* at a displacement (x+y) is

Options:

A. $E_1 + E_2$

B. $\sqrt{E_1^2 + E_2^2}$

C. $E_1 + E_2 + 2\sqrt{E_1E_2}$

D. $\sqrt{E_1E_2}$

Answer: C

Solution:

At displacement *x*, the potential energy of simple harmonic motion is

$E_1 = \frac{1}{2}\ m\omega^2 x^2$

$\sqrt{E_1} = x\sqrt{\frac{1}{2}m\omega^2}$ .....(i)

At displacement y, the potential energy of simple harmonic motion is

$E_2 = \frac{1}{2}\ m\omega^2 (y)^2$

$\sqrt{E_2} = y\sqrt{\frac{1}{2}m\omega^2}$ ......(ii)

At displacement (x + y), the potential energy of the simple harmonic motion is

$E = \frac{1}{2}\ m\omega^2 (x + y)^2$

$\sqrt{E} = (x + y)\sqrt{\frac{1}{2}m\omega^2}$ ......(iii)

From Eqs. (i), (ii) and (iii) it follows that

$\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$

On squaring both sides

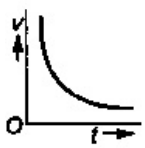
$(\sqrt{E})^2 = \left(\sqrt{E_1} + \sqrt{E_2}\right)^2$

$E = E_1 + E_2 + 2\sqrt{E_1 E_2}$

Question 40

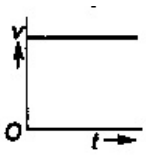
Which one of the following graph between velocity *versus* time for body falling in viscous fluid is correct

Options:

A. 

B. 

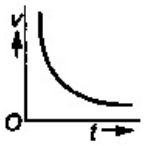
C. 

D. 

Answer: A

Solution:

When a body is falling in viscous fluid, from rest, after some time the body moves with constant terminal velocity. So initially the velocity of the body is decreasing but after some time its velocity becomes constant which is correctly represented in the graph (1).



Question 41

A body of mass M is kept on a rough horizontal surface (friction coefficient μ). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on A is F, where

Options:

A. $F = Mg$

B. $F = \mu MgF$

C. f $Mg \leq f \geq Mg\sqrt{1+\mu^2}$

D. $Mg \geq f \geq Mg\sqrt{1+\mu^2}$

Answer: C

Solution:

If R is normal reaction,
Maximum force by surface when friction force work
$F = \sqrt{f^2 + R^2} = \sqrt{\mu R^2 + R^2} \quad (\because f = \mu R)$
$F = R\sqrt{\mu^2 + 1}$
Maximum force = R, when there is no friction.
Hence, ranging from R to $R\sqrt{\mu^2+1}$
We get $Mg \leq f \leq mg\sqrt{\mu^2+1}$

Question 42

The ratio of acceleration due to gravity at a height *h* above the surface of the earth and at a depth *h* below the surface of the earth *h* << radius of earth.

Options:

A. is constant

B. increases linearly with *h*

C. decreases linearly with *h*

D. decreases parabolically with *h*

Answer: C

Solution:

Acceleration due to gravity height *h* above the surface of the earth is

$g_1 = g\left(1-\frac{2h}{R}\right)$

Acceleration due to gravity at depth *h* below the surface of the earth is

$g_2 = g\left(1-\frac{h}{R}\right)$

Ratio of the gravity at above height h to the at depth h is,

$$\frac{g_1}{g_2}=\frac{\left(1-\frac{2h}{R}\right)}{\left(1-\frac{h}{R}\right)}$$

$$\frac{g_1}{g_2}=\left(1-\frac{2h}{R}\right)\left(1-\frac{h}{R}\right)^{-1}$$

$$\frac{g_1}{g_2}=\left(1-\frac{2h}{R}\right)\left(1+\frac{h}{R}\right)$$

$$\frac{g_1}{g_2}=1+\frac{h}{R}-\frac{2h}{R}-\frac{2h^2}{R^2}$$

$$\frac{g_1}{g_2}=1-\frac{h}{R} \qquad (\text{ Since } h<<R\ )$$

$\therefore \frac{g_1}{g_2}$ decreases linearly with *h*.

Question 43

What is self inductance of a coil when a charge of current from 0 to 2A in 0.05 second induces an emf of 40 V in it?

Options:

A. 1 H

B. 2 H

C. 3 H

D. 4 H

Answer: A

Solution:

The Induced emf in the inductance is

$e = -L\frac{dI}{dt}$

$40V = L \times \frac{2}{0.05}$

$L = \frac{40 \times 0.05}{2}$

$L = 1\ \text{H}$

Question 44

Permanent magnet has properties retentivity and coercivity respectively?

Options:

A. high - high

B. low - low

C. low - high

D. high – low

Answer: A

Solution:

Coercivity is the intensity of the applied magnetic field required to reduce the magnetization of a given material to zero.

Retentivity is the capacity of an object to retain magnetism after the action of the magnetizing force has ceased.

The material for a permanent magnet should have high retentivity (so that magnet is strong) and high coercivity (so that magnetism is not wiped out by strong magnetic fields).

Question 45

The flux linked with a circuit is given by $\Phi = t^3 + 3t - 7$. The graph between time (x-axis) and induced emf (y-axis) will be

Options:

A. a straight line through the origin

B. straight line with positive intercept

C. straight line with negative intercept

D. parabola not through origin

Answer: D

Solution:

Given that the flux linked with the circuit is, $\Phi = t^3 + 3t - 7$

As we know that the induced emf is, $e = -\frac{d\phi}{dt}$

$$e = -\frac{d}{dt}\left(t^3 + 3t - 7\right)$$

$$e = -\left(3t^2 + 3\right)$$

$$e = -3\left(t^2 + 1\right)$$

From the above equation, the graph between e and t (along x axis) will be a parabola.

At t = e,

e = -3($\neq$0)

$\therefore$ The parabola would not pass through the origin.

Question 46

An inductor of 1 H is connected across a 220 V, 50 Hz supply. The peak value of the current is approximately.

Options:

A. 0.5 A

B. 0.7 A

C. 1 A

D. 1.4 A

Answer: C

Solution:

The peak value of the current in the circuit is

$I_o = \frac{E_o}{X_L}$

$I_o = \frac{E\sqrt{2}}{L\omega}$

$I_o = \frac{E\sqrt{2}}{L \times 2\pi f}$

$I_o = \frac{220\sqrt{2}}{1 \times 100\pi}$

$I_o \simeq 1 \text{ A}$

Question 47

Power supplied to a particle of mass 2 kg varies with time as $P = \frac{t^2}{2}$ watt, where *t* is in second. If the velocity of particle at *t*=0 is *v*=0, the velocity of particle at time *t*=2s will be

Options:

A. 1 m/s

B. 4 m/s

C. $2\sqrt{2}$ m/s

D. $2\sqrt{\frac{2}{3}}$ m/s

Answer: D

Solution:

According to the work-energy theorem,
Work done equals to the change in the kinetic energy

$$\int P\,dt = \triangle K$$

$$\int_0^2 \left[\frac{t^2}{2}\right] dt = K_{\text{final}} - K_{\text{initial}}$$

$$\frac{1}{2\times 3}\left[t^3\right]_0^2 = \frac{1}{2}mv^2 - 0$$

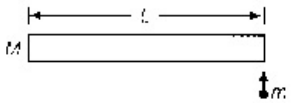
$$\frac{8}{2\times 3} = \frac{1}{2}\times 2\times v^2$$

$$v = \sqrt{\frac{4}{3}}$$

$$v = 2\sqrt{\frac{2}{3}}\ \text{m/s}$$

Question 48

A stick of length *L* and mass *M* lies on a frictionless horizontal surface on which it is free to move in any way. A ball of mass *m* moving with speed *v* collides elastically with the stick as shown in figure.
If after the collision the ball comes to rest, then what should be the mass of the ball?



Options:

A. $m = 2M$

B. $m = M$

C. $m = M/2$

D. $m = M/4$

Answer: D

Solution:

Given that the after collision ball is at rest so the velocity of the stick is V.
According to the conservation of momentum
Initial momentum = Final momentum
mv + M×0 = MV + m×0
mv = MV ……(i)
Conservation of angular momentum
$mv\left(\frac{L}{2}\right) = \left(\frac{mL^2}{12}\right)\omega$ ……(ii)

As the collision is elastic, we have

$\frac{1}{2}mv^2 = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$ .....(iii)

Substituting the values, we get

m = M / 4

Question 49

The refracting angle of a prism is *A* and the refractive index is cot $\left(\frac{A}{2}\right)$.

The angle of minimum deviation is

Options:

A. 180^0-A

B. 180^0-2A

C. 180^0-3A

D. 180^0-4A

Answer: B

Solution:

Given that the refractive index of prism is, $\mu = \cot\left(\frac{A}{2}\right)$

As we know that the refractive index of prism is

$$\mu = \frac{\sin\left(\frac{A+\vartheta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\cot\left(\frac{A}{2}\right) = \frac{\sin\left(\frac{A+\vartheta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$\cos\left(\frac{A}{2}\right) = \sin\left(\frac{A+\vartheta_m}{2}\right)$

$\text{Sin}\left(90^0 - \frac{A}{2}\right) = \sin\left(\frac{A+\vartheta_m}{2}\right)$

Which gives $\frac{A+\vartheta_m}{2} = 90^0 - \frac{A}{2}$

Or $\vartheta_m = 180^o - 2A$

Question 50

Two coherent light sources S_1 and S_2 (λ =6000Å) are 1mm apart from each other. The screen is places at a distance of 25 cm from the sources. The width of the fringes on the screen should be

Options:

A. 0.015 cm

B. 0.013 cm

C. 0.01 cm

D. 0.10 cm

Answer: A

Solution:

Given that the wavelength of light source is, λ = 6000Å = 6000 × 10^{-8} cm

D= 25 cm

Distance between the slits is, 2d = 1mm = 0.1 cm

According to Young's double slit experiment the fringe width is

$$\beta = \frac{D\lambda}{2d},$$

$$\therefore \beta = \frac{6000 \times 10^{-8} \times 25}{0.1}$$

β = 0.015 cm

Chemistry

Question 51

If a LPG cylinder contains mixture of butane and isobutane, then the amount of oxygen that would be required for combustion of 1 kg of it will be

Options:

A. 2.50×10^3 g

B. 4.50×10^3 g

C. 1.80×10^3 g

D. 3.58×10^3 g

Answer: D

Solution:

The moelcular formula for butane and isobutane is C_4H_{10}. The reaction for combustion of C_4H_{10} is as follows:

$C_4H_{10}(g) + \frac{13}{2} O_2(g) \rightarrow 4CO_2(g) + 5H_2O(l)$;

According to the given reaction

Amount of oxygen required for combustion of 58 g of butane

$$= \left(\frac{13}{2} \times 32 = 208 \text{ g}\right)$$

Amount of oxygen required for combustion of 1000 g of butane will be

$$= \frac{208}{58} \times 1000 \text{ g}$$

$$= 3.58 \times 10^3 \text{ g}$$

Question 52

A radioactive isotope having a half-life of 3 days was received after 12 days. It was found that there were 3 g of the isotope in the container. Find the initial weight of the isotope, where it was packed (antilog 1.203=16)

Options:

A. 12 g

B. 24 g

C. 36 g

D. 48 g

Answer: D

Solution:

No. Of half-lives,

$n = \frac{12}{3} = 4$

Amount left after n half-lives $= \frac{A_0}{2^n}$

$3g = \frac{A_0}{2^4}$

Or $3g = \frac{A_0}{16}$

$\therefore A^0 = 48$ g

Question 53

The pairs of compounds which cannot exists together in a solution is

Options:

A. $NaHCO_3$ and NaOH

B. Na_2CO_3 and NaOH

C. Na_2CO_3 and $NaHCO_3$

D. $NaHCO_3$ and NaCl

Answer: A

Solution:

$NaHCO_3$ is amphoteric, *i.e.*, it act as either an acid or a base. Dissolved in water, the pH will be slightly higher than 7, which is basic. Hence, combined with NaOH (a strong base), it gives carbonate and water.

$NaHCO_3 + NaOH \rightarrow Na_2CO_3 + H_2O$

That's why $NaHCO_3$ and NaOH cannot exist together in solution

Whereas, Na_2CO_3 is slightly basic and NaCl is neutral, thus doesn't react with $NaHCO_3$.

Question 54

The wave number of hydrogen atom in Lymen series is 82200 cm^{-1}. The electron goes from

Options:

A. $n_3 \rightarrow n_2$

B. $n_2 \rightarrow n_1$

C. $n_4 \rightarrow n_3$

D. none of these

Answer: B

Solution:

According to Rydberg formula,

$$\overline{v} = RZ^2\left[\frac{1}{{n_1}^2} - \frac{1}{{n^2}_2}\right]$$

Z corresponds to atomic number, R is Rydberg's constant .
for H atom, Z=1 so,

$$\bar{v} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here, $R = 109677 \cong 109600$ cm^{-1}

and $\bar{v}$ = 82200 cm^{-1}

$$\frac{\bar{v}}{R} = \left(\frac{1}{n_1 2} - \frac{1}{n_2{}^2}\right)$$

Thus,

$$\frac{82200}{109600} = \left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

or, $\frac{3}{4} = \left(1 - \frac{1}{n_2^2}\right)$

or, $\frac{1}{n_2^2} = 1 - \frac{3}{4} = \frac{1}{4}$

or $n_2 = 2$.

$\therefore$ The electron jumps from second orbit (n_2) to ground state (n_1).

Hence option(2) is correct.

Question 55

At room temperature, the reaction between NO and O_2 to give NO_2 is fast while that of between CO and O_2 is slow. It is because

Options:

A. the intrinsic energy of the reaction $2\,NO + O_2 \rightleftharpoons 2NO_2$ is less

B. CO is smaller in size than that of NO

C. CO is poisonous

D. the activation energy for the reaction $2\,NO + O_2 \rightleftharpoons 2NO_2$ is less

Answer: D

Solution:

Evidently, less is the activation energy, faster is the reaction or greater is the activation energy, slower is the reaction.

Question 56

If 5 L of H_2O_2 produces 50 L of O_2 at NTP, H_2O_2 is

Options:

A. 50 Volume

B. 10 Volume

C. 5 Volume

D. None of the above

Answer: B

Solution:

the decomposition reaction of H_2O_2 is:
$H_2O_2(aq) \rightarrow H_2O + O_2(g)$

According to the quetion,
5 L of H_2O_2 produces 50 L of O_2 .
so, 1 L of H_2O_2 produces=50/5 =10L of O_2

Thus,a solution of H_2O_2 labelled as 10ml volume actually means that 1 mL of such a solution of H_2O_2 on decomposition by heat produces 10 mL of oxygen at NTP. Hence, 5 L of 10 volume H_2O_2 will produced 50 L of O_2 at NTP.
Hence option (2) is correct.

Question 57

For the reaction,

the change of enthalpy at 27^0C is 9 kcal.
The value of *E*is

Options:

A. 21.2 kcal

B. 17.8 kcal

C. 18.4 kcal

D. 20.6 kcal

Answer: B

Solution:

The reaction is,
$A(g) + 2B(g) \rightarrow 2\ C(g) + 3D\ (g)$
Here $\triangle n_g = n_p - n, = 5 - 3 = 2$
$\triangle H = 19$ kcal $= 19 \times 10^3$ cal
$\therefore$ The value of internal energy
$\triangle E = \triangle H - \triangle n_g RT$
$= 19 \times 10^{-3} - (2 \times 2 \times 300)$
$= 17.8$ cal

Question 58

A chemist wishes to prepare a buffer solution of pH = 2.90 that efficiently resists a change in pH yet contains only small concentration of buffering agents. Which one of the following weak acid along with its salt would be best to use

Options:

A. *m*-chlorobenzoic acid ($pK_a = 3.98$)

B. Acetoacetic acid ($pK_a = 3.58$)

C. 2.5-dihydrobenzoic acid ($pK_a = 2.97$)

D. *p*-chlorocimanic acid ($pK_a = 4.41$)

Answer: C

Solution:

The buffer solution to be made should effectively resists a change in pH, *i.e.,* should have a higher buffer capacity.
Buffer capacity (Φ)
$$= \frac{\text{No. of moles of acid or base added to 1 L}}{\text{change in pH}}$$
By one of any two ways, we can obtain it.
(i) The higher the concentration of the components of a buffer mixture, the greater will be the buffer capacity. But, the condition is to maintain a small concentrations of buffering components.
(ii) Buffer capacity of a buffer is maximum when the concentration of the weak acid and its salt are equal, *i.e.*, [salt] = [acid]

$\therefore$ pH = pK_a + log $\frac{[salt]}{[acid]}$

= pK_a + log 1 (as [salt] = [acid])

Or pH = pK_a = 2.9

As, 2, 5-dihydrobenzoic acid has pK_a value = 2.297, hence it is the best choice for chemist to prepare buffer solution.

Question 59

The equilibrium constants for the reaction, $Br_2 \rightleftharpoons 2\ Br \longrightarrow 1$
At 500 K and 700 K are 1×10^{-10} and 1×10^{-5} respectively. The reaction is

Options:

A. endothermic

B. exothermic

C. fast

D. slow

Answer: A

Solution:

Chemical equilibrium constant for the reaction is
$Br_2 \rightleftharpoons 2\ Br$ is

$K_c = \frac{[Br]^2}{[Br_2]}$

K_c at 500 K is 1×10^{-10}. On increasing temperature (700K). The value of K_c is also increased, *i.e.*, concentration of product is increased and obuiously the increase of temperature will favour the forward reaction. Thus, the forward reaction is endothermic.

Question 60

1 mL of 0.01 N HCl is added to 999 mL solution of 0.1 N Na_2SO_4. The pH of the resulting solution will

Options:

A. 2

B. 7

C. 5

D. 1

Answer: C

Solution:

As Na_2SO_4 solution is neutral, it simply dilutes the HCL solution from 1 mL to 1000 mL.

Now $[H^+] = \frac{0.01}{1000} = 10^{-5}$ M

$\therefore$ pH = -log $[H^+]$

= -log 10^{-5}

= [-5log 10]

pH = 5

Question 61

The molarity of a solution in which 5.3 g Na_2CO_3 is dissolved in 500 mL will be

Options:

A. 1.0 M

B. 0.1 M

C. 0.20 M

D. 0.2 M

Answer: B

Solution:

Molecular mass of Na_2CO_3 = 106 g mol^{-1}

mass of Na_2CO_3 dissolvedd =5.3 g

$$number\ of\ moles = \frac{given\ mass}{molecular\ mass}$$

$$= \frac{5.3}{106}$$

=0.05 moles.

In 500 mL, 0.05 moles of Na_2CO_3 are dissolved.

$$so,\ in\ 1000ml,\ moles\ of\ Na_2CO_3\ dissolved = \frac{0.05}{500} \times 1000 \quad = 0.1M$$

Hence, option(2) is correct.

Question 62

An organic compound crystallizes in an ortho rhombic system with two molecules per unit cell. The unit cell dimensions are 12.05, 15.05 and 2.69Å. If the density of the crystal is 1.419 g cm^{-3}, then molar mass of compound will be

Options:

A. 207 g mol^{-1}

B. 209 g mol^{-1}

C. 308 g mol^{-1}

D. 317 g mol^{-1}

Answer: B

Solution:

Density, $\rho = \frac{Z \times M}{a^3 \times N_0}$

Given Orthorhombic system has different edge length, width and height

$\therefore\ a_n = 12.05Å = 12.05 \times 10^{-8}$ cm

$a_l = 15.05Å = 15.05 \times 10^{-8}$ cm

$a_w = 2.69Å = 2.69 \times 10^{-8}$ cm

$\because$ System has 2 molecules per unit cell,

$\therefore Z = 2$

And $N_0 = 6.023 \times 10^{23}$

Now, molecular mass will be

$$M = \frac{\rho \times a_h \times a_l \times a_w \times N_0}{Z}$$

$$= \frac{1.419 \times 12.05 \times 10^{-8} \times 15.05 \times 10^{-8} \times 2.69 \times 10^{-8} \times 6.023 \times 10^{23}}{2}$$

$= 2084.6 \times 10^{-1} = 209$ g mol^{-1}

Question 63

Which of the following has maximum bond energy?

Options:

A. C = C

B. C = O

C. O = O

D. N = O

Answer: B

Solution:

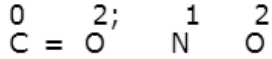
The amount of energy required to break one mole of bonds of a particular type between two atoms is called bond dissociation energy or bond energy.

Bond energy of C = O is 745 kJ mol^{-1}

Bond energy of N = O is 607 kJ mol^{-1}.

It can be explained as;

Higher is the bond length, lower will be its bond energy. Further, bond length depends upon the number of the lone pairs of electrons present on the bonded atoms. Greater the number of lone pairs of electrons present on the bonded atoms, greater is the repulsion between the atoms and hence less is the bond energy.



Whereas C = C and O = O are non-polar bonds thus have less bond energy than polar bonds.

Question 64

Among the following compounds both coloured and paramagnetic one is

Options:

A. $K_2Cr_2O_7$

B. $VOSO_4$

C. $(NH_4)_2$. $[TiCl_2]$

D. $K_3[Cu(CN)_4]$

Answer: B

Solution:

A. $_{K2}Cr_2O_7$ (potassium dichromate) forms orange – red triclinic crystals and dichromate ion $(Cr2O_7^{2-})$ in $K_2Cr_2O_7$ has two tetrahedral units shared through an oxygen atom. The central atom Cr is in +6 oxidation state which has electronic configuration [Ar]$3d^0 4s^0$. As it does not have any unpaired electron, It is diamagnetic.
B. $_{NOSO4}$ (vanadyl sul[hate) is a hygroscopic blue soild. It features VO^{2+} oin, which has vanadium in its +4 oxidation state. Since V^{4+} has electronic configuration [Ar] $3d^1 4s^0$, the salt is paramagnetic.
C. $_{(NH4)_2}$. $[TiCl_6].2H_2O$ (Ammonium chlorotitantate) is a yellow coloured crystal and Ti has +4 oxidation state. As electronic configuration of Ti^{4+} is [Ar] $3d^0 4s^0$, the salt is diamagnetic.
D. K3[Cu(CN)4] (Potassium tetracyanocuprate (l)0 is a white soluble complex. In it, Cu has +1 oxidation state and [Ar]$3d^{10}$ configuration. Hence, it is paramagnetic.

Question 65

The correct order of ionization energies is

Options:

A. Cu > Ag > Au

B. Cu > Au > Ag

C. Au > Cu > Ag

D. Ag > Au > Cu

Answer: C

Solution:

On moving down in a group, valence shell become for away from the nucleus and thus nucleus attraction towards test electron decreases which results in decrease in ionisation energy. So, the correct order should be
Cu > Ag > Au
However, in actual practice, the ionisation energy for Au is higher

than that of Cu and Ag. This is due to weak shielding effect of 4*f* electrons present in Au. Thus, a net result of effective nuclear charge and shielding effect favours for predominating trend of effective nuclear charge and therefore the correct order of ionisation energies will be
Au > Cu > Ag

Question 66

The shape of $[PtCl_3(C_2H_4)]$ and the hybridization of Pt respectively are

Options:

A. tetrahedral, sp^3

B. trigonal pyramidal, sp^3

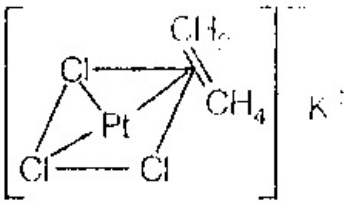
C. square planar, $d\,sp^2$

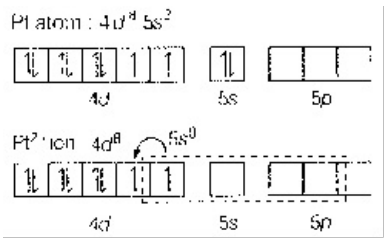
D. square planar, d^2sp^3

Answer: C

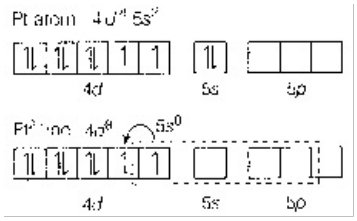
Solution:

Zeise's salt : $[K\,[PtCl_3(\eta^2\text{-}C_3H_4)]$ is a π-complex and square planar in structure with dsp^2 hybridisation.
(figure)
Hybridisation in $[PtCl_3(\eta^2 - C_2H_4)]$





As Cl^- ions and C_2H_4 approaches, the pairing of the unpaired electrons of *d* orbitals takes place.
∴ $[Pt\,Cl_3\,(C_2H_4)]$



Question 67

Which of the following is most acidic?

Options:

A. H_2O

B. H_2S

C. H_2Se

D. H_2Te

Answer: D

Solution:

The acidic nature of hydrides of group^{-16} elements increases from H_2O to H_2Te.

$H_2O < H_2S < H_2Se < H_2Te$

The increase in acidic nature of hydrides on moving down the group may be explained in terms of bond length of H-----M bond; larger is bong length, lesser is bond energy and thus easier is ionisation of H ----M bond or easier is proton donar nature.

Question 68

In Kjeldahl's method of estimation of nitrogen, $CuSO_4$ act as

Options:

A. oxidizing agent

B. reducing agent

C. catalytic agent

D. hydrolysis agent

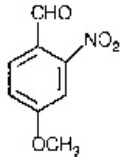
Answer: C

Solution:

Kjeldahl’s method is used for the estimation of nitrogen. The organic compound is heated with conc.
H_2SO_4 in presence of K_2SO_4 and a little $CuSO_4$ to convert all the nitrogen into $(NH_4)_2SO_4$. K_2SO_4 raises the boiling point of H_2SO_4 while CuSO4 catalyses the reaction.

Question 69

What is the correct IUPAC name of



Options:

A. 4-methoxy-2-nitrobenzaldehyde

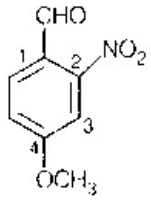
B. 4-formyl-3-nitroanisole

C. 4-methoxy-6-nitrobenzaldehyde

D. 2-formyl-5-methoxynitrobenzene

Answer: A

Solution:



As the principal functional group is –CHO, so numbering will start from the carbon atom containing –CHO group. According to lowest locant sum rule the IUPAC name of the compound is 4-methoxy-2-nitrobenzaldehyde.

Question 70

Zinc is used to protect iron from corrosion because

Options:

A. E_{red} of Zn<E_{red} of iron

B. $E_{oxidation}$ of Zn<$E_{oxidation}$ of iron

C. $E_{oxidation}$ of Zn=$E_{oxidation}$ of iron

D. Zinc is cheaper than iron

Answer: A

Solution:

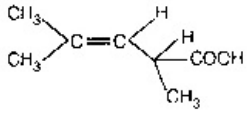
Zinc is used to protect iron from corrosion because it is more active metal than iron and it has the tendency to lose electrons with respect to iron. In other words, zinc has a negative reduction potential higher than that of iron.

$E^0_{zn2+,\ Zn1} = -0.76$ V

$E^0_{Fe2+,\ Fe} = -0.44$ V

Question 71

The structure shows



Options:

A. geometrical isomerism

B. optical isomerism

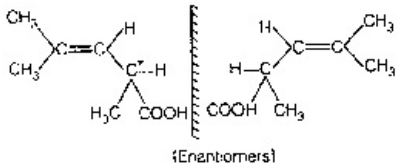
C. geometrical and optical isomerism

D. tautomerism

Answer: B

Solution:

The compound doesn't show geometrical isomerism because one carbon atoms attached to double bond has same substituent group (---Ch_3), however, it can show optical isomerism, because it has one optically active chiral carbon atom.



Question 72

50 g of saturated aqueous solution of potassium chloride at 30^0C is evaporated to dryness, When 13.2 g of dry KCI was obtained. The

solubility of KCI in water at 30^0C is

Options:

A. 35.87 g

B. 25.62 g

C. 28.97 g

D. 27.81 g

Answer: A

Solution:

The solubility of a solute in a solvent at any temperature is defined as the maximum amount of the solute in grams which can dissolve in 100 g of the solvent to form the saturated solution at that particular temperature.
Weight of solute (KCl) = 13.2 h
Weight of solvent = 50 – 13.2 = 36.8 g
$\therefore$ Solubility of KCl = $\frac{132}{36.3} \times 100 = 35.896$

Question 73

Which of the following compound is not aromatic?

Options:

A. 1, 3-cyclobutadiene

B. Pyridine

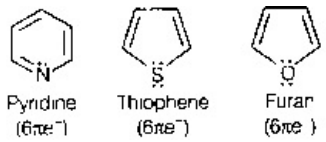
C. Furan

D. Thiophene

Answer: A

Solution:

According to Huckel's rule, the planar cyclically conjugated structures having $(4n+2)\pi$ electrons (where, n is o, 1, 2 etc) are considered as aromatic compounds.
However, cyclobutadien have $4n\pi$ electrons which are involved in cyclic delocalisation are referred to as anti aromatic compounds.



Question 74

Which are of the following is a false statement?

Options:

A. Cell fluid is an example of sol

B. Butter is an example of gel

C. Hair cream is an example of e mulsion

D. Cheese is an example of foam

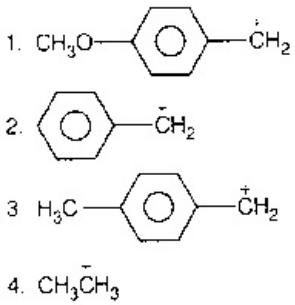
Answer: D

Solution:

Cheese is an example of gel in which dispersion medium is solid and dispersed phase is liquid.

Question 75

Out of



Relative stabilities order is

Options:

A. 4 < 2 < 3 < 1

B. 2 < 4 < 3 < 1

C. 4 < 2 < 1 < 3

D. 2 < 4 < 1 < 3

Answer: A

Solution:

Due to the presence of phenyl group the stabilities of the carbocations 1, 2 and 3 are greater than cation 4, (due to resonance). Further the presence of electron-donating groups on phenyl ring increases the stability of carbocation.
Hence, cation 1 and 3 are more stable than cation 2. Moreover – Ome group shows $+M$ effect ehich is more prominent than +/ effect of –Me group. Thus, cation 1 is more stable than cation 3. So, the correct order is, 4 < 2 < 3 < 1.

Question 76

In the extraction of copper, metal is formed is the Bessemer converter due to reaction

Options:

A. $2\ Cu_2S + 2Cu_2O \longrightarrow 6\ Cu + SO_2$

B. $2\ Cu_2O \longrightarrow 4\ Cu + O_2$

C. $Cu_2S \longrightarrow 2\ Cu + S$

D. $Fe + Cu_2O \longrightarrow 2\ Cu + FeO$

Answer: A

Solution:

In the extraction of copper, when matte (mostly Cu_2S) is transferred to the Bessemer converter and a blast of air is blown through the molten mass, a part of Cu_2S is oxidised which combines with remaining Cu_2S to form metal.

$2\ Cu_2S + 3O_2 \longrightarrow 2\ Cu_2O + 2SO_2$

$Cu_2S + 2Cu_2O \longrightarrow 6\ Cu + SO_2$

This is an example of auto-reduction.

Question 77

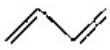
The cumulated alkadiene is

Options:

A. 

B. 

C. 

D. 

Answer: C

Solution:

(i) The alkadienes in which two double bonds are on adjacent carbon atoms are known as cumulated alkadienes.

Penta-1. 2-diene

(ii) The alkadienes, having alternate single and double bonds are known as conjugated dienes.

(iii) The alkadienes having two double bonds separated by more than one single bond are known as isolated dienes.

Penta-1, 4-diene

Question 78

One faraday of current was passed through the electrolytic cells placed in series containing solution of Ag^{+}, Ni^{2+} and Cr^{3+} respectively. The ratio of amounts of Ag, Ni and Cr deposited will be
(At. Wt. of Ag = 108, Ni = 59, Cr = 52)

Options:

A. 108 : 29.5 : 17.4

B. 17.4 : 29.5 : 108

C. 1 : 2 : 3

D. 108 : 59 : 52

Answer: A

Solution:

For deposition of Ag, reaction is

$Ag^{+} + e^{-} \longrightarrow Ag$
Thus, 1 F deposits Ag = 1 mol = 108 g
For deposition of Ni, the reaction is
$Ni^{2+} + 2e^{2} \longrightarrow Ni$
Thus, 2 F deposits Ni = 1 mol = 59 g
$\therefore$1 F deposits Ni = 0.5 mol = 29.5 g
For deposition of Cr, the reaction is,
$Cr^{3+} + 3e^{-} \longrightarrow Cr$
Thus, 3F deposits Cr = 1 mol
$\therefore$ 1 F deposits Cr = 0.33 mol = 17.4 g

Question 79

Oxidation number of chlorine atom in Ca(OCl)Cl is

Options:

A. -1

B. +1, -1

C. 0

D. -1, 0

Answer: B

Solution:

In $CaOCl_2$, according to the calculation, the oxidation states of both chlorine atoms are zero. But, it is their average oxidation state.
In actual practice, the structure of $CaOCl_2$ is
ClO ----Ca ----Cl.
Thus, in $CaOCl_2$, two types of anions ClO^{-} and Cl^{-} are present. Out of thee, the oxidation state of Cl^{-} is -1, but the oxidation state of chlorine in OCl^{-}ion is + 1 as oxygen is more electronegative than chlorine, hence oxidation state of oxygen remains – 2 while oxidation state of chlorine becomes + 1.

Question 80

At equilibrium, if K_p= 1, then

Options:

A. $\triangle G^0 > 1$

B. $\triangle G^0 < 1$

C. $\triangle G^0 = 0$

D. $\triangle G^0 = 1$

Answer: C

Solution:

At equilibrium, the relation between free energy of the reaction and equilibrium constant is as follows.

$\triangle G^0 = -RT \ln K_p$

If $K_p = 1$ at equilibrium

$\triangle G' = -RT \ln 1$

Or $\triangle G^0 = 0$.

Question 81

Manganese salt + PbO_2 + conc. $HNO_3 \rightarrow$ the solution acquires purple colour. The colour is due to

Options:

A. $Mn(NO_3)_2$

B. $Pb(NO_3)_2$

C. $HMnO_4$

D. MnO

Answer: C

Solution:

The manganese salt is MnO_2 :

$2\ MnO_2 + 4\ HNO_3 \longrightarrow 2\ Mn(NO_3)_2 + 2\ H_2O + O_2$

$2\ MN(NO_3)_2 + 5\ PbO_2 + 6\ HNO_3 \xrightarrow{\triangle} 2HMnO_4 + 5\ Pb(NO_3)_2 + 2H_2O$

Permangaic acid (violet pink)

Question 82

Which of the following pairs of ions cannot be separated by H_2S in dilute HCl?

Options:

A. Bi^3, Sn^{4+}

B. Al^{3+}, Hg^{2+}

C. Cu^{2+}, Zn^{2+}

D. Ni^{2+}, Cu^{2+}

Answer: A

Solution:

A. Bi^{3+} and Sn^{4+} cannot be separated by H_2S in dilute HCl, because both are present in second group and both are precipitated by H_2S in dil. HCl as Bi_2S_3 and SnS_2.
B. Al^{3+} is not precipitated by H_2S in dil HCl whereas Hg^{2+} is precipitated as HgS.
C. Zn^{2+} is not precipitated, but Cu^{2+} is precipitated as CuS.
D. Ni^{2+} is not precipitated, but Cu^{2+} is precipitated as mention earlier.

Question 83

A magnetic moment of 1.73 BM will be shown by which one among the following compounds

Options:

A. $[Cu(NH_3)_4]^{2+}$

B. $[Ni(CN)_4]^{2-}$

C. $TiCl_4$

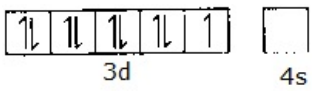
D. $[CoCl_6]^{4-}$

Answer: A

Solution:

Electronic configuration of Cu^{2+} ion in $[Cu(NH_3)_4]^{2+}$;

Cu^{2+} ion = [Ar] $3d^9$ $4s^0$.



Cu^{2+} ion has one unpaired electron

Magnetic moment of

$[Cu(NH_3)_4]^{2+}$ (μ) = $\sqrt{n(n+2)}$ BM

Where, n = no. Of unpaired electrons

$\mu = \sqrt{1(1+2)} = \sqrt{3} = 1.73$ BM

Whereas Ni^{2+} in $[Ni(CN)_4]^{2-}$, Ti^{4+} in $TiCl_4$ and Co^{2+} in $[COCl_6]^{4-}$ has 2, 0 and 3 unpaired electrons respectively.

Question 84

The ratio of the difference in energy between the first and second Bohr orbit to that between the second and third Bohr orbit is

Options:

A. $\frac{1}{2}$

B. $\frac{1}{27}$

C. $\frac{4}{9}$

D. $\frac{27}{5}$

Answer: D

Solution:

The expression for energy in case of H-atom :

$E_n = -\frac{2\pi^2 me^4}{n^2h^2}$

$\therefore E_n \propto -\frac{1}{n_2}$

(i) The difference of energies between first and second Bohr orbit =

$E_{n1} - E_{n2} = -\frac{1}{(1)^2} - \left[-\frac{1}{(2)^2}\right]$

$= -1 + \frac{1}{4} = -\frac{3}{4}$

(ii) The difference of energies between second and third Bohr orbit

$=$

$$E_{n2} - E_{n3} = -\frac{1}{(2)^2} - \left(-\frac{1}{(3)^2}\right)$$

$$= -\frac{1}{4} + \frac{1}{9} = -\frac{5}{36}$$

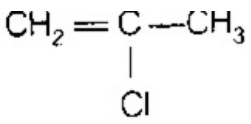
The ratio $= -\frac{3}{4} : -\frac{5}{36} = 27:5$

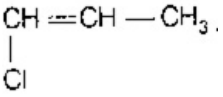
Question 85

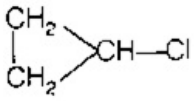
An organic compound C_3H_5Cl A. when treated with magnesium in dry ether gives B. which on treating with CO_2 followed by acid hydrolysis gives $C_4H_6O_2$ (*C*). C. is also obtained on oxidation of a hydrocarbon D, C_8H_{12}. Structure of *A* is

Options:

A. CH_2=CH-CH_2-Cl

B. 

C. 

D. 

Answer: D

Solution:

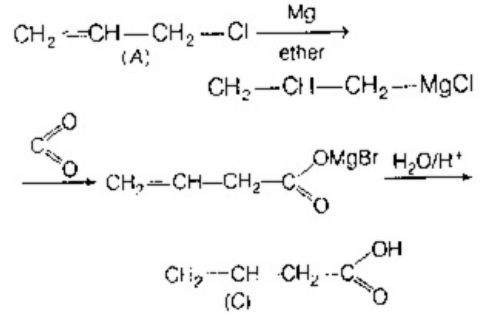
The reaction can be summerised as,

$$\underset{\text{A}}{C_3H_5Cl} + Mg \xrightarrow{\text{ether}} B \xrightarrow[\text{(ii) } H_2O/H^+]{\text{(i) } CO_2} \underset{\text{C}}{C_4H_6O_2}$$

$$\underset{\text{D}}{C_8H_{12}} \xrightarrow{\text{Oxidation}} \underset{\text{C}}{C_4H_6O_2}$$

Let us consider any one of *a, b* and *c* options as C_3H_5Cl and

proceeds the reaction

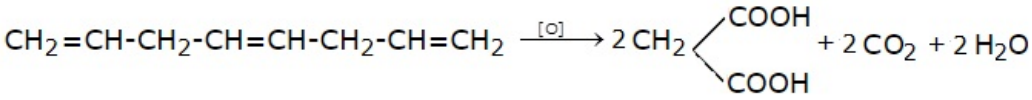


As it is known that carboxylic acid is also obtained on the oxidation of corresponding hydrocarbon

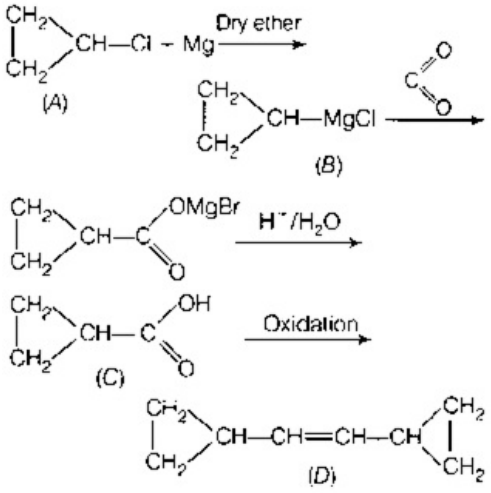
$$R\text{-}CH = CH\text{-}R \xrightarrow{[O]} RCOOH + RCOOH$$

Thus, by knowing the structure of C as CH_2=CH-CH_2-COOH, we can say that structure of C_8H_{12} will be CH_2 = CH-CH_2-CH=CH-CH_2-CH=CH_2

But this hydrocarbon on oxidation will not give acid C having molecular formula $C_4H_6O_2$ instead it will give malonic acid, CO_2 and water.



Thus, among *a, b, c* options none will give satisfactory explanation regarding the structure of hydrocarbon as they contain double bond hence, correct structure of A is cyclopropyl chloride.



Question 86

Which of the following is bactriostatic?

Options:

A. Penicillin

B. Erythromycin

C. Aminoglycoside

D. Ofloxacin

Answer: B

Solution:

Erythromycin is a bacteriostatic drug, *i.e.*, a drug which inhibits the growth of organisms whereas penicillin, aminoglycoside (streptomycin) and ofloxacin are bacteriocidal drugs, *i.e.*, drugs which kill the organisms in the body.

Question 87

Glucose is added to 1 L water to such an extent that $\triangle T_f/K_f$ becomes equal to 10^{-3}, the weight of glucose ($C_6H_{12}O_6$) added is

Options:

A. 180 g

B. 18 g

C. 1.8 g

D. 0.18 g

Answer: D

Solution:

Depression in freeing point,

$$\triangle T_f = \frac{100 K_1 w_2}{w_1 \times M_2}$$

Or $$w_2 = \frac{\triangle T_f w_q M_2}{100 K_f}$$

We are given,

w_1 (solvent) = 1 L = 1000 g

M_2 (solute) = 180 g mol^{-1}

$\frac{\triangle T_f}{K_f} = 10^{-3}$

W2 = ?

By putting these values in the formula, we get

$$W_2 = \frac{10^{-3} \times 1000 \times 180}{1000}$$

$W_2 = 0.18$ g

Question 88

Phenol + CCl_4 + KOH $\longrightarrow$ X;
Which of the following statement is true for *X*?
Topic ||| Chemistry || Organic Compounds Contaning Oxygen || Phenols and Reimer-Tieman Reaction

Options:

A. It gives effervescence with $NaHCO_3$

B. Gives silver mirror with Tollen's reagent

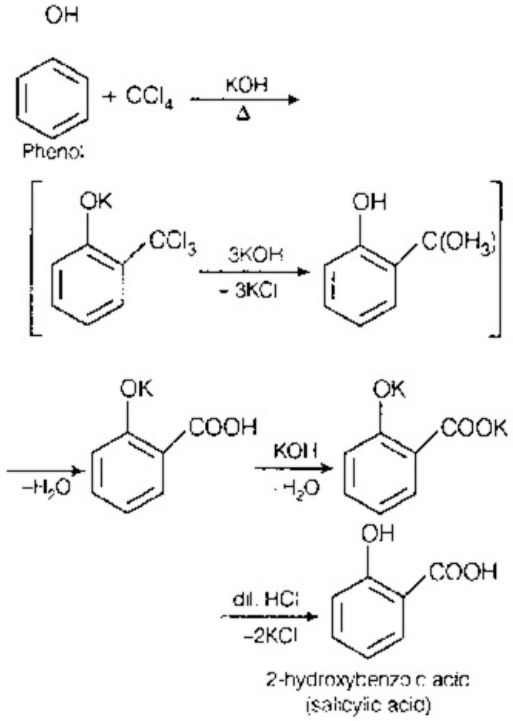
C. Does not give the red colour with $FeCl_3$

D. All of the above

Answer: A

Solution:

The given reaction is Reimer-TIemann reaction.



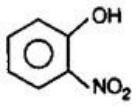
a) Salicylic acid gives effervescence with $NaHCO_3$, which is a characterstic feature of carboxylic acid. Thus, it ensures the presence of a carboxylic group in it.
b) It doesn't give silver mirror with Tollen's reagent because it

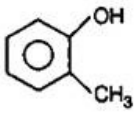
doesn’t have aldehydic group.
c) It gives red colour with neutral $FeCl_3$ as it contains enol group *i.e.*, C___OH group.

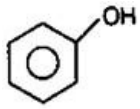
Question 89

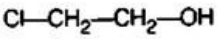
Which of the following compounds is most acidic?

Options:

A. 

B. 

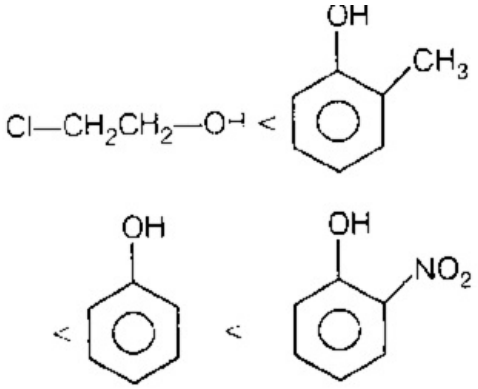
C. 

D. 

Answer: A

Solution:

Phenols are stronger acids than alcohols because the phenoxide ion left after the release of a proton is stabilised by resonance but the alkoxide ion is not. Further, the electron-withdrawing groups (-NO_2) which stabilise the phenoxide ion by dispersing the negative charge relative to phenol increase the acidic dtrngth of phenols, whereas electron donating group (__CH_3).
Which destabilise the phenoxide ion by intensifying the negative charge relative to phenol tend to decrease the acidity of phenols.
The correct order of acidic strength

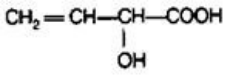


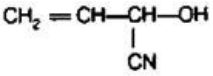
Question 90

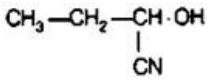
$CH_3CHO + HCHO \xrightarrow[\text{Heat}]{\text{dil.NAOH}} A \xrightarrow[H_3O^+]{\text{HCN}} B$

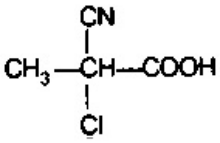
The structure of the compound *B* is

Options:

A. 

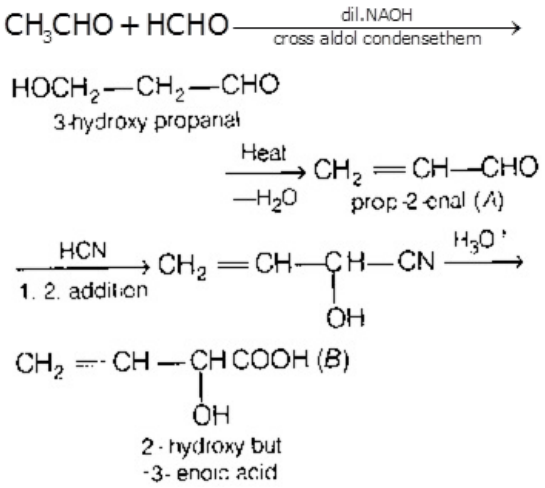
B. 

C. 

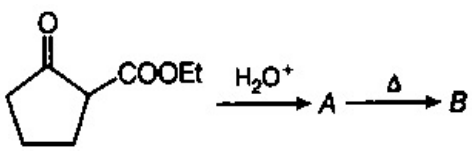
D. 

Answer: A

Solution:

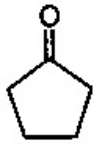


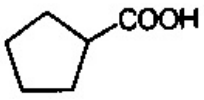
Question 91

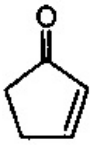


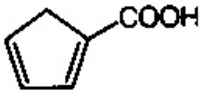
The compound B is

Options:

A. 

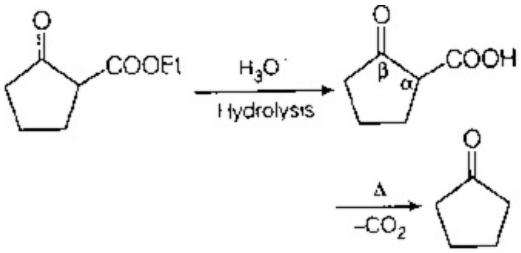
B. 

C. 

D. 

Answer: A

Solution:



Question 92

The dipole moment of HBr is 1.6×10^{-30} cm and inter atomic spacing is 1 Å. The % ionic character of HBr is

Options:

A. 7

B. 10

C. 25

D. 27

Answer: B

Solution:

Charge of electron (q) = 1.6×10^{-19}C

Dipole moment of HBr = 1.6×10^{-39} m

Inter-atomic spacing = 1Å =1×10^{-10} m

% of ionic character in HBr

$$= \frac{\text{dipole moment of HBr} \times 100}{\text{inter - spacing distance} \times q}$$

$$= \frac{1.6\times10^{-30}\times100}{1.6\times10^{-19}\times10^{-10}}$$

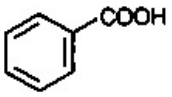
= 10%

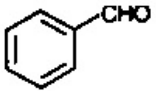
Question 93

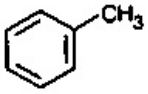
The reactant *X* in the reaction

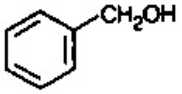
$X \xrightarrow[(CH_3CO)_2O]{CH_3COONa}$ Cinnamic acid is

Options:

A. 

B. 

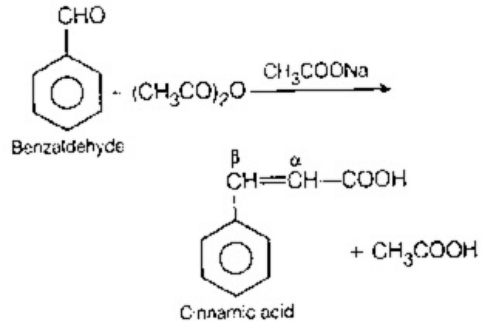
C. 

D.  s

Answer: B

Solution:

Perkin reaction is the condensation reaction in which aromatic aldehyde is heated with an anhydride of an aliphatic acid in the presence of sodium salt of the same acid to form α, β–unsaturated acid.



Question 94

Identify the final product *B* of the reaction

$$C_6H_5COOH + NaH\overset{*}{C}O3 \rightarrow \text{gas } (A) \xrightarrow[(ii)\ H_2O^+]{(i)CH_3MgBr} (B)$$

Here $\overset{*}{C} = C^{14}$

Options:

A. $CH_3\overset{*}{C}OOH$

B. $C_6H_5\overset{*}{C}OOH$

C. $CH_3\overset{*}{C}OOH$

D. $H\overset{*}{C}OOCH_3$

Answer: C

Solution:

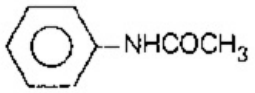
$$C_6H_5COOH + NaH\overset{*}{C}O3 \rightarrow C_6H_5COONa + H_2O + \overset{*}{C}O_2$$

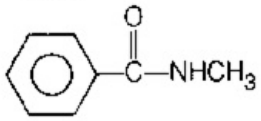
$$\overset{*}{C}O_2 \xrightarrow{1.CH_3MgBr} CH_3\overset{*}{C}OOMgBr \xrightarrow{H_3O^+} CH_3\overset{*}{C}OOH$$

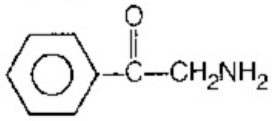
Please not that when carboxylic acids are treated with $NaHCO_3$ solution, CO_2 liberated comes from $NaHCO_3$.

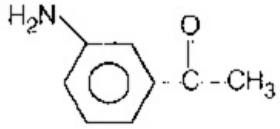
Question 95

The correct order of basic strength of the following are

1. 

2. 

3. 

4. 

Options:

A. 1>2>3>4

B. 4>2>3>1

C. 3>4>2>1

D. 3>2>4>1

Answer: C

Solution:

As it is known that electron-releasing group increases the basic

strength and electron withdrawing group decreases the basic strength of __NH_2 group.

As in 

Group is planked between two electron withdrawing groups, compound (1) is least basic among all. In compound (2), ___NH___is attached with one electron withdrawing group __CO ____and one electron releasing group ___CH_3, it is a bit basic than compound (1). In compound (4)_____$COCH_3$, group is attached to aniline ring, not directly with __NH_2 group.

Hence it is more basic than compound (2). Compound (3) is most basic among all compounds since in it, ____NH_2 is attached to only one electron releasing group (____CH_2___). Thus, the correct order of basic strength with be

3 > 4 > 2 > 1.

Question 96

The polypeptides are obtained by assembling the peptide units by

Options:

A. ionic bond

B. covalent bond

C. intermolecular H-bonding

D. covalent and H-bonding

Answer: C

Solution:

More than one polypeptide chains when assemble as a result of intermolecular hydrogen bonding between them, the polypeptides assume the secondary structure of proteins.

Question 97

A positive carbylamines test is given by
I. N, N-dimethylaniline
II. 2, 4-dimethylaniline

III. N-methyl-*o*- methylaniline
IV. *p*-methylbenzyl amine

Options:

A. (II) and (IV)

B. (I) and (IV)

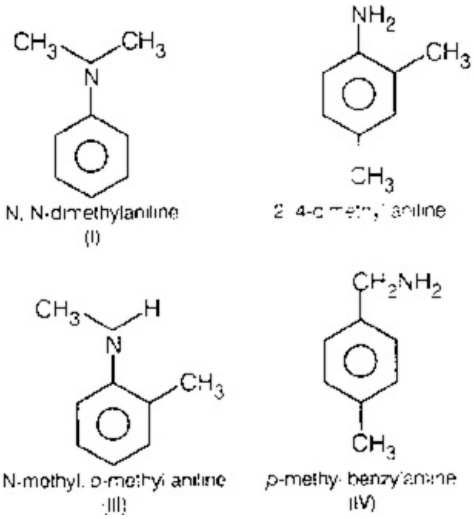
C. (II) and (III)

D. (I) and (II)

Answer: A

Solution:

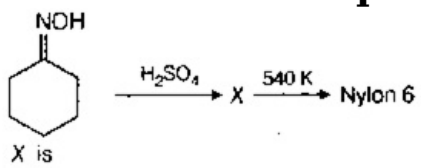
Carbylamine or isocyanide test is given by only primary amines, whether they are aliphatic or aromatic.



I, II and III are aromatic but compound IV is an aliphatic amine. Further, compound I is tertiary amine. II and IV are primary amines, thus give carbylamines test, whereas compound III is a secondary amine.

Question 98

In the reaction sequence,



***X* is**

Options:

A. cyclohexanone

B. caprolactum

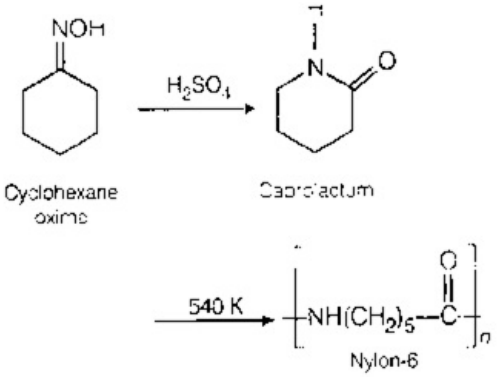
C. $HO(CH_2)_6NH_2$

D. Hexamethyline disocyanate

Answer: B

Solution:

Carpolactum is the monomer of nylon-6.



Question 99

Which of the following has the largest number of atoms?

Options:

A. 0.5 g atom of Cu

B. 0.635 g of Cu

C. 0.25 moles of Cu atoms

D. 1 g of Cu

Answer: A

Solution:

Gram atom is an amount of an element equal to its atomic weight expressed in grams which is nowadays replaced by mole.
One mole is defined as the amount of a substance that contains as many entities like atoms , molecules , ions as there are atoms exactly in 12g of Carbon-12. 1mole contains substance equal to Avogadro's number(6.023×10^{23})

A. 0.5 g atom or mole of Cu = $0.5\times6.023\times10^{23}$
= 3.011×10^{23} Cu atoms

$$number\ of\ moles\ of\ 0.635g\ Cu = \frac{given\ mass\ of\ Cu}{molecular\ mass\ of\ Cu}$$

B. $$or\ n_{Cu} = \frac{0.635}{63.5}$$

$$n_{Cu} = 0.01$$

0.635 g of Cu = n_{Cu}×6.023 × 10^{23}

= 0.01x 6.023 ×10^{21}

= 6.023 ×10^{21}

C. 0.25 moles of Cu= 0.25×6.023 ×10^{23}

= 1.5×10^{23} Cu atoms

$$number\ of\ moles\ in\ 1g\ Cu\ = \frac{given\ mass\ of\ Cu}{molecular\ mass\ of\ Cu}$$

$$= \frac{1}{63.5}$$

D. $= 0.016\ moles$ 1 g Cu =number of moles x 6.023 × 10^{23}

=0.016 × 6.023 × 10^{23}

= 9.48 × 10^{21} Cu atoms

Thus, 0.5 g atom of Cu has maximum number of Cu atoms.

Hence, option(1) is correct.

Question 100

Geometry of the molecule is distorted according to VSEPR theory for -
(I) H_2O (II) NH_3
(III) N_3H (IV) XeF_2

Options:

A. I, III, IV

B. II, III, IV

C. I, II, III, IV

D. I, II

Answer: C

Solution:

All molecules have distorted geometry

Maths

Question 101

The set $(A \cup B \cup C) \cap (A \cap B' \cap C') \cap C'$ is equal to

Options:

A. $B \cap C'$

B. $A \cap C$

C. $B' \cap C'$

D. None of these

Answer: A

Solution:

$$
\begin{aligned}
&(A \cup B \cup C) \cap (A \cap B' \cap C') \cap C' \\
&= (A \cup B \cup C) \cap (A' \cup B \cup C) \cap C' \\
&= [(A \cap A') \cup (B \cup C)] \cap C' \\
&= (\phi \cup B \cup C) \cap C' = (B \cup C) \cap C' \\
&= (B \cap C') \cup (C \cap C') \\
&= (B \cap C') \cup \phi = B \cap C'
\end{aligned}
$$

Question 102

If A = $\{(x, y) : x^2 + y^2 \leq 1; x, y \in R\}$ and B = $\{(x, y) : x^2 + y^2 \geq 4; x, y \in R\}$ then

Options:

A. $A - B = \phi$

B. $B - A = \phi$

C. $A \cap B \neq \phi$

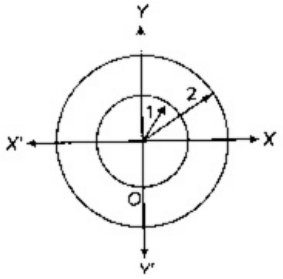
D. $A \cap B = \phi$

Answer: D

Solution:

A is the set of all points on the inner circle $x^2+y^2=1$. B is the set of

all points on the outer circle $x^2+y^2=4$.



$\therefore\ A - B = A,\ B - A = B,\ A \cap B = \phi$

Question 103

For real numbers *x* and *y*, we define *xRy* if *x-y*+$\sqrt{5}$ is an irrational number. The relation *R* is

Options:

A. reflexive

B. symmetric

C. transitive

D. None of these

Answer: A

Solution:

$x \in R \Rightarrow x - x + \sqrt{5} = \sqrt{5}$ is an irrational number

$\therefore\ (x, x) \in R$

Hence, R is reflexive.

$(\sqrt{5}, 1) \in R$ because $\sqrt{5} - 1 + \sqrt{5} = 2\sqrt{5} - 1,$ which is irrational umber.

$\therefore\quad (1, \sqrt{5}) \notin R$

Hence, R is not a symmetric.

We have, $(1, \sqrt{5}), (1, 2\sqrt{5}) \in R$ because $\sqrt{5} - 1 + \sqrt{5} = 2\sqrt{5} - 1,$ if

$1 - 2\sqrt{5} + \sqrt{5} = 1 - \sqrt{5}$ are irrational number.

$\therefore\quad (\sqrt{5}, 2\sqrt{5}) \notin R$

Hence,R is not transitive.

Question 104

If the real valued function $f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$ is even, then *n* is equal to

Options:

A. 2

B. $\frac{2}{3}$

C. $\frac{1}{4}$

D. 3

Answer: D

Solution:

$f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$ is even

$\therefore \quad f(-x) = f(x)$

i.e,. $\frac{a^x - 1}{(-x)^n(a^x + 1)} = \frac{a^x - 1}{x^n(a^x + 1)}$

$(-1)^n = -1$

Hence, $n = 3$ can satisfy this equation.

Question 105

If $(5 + 2\sqrt{6})^{(x^2-3)} + (5 - 2\sqrt{6})^{(x^2-3)} = 10$ **then** ***x*** **is equal to**

Options:

A. ± 3 or $\pm \sqrt{3}$

B. ± 5 or $\sqrt{5}$

C. ± 4 or $\sqrt{4}$

D. ± 2 or $\pm \sqrt{2}$

Answer: D

Solution:

Let $(\sqrt{5} + 2\sqrt{6})x^{2-3} = a$

We get, $(\sqrt{5} - 2\sqrt{6})x^{2-3} = \frac{1}{a}$

$\therefore \quad a + \frac{1}{a} = 10 \Rightarrow a^2 - 10a + 1 = 0$

$\Rightarrow a = \frac{10 \pm \sqrt{100 - 4}}{2} = 5 \pm 2\sqrt{6}$

$= (5 \pm 2\sqrt{6})^{1/(x^2-3)}$ (assumed)

$\Rightarrow x^2 - 3 = \pm 1$

$\Rightarrow x^2 = 4$ or 2

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

Or $x^2 = 2$

$\Rightarrow x = \pm 2$ or $\pm \sqrt{2}$

Question 106

The value of the expression $1.(2-\omega)(2-\omega^2) + 2.(3-\omega)(3-\omega^2) + \ldots.+ (n+1)(n-\omega)(n-\omega^2)$, where ω is an imaginary cube root of unity is

Options:

A. $\left\{\frac{n(n+1)}{2}\right\}^2$

B. $\left\{\frac{n(n+1)}{2}\right\}^2 - n$

C. $\left\{\frac{n(n+1)}{2}\right\}^2 + n$

D. None of the above

Answer: B

Solution:

$$\sum_{k=2}^{n}(k-1)(k-\omega)(k-\omega^2) = \sum_{k=2}^{n}\left(k^3-(1+w+w^2)k^2+(w+w^3+w^2)k-(w^3)\right)$$

$$=\sum_{k=2}^{n}(k^3-1)$$

$$=\left(\frac{n(n+1)}{2}\right)^2-1-n+1$$

$$=\left(\frac{n(n+1)}{2}\right)^2-n$$

Question 107

A series, whose n^{th} term is (n/x)+y, then the sum of r terms will be

Options:

A. $\left\{\frac{r(r+1)}{2x}\right\}+ry$

B. $\left\{\frac{r(r-1)}{2x}\right\}$

C. $\left\{\frac{r(r-1)}{2x}\right\}^2-xy$

D. $\left\{\frac{r(r+1)}{2x}\right\}-ry$

Answer: A

Solution:

Given, a_n = (n/x) + y

$\Rightarrow$ a_1 = (1/x) + y

and a_2 = (2/x) + y

$\therefore$ $d = a_2 - a_1$ = (1/x)

Where, d is common difference of given AP.

$\therefore$ S_r =(r/2) [2a_1 + (r – 1)d]

= r(r+1) / 2x + ry

Question 108

Let S_1, S_2, ...S_n be squares such that for each n$\geq$ 1, the length of a side of S_n equals the length of the diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then the least value of *n* for which the area of S_n less than 1 sq cm

Options:

A. 7

B. 8

C. 9

D. 10

Answer: B

Solution:

Given, length of a side of S_n

= Length of a diagonal of S_{n+1}

$\Rightarrow$ Length of side of $\sqrt{2}$

$= \sqrt{2}$ (Length of a side of S_{n+1})

$\Rightarrow \frac{\text{Length of a side of } S_{n+1}}{\text{Length of a side of } S_n}$

$= \frac{1}{\sqrt{2}}$ for all $n \geq 1$

So, the side of S_1, S_2, ..., S_n from a GP, ith common ratio is $\frac{1}{\sqrt{2}}$ and first term 10.

$\therefore$ Side of $S_n = 10\left(\frac{1}{\sqrt{2}}\right)^{n-1} = \frac{10}{2^{\frac{(n-1)}{2}}}$

Since, area of $S_n < 1$ (given)

$\Rightarrow \quad \frac{100}{2^{n-1}} < 1$

$\Rightarrow 2^{n-1} > 100$

$\Rightarrow n - 1 \geq 7 \Rightarrow n \geq 8$

Question 109

If p, q and r are positive and are in AP, the roots of the quadratic equation $px^2 + qx + r = 0$ are real for

Options:

A. $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$

B. $\left|\frac{p}{r} - 7\right| \leq 4\sqrt{3}$

C. all p and r

D. no p and r

Answer: A

Solution:

Given p, q and r are in AP

$\therefore 2q = p + r$

The roots of $px^2 + qx + r = 0$ are real, if $q^2 - 4pr \geq 0$

$\Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$

$$\left(\frac{p\left(1+\frac{r}{p}\right)}{2}\right)^2 - 4pr = 0$$

$$p^2\left(\frac{\left(1+\frac{r}{p}\right)^2}{4} - 4\frac{r}{p}\right) = 0$$

$$\left(1+\frac{r}{p}\right)^2 - 16\frac{r}{p} = 0$$

$$\left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 = 0$$

$$\left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 + 49 - 49 \geq 0$$

$$\left(\frac{r}{p} - 7\right)^2 - 48 \geq 0$$

$$\left(\frac{r}{p} - 7\right)^2 \geq 48$$

$$\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$$

Question 110

The value of *x*, for which the 6th term in the expansion $\left\{2^{\log_2\sqrt{(9^{x-1}+7)}} + \frac{1}{2^{\left(\frac{1}{5}\right)\log_2(3^{x-1}+1)}}\right\}^7$ of 84 is equal to

Options:

A. 4

B. 3

C. -2

D. 1

Answer: D

Solution:

Given Expression

$$\left[\sqrt{9^{x-1}+7}\,\frac{1}{(3^{x-1}+1)^{1/5}}\right]^7$$

$\therefore\ T_6 = T_{5+1}$

$= {}^7C_5(\sqrt{9^{x-1}+7})^{7-5}\left\{\frac{1}{(3^{x-1}+1)^{1/5}}\right\}^5$

$= 21(9^{x-1}+7).\frac{1}{3^{x-1}+1}$

$= 21.\frac{3^{2x-2}+7}{3^{x-1}+1} = 84$ (given)

$\Rightarrow 3^{2x-2} + 7 = 4(3^{x-1} + 1)$

$\Rightarrow y^2 + 7 - 4y - 4 = 0$

Where $y = 3^{x-1}$

$\Rightarrow y = 3$

$\Rightarrow 3^{x-1} = 3$

$\Rightarrow x - 1 = 1$

$\Rightarrow x = 2$ and $y = 1$

$\Rightarrow 3^{x-1} = 3^0$

$\Rightarrow x - 1 = 0 \Rightarrow x = 1$

Question 111

$\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ **and** $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ **are the given determinants, then**

Options:

A. $\Delta_1 = 3(\Delta_2)^2$

B. $\frac{d}{dx}(\Delta_1) = 3\Delta_2$

C. $\frac{d}{dx}(\Delta_1) = 3\Delta(\Delta_2)^2$

D. $\Delta_1 = 3\Delta_2^{3/2}$

Answer: B

Solution:

$\Delta_1 = x(x^2 - ab) - b(ax - ab) + b(a^2 - ax)$

$= x^3 - 3abx + ab^2 + a^{2b}$

$\frac{d}{dx}(\Delta_1) = 3x^2 - 3ab = 3(x^2 - ab) = 3\Delta_2$

Question 112

If A and *B* are square matrices of the same order and *A* is non-singular, then for a positive integer *n*, $(A^{-1}BA)^n$ is equal to

Options:

A. $A^{-1}B^nA^n$

B. $A^nB^nA^{-n}$

C. $A^{-1}B^nA$

D. $n(A^{-1}BA)$

Answer: C

Solution:

$(A^{-1}BA)^2 = (A^{-1}BA)\ (A^{-1}BA)$
$= A^{-1}B = (AA^{-1})BA$
$= A^{-1}B/BA = A^{-1}B^2A$
$(A^{-1}BA)^3 = (A^{-1}B^2A)\ (A^{-1}BA)$
$= A^{-1}B^2(AA^{-1})BA$
$= A^{-1}B^3A$ and so on.
$\therefore\ (A^{-1}BA)^n = A^{-1}B^nA$

Question 113

The function $f(x)$ =

$$\begin{cases} x^2/a, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^2}, & \sqrt{2} \le x < \infty \end{cases}$$

is continuous for $0 \le x < \infty$, then the most suitable values of a and b are

Options:

A. $a = 1, b = -1$

B. $a = -1, b = 1 + \sqrt{2}$

C. $a = -1, b = 1$

D. None of the above

Answer: C

Solution:

Since, f is continuous for $0 \le x < \infty$, f is continouos ar x = 1.
$\therefore\ a = \pm 1$
Since, f is continuous ar x = $\sqrt{2}$

$\therefore \frac{2b^2 - 4b}{2} = a$

$\Rightarrow b^2 - 2b = a$

When, $a = 1$, $b^2 - 2b = 1$

$\Rightarrow b = 1 \pm \sqrt{2}$

When, $a = 1$, $b^2 - 2b = 1$

$\Rightarrow (b-1)^2 = 0 \Rightarrow b = 1$

Hence, $a = -1$ and $b = 1$ are most suitable values.

Question 114

If $f(x) = \begin{cases} 1, & x < 0 \\ 1 + \sin x, & 0 \le x < \frac{\pi}{2}, \end{cases}$ then at $x = 0$ the derivative $f'(x)$ is

Options:

A. 1

B. 0

C. infinite

D. not defined

Answer: D

Solution:

$f(0) = 1 + \sin 0 = 1$

RHD $= f'(0^+) = \lim_{h \to 0} \frac{1 + \sin(0+h) - 1}{h}$

$= \lim_{h \to 0} \left(\frac{\sin h)}{h} \right) = 1$

RHD $= f'(0^-) = \lim_{h \to 0} \left(\frac{1 - 1}{-h} \right) = 0$

Hence, $f'(0)$ does not exist.

Question 115

If $\lim_{x \to 0} \Phi(x) = a3$, $a \neq 0$, then $\lim_{x \to 0} \Phi \left(\frac{x}{a} \right)$ is

Options:

A. a^2

B. $1/a^3$

C. $1/ a^2$

D. a^3

Answer: D

Solution:

$$\lim_{x\to 0}\phi\left(\frac{x}{a}\right)=\lim_{at\to 0}\phi\left(\frac{at}{a}\right)$$
$$=\lim_{t\to 0}\phi(t)=\lim_{x\to 0}\phi(x)=a^3$$

Question 116

If x = secθ -cosθ and y =sec^{n}θ-cos^{n}θ, then $\left(\frac{dy}{dx}\right)^2$ is equal to

Options:

A. $\frac{n^2(y^2+4)}{x^2+4}$

B. $\frac{n^2(y^2-4)}{x^2}$

C. $n\left(\frac{y^2-4}{x^2-4}\right)$

D. $\left(\frac{ny^2}{x}\right)-4$

Answer: A

Solution:

$\frac{dy}{d\theta}$ = $n$sec$^{n\text{-}1}$θ.secθ.tanθ - n.cos$^{n\text{-}1}$(-sinθ)

= n tanθ (sec^{n}θ+ cos^{n}θ)

$\frac{dx}{d\theta}=\frac{\sin\theta}{\cos\theta}$ (secθ+ cosθ)

= tanθ(secθ+ cosθ)

$$\therefore\ \frac{dy}{dx}=\frac{n\tan\theta(sec^n\theta+\cos^n\theta)^2}{\tan\theta(sec\,\theta+\cos\theta)^2}$$

$$\left(\frac{dy}{dx}\right)^2=\frac{n^2(sec^n\theta+\cos^n\theta)^2}{(sec\,\theta+\cos\theta)^2}$$

$$=\frac{n^2\{(sec^n\theta+\cos^n\theta)^2+4\}}{(sec\,\theta+\cos\theta)^2+4}=\frac{n^2(y^2+4)}{x^2+4}$$

Question 117

Let $P(x) = a_0+a_1x^2 + a_2x^4 + a_3x^6 + ... + a_nx^{2n}$ be a polynomial in a real variable x with $0< a_0< a_1 < a_2 <....<a_n$. The function $P(x)$ has

Options:

A. neither a maxima nor a minima

B. only one maxima

C. both maxima and minima

D. only one minima

Answer: D

Solution:

$p(x) = 2x(a_1 + 2a_2x^2 + ...+ na_nx2^{n-2})$

$\therefore\ p'(x) =0$

$\Rightarrow\ X=0$

$P''(x) =2[a_1 + 6a_2x^2 + ...+ n(2n-1)a_n.x^{2n-2}]$

$[P''(x)]_{x=0} =2a_1 >0$

Hence, $p(x)$ has only one minimum at $x=0$.

Question 118

$\int \frac{dx}{9+16\sin^2 x}$ **is equal to**

Options:

A. $\frac{1}{3}\tan^{-1}\left(\frac{3\tan x}{5}\right)+c$

B. $\frac{1}{5}\tan^{-1}\left(\frac{\tan x}{15}\right)+c$

C. $\frac{1}{15}\tan^{-1}\left(\frac{\tan x}{5}\right)+c$

D. $\frac{1}{15}\tan^{-1}\left(\frac{5\tan x}{3}\right)+c$

Answer: D

Solution:

$$\int \frac{dx}{9+16\sin^2 x} = \int \frac{dx}{9\cos^2 x + 25\sin^2 x}$$

$$= \int \frac{\sec^2 x}{9+25\tan^2 x}\,dx$$

Put $\tan x = t$

$$= \int \frac{dt}{9+16t^2} = \frac{1}{15}\tan^{-1}\left(\frac{5t}{3}\right) + C$$

$$= \frac{1}{15}\tan^{-1}\left(\frac{5\tan x}{3}\right) + C$$

Question 119

$\int \frac{x^2 dx}{(x\sin x + \cos x)^2}$ **is equal to**

Options:

A. $\frac{\sin x + \cos x}{x\sin x + \cos x} + c$

B. $\frac{x\sin x - \cos x}{x\sin x + \cos x} + c$

C. $\frac{\sin x - x\cos x}{x\sin x + \cos x} x$

D. None of these

Answer: D

Solution:

$$\frac{d}{dx}(x\sin x + \cos x) = x\cos x$$

$$= \int \frac{x.\cos x}{(x\sin x + \cos x)^2} \cdot \frac{x}{\cos x}\,dx$$

$$= \int \frac{x}{\cos x} \cdot \frac{x.\cos x}{(x\sin x + \cos x)^2}\,dx$$

On integrating by parts,

$$= \int \frac{x}{\cos x} \cdot \left(\frac{1}{x\sin x + \cos x}\right)$$

$$-\int \frac{\cos x - x.(-\sin x)}{\cos^2 x} - \frac{-1dx}{x\sin x + \cos x}$$

$$= \frac{-x}{\cos x(x\sin x + \cos x)} + \int \sec^2 x dx$$

$$= \frac{-x}{\cos x(x\sin x + \cos x)} + \tan x + C$$

Question 120

If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and, $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, then A and B are

Options:

A. $\frac{\pi}{2}, \frac{\pi}{2}$

B. $\frac{2}{\pi}, \frac{3}{\pi}$

C. 0, -4π

D. $\frac{4}{\pi}, 0$

Answer: D

Solution:

$f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$

$\Rightarrow f'(x) = \frac{A\pi}{2}\cos\left(\frac{\pi x}{2}\right)$

$\Rightarrow f'\frac{1}{2} = \frac{A\pi}{2}\cos\frac{\pi}{4} = \frac{A\pi}{2\sqrt{2}}$

$\Rightarrow \frac{A\pi}{2\sqrt{2}} = \sqrt{2} \quad \left\{\because f'\frac{1}{2} = \sqrt{2}\right\}$

$\Rightarrow A = \frac{4}{\pi}$

Now, $\int_0^1 f(x)dx = \frac{2A}{\pi}$

$\Rightarrow \int_0^1 A\sin\left(\frac{\pi x}{2}\right) + B.dx = \frac{2A}{\pi}$

$\Rightarrow \left[\frac{-2A}{\pi}\cos\left(\frac{\pi x}{2}\right) + Bx\right]_0^1 = \frac{2A}{\pi}$

$\Rightarrow B + \frac{2A}{\pi} = \frac{2A}{\pi}$

$\therefore B = 0$

Question 121

Let $g(x) = \int_0^x f(t)\,dt$, where f is such that $\frac{1}{2} \le f(x) \le 1$ for $t \in [0,1]$ and $0 \le f(t) \le \frac{1}{2}$ for $t \in [1,2]$. Then, $g(2)$ satisfies the inequality

Options:

A. $-\frac{3}{2} \le g(2) < \frac{1}{2}$

B. $0 \le g(2) < 2$

C. $\frac{1}{2} \le g(2) < \frac{3}{2}$

D. $2 < g(2) < 4$

Answer: C

Solution:

$g(2) = \int_0^1 f(t)\,dt \int_0^1 = f(t)\,dt + \int_1^2 f(x)\,dt$

$\because \frac{1}{2} \le f(t) \le 1$ for $t \in [0,1]$

$\therefore \frac{1}{2} .1 \le I_2 \le 1$

Again, $0 \le f(t) \le \frac{1}{2}, t \in [1,2]$

$\Rightarrow \frac{1}{2} \le I_1 + I_2 \le 1 + \frac{1}{2}$

$\Rightarrow \frac{1}{2} \le g(2) \le \frac{3}{2}$

Question 122

For which of the following value of *m*, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals to $\frac{9}{2}$?

Options:

A. -4

B. -3

C. 2

D. 4

Answer: D

Solution:

Area $= \int_0^{1-m} (x - x^2 - mx)\,dx$

$= \left[(1-m).\frac{x^2}{2} - \frac{x^3}{3}\right]^{1-m}$

$= (1-m)^3\left(\frac{1}{2} - \frac{1}{3}\right) = \pm\frac{9}{2}$ (given)

Taking +ve sign, $(1-m)^3 = 27 \Rightarrow m = -2$

Taking -ve sign, $(1-m)^3 = -27 \Rightarrow m = 4$

Question 123

Solution of $2y\sin x.\frac{dy}{dx} = 2\sin x.\cos x - y^2\cos x$, $x = \frac{\pi}{2}$, $y = 1$ is given by

Options:

A. $y^2 = \sin x$

B. $y = \sin^2 x$

C. $y^2 = \cos x + 1$

D. None of these

Answer: A

Solution:

On dividing by $\sin x$,

$2y\frac{dy}{dx} + y^2\cot x = 2\cos x$

Put $y^2 = v \Rightarrow \frac{dv}{dx} + v\cot x = 2\cos x$

IF $= e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

$\therefore$ Solution is,

$v.\sin x = \int \sin x(2\cos x)\,dx + C$

$\Rightarrow y^2.\sin x = \sin^2 x + C$

When $x = \frac{\pi}{2}$, $y = 1$

then $C = 0$

$\therefore y^2 = \sin x$

Question 124

The solution of $x^2\frac{dy}{dx} - xy = 1 + \cos\frac{y}{x}$ is

Options:

A. $\tan\frac{y}{2x} = C - \frac{1}{2x^2}$

B. $\tan = \frac{y}{x}C + \frac{1}{x}$

C. $\cos\left(\frac{y}{x}\right) = 1+\frac{C}{x}$

D. $x^2 = (C+x^2)\tan y/x$

Answer: A

Solution:

$\frac{dx}{dy} - \frac{1}{x}.y = \frac{1}{x^2} + \frac{1}{x^2}\cos\frac{y}{x}$ ... (i)

Put y = $vx \Rightarrow \frac{dx}{dy} = v + x\frac{dv}{dx}$

$\therefore$ Eqs. (i) becomes

V= $x\frac{dv}{dx} - v = \frac{1}{x^2} + \frac{1}{x^2}\cos v$

$\Rightarrow \frac{dv}{1+\cos v} = \frac{dx}{x^3} \Rightarrow \int \frac{1}{2}\sec^2\frac{v}{2}dv = \frac{-x^{-2}}{2} + C$

$\Rightarrow \tan\frac{v}{2} = -\frac{1}{2x^2} + C$

$\Rightarrow \tan\frac{y}{2x} = C - \frac{1}{2x^2}$

Question 125

A rectangle *ABCD* has its side *AB* parallel to line *y* = *x* and vertices *A, B* and *D* lie on *y* = 1, *x* = 2 and *x* = -2, respectively. Locus of vertex *'C'* is

Options:

A. $x = 5$

B. $x - y = 5$

C. $y = 5$

D. $x + y = 5$

Answer: C

Solution:

Let the equation of side BC be $y = x + a$.

$\Rightarrow A = (1 - a, 1), B = (2, 2 + a)$

Equation of side AD is

$y - 1 = -[(x-(1 - a)]$

$\Rightarrow D = (-2, 4 - a)$

Let $C = (h, k) \Rightarrow h + 1 - a = 2 - 2$

$\Rightarrow h = a - 1$ and $k - 1 = 2 + a + 4 - a \Rightarrow k = 5$

Thus, the locus of C is $y = 5$

Question 126

The value of λ, for which the equation $6x^2 + 11xy - 10y^2 + x + 31y + \lambda = 0$ represents a pair of straight lines, is

Options:

A. -15

B. 0

C. 2

D. None of these

Answer: A

Solution:

Given, equation will represent a pair of straight lines, if

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$H^2 - ab = 0$

Here, $a = 6$, $h = \frac{11}{2}$, $b = -10$, $g = \frac{1}{2}$,

$f = \frac{31}{2}$, C = λ

$\therefore \Delta = 0$

$\Rightarrow \lambda = \frac{-5415}{361} = -15$

Also, $h^2 - ab = \frac{121}{4} + 60 = \frac{361}{4} > 0$

Hence, λ = -15 is valid.

Question 127

The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation

Options:

A. $x^2 - 6x - 10y + 14 = 0$

B. $x^2 - 10x - 6y + 14 = 0$

C. $y^2 - 6x - 10y + 14 = 0$

D. $y^2 - 10x - 6y + 14 = 0$

Answer: D

Solution:

$c = f^2$

And $(g - 3)^2 + (f - 3)^2$

$= [\sqrt{9 + 9 - 14} + \sqrt{g^2 + f^2 - c^2}]^2$

$\Rightarrow g^2 + f^2 - 6g - 6f + 18 = (2 + g)2; asc = f^2$

$\Rightarrow f^2 - 10g - 6f + 14\ 0$

$\therefore$ Locus of centre (g, f) is

$y^2 - 10x - 6y + 14 = 0$

Question 128

The range of value of r, for which the point $\left(-5 + \frac{r}{\sqrt{2}}, -3 + \frac{r}{\sqrt{2}}\right)$ is an interior point of the major segment of the circle $x^2 + y^2 = 16$ cut-off by the line $x+y = 2$ is

Options:

A. $(\infty, 5\sqrt{2})$

B. $(4\sqrt{2} - \sqrt{14}, 5\sqrt{2})$

C. $(4\sqrt{2} - \sqrt{14}, 4\sqrt{2} + \sqrt{14})$

D. None of the above

Answer: B

Solution:

Since, the given point is an interior point.

Then, $\left(-5 + \frac{r}{\sqrt{2}}\right)^2 + \left(-3 + \frac{r}{\sqrt{2}}\right)^2 - 16 < 0$

$\Rightarrow r^2 - 8\sqrt{2}r + 18 < 0$

$\Rightarrow 4\sqrt{2} - \sqrt{14} < r < 4\sqrt{2} + \sqrt{14}$

The point is on the major segment.

The centre and the point are on the same side of the line $= x + y = 2$.

$\therefore -5 + \frac{r}{\sqrt{2}} - 3 + \frac{r}{\sqrt{2} - 2} < 0$

$\Rightarrow r < 5\sqrt{2}$, so $4\sqrt{2} - \sqrt{14} < r < 5\sqrt{2}$

Question 129

A line L passing through the focus of the parabola $y^2=4(x-1)$, interseds the parabola in two distinet points. If ‘m’ be the slope of the line ‘L’ then

Options:

A. $-2 < m < 1$

B. $m < -1$ or $m > 1$

C. $m \in R$

D. None of the above

Answer: D

Solution:

Let $y = Y$, $x - 1 = X$

Then, the equation becomes $Y^2 = 4X$.

So, the focus = (2, 0)

Any line through the focus is $y = m(x-2)$.

On solving this with $y^2 = 4(x-1)$

$m^2 (x-2)^2 = 4 (x - 1)$

$\Rightarrow m^2x^2 - 4 (m^2 + 1)x + 4(m^2 + 1) = 0$

If $m \neq 0$, $D = 16(m^2 + 1)^2 - 16 m^2(m^2+1)$

$=16(m^2+1) > 0$, for all m

But, if $m = 0$, then x does not have two real distinct values.

So, $m \in R$ except m = 0

$\therefore m \in R - \{0\}$

Question 130

Tangent are drawn from the points on the line $x - y - 5 = 0$ $x^2 + 4y^2 = 4$, then all the chords of contact pass through a fixed point, whose coordinate are

Options:

A. $\left(\frac{4}{5}, -\frac{1}{5}\right)$

B. $\left(\frac{4}{5}, \frac{1}{5}\right)$

C. $\left(-\frac{4}{5}, \frac{1}{5}\right)$

D. None of these

Answer: A

Solution:

Let $A(x_1, x_1 - 5)$ be a point on $x - 5 = 5$, then the chord of contact of $x^2 + 4y^2 = 4$ with respect to A is

$x.x_1 + 4y\,(x_1\text{-}5) = 4$

$\Rightarrow (x+4)\, x_1 - (20y + 4) = 0$

Since, it passes through a fixed point.

$\therefore x+4y = 0$

and$20y + 4 = 0$ (from $P + \lambda\, q = 0$)

$\Rightarrow y = -\frac{1}{5}$ and $x = \frac{4}{5}$

So, the coordinates of fixed point is $\left(\frac{4}{5}, -\frac{1}{5}\right)$

Question 131

The coordinates of a point are $a\tan(\theta+\alpha)$ and $b\tan(\theta+\beta)$, where θ is variable, then locus of the point is

Options:

A. hyperbola

B. rectangular hyperbola

C. ellipse

D. None of the above

Answer: A

Solution:

Given that,

$x = a\tan(\theta + \alpha)$

And $y = b\tan(\theta + \beta)$

or $\tan^{-1}\frac{x}{a} = \theta + \alpha$ ...(i)

And $\tan^{-1}\frac{y}{b} = \theta + \beta$ .....(ii)

To get the required focus, we have to eliminateθ from Eqs. (i) and (ii).

On subtracting Eq. (ii) from Eq. (i), we get

$\tan^{-1}\left(\frac{x}{a}\right)-\tan^{-1}\left(\frac{y}{b}\right)=\alpha - \beta$

$$\Rightarrow \tan^{-1}\left\{\frac{\frac{x}{a}-\frac{y}{b}}{1+\frac{x}{a}.\frac{y}{b}}\right\} = \alpha - \beta$$

$$\Rightarrow \frac{\frac{x}{a}-\frac{y}{b}}{1+\frac{x}{a}.\frac{y}{b}} = \tan(\alpha - \beta)$$

Simplifying, we get the required locus as $xy + ab = (bx - ay) \cot(\alpha - \beta)$

Which is a hyperbola.

Question 132

If $\sin x + \sin^2 x + \sin^3 x = 1$, $\cos^6 x - 4\cos^4 x + 8\cos^2 x$ is equal to

Options:

A. 4

B. 1

C. 0

D. None of these

Answer: A

Solution:

$\sin x + \sin^2 x + \sin^3 x = 1$

$\Rightarrow \sin x (1+\sin^2 x) = 1 \sin^2 x$

$\Rightarrow \sin x (2+\cos^2 x) = \cos^2 x$

Squaring on both sides, we get

$\sin^2 x (2+\cos^2 x)^2 = \cos^4 x$

$\Rightarrow (1 - \cos^2 x) (4-4\cos^2 x + \cos^4 x)$

$= \cos^4 x$

$\Rightarrow \cos^6 x - 4\cos^4 x + 8\cos^2 x = 4$

Question 133

If sin x + sin y = 3(cos y – cos x), then the value of $\frac{\sin 3x}{\sin 3y}$ is

Options:

A. 1

B. -1

C. 0

D. 3

Answer: B

Solution:

We have, sin x + sin y = 3 (cos y – cos x)

$\Rightarrow \sin x + 3\cos x = 3\cos y - \sin y$ ..(i)

$\Rightarrow r\cos(x - \alpha) = r\cos(y + \alpha)$

Where $r = \sqrt{10}$ and $\tan\alpha = \frac{1}{3}$

$\Rightarrow x - \alpha = \pm(y + \alpha)$

$\Rightarrow x = -y$ or $x - y = 2\alpha$

Thus, $x = -y$ satisfied the relation (i).

Therefore, $\frac{\sin 3x}{\sin 3y} = \frac{-\sin 3y}{\sin 3y} = 1$

Question 134

If sin(π cosθ)= cos (π sinθ), then cos $\left(\theta \pm \frac{\pi}{4}\right)$ is equal to

Options:

A. cos $\frac{\pi}{4}$

B. $\frac{1}{2}$ cos $\frac{\pi}{4}$

C. cos $\frac{\pi}{8}$

D. None of these

Answer: B

Solution:

We have, $\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$

$\Rightarrow \sin(\pi\cos\theta) = \sin\left(\frac{\pi}{2} \pm \pi\sin\theta\right)$

$\Rightarrow \pi\cos\theta = \frac{\pi}{2} \pm \pi\sin\theta$

$\Rightarrow \cos\theta \pm \sin\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta \pm \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2}\cos\frac{\pi}{4}$

Question 135

The number of roots of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval [0, 2π] is

Options:

A. 1

B. 2

C. 3

D. infinite

Answer: C

Solution:

We have, $x + 2\tan x = \frac{\pi}{2}$

$\Rightarrow \tan x = \frac{\pi}{4} - \frac{x}{2}$

Let, $y \tan x$ and $y = \frac{\pi}{4} - \frac{x}{2}$

The curves y = tan x and $y = \frac{\pi}{4} - \frac{x}{2}$ in interval [0, 2π], intersect at three points. The abscissa of these three points are the roots of the equation.

Question 136

The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one, then the sides are

Options:

A. 6, 7, 8

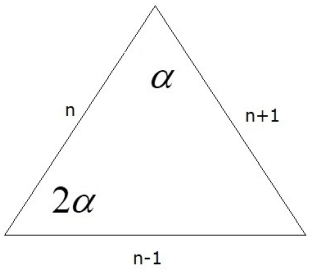
B. 4, 5, 6

C. 1, 2, 3

D. 3, 4, 5

Answer: B

Solution:



The side corresponding to the largest angle is the largest side and the side corresponding to the smallest angle is the smallest side. Thus, the side corresponding to 2α is n+1 and the side corresponding to α is n-1.

Using sine rule

$$\frac{\sin\alpha}{n-1}=\frac{\sin 2\alpha}{n+1}$$

$$\Rightarrow \frac{\sin\alpha}{n-1}=\frac{2\sin\alpha\cos\alpha}{n+1}$$

$$\Rightarrow \frac{1}{n-1}=\frac{2\cos\alpha}{n+1}$$

$$\Rightarrow \cos\alpha=\frac{(n+1)}{2(n-1)}$$

Using cosine rule

$$\cos\alpha = \frac{n^2+(n+1)^2-(n-1)^2}{2n(n+1)}$$

$$\cos\alpha = \frac{n^2+n^2+1+2n-n^2-1+2n}{2n(n+1)}$$

$$\cos\alpha = \frac{n^2+4n}{2n(n+1)}$$

$$cos\alpha = \frac{n+4}{2(n+1)}$$

Thus,

$$\frac{(n+1)}{2(n-1)} = \frac{(n+4)}{2(n+1)}$$

$(n+1)^2 = (n+4)(n-1)$

$n^2+1+2n = n^2+3n-4$

$n=5$

Hence, lengths of the side of the triangle are 4, 5 and 6.

Question 137

If in △ ABC, cos *A* + cos *B* + cos *C* = 3/2, then triangle △ is

Options:

A. right angled

B. isosceles

C. acute

D. equilateral

Answer: D

Solution:

$$\because \cos A + \cos B + \cos C = \frac{3}{2}$$

$$\Rightarrow \frac{b^2+c^2-a^2}{2bc} + \frac{b^2+c^2-b^2}{2ac} + \frac{a^2+b^2-c^2}{2ab} = \frac{3}{2}$$

$\Rightarrow (a+ b- c)\,(a - b)^2 + (b + c - a)\,(b - c)^2$

$+ (c + a - b)\,(c - a)^2 = 0$

As we known,

$a + b > c,\ b + c > a,\ c + a > b$

$\Rightarrow\ a = b = c$

Hence, triangle is an equilateral.

Question 138

The top of a hill observed from the top and bottom of a building of height 'h' is at angles of elevation p and q. Respectively. The height of hill

Options:

A. $\frac{h\cot q}{\cot q - \cot p}$

B. $\frac{h\cot p}{\cot p - \cot q}$

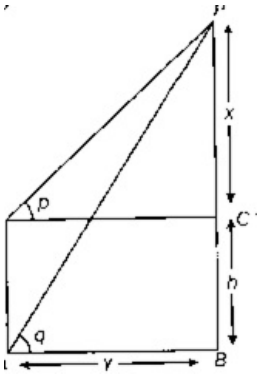
C. $\frac{h\cot p}{\tan p - \tan q}$

D. None of these

Answer: B

Solution:

Let $AD = h$ is the height of the building and PB is height of the hill.



And tan $q = \frac{h+x}{y}$

And tan $p = \frac{x}{y}$

$\Rightarrow y = x\cot p$

Now, tan $q = \frac{h+x}{x\cot p}$

$\Rightarrow x\cot p = (h + x)\cot q$

$\Rightarrow x = \frac{h\cot q}{\cot p - \cot q}$

$\therefore h + x = h + \frac{h\cot q}{\cot p - \cot q}$

$= \frac{h\cot q}{\cot p - \cot q}$

Question 139

If a and b are unit vectors and θ is the angle between them, then $|a+b| < 1$,

if

Options:

A. $\theta = \frac{\pi}{2}$

B. $\theta = \frac{\pi}{3}$

C. $\pi \geq \theta \geq \frac{2\pi}{3}$

D. $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$

Answer: C

Solution:

$\left|\hat{a}+\hat{b}\right| < 1 \Rightarrow \left|\hat{a}+\hat{b}\right|^2 < 1^2$

$\Rightarrow \ (\hat{a}+\hat{b}).(\hat{a}+\hat{b}) < 1^2$

$\Rightarrow \ \left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 + 2\left|\hat{a}\right|+\left|\hat{b}\right|\cos\theta < 1$

$\Rightarrow$ 1 + 1 + 2 cos θ<1

$\Rightarrow$ 2 cos θ < -1 $\Rightarrow$ cos θ < $-\frac{1}{2}$

$\Rightarrow$ $-1 \leq \cos\theta < -\frac{1}{2}$

($\because -1 \leq \cos\theta < 1$, if $\theta \in R$)

$\Rightarrow \cos\pi \leq \cos\theta < \cos\frac{2\pi}{3}$

$\Rightarrow \pi \geq \theta > \frac{2\pi}{3}$

Question 140

The points, whose position vectors are 60 i + 3 j, 40 i – 8 j and *a*i – 52 j collinear, if

Options:

A. *a* = 40

B. *a* = -40

C. *a* = 20

D. *a* = -20

Answer: B

Solution:

(40i – 8j)- (60i – 3j)=λ [(ai – 52j) - (40i – 8j)]

For some scalar λ,

-20 =λ (a – 40) and -11 = -44λ

$\Rightarrow$ a – 40= $\frac{-20}{\lambda}$ and λ = $\frac{1}{4}$

$\Rightarrow$ a = 40 - $\frac{20}{1}$ × 4 = -40

Question 141

In a $\triangle ABC$, AB = ri + j, AC = si – j if the area of triangle is of unit magnitude, then

Options:

A. $|r - s| = 2$

B. $|r + s| = 1$

C. $|r + s| = 2$

D. $|r - s| = 1$

Answer: C

Solution:

Area of $\triangle$ABC

$= \frac{1}{2}\left|\overline{AB} \times \overline{AC}\right|$ = 1 (given)

$\Rightarrow \quad \left|\overline{AB} \times \overline{AC}\right| = 2$

$\Rightarrow \quad \left|(-r)(\hat{i} \times \hat{j}) \times (s\hat{i} - \hat{j})\right| = 2$

$\Rightarrow \quad \left|(-r)(\hat{i} \times \hat{j}) + s(\hat{j} \times \hat{i})\right| = 2$

$\Rightarrow \quad \left|-(r + s)\hat{k}\right| = 2$

$\Rightarrow \quad |r + s| = 2$

Question 142

If a = i-j+k, a.b = 0, a×b = c, where c^- = -2i-j+k, then b^- is equal to

Options:

A. (1, 0, -1)

B. (0, 1, 1)

C. (-1, -1, 0)

D. (-1, 0, 1)

Answer: B

Solution:

Let $b = (x, y, z)$, then
$a.b = 0$
$x + y + z = 0$ ..(i)
Also, a×b = c

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ x & y & z \end{vmatrix} = -2\hat{i} - \hat{j} + \hat{k}$$

$$\Rightarrow (-z - y)\hat{i} + (x - z)\hat{j} + (y + x)\hat{k}$$

$$= -2i - \hat{j} + \hat{k}$$

$\Rightarrow$ $-z-y=-2$, $x-z=-1$, $y+x=1$ ...(ii)
From Eqs. (i) and (ii), we get
$x = 0$, $y = 1$, $z = 1$

Question 143

If P A. =65, P (B)= 80, then P $(A \cap B)$ lies in the interval

Options:

A. [.30, .80]

B. [.35, .75]

C. [.4, .70]

D. [.45, .65]

Answer: D

Solution:

$P(A \cap B) \leq \min \{P(A), P(B)\} = \min \{.65, .80\} = 0.65$
$\therefore$ $P(A \cap B) \leq .65$
Also, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$\geq .65 + .80 - 1 = 0.45$
$\therefore$ $45 \leq P(A \cap B) \leq 0.65$

Question 144

For any two independent events E_1 and E_2 in a space S, $P^{[(E_1 \cup E_2) \cap (E_1 \cup E_2)]}$ is equal to

Options:

A. $\leq \frac{1}{4}$

B. $> \frac{1}{4}$

C. $\geq \frac{1}{2}$

D. $> \frac{1}{2}$

Answer: A

Solution:

$P^{\{(E_1 \cup E_2) \cap (\overline{E}_1 \cup \overline{E}_2)\}}$

$P^{\{(E_1 \cup E_2) \cup (\overline{E_1 \cup E_2})\}} = P(\Phi) = 0 \leq \frac{1}{4}$

Question 145

The resultant of P and Q is R. If Q is also doubked and if Q is reversed, R is again doubled. Then, $P^2 : Q^2 : R^2$ given by

Options:

A. 2 : 2 : 3

B. 3 : 2 : 2

C. 2 : 3 : 2

D. 2 : 3 : 1

Answer: C

Solution:

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$

$4R^2 = P^2 + 4Q^2 - 2PQ\cos\theta$

On solving,

$\frac{P^2}{-6} = \frac{Q^2}{-9} = \frac{R^2}{-6}$

$\Rightarrow$ $P^2 : Q^2 : R^2$ as 2 : 3 : 2

Question 146

Forces of magnitudes 3, *P*, 5, 10 and *Q* are respectively acting along the sides *AB*, *BC*, *CD*, *Ad* and the diagonal *CA* of a rectangle *ABCD*, where *AB* =4m and *BC* =3m. If the resultant is a single force along the other diagonal *BD*, then P, *Q* and the resultant are

Options:

A. 4, $10\frac{5}{12}, 12\frac{11}{12}$

B. 5, 6, 7

C. $3\frac{1}{2}, 8, 9\frac{1}{2}$

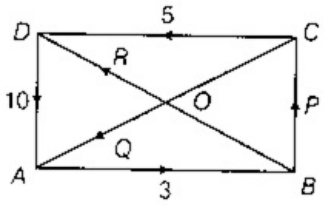
D. None of the above

Answer: A

Solution:

ABCD is a rectangle in which *AB* = 4 m and *BC* = 3 m.

Then, tanθ = $\frac{BC}{AB} = \frac{3}{4}$



$R\cos\theta = Q\cos\theta + 5 - 3$

$= Q\cos\theta + 2$ ....(i)

and $R\sin\theta = P + 10 - Q\sin\theta$...(ii)

and $Q\,AB\sin\theta + 5.BC = 10.AB$

or $Q\,4.\,(3/5) + 5\times3 = 10\times4$

$\left(\because \sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5}\right)$

$\Rightarrow \frac{12}{5} Q = 40 - 15 = 25$

$\Rightarrow Q = \frac{125}{5}$ Newton

Then, find *R* from Eq. (i) and *P* from Eq. (ii)

$\frac{4R}{5} = \frac{125}{5} \times \frac{4}{5} + 2$

$\Rightarrow 4R = \frac{125}{3} + 10 = \frac{155}{3}$

$\Rightarrow R = \frac{155}{12}$

and $\frac{155}{12} \times \frac{3}{5} = P + 10 - \frac{125}{12} \times \frac{3}{5}$

$\frac{155}{4} = 5P + 50 - \frac{125}{4}$

$\Rightarrow 5P = \frac{155}{4} + \frac{125}{4} - 50$

$\Rightarrow 5P = \frac{280}{4} - 50 = 70 - 50 = 20$

$\Rightarrow P = 4$

The forces 3, P, 5, 10 and Q newtons have the resultant R Newton as shown in the figure.

Question 147

A stone is dropped from a certain height which can reach the ground in 5 s. If the stone is stopped after 3 s of its fall and then allowed to fall again, then the time taken by the stone to reach the ground for the remaining distance is

Options:

A. 2 s

B. 3 s

C. 4 s

D. None of these

Answer: C

Solution:

Let h be the height, $t = 5$ s

$\therefore h = \frac{1}{2} gt^2 = (9.8)\ (25) = 122.5$ m

Distance covered in 3 sec = $\frac{1}{2}$ $(9.8)\ (3)^2 = 44.1$ m

Remaining distance = 122.5 – 44.1 = 78.4 m

If t s is the required time, then

$78.4 = 0 + \frac{1}{2} gt^2$

$\Rightarrow t^2 = \frac{784}{49} = 16$

$\Rightarrow t = 4$ s

Question 148

A gun projects a ball at the angle of 45^0 with the horizontal. If the horizontal range is 39.2 m, then the ball will rise to

Options:

A. 9.8 m

B. 4.9 m

C. 2.45 m

D. 19.6 m

Answer: A

Solution:

Horizontal range

$= \frac{u^2 \sin^2 \alpha}{g}$ 39.2

$\Rightarrow \frac{u^2 \sin^2 90^0}{9} = 39.2$

$\Rightarrow \frac{u^2}{g} = 39.2$

$\therefore$ Greatest height reached $= \frac{u^2 \sin^2 \alpha}{2g}$

$\frac{39.2}{2} = \sin^2 45^0 = (19.6). \left(\frac{1}{2}\right) = 9.8$ m

---

Question 149

If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then x is equal to

Options:

A. -1

B. 1

C. 0

D. None of the above

Answer: A

Solution:

We have,

$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$

$\Rightarrow (\tan^{-1}x + \cot^{-1}x)^2 - 2\tan^{-1}x.\cot^{-1}x = \frac{5\pi^2}{8}$

$\Rightarrow (\tan^{-1}x + \cot^{-1}x)^2 - 2\tan^{-1}x.\cot^{-1}x = \frac{5\pi^2}{8}$

$$\Rightarrow \left(\frac{\pi}{2}\right)^2 - 2\tan^{-1}(x)\left(\frac{\pi}{2} - \tan^{-1}(x)\right) = \frac{5\pi^2}{8}$$

$$\Rightarrow \frac{\pi^2}{4} - \pi\tan^{-1}(x) + 2\left(\tan^{-1}(x)\right)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow 2(\tan^{-1}x)^2 - \pi(\tan^{-1}x) - \frac{3\pi^2}{8} = 0$$

Substitute $\tan^{-1}x = p$

So,

$2p^2 - \pi p - 3\pi^2/8 = 0$

$\Rightarrow p^2 - (\pi/2)p - (3\pi^2/16) = 0$

$\Rightarrow p^2 + (\pi/4 - 3\pi/4)p + (\pi/4)(-3\pi/4) = 0$

$\Rightarrow (p + \pi/4)(p - 3\pi/4) = 0$

$\Rightarrow p = -\pi/4, 3\pi/4$

$\Rightarrow \tan^{-1}x = -\pi/4, 3\pi/4$

$\Rightarrow x = -1$

Question 150

$$\frac{\alpha^3}{2}\csc^2\left(\frac{1}{2}\tan^{-1}\left(\frac{\alpha}{\beta}\right)\right) + \frac{\alpha^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\left(\frac{\beta}{\alpha}\right)\right)$$ **is equal to**

Options:

A. $(\alpha - \beta)(\alpha + \beta)^2$

B. $(\alpha + \beta)(\alpha^2 - \beta^2)$

C. $(\alpha + \beta)(\alpha^2 + \beta^2)$

D. None of the above

Answer: C

Solution:

We have,

$$\frac{\alpha^3}{2\sin^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right)} + \frac{\beta^3}{2\cos^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right)}$$

$$= \frac{\alpha^3}{1-\cos\left(\tan^{-1}\frac{\alpha}{\beta}\right)} + \frac{\beta^3}{2\cos^2\left(\tan^{-1}\frac{\beta}{\alpha}\right)}$$

$$= \frac{\alpha^3}{1-\cos\left(\cos^{-1}\frac{\beta}{\sqrt{\alpha^2+\beta^2}}\right)} + \frac{\beta^3}{1+\cos\left(\cos^{-1}\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}\right)}$$

$$= \frac{\alpha^3}{1-\frac{\beta}{\sqrt{\alpha^2+\beta^2}}} + \frac{\beta^3}{1+\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}}$$

$$= \sqrt{\alpha^2+\beta^2}\left[\frac{\alpha^3}{\sqrt{\alpha^2+\beta^2}-\beta} + \frac{\beta^3}{\sqrt{\alpha^2+\beta^2}+\alpha}\right]$$

$$= \sqrt{\alpha^2+\beta^2}\left[\frac{\alpha^3\left(\sqrt{\alpha^2+\beta^2}+\beta\right)}{\alpha^2+\beta^2-\beta^2} + \frac{\beta^3\left(\sqrt{\alpha^2+\beta^2}-\alpha\right)}{\alpha^2+\beta^2-\alpha^2}\right]$$

$$= \sqrt{\alpha^2+\beta^2}[\alpha(\sqrt{\alpha^2+\beta^2}+\beta) + \beta(\sqrt{\alpha^2+\beta^2}-\alpha)]$$

$$= \sqrt{\alpha^2+\beta^2}\left[\alpha(\sqrt{\alpha^2+\beta^2}) + \beta(\sqrt{\alpha^2+\beta^2})\right]$$

$$= (\alpha + \beta)(\alpha^2 + \beta^2)$$
