Chapter - 2
Polynomial

Exercise 2.1

Multiple Choice Questions:

Question 1:
Choose the correct answer from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial $(k-1)x^2+kx+1$ is –3, then the value of *k* is

(A) $\frac{4}{3}$

(B) $\frac{-4}{3}$

(C) $\frac{2}{3}$

(D) $\frac{-2}{3}$

Solution:

(A) $\frac{4}{3}$

It is given in the question,
-3 is one of the zeros of quadratic polynomial $(k-1)x^2+kx+1$.

Putting -3 in the given polynomial,

$(k-1)(-3)^2+k(-3)+1=0$

$(k-1)9+k(-3)+1=0$

$9k-9-3k+1=0$

$6k-8=0$

$k=8/6$

Or,
$k=\frac{4}{3}$

2. A quadratic polynomial, whose zeroes are –3 and 4, is

(A) $x^2 - x + 12$

(B) $x^2 + x + 12$

(C) $\frac{x^2}{2} - \frac{x}{2} - 6$

(D) $2x^2 + 2x - 24$

Solution:

(C) $\frac{x^2}{2} - \frac{x}{2} - 6$

Justification:
Sum of zeroes, $\alpha + \beta = -3+4 = 1$

Product of Zeroes, $\alpha\beta = -3\times 4 -12$

So, the quadratic polynomial becomes,
x^2- (sum of zeroes) x + (product of zeroes)
$= x^2 - (\alpha+\beta)x + (\alpha\beta)$
$= x^2 - (1)x + (-12)$
$= x^2 - x - 12$
$= \frac{x^2}{2} - \frac{x}{2} - 6$

3. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and –3, then

(A) $a = -7, b = -1$

(B) $a = 5, b = -1$

(C) $a = 2, b = -6$

(D) $a = 0, b = -6$

Solution:
(D) a = 0, b = – 6

The zeroes of the polynomial = 2 and -3,

Putting, x = 2 in $x^2 + (a+1)x + b$
$2^2 + (a+1)(2) + b = 0$
$4 + 2a + 2 + b = 0$
$6 + 2a + b = 0$
$2a + b = -6$ …. (i)

Now Putting x = -3 in equation.
$(-3)^2 + (a+1)(-3) + b = 0$

9 – 3a-3 + b = 0
6 – 3a+b = 0
-3a+b = -6 …. (ii)

Subtracting equation (ii) from (i)
2a+b – (-3a+b) = -6-(-6)
2a+b+3a-b = -6+6
5a = 0
a = 0

Putting the value of ‘a’ in equation (i),
2a + b = -6
2(0) +b = -6
b = -6

4. The number of polynomials having zeroes as –2 and 5 is

(A) 1
(B) 2
(C) 3
(D) more than 3

Solution:

(D) More than 3

Explanation:

It is given in the question, the zeroes of the polynomials are -2 and 5.

The polynomial is the form of

p(x) = $ax^2 + bx + c$.

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2.

Sum of the zeroes = $-\frac{b}{a}$

$-2 + 5 = -\frac{b}{a}$

$3 = -\frac{b}{a}$

b = – 3

and

a = 1

Also,

Product of zeroes = $\frac{c}{a}$

(- 2)x(5) = $\frac{c}{a}$

– 10 = c (as, a=1)

Putting the values of a, b and c in the polynomial p(x) = $ax^2 + bx + c$. We get,

$x^2 – 3x – 10$

Hence, we can conclude that x can have any value.

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(A) $-\frac{c}{a}$

(B) $\frac{c}{a}$

(C) 0

(D) $-\frac{b}{a}$

Solution:

(B) $\frac{c}{a}$

Justification:

We have the polynomial,
$ax^3 + bx^2 + cx + d$

Sum of product of roots of a cubic equation is given by $\frac{c}{a}$.

It is given that one root = 0

Now, let the other roots be α, β

$\alpha\beta + \beta(0) + (0)\alpha = \frac{c}{a}$

$\alpha\beta = \frac{c}{a}$

Hence the product of other two roots is $\frac{c}{a}$

6. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is

(A) $b-a+1$

(B) $b-a-1$

(C) $a-b+1$

(D) $a-b-1$

Solution:

(A) $b-a+1$

Taking,
f(x) = x^3+ ax^2+ bx + c

Also,
Zero of f(x) is – 1 so f(-1) = 0
$(-1)^3 + a(-1)^2 + b(-1) + c = 0$
-1 + a – b + c = 0
a – b + c = 1
c = 1 + b – a

Now,
$\alpha \cdot \beta \cdot \gamma = \frac{-d}{a}$ [∵ c = b, d = c]

$-1\beta\gamma = \frac{-c}{1}$

$\beta\gamma = c$

$\beta\gamma = 1 + b - a$

7. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(A) **both positive**

(B) **both negative**

(C) **one positive and one negative**

(D) **both equal**

Solution:

(b) both negative

Taking,
f(x) = x^2 + 99x + 127

Now,
$b^2 - 4ac = (99)^2 - 4(1)\ 127$ (a = 1, b = 99, c = 127)

$b^2 - 4ac = 9801 - 508$

$\sqrt{b^2 - 4ac} = \sqrt{9293}$

$\sqrt{b^2 - 4ac} = 96.4$

Now,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-99 \pm 96.4}{2}$$

Therefore, both roots will be negative as 99 > 96.4

8. The zeroes of the quadratic polynomial $x^2 + kx + k, k \neq 0$,

(A) cannot both be positive

(B) cannot both be negative

(C) are always unequal

(D) are always equal

Solution:

(A) cannot both be positive

Taking,

$f(x) = x^2 + kx + k$

To find the zeroes of f(x), we take,

$f(x) = 0$

$x^2 + kx + k = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-k \pm \sqrt{k^2 - 4k}}{2}$$

$$x = \frac{-k \pm \sqrt{k(k-4)}}{2}$$

For real roots,

$b^2 - 4ac > 0$

$k(k - 4) > 0$

So, solution $k(k - 4) > 0$.

Let,

k = -4 be any point on number line,

$$x = \frac{-k \pm \sqrt{k(k-4)}}{2}$$

$$x = \frac{-(-4) \pm \sqrt{-4(-4-4)}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{2}}{2}$$

$$x = 2(-1 \pm \sqrt{2})$$

$$x_1 = 2(-1 + \sqrt{2})$$

$$x_2 = 2(-1 - \sqrt{2})$$

Here one root is positive, and the other root is negative.
So, the roots cannot be both positive.

9. If the zeroes of the quadratic polynomial $ax^2 + bx + c, c \neq 0$ are equal, then

(A) *c* and *a* have opposite signs
(B) *c* and *b* have opposite signs
(C) *c* and *a* have the same sign
(D) *c* and *b* have the same sign

Solution:

(C) *c* and *a* have the same sign

For equal roots $b^2 - 4ac = 0$
As, b^2 is always positive so 4ac must be positive or we can say product of a and c must be positive i.e.,a and c must have same sign either positive or negative.

10. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(A) Has no linear term and the constant term is negative.
(B) Has no linear term and the constant term is positive.
(C) Can have a linear term but the constant term is negative.
(D) Can have a linear term but the constant term is positive.

Solution:

(A) Has no linear term and the constant term is negative.

Taking,
$f(x) = x^2 + ax + b$
Let, α, β are the roots of it.

Then,
$\beta = -\alpha$ (Given)

$\alpha + \beta = \dfrac{-b}{a}$

and

$\alpha \cdot \beta = \dfrac{c}{a}$

Putting β = - α in equation $\alpha + \beta = \dfrac{-b}{a}$.

$\alpha - \alpha = \dfrac{-a}{1}$

$0 \quad = -a$

Also,

$\alpha(-\alpha) = \dfrac{b}{1}$

$-\alpha^2 = b$

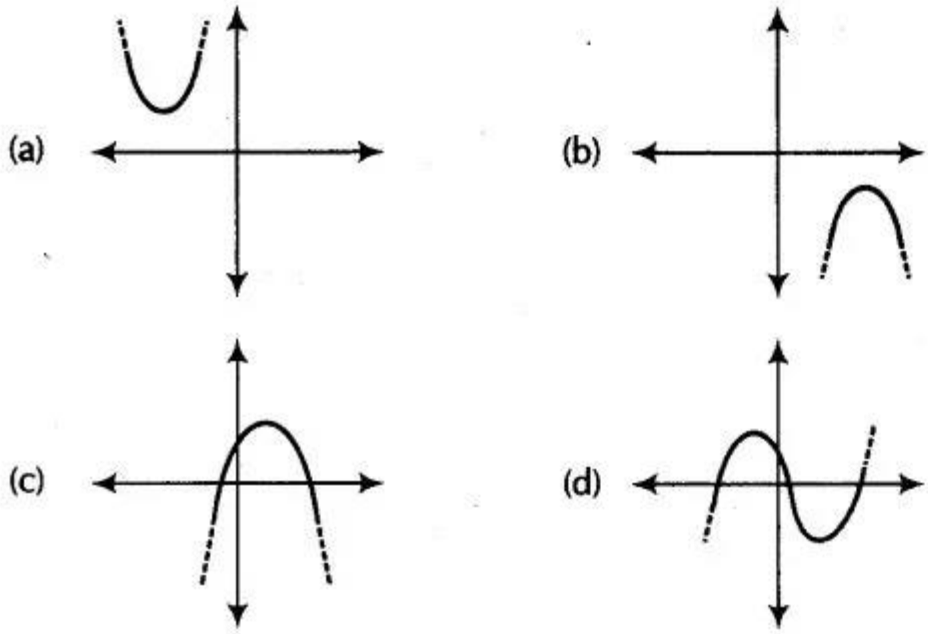
So,

a = 0,

b < 0 or b is negative

Therefore, $f(x) = x^2 + b$ shows that it has no linear term and constant term is negative.

11. Which of the following is not the graph of a quadratic polynomial?



Solution:

(D)

Graph ‘D’ intersect at three points on X-axis so the roots of polynomial of graph is three, so it is cubic polynomial. Other graph are of quadratic polynomial.

Exercise No: 2.2

Short Answer Questions with Reasoning:

Question 1:
Answer the following and justify:

i. **Can x^2-1 be the quotient on division of x^6+2x^3+x-1 by a polynomial in *x* of degree 5?**

ii. **What will the quotient and remainder be on division of ax^2+bx+c by $px^3+qx^2+rx+s, p\neq 0$?**

iii. **If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?**

iv. **If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?**

v. **Can the quadratic polynomial x^2+kx+k have equal zeroes for some odd integer $k > 1$?**

Solution:
(i)

No, x^2-1 cannot be the quotient on division of x^6+2x^3+x-1 by a polynomial in x of degree 5.

Explanation:
When a polynomial with degree 6 is divided by degree 5 polynomial, the quotient will be of degree 1.
Assuming that $(x^2 – 1)$ divides the degree 6 polynomial and the quotient obtained is degree 5 polynomial.

As a = bq+r, so,
(Degree 6 polynomial) = $(x^2 – 1)$(degree 5 polynomial) + r(x)
= (degree 7 polynomial) + r(x)
[As, (x^2 term × x^5 term = x^7 term)]
= (degree 7 polynomial)
So, this contradict our assumption.
Hence, $x^2 – 1$ cannot be the quotient on division of $x^6 + 2x^3 + x – 1$ by a polynomial in x of degree 5

(ii)
Solution:

Degree of the polynomial $px^3 + qx^2 + rx + s = 3$

Degree of the polynomial $ax^2 + bx + c = 2$

Here, Degree of $px^3 + qx^2 + rx + s$ is greater than degree of the $ax^2 + bx + c$

Hence, the quotient would be zero,
Therefore, the remainder would be the dividend = $ax^2 + bx + c$.

(iii)
Solution:

We have, $p(x)= g(x) \times q(x)+r(x)$

As given in the question,
$q(x) =0$
When $q(x)=0$,
$r(x)$ is also = 0
Here, when we divide $p(x)$ by $g(x)$,

Then, $p(x)$ should be = 0
Therefore, the relation between the degrees of p (x) and g (x) is the degree $p(x) <$ degree $g(x)$.

(iv)
Solution:

To divide $p(x)$ by $g(x)$

We have,
Degree of $p(x) >$ degree of $g(x)$

or
Degree of $p(x)$ = degree of $g(x)$

Hence, the relation between the degrees of p (x) and g (x) is degree of $p(x) >$ degree of $g(x)$

(v)

Solution:

A Quadratic Equation has equal roots when:
$b^2 – 4ac = 0$

Given,
$x^2 + kx + k = 0$
$a = 1$,
$b = k$,
$x = k$

Putting values in the equation we get,
$k^2 - 4 (1) (k) = 0$
$k^2 - 4k = 0$
$k (k - 4) = 0$
$k = 0$,
$k = 4$

Here, it is given that k is greater than 1. So, the value of k is 4 if the equation has common roots.

If $k = 4$, then the equation $(x^2 + kx + k)$ will have equal roots.

Question 2:

Are the following statements 'True' or 'False'? Justify your answers.

- **i.** **If the zeroes of a quadratic polynomial ax^2+bx+c are both positive, then *a, b* and *c* all have the same sign.**
- **ii.** **If the graph of a polynomial intersects the *x*-axis at only one point, it cannot be a quadratic polynomial.**
- **iii.** **If the graph of a polynomial intersects the *x*-axis at exactly two points, it need not be a quadratic polynomial.**
- **iv.** **If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.**
- **v.** **If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.**
- **vi.** **If all three zeroes of a cubic polynomial x^3+ax^2-bx+c are positive, then at least one of *a, b* and *c* is non-negative.**
- **vii.** **The only value of *k* for which the quadratic polynomial kx^2+x+k has equal zeros is $\frac{1}{2}$.**

Solution:

(i)

False

Taking α and β as the roots of the quadratic polynomial. If α and β are positive then

$\alpha + \beta = \frac{-b}{a}$. For making sum of roots positive either b or a must be negative.

(ii)

False

The statement is false, because when two zeroes of a quadratic polynomial are equal, then two intersecting points coincide to become one point.

(iii)

True

If a polynomial of degree more than two has two real zeroes and other zeroes are not real or are imaginary, then graph of the polynomial will intersect at two points on x-axis.

(iv)

True

Taking,

$\beta = 0$,

$\gamma = 0$

$f(x) = (x - \alpha)(x - \beta)(x - \gamma)$

$= (x - \alpha)\, x \cdot x$

$f(x) = x^3 - \alpha x^2$

So, it has no linear (coefficient of x) and constant terms**.**

(v)

True

α, β and γ are all negative for cubic polynomial $ax^3 + bx^2 + cx + d$.

$\alpha + \beta + \gamma = \frac{-b}{a}$ ... (i)

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ ....(ii)

$\alpha\beta\gamma = \frac{-d}{a}$ ...(iii)

As α, β, γ are all negative so,

$\alpha + \beta + \gamma = -x$ (Any negative number)

$\frac{-b}{a} = -x$ [From (i)]

$\frac{-b}{a} = x$

As, a, b have same sign and product of any two zeroes will be positive.

So, $\alpha\beta + \beta\gamma + \gamma\alpha = +y$ (Any positive number)

$\frac{c}{a} = +y$ [From (ii)]

c and a have same sign.

$\alpha\beta\gamma = -z$ (Any negative number)

$\frac{-d}{a} = -z$ [From (iii)]

$\frac{-d}{a} = z$

Here, d and a will have same sign.

So, sign of b, c, d are same as of a.

Signs of a, b, c, d will be same either positive or negative.

(vi)

False: As all zeroes of cubic polynomial are positive
Let $f(x) = x^3 + ax^2 - bx + c$

$\alpha + \beta + \gamma$ = positive,
(say + x)
$\frac{-b}{a} = x$
a and b has opposite signs ...(i)

$\alpha\beta + \beta\gamma + \gamma\alpha = +y$
$\frac{c}{a} = y$

So, signs of a and c are same. ... (ii)

Now, $\alpha\beta\gamma$ = positive = +z
$\frac{-d}{a} = z$

a and d have opposite signs.

Therefore, we can conclude that,
From (i) if a is positive, then b is negative.
From (ii) if a is positive, then c is also positive.
From (iii) if a is positive, then d is negative.

So, if zeroes α, β, γ of cubic polynomial are positive then out of a, b, c at least one is negative.

(vii)

False.

$f(x) = kx^2 + x + k$
a = k,
b = 1,
c = k

Condition of equal roots,
$b^2 - 4ac = 0$
$(1)^2 - 4(k)(k) = 0$
$4k^2 = 1$
$k^2 = 1/4$

$k = \pm\frac{1}{2}$

So, the values of k are $\pm\frac{1}{2}$ so that the given equation has equals roots.

Exercise 2.3

Short Answer Questions:

Find the zeroes of the following polynomials by factorization method.

1. $4x^2 - 3x - 1$
2. $3x^2 + 4x - 4$
3. $5t^2 + 12t + 7$
4. $t^3 - 2t^2 - 15t$
5. $2x^2 + \frac{7}{2}x + \frac{3}{4}$
6. $4x^2 + 5\sqrt{2}x - 3$
7. $2s^3 - \left(1 + 2\sqrt{2}\right)s + \sqrt{2}$
8. $v^2 + 4\sqrt{3}v - 15$
9. $y^2 + \frac{3}{2}\sqrt{5}y - 5$
10. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Solution:

1.

$4x^2 - 3x - 1$

By splitting the middle term,
$4x^2-4x+1x-1$

Now, taking out the common factors,
$4x(x-1) +1(x-1)$
$(4x+1)(x-1)$

The zeroes are,
$4x+1= 0$
$4x = -1$
$x = \frac{-1}{4}$

Also,
$(x-1) = 0$
$x = 1$
Therefore, zeroes are $\frac{-1}{4}$ and .

2. $3x^2 + 4x - 4$

Solution:

$3x^2 + 4x - 4$

By splitting the middle term, we get,

$3x^2 + 6x - 2x - 4$

$3x(x+2) -2(x+2)$

$(x+2)(3x-2)$

Either,

$x+2 = 0$

$x = -2$

$3x-2=0$

$3x =2$

$x = \frac{2}{3}$

Therefore, zeroes are $\frac{2}{3}$ and -2.

3. $5t^2 + 12t + 7$

Solution:

$5t^2 + 12t + 7$

By splitting the middle term, we get,

$5t^2 +5t + 7t + 7$

$5t (t+1) +7(t+1)$

$(t+1)(5t+7)$

So, the zeroes are,

$t+1= 0$

$y = -1$

$5t+7=0$

$5t = -7$

$t = -\frac{7}{5}$

So, the zeroes are $\frac{7}{5}$ and -1

4. $t^3 - 2t^2 - 15t$

Solution:

$t^3 – 2t^2 – 15t$
$t (t^2 -2t -15)$

Splitting the middle term of the equation $t^2 -2t -15$, we get,
$t(t^2 -5t + 3t -15)$
$t (t (t-5) +3(t-5)$
$t (t+3)(t-5)$

The zeroes are,
$t = 0$
$t+3= 0$
$t = -3$

$t -5 = 0$
$t =5$

So, zeroes are 0, 5 and -3.

5. $2x^2+\frac{7}{2}x+\frac{3}{4}$

Solution:

$2x^2+\frac{7}{2}x+\frac{3}{4}$

We can write this equation as,
$8x^2+14x+3$

Now, splitting the middle term, we get,
$8x^2+12x+2x+3$
$4x (2x+3) +1(2x+3)$
$(4x+1)(2x+3)$

The zeroes are,
$4x+1= 0$
$x = \frac{-1}{4}$

$2x+3=0$
$x = \frac{-3}{2}$

Therefore, zeroes are $\frac{-1}{4}$ and $\frac{-3}{2}$.

6. $4x^2+5\sqrt{2}x-3$

Solution:

By splitting middle term, we get,

$4x^2+5\sqrt{2}x-3$

$4x^2+6\sqrt{2}x-\sqrt{2}x-3$

$2\sqrt{2}x(\sqrt{2}x+3)-1(\sqrt{2}x+3)$

$(2\sqrt{2}x-1)(\sqrt{2}x+3)$

Therefore,

$x=\frac{1}{2\sqrt{2}}$

or,

$x=\frac{-3}{\sqrt{2}}$

7. $2s^3-\left(1+2\sqrt{2}\right)s+\sqrt{2}$

Solution:

By splitting middle term, we get,

$2s^3-\left(1+2\sqrt{2}\right)s+\sqrt{2}=0$

$2s^3-1s-2\sqrt{2}s+\sqrt{2}\ =0$

$s(2s-1)-\sqrt{2}(2s-1)=0$

$(2s-1)(s-\sqrt{2})=0$

$s=\frac{1}{2}$

$s=\sqrt{2}$

8.

Solution:

By splitting middle term, we get,

$v^2+4\sqrt{3}v-15$

$v^2+5\sqrt{3}v-\sqrt{3}v-15$

$v(v+5\sqrt{3})-\sqrt{3}(v+5\sqrt{3})$

$(v+5\sqrt{3})(\mathrm{v}-\sqrt{3})$

$v=-5\sqrt{3}$ or $v=\sqrt{3}$

9. $y^2+\frac{3}{2}\sqrt{5}y-5$

Solution:

By splitting middle term, we get,

$$y^2+\frac{3}{2}\sqrt{5}y-5=0$$
$$2y^2+3\sqrt{5}y-10=0$$
$$2y^2+4\sqrt{5}y-1\sqrt{5}-10=0$$
$$2y(y+2\sqrt{5})-\sqrt{5}(y+2\sqrt{5})=0$$
$$(y+2\sqrt{5})(2y-\sqrt{5})=0$$
$$y=-2\sqrt{5}\text{ or,y}=\frac{\sqrt{5}}{2}$$

10. $7y^2-\frac{11}{3}y-\frac{2}{3}$

Solution:

By splitting middle term, we get,

$7y^2-\frac{11}{3}y-\frac{2}{3}=0$

We can write this equation as,

$21y^2-11y-2=0$

$21y^2-14y+3y-2=0$

$7y(3y-2)+1(3y-2)=0$

$(7y+1)(3y-2)=0$

$y=\frac{-1}{7}\,or\;\;y=\frac{2}{3}$

Exercise 2.4

Long Answer Questions:

1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorization.

i. $\frac{-8}{3}, \frac{4}{3}$

ii. $\frac{21}{8}, \frac{5}{16}$

iii. $-2\sqrt{3}, -9$

iv. $\frac{-3}{2\sqrt{5}}, \frac{-1}{2}$

Solution:

(i)

Sum of the zeroes = $\frac{-8}{3}$

Product of the zeroes = $\frac{4}{3}$

P(x) = x^2 – (sum of the zeroes) x + (product of the zeroes)

So,

P(x) = $x^2 - x\left(\frac{-8}{3}\right) + \frac{4}{3}$

P(x) = $3x^2 + 8x + 4$

Splitting the middle term, we get,

$3x^2 - 8x + 4 = 0$

$3x^2 + 6x + 2x + 4 = 0$

$3x(x + 2) + 2(x + 2) = 0$

$(x + 2)(3x + 2) = 0$

$x + 2 = 0$

Or

$3x + 2 = 0$

$x = -2, \frac{-2}{3}$

(ii)

Sum of the zeroes $= \frac{21}{8}$

Product of the zeroes = $\frac{5}{16}$

$P(x) = x^2$ – (sum of the zeroes) x + (product of the zeroes)

$P(x) = x^2 - x\left(\frac{21}{8}\right) + \frac{5}{16}$

$P(x) = 16x^2 - 42x + 5$

Splitting the middle term,

$16x^2 - 42x + 5 = 0$

$16x^2 - (2x + 40x) + 5 = 0$

$16x^2 - 2x - 40x + 5 = 0$

$2x(8x - 1) - 5(8x - 1) = 0$

$(8x - 1)(2x - 5) = 0$

$x = \frac{1}{8}, \frac{5}{2}$

(iii)

Sum of the zeroes $= -2\sqrt{3}$

Product of the zeroes $= -9$

$P(x) = x^2$ – (sum of the zeroes) x + (product of the zeroes)

$P(x) = x^2 - 2\sqrt{3}x - 9$

Splitting the middle term,

$x^2 - 2\sqrt{3}x - 9 = 0$

$x^2 - (-\sqrt{3}x + 3\sqrt{3}x) - 9 = 0$

$x^2 + \sqrt{3}x - 3\sqrt{3}x - 9 = 0$

$x(x + \sqrt{3}) - 3\sqrt{3}(x + \sqrt{3}) = 0$

$(x + \sqrt{3})(x - 3\sqrt{3}) = 0$

$x = -\sqrt{3}, 3\sqrt{3}$

(iv)

Sum of the zeroes $= \frac{-3}{2\sqrt{5}}$

Product of the zeroes $= -\frac{1}{2}$

$P(x) = x^2$ – (sum of the zeroes) x + (product of the zeroes)

$P(x) = x^2 - (\frac{-3}{2\sqrt{5}})x - \frac{1}{2}$

$P(x) = 2\sqrt{5}\,x^2 + 3x - \sqrt{5}$

Splitting the middle term,

$2\sqrt{5}\,x^2 + 3x - \sqrt{5} = 0$

$2\sqrt{5}\,x^2 + (5x - 2x) - \sqrt{5} = 0$

$2\sqrt{5}\,x^2 + 5x - 2x - \sqrt{5} = 0$

$\sqrt{5}\,x\,(2x + \sqrt{5}) - (2x + \sqrt{5}) = 0$

$(2x + \sqrt{5})(\sqrt{5}\,x - 1) = 0$

$x = \frac{-\sqrt{5}}{2}$ and $x = \frac{1}{\sqrt{5}}$

2. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form *a*, *a* + *b*, *a* + 2*b* for some real numbers *a* and *b*, find the values of *a* and *b* as well as the zeroes of the given polynomial.

Solution:

We have, a, a+b, a+2b are roots of given polynomial $x^3-6x^2+3x+10$.

Sum of the roots = a+2b+a+a+b

$$= -\frac{\textbf{coefficient of } x^2}{\textbf{coefficient of } x^3}$$

$3a+3b = -(\frac{-6}{1})$

$= 6$

$3(a+b) = 6$

$a+b = 2$ ....(i)

$b = 2- a$

Product of roots = (a+2b)(a+b)a

$$= -\frac{\textbf{constant}}{\textbf{coefficient of } x^3}$$

$(a+b+b)(a+b)a = -\frac{10}{1}$

Putting the value of a + b = 2, we get,
(2+b)(2) a = -10
(2+b) 2a = -10
(2+2-a) 2a = -10
(4-a)2a = -10
$4a-a^2 = -5$
$a^2-4a-5 = 0$
$a^2-5a+a-5 = 0$

(a-5)(a+1) = 0
a-5 = 0   or a+1 = 0

a = 5 and a = -1

Putting values of a in equation (i),

When a = 5,
5+b = 2
b = -3

When a = -1,
-1+b = 2
b = 3

3. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$, find its other two zeroes.

Solution:

According to question, $\sqrt{2}$ is one of the zero of the cubic polynomial.

So,
(x-$\sqrt{2}$) is one of the factor of the given polynomial,
p(x) = $6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$

So, by dividing p(x) by x-$\sqrt{2}$

$$
\begin{array}{r|l}
 & \quad\quad\quad\quad 6x^2 + 7\sqrt{2}x + 4 \\
x-\sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & 6x^3 - 6\sqrt{2}x^2 \\
 & - \quad - \\ \hline
 & \quad 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & \quad 7\sqrt{2}x^2 - 14x \\
 & \quad - \quad\quad + \\ \hline
 & \quad\quad\quad\quad 4x - 2\sqrt{2} \\
 & \quad\quad\quad\quad 4x - 2\sqrt{2} \\
 & \quad\quad\quad\quad - \quad + \\ \hline
 & \quad\quad\quad\quad\quad\quad 0
\end{array}
$$

Therefore,

$6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$ = (x-$\sqrt{2}$) (6x² +7$\sqrt{2}$ x + 4)

Splitting the middle term, we have,
(x-$\sqrt{2}$) (6x² + 4$\sqrt{2}$ x + 3$\sqrt{2}$ x + 4)
(x-$\sqrt{2}$) [2x(3x+2$\sqrt{2}$) + $\sqrt{2}$ (3x+2$\sqrt{2}$)]
(x-$\sqrt{2}$) (2x+$\sqrt{2}$) (3x+2$\sqrt{2}$)

For finding the zeroes of p(x),

Take p(x) = 0
(x-$\sqrt{2}$) (2x+$\sqrt{2}$) (3x+2$\sqrt{2}$) = 0
x = $\sqrt{2}$,
x = $\frac{-\sqrt{2}}{2}$,
x = $\frac{-2\sqrt{2}}{3}$

On simplifying,
x = $\sqrt{2}$,
x = $\frac{-1}{\sqrt{2}}$,
x= $\frac{-2\sqrt{2}}{3}$

So, the other two zeroes of p(x) are $\frac{-1}{\sqrt{2}}$ and $\frac{-2\sqrt{2}}{3}$.

4. Find *k* so that x^2+2x+k is a factor of $2x^4+x^3-14x^2+5x+6$. Also find all the zeroes of the two polynomials.

Solution:

By factor theorem and Euclid's division algorithm, we get
f(x) = g(x) × q (x) + r(x)

Taking,
f(x) = $2x^4 + x^3 - 14x^2 + 5x + 6$ ...(i)

and g(x) = $x^2 + 2x + k$

$$
\begin{array}{r|l}
 & x^2-3x-8-2k \\
\hline
x^2+2x+k & 2x^4+x^3-14x^2+5x+6 \\
 & 2x^4+4x^3-2kx^2 \\
 & -\quad -\quad - \\
\hline
 & -3x^3-14x^2-2kx^2+5x+6 \\
 & -3x^3-6x^2\qquad -3kx \\
 & +\quad +\qquad + \\
\hline
 & -8x^2-2kx^2+5x+3kx+6 \\
 & -8x^2\quad -16x\quad -8k \\
 & +\quad +\quad + \\
\hline
 & -2kx^2+21x+3kx+8k+6 \\
 & -2kx^2\quad -4kx-2k^2 \\
 & +\quad +\quad + \\
\hline
 & 21x+7kx+2k^2+8k+6
\end{array}
$$

But, r(x) = 0

$(21 + 7k)x + 2k^2 + 8k + 6 = 0x + 0$

So,
$21 + 7k = 0$
$k = \dfrac{-21}{7}$

And

$2k^2 + 8k + 6 = 0$

Splitting the middle term, we get,
$2k^2 + 6k + 2k + 6 = 0$
$2k(k + 3) + 2(k + 3) = 0$
$(k + 3)(2k + 2) = 0$
$k + 3 = 0$ or $2k + 2 = 0$

Therefore, $k = -3, -1$.

Common solution is $k = -3$.
$q(x) = 2x^2 - 3x - 8 - 2(-3)$
$= 2x^2 - 3x - 8 + 6$
$q(x) = 2x^2 - 3x - 2$

$f(x) = g(x)\,q(x) + 0$
$= (x^2 + 2x - 3)(2x^2 - 3x - 2)$
$= (2x^2 - 4x + 1x - 2)(x^2 + 3x - 1x - 3)$
$= [2x(x - 2) + 1(x - 2)][x(x + 3) - 1(x + 3)]$

$f(x) = (x - 2)(2x + 1)(x + 3)(x - 1)$

For zeroes of $f(x)$, $f(x) = 0$
$(x - 1)(x - 2)(x + 3)(2x + 1) = 0$

$(x - 1) = 0$,
$(x - 2) = 0$,
$(x + 3) = 0$

and
$2x + 1 = 0$

$x = 1$,
$x = 2$,
$x = -3$ and $x = \frac{-1}{2}$

Hence, zeroes of $f(x)$ are 1, 2, -3 and $\frac{-1}{2}$. And, the zeros of x^2+2x-3 is 1, -3.

5. Given that $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.

Solution:

Let f(x) = $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$ and g(x) = $x - \sqrt{5}$.
g(x) is a factor of f(x) so f(x) = q(x) $x - \sqrt{5}$.

On dividing f(x) by g(x) we get,

$$
\begin{array}{r|l}
 & x^2 - 2\sqrt{5}x + 3 \\
\hline
x - \sqrt{5} & x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
 & x - \sqrt{5}x^2 \\
 & - \quad + \\
\hline
 & \quad -2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
 & \quad -2\sqrt{5}x^2 + 10x \\
 & \quad + \qquad - \\
\hline
 & \qquad\qquad 3x - 3\sqrt{5} \\
 & \qquad\qquad 3x - 3\sqrt{5} \\
 & \qquad\qquad - \quad + \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

f(x) = q(x) g(x)

$f(x) = (x^2 - 2\sqrt{5}x + 3)(x - \sqrt{5})$
$f(x) = (x - \sqrt{5} - \sqrt{2})(x - \sqrt{5} + \sqrt{2})(x - \sqrt{5})$
So,
$f(x) = 0$
Therefore,
$(x - \sqrt{5}) \quad = 0$
$(x - \sqrt{5} - \sqrt{2}) = 0$
$(x - \sqrt{5} + \sqrt{2}) = 0$

Therefore, zeroes of polynomial are:
x = $\sqrt{5}$
x = $(\sqrt{5} + \sqrt{2})$
x= $\sqrt{5} - \sqrt{2})$

6. For which values of *a* and *b*, are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of *p*(*x*) are not the zeroes of *q*(*x*)?

Solution:

We have, factor theorem and Euclid's division lemma to solve this question,

Using factor theorem if q(x) is a factor of p(x), then r(x) must be zero.
$p(x) = x^5 – x^4 – 4x^3 + 3x^2 + 3x + b$
$q(x) = x^3 + 2x^2 + a$

$$
\begin{array}{r|l}
 & \qquad\qquad\qquad\qquad x^2 - 3x + 2 \\
\cline{2-2}
x^3+2x^2+a & x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \\
 & x^5 + 2x^4 \qquad + ax^2 \\
 & - \quad - \qquad\quad - \\
\cline{2-2}
 & -3x^4 - 4x^3 - ax^2 + 3x^2 + 3x + b \\
 & -3x^4 - 6x^3 \qquad\qquad - 3ax \\
 & + \quad\; + \qquad\qquad\quad + \\
\cline{2-2}
 & \quad -2x^3 - ax^2 + 3x^2 + 3x + 3ax + b \\
 & \quad -2x^3 \qquad + 4x^2 \qquad\qquad + 2a \\
 & \quad + \qquad\quad - \qquad\qquad\quad - \\
\cline{2-2}
 & -ax^2 - x^2 + 3x + 3ax - 2a + b
\end{array}
$$

So, by factor theorem remainder must be zero so,
$r(x) = 0$
$-(a + 1)x^2 + (3a + 3)x + (b – 2a) = 0x^2 + 0x + 0$

Comparing the coefficients of x^2, x and constant on both sides, we get,
$- (a + 1) = 0$, $3a + 3 = 0$ and $b – 2a = 0$.

So,
$a = -1$ and
$b – 2(-1) = 0$
$b = -2$

For a = -1 and b = -2, zeroes of q(x) will be zeroes of p(x).

For zeroes of p(x),
$p(x) = 0$

$(x^3 + 2x^2 + a)(x^2 – 3x + 2) = 0$ $\quad [\because a = -1]$
$[x^3 + 2x^2 – 1][x^2 – 2x – 1x + 2] = 0$
$(x^3 + 2x^2 – 1)[x(x – 2) – 1(x – 2) = 0$
$(x^3 + 2x^2 – 1) (x – 2) (x – 1) = 0$

So, $x = 2$ and 1 are not the zeroes of $q(x)$.