Number System and Its Operations

Numbers are the symbolic representation of counted objects. There are infinite counting numbers from 1. Some are divisible by another whereas some are not divisible. Numbers are differentiated according to their divisibility and factors. A numeral system is a writing system for expressing numbers. The most commonly used system of numerals is Hindu-Arabic numeral system. In this chapter, we will learn about various numeral systems, types of numbers and operation on numbers.

Indian or Hindu-Arabic Number System

This number system was introduced by Indians, and is therefore, called Indian Number System. In this number system 10 is considered as the base.
10 ones = 10, 10 tens = 1 hundred, 10 hundreds = 1 thousand
Hindu - Arabic number system is based on the place value of digits in number.

Indian Place Value Chart

Crores	Ten Lakhs	Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Ones
		2	9	8	7	3	5

The number two lakh ninety-eight thousand seven hundred and thirty- five is written by placing 2 at the place of "lakhs", 9 at the place of "Ten Thousands", 8 at "Thousands", 7 at "Hundreds", 3 at "Tens" and 5 at "Ones".

Place Value

If a number contains more than one digit then the place occupies by each digit is its place value. In the number 732 the number 7 occupies the place of hundreds, therefore, the place value of 7 is seven hundred.

Face Value

The face value of a number does not change regardless of the place it occupies. Therefore, the face value of a number is the number itself.

International Place Value Chart

Billions		**Millions**			**Thousand**					
Ten Billion	Billion	Hundred Million	Ten Million	Million	Hundred Thousand	Ten	Thousand	Hundred	Tens	Ones
	5	6	8	4	3	2	5	4	3	1

The above chart is the international place value chart. The number 5,684,325,431 is read as five billion, six hundred eighty-four million, three hundred twenty-five thousand, four hundred thirty-one.

Comparison between Indian and International Number System

International	Hundred Billion	Ten billion	Billion	Hundred Million	Ten Million	Million	Hundred Thousand	Ten Thousand	Thousand	Hundred	Ten	Ones

Indian	Kharab	Ten Arrab	Arab	Ten Crore	Crore	Ten Lakh	Lakh	Ten Thousand	Thousand	Hundred	Ten	Ones

Example:
According to the place value of International number system, which one is six million five hundred fifteen thousand two hundred twenty-one?
(a) 60515221 (b) 6515221
(c) 65150221 (d) 600515221
(e) None of these
Answer (b)

Example:
What is the reduced form of the following expanded form:
$7\times10000+5\times100+4\times10+6.$
(a) 70546 (b) 7546
(c) 75460 (d) 07546
(e) None of these
Answer (a)

Explanation: $7\times10000+5\times100+4\times10+6$
$=70000+500+40+6=70546$

Roman Numerals

This numeric system is represented by Roman numerals which are the combinations of letters from the latin alphabet. Roman numerals are based on seven symbols as given below:

Symbol	I	V	X	L	C	D	M
Value	1	5	10	50	100	500	1000

The numbers I, 2, 3, 4, 5, 6, 7, 8, 9 and 10 are expressed in Roman numerals as I, II, III, IV, V, VI VII, VIII, IX, X
The rules for this system are:

- Repeating a numeral upto three times represents addition of the numbers, eg. III represent $1+1+1=3.$

Note: Only I, X, C and M can be repeated. But the symbols V, L and D are never repeated.

- Numerals that decrease from left to right denotes addition of the numbers, eg. XVI represents $10+5+1=16.$

- Writing a smaller numeral to the left of a larger numeral denotes subtraction, eg. IX represents $10-1=9 \text{ and } XC=100-10=90$

Note:
(a) The symbols V, Land Dare never written to the left of a symbol of greater value ie. V, L and D are never subtracted.
(b) The symbol 1 can be subtracted from V and X only.
(c) The symbol X can be subtracted from L, M and C only.

- To represent larger numbers, a bar over a numeral means to multiply the number by 1000.

	5000	10000	50000	100000	500000	1000000
Value	$\overline{V}$	$\overline{X}$	$\overline{L}$	$\overline{C}$	$\overline{D}$	$\overline{M}$

Example:
Arrange the following Roman numerals in descending order.
MD, MDCCC, DCCCXC, CM
(a) MDCCOMD> DCCCXC > CM
(b) MDCCC > DCCCXC > MD > CM
(c) MDCCC > CM > DCCCXC > MD
(d) MDCCC > MD > CM > DCCCXC
(e) None of these
Answer (d)

Explanation: Clearly, MDCCC = 1800, MD = 1500, CM = 900 and DCCCXC = 890
∴ Option (d) is correct.

Types of Numbers

Natural Numbers

Every counting number is called a natural number. 1, 2, 3, 4, 5, etc. are natural numbers.
Zero is excluded from the natural numbers.
Natural numbers are represented by N (First capital letter of its name).

Whole Numbers

When 0 is included with counting numbers, it becomes whole number. Whole number is represented by W (First letter of its name). Whole numbers (W) = {0, 1, 2, 3, 4 .....}.

Prime Numbers

The numbers which have only two factors, 1 and the number itself are called prime numbers.
The numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, etc are prime numbers.

Composite Numbers

The numbers which have more than two factors are called composite numbers.
Factors of 4 and 6 are: 1, 2, 4 and 1, 2, 3, 6 respectively. Hence, these are composite numbers.

Co-prime

If H.C.F. (Highest Common Factor) of two numbers is 1 then the numbers are called co-prime. The numbers 21 & 22 are co-prime as both have no common factors other than 1.

Perfect Numbers

The natural numbers whose sum of positive divisors (excluding the number itself) is equal to the number itself are called perfect numbers. The first perfect number is 6.

Successor

Successor of every number comes just after the number. Successor of $25 = 25 + 1 = 26$.
Successor of $4573 = 4573 + 1 = 4574$.

Predecessor

Predecessor of every number comes just before the number. Predecessor of 23 is obtained by subtracting 1 from the number. Predecessor of $23 = 23 - 1 = 22$.

Example: Choose the composite numbers from the following numbers:
87,67,45,34,23,27,33.
(a) 45, 87, 34, 27, 33 (b) 45, 87, 67, 33
(c) 33, 27, 23, 34 (d) All the above
(e) None of these

Answer (a)

Explanation: In the given numbers 23 and 67 are prime numbers and rest are composite numbers.

Example:
How many prime numbers are there between 10 to 50?
(a) 10 (b) 11
(c) 13 (d) 18
(e) None of these
Answer (b)

Explanation: Prime numbers between 10 and 50 are:
11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.

Properties of Addition

Closure Property

The sum of two whole numbers is always a whole number. If a and b are two whole numbers, then their addition (a + b) is also a whole number.

Commutative Property

The sum of two and more whole numbers remains same even if the order of the numbers are changed.

Associative Property

The addition of a set of numbers is same regardless of how the numbers are grouped. The associative property involves 3 or more numbers.

Additive Identity

Zero (0) is called the additive identity of every whole number. When 0 is added to the whole number its identity does not change or number remains unchanged.

Additive Inverse

Additive inverse of a is $-a$ and additive inverse of $-a$ is a, therefore, the sum of number with its additive inverse is always zero.

Properties of Subtraction

- Subtraction is inverse process of addition, and subtraction of two whole numbers is always a whole number.
- If a < b then their subtraction, $a-b$ is not a whole number.
- Subtraction is neither commutative nor associative, i.e. $8-(6-5) \neq (8-6)-5$.

Properties of Multiplication

Closure Property

If a and b are whole numbers, then their product a x b is also a whole number. Let us consider two whole numbers 3 and 4, their product is 12, which is also a whole number.

Commutative Property

The product of the whole numbers remains same even if the order of the multiplication is changed. In other words if a and b are whole numbers, their product $a \times b = b \times a$.

For example: $5 \times 6 = 30$, on changing their order, $6 \times 5 = 30$, Thus $5 \times 6 = 6 \times 5$.
Similarly, $10 \times 15 = 150$ and $15 \times 10 = 150$, thus, $10 \times 15 = 15 \times 10$

Associative Property

The product of more than two numbers remains same by changing the groups of the numbers.
If a, b and c are three numbers then their product $(a \times b) \times c = a \times (b \times c)$.

For example:

$(4 \times 5) \times 6 = 20 \times 6 = 120$ and $4 \times (5 \times 6) = 4 \times 30 = 120$.

Thus $(4 \times 5) \times 6 = 4 \times (5 \times 6)$.

Multiplicative Identity

The product of every whole number with 1 is the number itself.

If a is a whole number then, $a \times 1 = 1 \times a = a$

Multiplication of a number by 1 is the number itself, therefore, the identity of the whole number does not change thus 1 is called multiplicative identity of the whole number, i.e.

$5 \times 1 = 1 \times 5 = 5$ and $10 \times 1 = 1 \times 10 = 10$.

Multiplication of Whole Numbers with 0:

When a whole number is multiplied by 0, it becomes equal to zero.

In other words $0 \times a = a \times 0 = 0$. i.e. $0 \times 11 = 0$ and $11 \times 0 = 0$.

Multiplicative Inverse

Multiplicative inverse of a number a is $\frac{1}{a}$ and multiplication of a and $\frac{1}{a}$ is 1.

Let us consider a number 4 and its multiplicative inverse $\frac{1}{4}$.

Hence, its multiplication $= 4 \times \frac{1}{4} = 1$.

Properties of Division

- If a and b are two whole numbers in the form of $a \div b$ then it can be expressed by $\frac{a}{b}$.

  The division of a by b may not be a whole number. i.e. $10 \div 5 = \frac{10}{5} = 2$ is a whole number. The division of $25 \div 15 = \frac{25}{15} = \frac{5}{3} = 1.666$ is not a whole number.

- For every number a, $a \div 1 = \frac{a}{1} = a$ and $a \div a = \frac{a}{a} = 1$, therefore, the division of every number by 1 is the number itself.

  i.e. $6 \div 1 = \frac{6}{1} = 6, 6 \div 6 = \frac{6}{6} = 1, 10 \div 1 = \frac{10}{1} = 10, 10 \div 10 = \frac{10}{10} = 1.$

- If a is a number whereas, $a \neq 0$ then $0 \div a = 0$ and $a \div 0$ cannot be defined. The division of 0 by 4 or $0 \div 4 = 0$ and $4 \div 0$ cannot be defined.
- If a, b and c are whole numbers then $(a \div b) \div c \neq a \div (b \div c)$ therefore, division is not associative, i.e. $(16 \div 4) \div 2 \neq 16 \div (4 \div 2)$
- If a, b, c, d are whole numbers in the form of $a\overline{)\frac{c}{b}}$ d then a is called divisor, b is dividend, c is quotient and d is remainder. Whereas, Dividend = divisor × quotient + remainder.

BODMAS Rule

When a single expression contains many mathematical operations then BODMAS rule is used for the simplification of the expression. The word BODMAS has been arranged according to the priority of the operations.

The letters of **BODMAS** express the following operations:

B Stands for **Bracket**
O Stands for **of**
D Stands for **Division**
M Stands for **Multiplication**
A Stands for **Addition**
S Stands for **Subtraction**

Example:

Evaluate $[\{(563\times 8)+70\}-4384]\div 10$.

(a) 60 (b) 45
(c) 19 (d) 24
(e) None of these

Answer (c)

Explanation: $[\{(563\times 8)+70\}-4384]\div 10$

$=(4504+70-4384)\div 10=190\div 10=19$

Example:

Product of 23 and 13 is added with 56 then subtracted from 675. Which one of the following is correct for the arrangement of given statement using brackets?

(a) $675-\{(23\times 13)+56\}$
(b) $\{(23\times 13)+56\}-675$
(c) $(675+56)-(23\times 13)$
(d) All the above
(e) None of these

Answer (a)