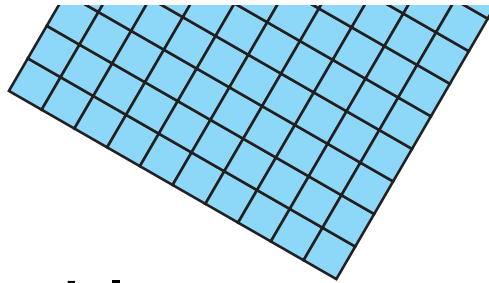


Activity 26



Factorisation of polynomials

Objective

To interpret geometrically the factors of a quadratic expression of the type $x^2 + bx + c$ using square grids, strips and paper slips.

Pre-requisite knowledge

1. Splitting the middle term of a quadratic polynomial.
2. Area of a rectangle

Material Required

Square grids, strips and paper slips.

Procedure

Case I

Take $b = 5$, $c = 6$

Polynomial is $x^2 + 5x + 6$

Now find two numbers whose sum is 5 and product is 6 i.e. 3 and 2.

therefore, $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

1. Take a square grid of dimension (10×10) . It represents x^2 as shown in Fig 26 (a).
2. Add 3 strips of each dimensions $x \times 1$ as shown in Fig 26 (b).
3. The area of rectangle formed in Fig 26 (b) represents $x^2 + 3x$.
4. Add 2 strips of dimensions $2 \times x$ as shown in Fig 26 (c). Now the total area = $x^2 + 3x + 2x$.
5. Add 6 slips of dimensions (1×1) i.e. 6×1 to complete the rectangle as shown in Fig 26 (d).

Observations

The students will observe that

Area of new rectangle = $x^2 + 3x + 2x + 6$

$(x + 3)(x + 2) = x^2 + 3x + 2x + 6$

therefore, $x^2 + 5x + 6 = (x + 3)(x + 2)$

Case II

Take $b = 1$, $c = -6$

Polynomial is $x^2 + x - 6 = x^2 + 3x - 2x - 6$

1. Take a square grid of dimension (10×10) . It represents x^2 as shown in Fig 26 (e).
2. Add 3 strips of dimensions $1 \times x$ as shown in Fig 26 (f).
3. The area of rectangle formed in Fig 26 (f) represents $x^2 + 3x$.

4. Shade 2 strips of dimensions $1 \times x$ as shown in Fig 26 (g).
5. Remove 6 slips of dimensions 1×1 so as to complete the rectangle. We have new rectangle of dimensions $(x - 2) \times (x + 3)$ as shown in Fig 26 (h).

Observations

The students will observe that

$$\text{Area of new rectangle} = x^2 + 3x - 2x - 6 = (x + 3)(x - 2)$$

$$\text{therefore, } x^2 + x - 6 = (x + 3)(x - 2)$$

Learning Outcomes

The students learn the geometrical meaning of the process of factorization of a quadratic expression. The three terms in the polynomial $x^2 + bx + c$ correspond to a square and two rectangles. The polynomial is factorisable if the three figures can be arranged to form a single rectangle whose sides are the factors of the given polynomial.

Remark

1. Teacher may choose polynomial of the type $x^2 + bx + c$ taking other suitable values of b and c for the activity (where b or c is negative).
2. 10 slips = 1 strip, 10 strips = 1 grid.

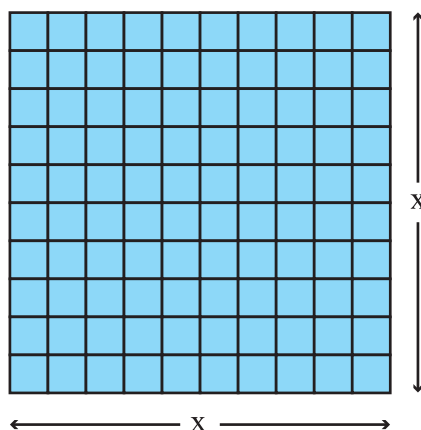


Fig 26 (a)

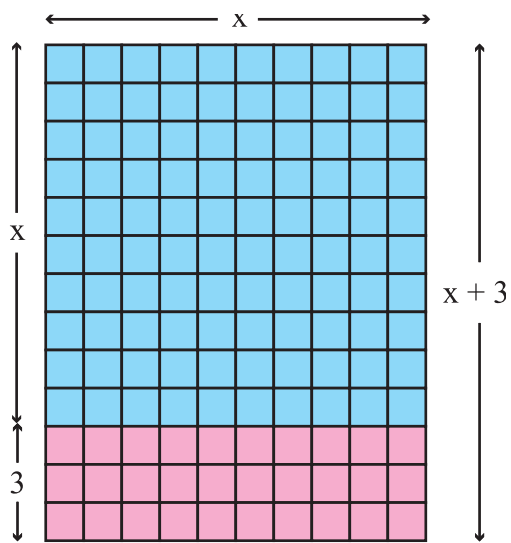


Fig 26 (b)

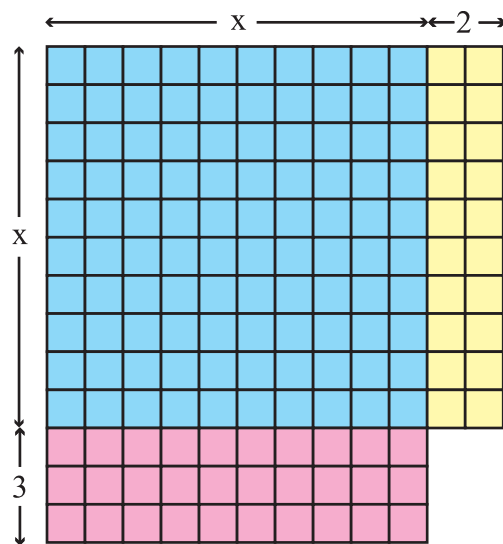


Fig 26 (c)

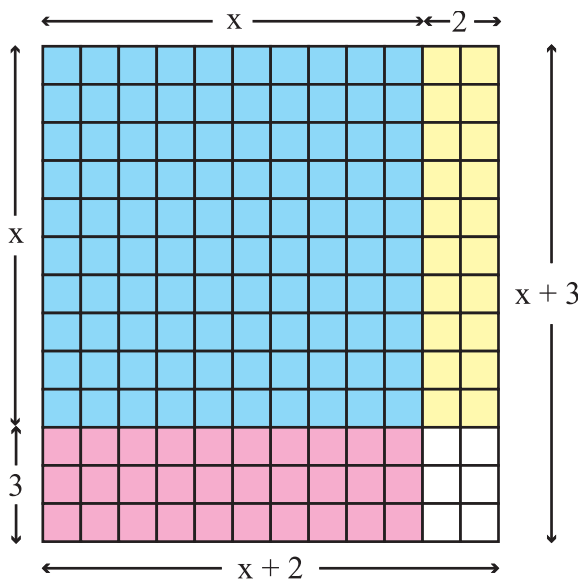


Fig 26 (d)

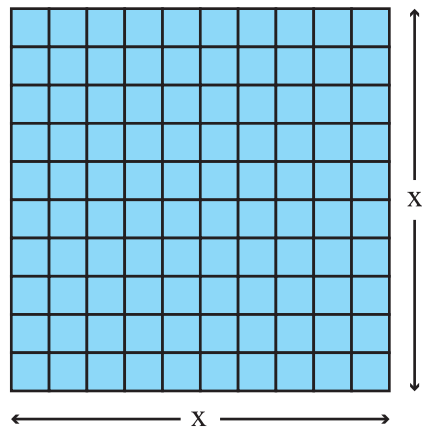


Fig 26 (e)

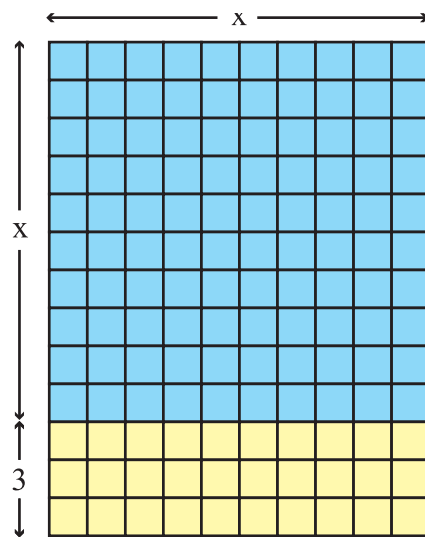


Fig 26 (f)

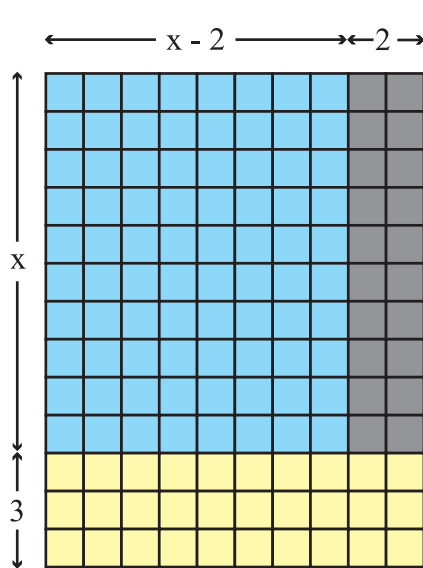


Fig 26 (g)

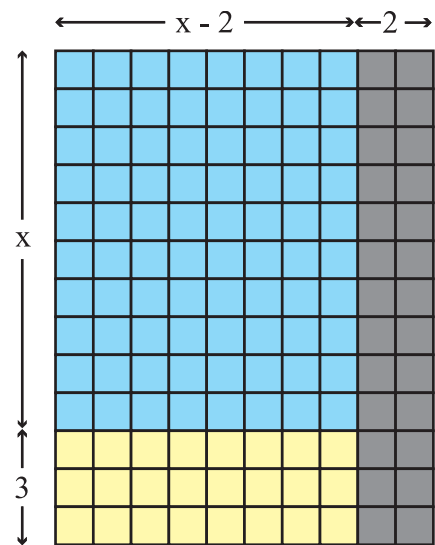


Fig 26 (h)