Sample Paper-01
Physics (Theory)
Class – XI

Time allowed: 3 hours **Maximum Marks: 70**

General Instructions:

a) All the questions are compulsory.
b) There are **26** questions in total.
c) Questions **1** to **5** are very short answer type questions and carry **one** mark each.
d) Questions **6** to **10** carry **two** marks each.
e) Questions **11** to **22** carry **three** marks each.
f) Questions **23** is value based questions carry **four** marks.
g) Questions **24** to **26** carry **five** marks each.
h) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions in five marks each. You have to attempt only one of the choices in such questions.
i) Use of calculators is **not** permitted. However, you may use log tables if necessary.
j) You may use the following values of physical constants wherever necessary:

$c = 3x10^8 m/s$

$h = 6.63x10^{-34} Js$

$e = 1.6x10^{-19} C$

$\mu_o = 4\pi x10^{-7} TmA^{-1}$

$\frac{1}{4\pi\varepsilon_0} = 9x10^9 Nm^2C^{-2}$

$m_e = 9.1x10^{-31} kg$

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1. What is the slope of stress-strain body within the elastic limit? Comment: "A body at higher temperature contains more heat".
2. What is apparent weight of the floating block, when a block of wood is floating in a lake?
3. Fill in the blanks:
   (a) 6 ms^{-2} = -------------- kmh^{-2}
   (b) 1 m = ----------------- light year
4. What happens to surface tension when impurity is mixed in liquid?
5. Is it possible for the relative velocity of two bodies moving in opposite direction to be greater than the absolute velocity of either?
6. If the velocity at the maximum height of a projectile is half its initial velocity of projection u, then find its range on the horizontal plane.
7. What fraction of its mechanical energy is lost in each bounce, if a ball bounces to 80% of its original height?
8. What is the error in the estimation of g if the length and time period of an oscillating pendulum have errors of 1% and 2%?
9. Give reason: "One should take short steps rather than long steps when walking on ice".
10. A solid sphere of radius 10 cm is subjected to a uniform pressure equal to $5\times10^8 Nm^{-2}$. Calculate the change in volume. [Given: Bulk modulus of the material of the sphere is $3.14\times10^{11} Nm^{-2}$]

Or

A group of boys went for boating as a picnic. They were dancing and singing. Suddenly, their boat lost its balance and the boys fell into the river. By seeing this, many fishermen rushed for their help and provided them pieces of wood so that they could float and save themselves. The alertness of the fishermen saved the life of all the boys.

(a) What would be the density of water if the boys saved themselves with a wood that floats with $\frac{1}{4}th$ of its volume above the water surface?

(b) What is the use of life saving jackets while going on a boat?

11. A Carnot engine whose heat sink is at $27°C$ has an efficiency of 40%. By how many degrees should the temperature of source be changed to increase the efficiency by 10% of the original efficiency?

Or

A flask contains argon and chlorine in the ratio 2:1 by mass. The temperature of the mixture is $27°C$. Obtain the ratio of

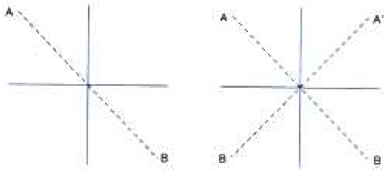
(i) Average K.E. per molecule

(ii) Root mean square speed v_{max} of the molecules of the two gases.

Given: Atomic mass of argon = 39.9 u; Molecular mass of chlorine = 70.9 u.

12. Find the pressure required to compress a gas adiabatically at atmospheric pressure to one fifth of its volume (Given: γ = 1.4)
13. If a block of mass M is placed on a frictionless, inclined plane of angle θ. Determine
    (i) The acceleration of the block after it is released
    (ii) The force exerted by the incline on the block
14. Calculate the rms speed of oxygen molecules at 1092 K, if the density of oxygen at STP = 1.424 kg m^{-3}.
15. Find the centre of mass of the remaining disc, if a circular hole of radius 1 m is cut off from a disc of radius 6 m and the centre of the hole is 3 m from the centre of the disc.
16. If a block of mass 2 kg is pulled up on a smooth incline of angle $30°$ with horizontal and the block moves with an acceleration of 1 m/s^2, then
    (a) Find the power delivered by the pulling force at a time 4 seconds after motion starts.
    (b) What is the average power delivered during these four seconds after the motion starts?
17. Show the variation of potential energy, K.E and the total energy of a body freely on earth from a height 'h' by using a graph.
18. An automatic manufacturer claims that its super-deluxe sports car will accelerate from rest to a speed of 42.0 ms^{-1} in 8.0 s assuming that the acceleration is constant.
    (a) Determine the acceleration of car in ms^{-2}
    (b) Find the distance the car travels in 8.0 s
    (c) Find the distance the car travels in 8th second.
19. A monkey of mass 40 kg climbs on a rope which stands a maximum tension of 600 N. In which of the following cases will the rope break.
    (i) When the monkey climbs up with an acceleration of 6 ms^{-2}
    (ii) When the monkey climbs down with an acceleration 4 ms^{-2}
    (iii) When the monkey climbs up with a uniform speed of 5 ms^{-1}
    (iv) When the monkey falls down the rope nearly freely under gravity

20. Find the moment of inertia of the system about the bisector line AB when two uniform thin identical rods, each of mass m and length L are joined so as to form a cross as shown in the diagram?



21. Deduce an expression for the orbital velocity of a satellite revolving around the earth in a circular orbit at a height ‘h’ above earth surface.
22. A thermodynamic system is taken an original state to an intermediate state by the linear process shown in the diagram. If its volume is then reduced to the original value from E to F by an isobaric process, then calculate the total work done by the gas from D to E to F.
23. Vinita went to her grandfather’s village for vacation. She saw a bullock cart got struck in wet mud and the driver was not able to push it out by himself. Vinita ran to his help and together they pushed it out, but the iron rim of the wheel came out. They tried to put it on the wheel but it was smaller than diameter of wheel. Suddenly she collected some wood and set them on fire and heated the rim and it slipped on the wheel.
    (a) What nature is shown by Vinita?
    (b) Name the property of solid used here?
    (c) To what temperature had Vinita heated the ring so as to fit the rim of the wheel if the diameter of the rim and ring were 6.243 m and 6.231 m respectively at $27°$? [Coefficient of linear expansion of iron = $1.20\times10^{-5}k^{-1}$
24. Find the expression for time period of motion of a body suspended by two springs connected in parallel and series.

Or

Calculate the frequency of the first and last fork if a set of 24 tuning forks is arranged in series of increasing frequencies. If each fork gives with the preceding one and the last fork is octave of the first.

25. Explain the kinematic equation for uniformly accelerated motion.

Or

A particle is thrown over a triangle from one end of a horizontal base that grazing h vertex fall on the other end of the base. If α and β be the same angles and θ the angle of projection, prove that $\tan\theta = \tan\alpha + \tan\beta$.

26. If a stone is dropped from the top of a mountain and n second later another stone is thrown vertically downwards with a velocity of u m/s, then how far below the top of the mountain will be the second stone overtake the first?

Or

A particle is projected horizontally with a speed u from top of a plane inclined at an angle θ with the horizontal direction. How far from the point of projection will the particle strike the plane?

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Answers

1. This is not always true because heat content of a body depends on mass of the body, its specific heat and temperature.
2. The apparent weight of the floating block is equal to zero because the weight of the block acting vertically downwards is balanced by the buoyant force acting on the block upwards.
3. (a) 7.776×10^{4}

   (b) 1.06×10^{-16}
4. If impurity is mixed in liquid, the surface tension of the liquid decreases.
5. Yes. When two bodies move in opposite direction then the relative velocity of each is greater than the individual velocities.
6. $u\cos\theta=\frac{u}{2}$

   $\cos\theta=\frac{1}{2}\Rightarrow\theta=60°$

   Horizontal range =R = $\frac{u^2\sin2\theta}{g}$

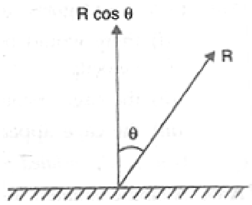
   R = $\frac{u^2\sin2\times60°}{8}$

   $=\frac{u^2\sin120°}{g}=\frac{\sqrt{3}u^2}{2g}$
7. Let the ball fall from height h then,
   Kinetic energy of ball at the time of just striking the ground = Potential energy of ball at height h,
   K = mgh
   Similarly, on rebounding the ball moves to a maximum height h', then kinetic energy will be K' = mgh'
   Loss of kinetic energy K – K' = mgh – mgh' = mg (h –h')
   = mg (h – 80/100h) = mgh x (0.2)
   Fractional loss in K.E. of ball in each re-bounce = K –K'/K
   = mgh x (0.2)/mgh = 0.2
   = 0.2 x 100% = 20%
8. $T=2\pi\sqrt{\frac{l}{g}}$

   $g=4\pi\frac{l}{T^2}$

   $\frac{\Delta g}{g}=\frac{\Delta l}{l}+2\frac{\Delta T}{T}$

   % of error in g = 1% + 2 x 2% = 5%
9.



Let R represents the reaction offered by the ground. The vertical component $R\cos\theta$ will balance the weight of the person and the horizontal component $R\sin\theta$ will help the person to walk forward.

Normal reaction = $R\cos\theta$

Friction force = $R\sin\theta$

Coefficient of friction

$$\mu = \frac{R\sin\theta}{R\cos\theta} = \tan\theta$$

In along step, θ is more and tan θ is more. But μ has a fixed value. So, there is danger of shipping in along step.

10. $K=\frac{PV}{\Delta V}$

$\Delta V=\frac{PV}{V}$

P= $5\times10^{8}\,Nm^{-2}$

$V=\frac{4}{3}\pi r^{3}=\frac{4}{3}\pi(0.1)^{3}m^{3}$

$V=4.19 \times 10^{-3}m^{3}$

$K=3.14 \times 10^{11}Nm^{-2}$

$\Delta V=\frac{5 \times 10^{8} \times 4.19 \times 10^{-3}}{3.14 \times 10^{11}}=6.67 \times 10^{-6}m^{3}$

Or

(a) Here,

Volume of wood body outside water = $\frac{v}{4}$

Volume of wood body inside water = $V - \frac{V}{4}$

Now, weight of water displaced by wood = $\frac{3V}{4} \times 10^{3}g$

Therefore, $V\rho g = \frac{3V}{4} \times 10^{3}g$

Then, ρ = 750 kg/m^{3}

(b) This is because the lives saving jackets have air in them which keeps us afloat if we accidently fall into water.

11. $T_2 = 27°C$ = 27 + 273 = 300 K

$\eta = 40\%$, T_2 =?

$\eta = 1 - \frac{T_2}{T_1}$

$$\frac{T_2}{T_1} = 1 - \eta = 1 - \frac{40}{100} = \frac{60}{100} = \frac{3}{5}$$

$$T_1 = \frac{5}{3},\ T_2 = \frac{5}{3} \times 100$$

=500K

Increase in efficiency = 10% of 40 =4%

New efficiency η' = 40 + 4 = 44%

Let T_1' be the new temperature of the source,

$$\eta' = 1 - \frac{T_2}{T_1'}$$

$$\frac{T_2}{T_1'} = 1 - \eta' = 1 - \frac{44}{100} = \frac{56}{100}$$

$$T_1' = \frac{100}{56} T_2 = \frac{100}{56} \text{ x } 300 = 535.7\text{K}$$

Increase in temperature of source = 535.7 – 500 = 35.7 K

Or

The important point to remember is that the average K.E of any gas is always equal to $\left(\frac{3}{2}\right) k_B T$.

It depends only on temperature and is independent of the nature of the gas.

(i) Since argon and chlorine both the same temperature in the flask, the ratio of average K.E of the two gases is 1:1

(ii) Now $\frac{1}{2} m v_{\max^2}$ = average K.E per molecule = $\left(\frac{3}{2}\right) k_B T$, where m is the mass of molecule of the gas.

$$\frac{\left(V_{rms}^2\right)_{Ar}}{\left(V_{rms}^2\right)_{Cl}} = \frac{(m)_{Cl}}{(m)_{Ar}} = \frac{(M)_{Cl}}{(M)_{Ar}} = \frac{70.9}{39.9} = 1.77$$

Where M denotes the molecular mass of the gas, taking square root on both sides,

$$\frac{\left(V_{rms}\right)_{Ar}}{\left(V_{rms}\right)_{Cl}} = 1.33$$

Note that the composition of the mixture by mass is quite irrelevant to the above calculation. Any other proportion by mass of argon and chlorine would give the same answer (i) and (ii) provided the temperature remains unaltered.

12. P_1 = 1 atm.

V_1 = x cc and $V_2 = \frac{x}{5} cc$

γ = 1.4 and P_2 =?

Using the relation $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$P_2 = P_1 = \left(\frac{V_1}{V_2}\right)^\gamma$$

$$=1\left(\frac{x}{\frac{x}{5}}\right)^{1.4} = (5)^{1.4}$$

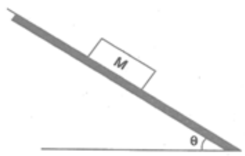
Taking log both sides, we get

$Log P_2$ = 1.4 log 5 = 1.4×0.6990

= 0.97860

P_2 = 9.519 atm.

13. When the block is released, it will move down the incline.
Let its acceleration be a.
As the surface is frictionless, so the contact force will be normal to the plane. Let it be N.



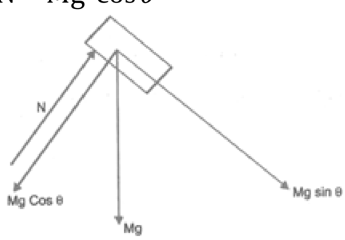
Here for the block we can apply equation for motion along the plane and equation for equilibrium perpendicular to the plane.

Mg $\sin\theta$ = Ma

a = g $\sin\theta$

Mg $\cos\theta$ - N = 0

N = Mg $\cos\theta$



14. First calculate the root-mean square speed of oxygen at STP

P_0 = 0.76 m of Hg = $1.01\times10^5\,Nm^{-2}$

P_0 = 1.424 kg m^{-3}

The root-mean square speed at $0°C$ is given by,

$$c_0 = \sqrt{\frac{3P_0}{\rho_0}}$$

$$c_0 = \sqrt{\frac{3 \text{ x } 1.01 \text{ x } 10^5}{1.424}}\text{ms}^{-1} = 4.61\times10^2\,ms^{-1}$$

$$c_{rms} = \sqrt{\frac{3kT}{m}}$$

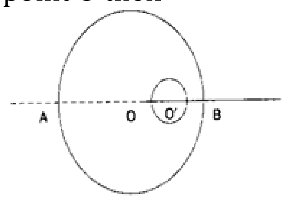
$$\frac{c_{ms}}{c_0} = \sqrt{\frac{T}{T_0}}$$

T_0 = 273 K and T = 1092 K

$$c_{rms} = c_0\sqrt{\frac{T}{T_0}} = 4.61\text{x }10^2 \text{ x}\sqrt{\frac{1092}{273}} = 9.22 \text{ x } 10^2\text{ms}^{-1}$$

15. Let O be the centre of the disc and O' that of the hole. To find the centre of mass, we use the fact that a body balances at this point. The algebraic sum of the moments of the weights about the

centre of gravity is zero. The weight W_1 of the disc acts at point O. The hold can be regarded as a negative weight W_2 acting at O’. If X is distance of the centre of gravity of the combination from point O then



$$x = \frac{W_1 x\ O + (-W_2) x 3}{W_1 + (-W_2)}$$

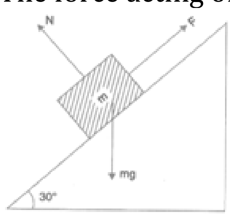
$W_1 = \rho\pi x (6)^2 = 36\rho\pi$

$W_2 = \rho\pi\ x\ (1)^2 = \rho\pi$

where ρ is the mass per unit area of the disc. by passing the value of W_1 and W_2 we get,

$$x = \frac{-\rho\pi\ x\ 3}{36\ \rho\pi - \rho\pi} m = \frac{-3}{35} m$$

16. The force acting on the block are presented in the diagram



Resolving the forces parallel to incline

F – mg $\sin\theta$ = ma

F = mg $\sin\theta$ + ma

= $2\times 9.8\times \sin 30° + 2\times 1 = 11.8$ N

The velocity after 4 seconds = u + at

$= 0 + 1\times 4 = 4\ m/s$

Power delivered by force at t =4 seconds

= $force \times velocity$

= $11.8 N \times 4 s$ = 47.2 W

The displacement during 4 seconds is given by

$v^2 = u^2 + 2as$

$= 0 + 2\times 1\times s$

s =8 m

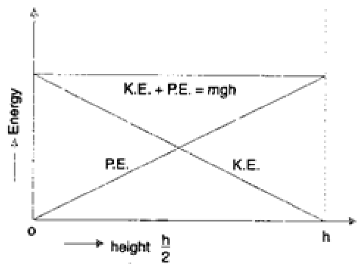
The work done in 4 seconds = $force \times dis\tan ce$

$11.8\times 8 = 94.4$ J

Average power delivered = $\frac{work\ done}{time}$

= 94.4 / 4 = 23.6 V

17.



Graphs depicting variations of (i) gravitational potential energy (P.E) (ii) K.E and (iii) the total sum of potential and Kinetic energies for a freely falling body are shown in the diagram.

(i) Gravitational potential energy decease as the body falls downwards and is zero at earth.

(ii) Kinetic energy increase as the body falls downwards and will be at maximum when the body just strikes the ground.

(iii) The sum of kinetic and potential energy remains constant at all during its free fall.

18. (a) Given that u =0 and velocity after 8 s is 42 m/s. So, acceleration

$$a = \frac{v - u}{t}$$

$$= \frac{42.0 - u}{8.0} = 5.25 ms^{-2}$$

(a) Distance travelled in 8 s

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times 5.25 \times 8^2$$

$$= 168 \text{ m}$$

(b) Distance travelled in 8^{th} second

$$S_n = u + (2n - 1)\frac{a}{2}$$

$$= (2 \times 8 - 1) \times \frac{5.25}{2}$$

$$= 39.375 \text{ m}$$

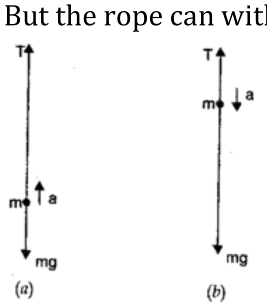
19. (i) When the monkey climbs up with an acceleration a, then T - mg = ma

Where T represents the tension

T = m g + ma = m (g + a)

T' = 40kg (10 + +6) ms^{-2} = 640 N

But the rope can withstand a maximum tension of 600 N. so the rope will break.



(ii) When the monkey is climbing down with an acceleration then mg – T = ma

T = mg – ma = m (g – a)

T = $40kg \times (10 - 4) ms^{-2}$ = 240 N

The rope will not break

(iii) When the monkey climbs up with uniform speed

T = mg = $40kg \times 10ms^{-2}$ = 400 N

The rope will not break.

(iv) When the monkey is falling freely, it would be state of weightlessness. So tension will be zero and the rope will not break.

20. Take a bisector line A'B' perpendicular to bisector line AB.

Moment of inertia about an axis perpendicular to the plane and passing through the point of intersection is

$$2x\frac{ML^2}{12} or \frac{ML^2}{6}$$

Applying theorem of perpendicular axis we get

$$\frac{ML^2}{6} = I_{AB} + I_{A'B'}$$

$$2I = \frac{ML^2}{12}$$

21. Consider the satellite of mass m revolving around the earth at a height h from its surface so that radius of its orbit r = R + h. If v_0 be the orbital velocity of satellite then centripetal force needed by it for its uniform circular motion is $F = \frac{mv_0^2}{r}$

This value of centripetal force is provided by the gravitational pull of the earth acting on the satellite.

$$F = \frac{GMm}{r^2}$$

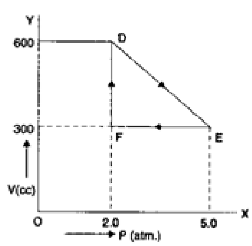
$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$$

$$g = \frac{GM}{R^2}$$

$$v_0 = \sqrt{\frac{gR^2}{(R+h)}} = R\sqrt{\frac{g}{(R+h)}}$$

22.



Change in pressure ΔP = EF = 5.0 – 2.0 = 3.0 atm = $3.0 \times 10^5 Nm^{-2}$

Change in volume ΔV = DF = 600 – 300 = 300 cc = $300 \times 10^{-6} m^3$

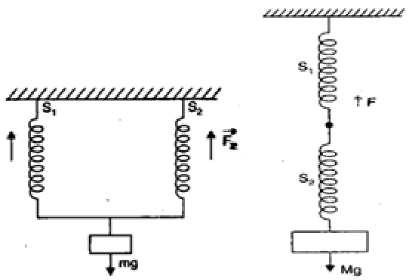
Work done by the gas from D to E to F = area of ΔDEF

$$W = \frac{1}{2} \times DF \times EF$$

$= \frac{1}{2} \times (300 \times 10^{-6}) \times (3.0 \times 10^{5}) = 45$ J

23. (a) She has presence of mind and is helpful in nature.
(b) She has used linear expansion of solids.
(c) Here,
$L_1 = 6.231$ m, $L_2 = 6.243$ m, $T_1 = 27^0C$
Using the formula, $\alpha = \frac{L_2 - L_1}{L_1(T_2 - T_1)}$
We get $T_2 = \frac{6.243 - 6.231}{6.243 \text{ x } 1.2 \text{ x } 10^{-5}} + 27 = 187^0C$

24.



Consider the body of mass M suspended by two springs connected in parallel. Let K_1 and K_2 be the spring constants of two springs.
Let the body be pulled down so that each spring is stretched through a distance y. Restoring F_1 and F_2 will be developed in the springs S_1 and S_2.
According to Hooke's law $F_1 = - K_1y$ and $F_2 = - K_2y$
Since both the forces acting in the same direction, total restoring force acting on the body is given by
$F = F_1 + F_2 = - K_1 y – K_2 y = - (K_1 + K_2)y$
Acceleration produced in the body is given by
$a = \frac{F}{M} = - \frac{(K_1+K_2)Y}{M}$ ………………………….. (i)
Since $\frac{(K_1+K_2)}{M}$ is constant a = - y
Hence motion of the body is SHM
The time period of body is given by
$T = 2\pi\sqrt{\frac{y}{|a|}} = 2\pi\sqrt{\frac{M}{K_1+K_2}}$ ……………………… (ii)
$K_1 = K_2 = K$
$T = 2\pi\sqrt{\frac{M}{2K}}$
For series:
Consider the body of mass M suspended by two springs S_1 and S_2 which are connected in series. Let k_1 and k_2 be the spring constants of spring S_1andS_2.
At any instant the displacement of the body from equilibrium position is y in the downward direction. If y_1 and y_2 be the extension produced in the springs S_1 and S_2.
$y = y_1 + y_2$ ………………………….. (i)

Restoring the forces developed in S_1 and S_2 are given by,
$F_1 = -k_1 + y_1$ ……………………………… (ii)
$F_2 = -k_2 + y_2$ …………………………….. (iii)
Multiplying the equation (ii) by k_2 and equation (iii) by k_1 and adding we get,
$K_2F_1 + k_1F_2 = -k_1k_2(y_1+y_2) = -k_1k_2\, y$
Since both the springs are connected in series.
$F_1 = F_2 = F$
$F\,(k_1 + k_2) = -k_1k_2y$

$$F = \frac{k_1k_2}{(k_1 + k_2)\,y}$$

If 'a' be the acceleration produced in the body of mass 'M' then,

$$a = \frac{F}{M} = \frac{k_1k_2y}{(k_1 + k_2)M} \text{ ………………………… (iv)}$$

Time period of the body is given by,

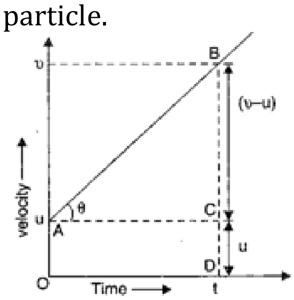
$$T = 2\pi\sqrt{\frac{y}{|a|}} = 2\pi\sqrt{\frac{(k_1+k_2)M}{k_1k_2}}$$

$$T = 2\pi\sqrt{\left(\frac{1}{k_1}+\frac{1}{k_2}\right)M}$$

Or

Let the frequency of the first tuning fork be n. as last fork is octave of the first therefore frequency of the last fork = 2n
As each fork gives 4 beats / seconds with the preceding one,
Therefore, frequency of 2nd fork = n + 4
Frequency of the 3rd fork = n +8 = n +4(2) = n + 4(3-1)
Proceeding in this way, frequency of 24th fork = n + 4(24 -1) = n +92
2n = n + 92 or n = 92 Hz.
Frequency of the last fork = 2n = 2 x 92 = 184 Hz.

25. Uniformly accelerated motion, we can drive some simple equation that relate displacements (x), time taken (f), initial velocity (u), final velocity (v) and acceleration (a).
(i) Velocity attends after time t: the velocity-time graph for positive constant acceleration of a particle.



Let u be the initial velocity of the particle at l =0 and v is the final velocity of the particle after time t. consider two points A and B on the curve corresponding to t=0 and t =t.
Draw ZBD perpendicular to time axis. Also draw AC perpendicular to BD.
OA = CD = u
BC = (v – u) and OD = t
Now,
Slope of v – t graph = acceleration (a)
a = slope of v – t graph

$\tan\theta = \frac{BC}{AC} = \frac{BC}{OD}$

$a = \frac{v - u}{t}$

v –u = at

v = u + at

(ii) Distance travelled in time t will be,

x_0 = position of the particle at t =0 from the origin

x = position of the particle at t =t from the origin

$(x - x_0)$ = S = distance travelled by the particle in the time interval (t -0) =t

Distance travelled by a particle in the given time

Interval = area under velocity-time graph

$(x - x_0)$ = area OABD

= area of Trapezium OABD

= ½ [Sum of parallel sides **x** perpendicular distance between parallel sides]

= ½ (OA + BZD) x AC

= ½ (u + v) x t

v =u +at

$(x - x_0)$ = ½ (u + u + at) x t

= ½ (2u + at) x t

= ut + ½ at^2

$x - x_0 = S$

S = ut + ½ at^2

(iii) Velocity attained after travelling a distance S:

Distance travelled by a particle in time t is equal to the area under velocity-time graph. The distance (s) travelled by a particle during time interval t is given by

S = area under v – t graph

S = area of Trapezium OABD

= ½ (sum of parallel sides) x perpendicular distance between these parallel sides

S = ½ (OA + OD) x AC ……………………… (i)

Acceleration a = slope of v – t graph

$A = \frac{BC}{AC} = \frac{BD - CD}{AC} = \frac{v - u}{AC}$

$AC = \left(\frac{v - u}{a}\right)$ ……………………… (ii)

OA = u and BD = v ………………………….. (iii)

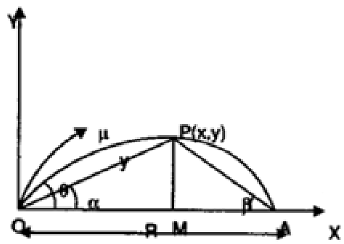
From equation (i), (ii) and (iii) we get

$S = \frac{1}{2}(v+u)\frac{(v-u)}{a}$

$S = \frac{v^2 - u^2}{2a}$

$v^2 - u^2 = 2aS$

Or



From the $\tan\alpha = \frac{y}{x}$ where R is horizontal range.

$\tan\beta = \frac{y}{MA} = \frac{y}{R - x}$

$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R - x}$

$= \frac{(R - x + x)y}{x(R - x)} = \frac{yR}{x(R - x)}$

$\tan\alpha + \tan\beta = \frac{yR}{x(R - x)}$ ………………………………… (i)

$x = (u\cos\theta)t$ ……………………………………… (ii)

$y = (u\sin\theta)t - \frac{1}{2}gt^2$ ……………………………… (iii)

From equation (ii) and (iii),

$y = x\tan\theta\left[1 - \frac{xg}{2u^2\cos^2\theta\tan\theta}\right]$

Substituting, $R = \frac{2u^2\sin\theta\cos\theta}{g}$

$y = x\tan\theta\left[1 - \frac{xg}{2u^2\cos\theta\sin\theta}\right]$

$y = x\tan\theta\left[1 - \frac{x}{R}\right]$

$\frac{y}{x} = \tan\theta\left(\frac{R - x}{R}\right)$ …………………………… (iv)

Putting (iv) in (i) we get,

$\tan\alpha + \tan\beta = \frac{yR}{x(R - X)} = \tan\theta$

$\tan\alpha + \tan\beta = \tan\theta$

26. The second stone will catch up with the first stone when the distance covered by it in (t – n) second will equal the distance covered by the first stone in t second.
The distance covered by the first stone in t second = $\frac{1}{2}gt^2$ and distance covered by the second stone in (1 –n) second.
$u(t - n) + \frac{1}{2}g(t - n)^2$
$\frac{1}{2}gt^2 = u(t - n) + \frac{1}{2}g(t - n)^2$
$\frac{1}{2}g[t^2 - (t - n)^2] = u(t - n)$
$\frac{1}{2}g[(2t - n)n] = u(t - n)$
$gnt - \frac{1}{2}gn^2 = ut - un$
$t(gn - u) = (\frac{1}{2}gn - u)n$

$t = \frac{n\left(\frac{1}{2}gn - u\right)}{(gn - u)}$

The distance covered by the first stone in this time

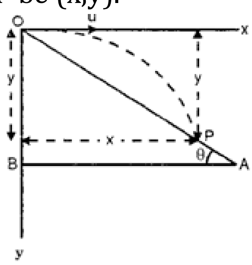
$$h = \frac{1}{2}gt^2 = \frac{1}{2}g\left[\frac{n\left(\frac{1}{2}gn - u\right)}{(gn - u)}\right]$$

Thus the second stone will overtake the first at distance

$$\frac{1}{2}g\left[\frac{n\left(\frac{gn}{2} - u\right)}{(gn - u)}\right]^2$$

Or

Let the particle projected from O strike the inclined plane OA at P after time t and coordinates of P be (x,y).



Taking motion of projectile from O to P along x-axis we have

$x_0 = 0$, $x = x$, $u_x = u$, $a_x = 0$, $t=t$

Using the relation $x = x_0 + u_x t + ½ a_x t^2$

We get x = ut or i =x/u

Taking motion of projectile along y – axis

$y_0 = y$, $y = y$, $u = 0$, $a_y = g$, $t = t$

Using the relation $y = y_0 + u_y t + ½ a_y t^2$

$$y = 0 + 0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 = \frac{1}{2}g\frac{x^2}{u^2}$$

$y = x \tan \Theta$, so $gx^2 / 2u^2 = x \tan\Theta$

$$x = \frac{2u^2\tan\theta}{g}$$

And $y = x \tan\Theta = \dfrac{2u^2\tan^2\theta}{g}$

Distance OP $= \sqrt{x^2 + y^2}$

$$= \frac{2u^2 \tan\theta}{g}\sqrt{1+\tan^2\theta}$$

$$= \frac{2u^2 \tan\theta \sec\theta}{g}$$