Vector Addition

Scalar and vector quantities

Physical quantities, having only magnitude, are called scalar quantities or scalars.
Examples : Distance, speed, mass, etc.

Physical quantities, having both magnitude and direction and obey vector algebra are called vector quantities or vectors.
Examples : Displacement, velocity, weight, etc.

Geometrical vector and its representation

A geometrical vector is a straight line with an arrow at one end called tip (or head), the other end is called tail. This arrow represents a vector quantity.
The length of the line represents the magnitude of the vector quantity on some chosen scale.

The direction of the arrow represents the direction of the quantity.

A letter used as a symbol for a vector quantity has an arrow drawn over it. As an example, displacement has symbol $\vec{S}$, velocity has symbol $\vec{v}$, acceleration has symbol $\vec{a}$, weight has symbol $\vec{W}$ and force has symbol $\vec{F}$.

Addition of vectors

Combining the effect of many simultaneous vectors into a single vector is called addition or composition of vectors. The single vector so obtained, is called resultant of the many vectors. The many added vectors, are called the components of the single vector.

Parallelogram law of addition o two vectors

Statement. If two vectors acting simultaneously on a particle be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is completely represented in magnitude and direction by the diagonal of that parallelogram drawn from that point.

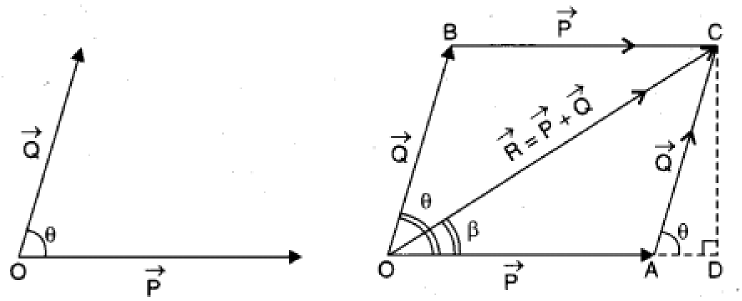

Fig. 5.01. Parallelogram law of addition of two vectors.

Explanation. Let two vectors $\vec{P}$ and $\vec{Q}$ act simultaneously on a particle O at an angle θ. They are represented in magnitude and direction by the adjacent sides OA and OB of a parallelogram OACB drawn from a point O (Fig. 5.01).

Then the diagonal OC passing through O, will represent the resultant $\vec{R}$ in magnitude and direction

i.e.,
$$\vec{R} = \vec{P} + \vec{Q}$$

Calculation. Drawing CD perpendicular to OA produced.

Then in ΔCAD, AC = OB = Q

$$\angle CAD = \text{corresponding } \angle BOA = \theta$$

Hence,
$$\frac{CD}{AC} = \sin\theta \quad \text{or} \quad CD = AC\sin\theta = Q\sin\theta$$

and
$$\frac{AD}{AC} = \cos\theta \quad \text{or} \quad AD = AC\cos\theta = Q\cos\theta$$

Now in ΔCOD,

$$(OC)^2 = (OD)^2 + (DC)^2 = (OA + AD)^2 + (DC)^2$$
$$= (OA)^2 + 2OA\,.\,AD + (AD)^2 + (DC)^2$$
$$= (OA)^2 + [(AD)^2 + (DC)^2] + 2OA\,.\,AD$$
$$= (OA)^2 + (AC)^2 + 2OA\,.\,AD$$

or $$R^2 = P^2 + Q^2 + 2P \cdot Q \cos\theta$$

i.e., $$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$, which gives **magnitude** of resultant vector.

Let OC = R make $\angle\beta$ with OA = P.

Then in ΔCOD,

$$\tan\beta = \frac{CD}{OD}$$

$$= \frac{CD}{OA + AD}$$

or $$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$$, which gives **direction** of resultant vector.

Triangle law of addition of two vectors

Statement. If two vectors acting simultaneously on a particle be represented in magnitude and direction by the two sides of a triangle taken in same order, then their resultant will be completely represented in magnitude and direction by the third side of the triangle taken in the opposite order.

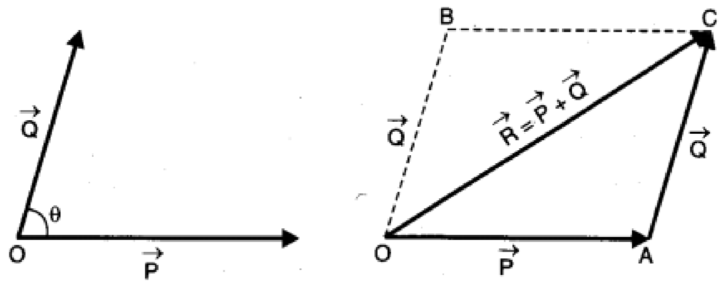

Fig. 5.02. Triangle law of addition of two vectors.

Explanation. Let two vectors $\vec{P}$ and $\vec{Q}$ act simultaneously on a particle O at an angle θ. They are represented in magnitude and direction by the sides OA and AC of a Δ OAC, taken in same order (*i.e.,* from O to A and then A to C) (Fig. 5.02).

Then the third side OC will represent the resultant $\vec{R}$ in magnitude and direction, taken in opposite order (*i.e.,* O to C, not C to O)

i.e., $$\vec{R} = \vec{P} + \vec{Q}$$

Proof. Completing the parallelogram OACB. Then OB || AC represents Q.

OC becomes diagonal of parallelogram OACB.

Hence from parallelogram law,

$$\vec{R} = \vec{P} + \vec{Q} \quad \textbf{(Proved)}$$

Equilibrium of vectors and equilibrant vector

Two or more vectors are said to be in equilibrium if their resultant is zero. In this case, each single vector balances the other remaining vectors.

A single vector which balances other vectors, is called an equilibrant of other vectors. It must be equal and opposite of the resultant of other vectors.